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ON THE

SENSATIONS OF TONE

AS A PHYSIOLOGICAL BASIS FOR THE

THEORY OF MUSIC.

BY

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TRANSLATED WITH THE AUTHOR'S SANCTION FROM THE THIRD GERMAN EDITION,
WITH ADDITIONAL NOTES AND AN ADDITIONAL APPENDIX,

BY

ALEXANDER J. ELLIS,

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174. e. 31.

The great feature of this work is the proof that all musical sounds, whether proceeding from one or many sources, are heard by the ear as if they came from one or more distinct sources of a particular simple kind, combined by a well-known and definite law so long as the excursions of the particles of air are small in comparison with the length of the wave, and by another and different law when those excursions are larger. From this flows the fact that any individual tone may be considered as compounded of the partial simple tones of which it is audibly composed, and the several combinations of such compound tones can be reduced to the combinations of the simple tones of which they are compounded, and of the other simple tones, which result when the excursions of the particles of air are larger than usual. Hence the whole theory of combined musical sounds is reduced to the theory of combined simple sounds. From this flows with great ease and simplicity, the nature of consonance and dissonance, on which must in the last resort depend every constructible theory of harmony, and the laws which regulate it. From this also are seen to flow the principles which determined those degrees of musical sounds that were esthetically selected by various nations at various times, and the harmonic capabilities or incapabilities of the various scales thus produced. And again, the same principle furnishes the explanation, so long and vainly sought, not only of the simple ratios of the lengths of strings which produce consonant tones, and of the limitation of the number of these ratios, but also (and this was the puzzle of puzzles) of their various endurable imitations by other extremely complex ratios, with a perfect and simple means of estimating and comparing the resulting roughness or smoothness, and the cause of its variability from instrument to instrument; in short, of the value of different 'temperaments' on different instruments. Anything more beautiful than the explanation of these phenomena, which have perplexed philosophers and musicians from the time of Pythagoras in Greece, and thousands of years before Pythagoras in China, to the year 1862, when Prof. Helmholtz first gave his results to the world in a connected form, it is scarcely possible to conceive; and the gradual, simple, and popular manner in which he has laid those results before the reader in his text, so that all that have any interest in music may peruse them with pleasure, and understand them without difficulty, while the lover of exact and abstruse mathematical reasoning is fully satisfied by the investigations in his Appendices, evinces a master's hand directed by a master's mind.

The two first parts of this work are indispensable to all who would understand the nature of sound, how its various qualities of tone are distinguished by the ear, and how they react on each other when combined. While, therefore, they convey information which is of special importance to the physicist, the phonetist, and the musician, they are as well adapted for the purposes of general education. The third part is more

strictly musical. It enters upon the history of the different kinds of music, and then, taking up the harmonies dependent on Octaves, Fifths, and Thirds, and their inversions and extensions, determines the proper method of tuning for producing their best results, that is, just intonation, and traces the effect of altering the positions of notes in a chord, the introduction and resolution of discords, the laws of progression of parts, and all those troublesome points on which ordinary treatises are prolific in rules, and, when not absolutely silent, scanty or inaccurate in explanation. This part is as necessary to musicians as grammar is to writers. Armed with this knowledge, which could not have been previously presented because its very basis (the composition of musical tones) was unknown, the mere practical executant sees into the meaning of the composer, and the composer is endowed with a new power of realising his conceptions. But here the work stops. Into composition it does not enter. It gives the composer a knowledge of his material[1] and its laws, and then leaves his genius to do the rest.

In now fulfilling a wish which I had entertained from the first appearance of the original German work, and which only grew stronger during more than ten years of familiarity with its contents, I have endeavoured

[1] That is, musical sound in general, and its particular esthetic selections. 1) Pythagorean intonation with perfect Fifths, and sharpened major Thirds, formed by taking four Fifths up and two Octaves down, give one peculiar material worked out by the Greeks and Chinese with great fineness of detail, but incapable of the effects of modern harmony. 2) Mean intonation, with perfect major Thirds, but Fifths flattened to obey the Greek law, forming a totally different material, first rendered harmony possible and founded almost all its rules. 3) Equal intonation, with Fifths so slightly flattened that 12 of them should make 7 Octaves, but having its sharpened major Thirds formed by the above rule, is a third and very different material, but one which only some very modern ears of Western Europe have ever considered to be capable of harmony. 4) Skhismic or Arabic intonation, in which the Fifths are perfect, but the flattened major Thirds are formed by taking 8 Fifths down and 5 Octaves up, is quite a new material, suitable for harmony, but not yet used for harmony. 5) Skhistic or Helmholtzian intonation, flattening the Fifths in the last case imperceptibly, and thus making the major Thirds perfect, is another and perfectly new material eminently suitable for harmony, but with totally different laws from the second or third. 6) Mr. Bosanquet's cyclic intonation is another variety of the fourth class, which in addition to the laws of the fifth possesses the cyclic power of infinite modulation; it has also sensibly perfect Fifths, and almost insensibly flattened major Thirds; this offers quite a new material to composers. 7) Just intonation with absolutely perfect Fifths and Thirds, and, if need be, perfect harmonic Sevenths, is again a new material, sufficiently represented in practice by the fifth or sixth, to which keyed instruments can be adapted. It is at present executable only by the voice, bowed instruments, and trombones, but its laws are the foundation of all harmony, and, as it is the only system of acoustic consonance, it is the only easy system for singers. How these different materials are acoustically derived and related, and how far compositions written for the second or third, are executable in the fifth, sixth, or seventh class, or conversely, will be seen in the work itself, or the additional Appendix.

to render the thoughts of the Author faithfully into the most appropriate idiomatic English. Any changes, however slight, have been duly indicated. The additional notes which I have subjoined, especially to the third, or strictly musical, part of the work, were suggested to me by the difficulties which I myself experienced when first studying the subject. In an additional Appendix XIX. I have endeavoured to bring together the whole of the matter bearing on just and tempered intonation, with the necessary tables, so as to complete, in a very condensed form, the admirable introduction in the text. I felt that unless musicians had at hand the information and results there collected, the scientific statements in the text would not be duly appreciated in this country.

The remarks which I have heard made by musicians of high standing confirm me in this view. Although the present system of tuning is so recent that it was not adopted till the year 1867 in the great organ of St. George's Hall, Liverpool, yet one century of ordinary use has led even composers of great repute to identify music itself with 'equal temperament.' In a *brochure* printed in 1860 (just two years before the publication of the first German edition of this work), and signed by 'AUBER (de l'Institut), CARAFA (id.), CLAPISSON (id.), ERMEL, Victor FOUCHER, président, Casimir GIDE, Charles GOUNOD, F. HALÉVY (de l'Institut), JOMARD (id.), Général MELLINET, Édouard MONNAIS, NIEDERMEYER, Édouard RODRIGUES, vice-président, Ambroise THOMAS (de l'Institut), VARCOLLIER, *membres de la commission de surveillance de l'enseignement du chant dans les écoles communales de Paris*,—H. BERLIOZ (de l'Institut), DIETSCH (chef d'orchestre de l'Opéra), Georges KASTNER (de l'Institut), J. D'ORTIGUE (directeur-rédacteur en chef de la *Maîtrise*)—PASDELOUP, F. BAZIN (directeurs de l'Orphéon de Paris),' who may be considered to represent the musical feeling of France in that year, I find the following explicit and emphatic declarations[1]: 'Il est certain, évident, incontestable, c'est un fait, qu'il n'y a réellement dans les vingt

[1] In English. 'It is certain, evident, indisputable, it is a fact, that the 21 signs of the enharmonic scale [that is, the 7 naturals, 7 sharps, and 7 flats] contain really nothing more or less than the 12 degrees of the chromatic scale,' [that is, 1 *b*♯ = *c*, 2 *c*♯ = *d*♭, 3 *d*, 4 *d*♯ = *e*♭, 5 *e* = *f*♭, 6 *e*♯ = *f*, 7 *f*♯ = *g*♭, 8 *g*, 9 *g*♯ = *a*♭, 10 *a*, 11 *a*♯ = *b*♭, 12 *b* = *c*♭, so that the 7 double sharps and 7 double flats have been overlooked]; and again: 'D♭ is neither flatter nor sharper than C♯,—these notes have the same meaning, they express the same sound.—Sameness of meaning for different signs, that is, enharmonic equivalence, is the source of all the wealth of modern art.—Suppress the tempered system, suppress enharmonic equivalence, the sameness of meaning for different signs, and by the same act you suppress the most beautiful inspirations of composers, you suppress music itself.' The title of this pamphlet is: 'Observations de quelques Musiciens et de quelques Amateurs sur la méthode de Musique de M. le docteur Émile CHEVÉ. Paris, Imprimerie de J. Claye, Rue St. Benoit, 7. 1860.' It does not appear to have been published, but there is a copy in the British Museum, catalogued under *Chevé*, press mark 7896, d. M. Chevé in quoting it ('Méthode Élémentaire de Musique Vocale,' preface to the edition of 1863–4, p. xxxvi.) adds to the above list the names of 'Verdy [sic], Meyerbeer.'

et un signes de la gamme enharmonique que les douze échelons de la gamme chromatique' (p. 9). 'Le *ré bémol* [D♭] n'est ni plus grave, ni plus aigu que l'*ut dièse* [c♯],—ces notes sont synonymes, elles expriment le même son.—La synonymie, c'est-à-dire l'enharmonie, est la source de toutes les richesses de l'art moderne.—Supprimez le système tempéré, supprimez l'enharmonie, la synonymie, vous supprimez du même coup les plus belles inspirations des maîtres, vous supprimez la musique même' (p. 16). Hence these eminent men consider that there was no 'wealth of modern art,' no 'beautiful inspirations of composers,' in fact no 'music,' till about a century ago. They declare their convictions that 'music itself would be suppressed' by altering a system of tuning, unknown to Palestrina (1524-94), unused by Handel (1685-1759),[1] supposed to have been first employed for harmony by Sebastian Bach (1685-1750),[2] but at any rate first conspicuously maintained for the piano in 1753 by his son Emanuel Bach, long contested by Rameau (1683-1764), characterised by our first scientific writer on harmony, Dr. Robert Smith, who would not employ it, as 'that inharmonious system of 12 hemitones,' producing a 'harmony . . . extremely coarse and disagreeable,'[3] not brought into use till long after the present musical notation was perfected, and still without a notation adapted to its own peculiarities.

Similarly Mr. John Hullah, Her Majesty's Inspector of Singing in Training Colleges and Schools, where are taught the teachers of primary schools in which the children have to sing mainly without any accompaniment, in a memorandum appended to his Report for 1872, and reprinted in his 'Time and Tune in the Elementary School, a New Method of Teaching Vocal Music,' 1875, says: 'Every musician knows that C♯ and D♭ are not theoretically identical.[4] For all practical purposes they

[1] Handel gave to the Foundling Hospital an organ, with stops arranged to play from four flats to five sharps (that is, with 16 mean-tone notes to the Octave). These stops having long become utterly useless, because not understood, were nailed up (!), and the organ afterwards rebuilt, but it retained its old intonation under all changes. Some time after 1848 it was enlarged (as detailed in 'Hopkins on the Organ'), and tuned on the equal temperament. The organist (Mr. C. Edwin Willing) who obligingly gave me this information in 1863, adds that to his ear 'unequal temperament is intolerable,' meaning the mean-tone temperament reduced to 12 notes. These facts, however, establish not only that Handel used it, but how he proposed to remedy its imperfection. And Handel is commonly supposed to have written 'music.'

[2] See below, p. 500, and observe that there is no guarantee for his having attempted a theoretically perfect equal temperament. Silberman, the great organ builder of his time, maintained the mean-tone temperament, and there are anecdotes of S. Bach's driving him to distraction by using it for unsuitable keys, as F minor.

[3] 'Harmonics, or the Philosophy of Musical Sounds,' 2nd edn. 1759, pp. 166-7.

[4] But very few indeed are aware of what the 'theoretical' difference is, or that to use one for the other on systems of tuning where they are distinguished (as the mean-tone system, where the difference is about two-fifths of an equal Semitone) produces hideous discords, and that it was these very 'wolves' which determined the present

are regarded as such, in pitch, by the greatest composers. The system of "equal temperament" on which all pianofortes and most organs are now tuned, is universally accepted, even by the most skilful and refined performers on stringed instruments, capable as these are of infinite variety of intonation. We may confidently adopt a mode of tuning, even though it be theoretically incorrect, which satisfies the ears of a Joachim [violinist] or a Piatti [violoncellist].'

The results of the calculations in Table VI., division D (p. 781), founded on the acoustical and physiological theories of this work, and fully confirmed by actual experiment, shew what the universal acceptance of equal temperament would compel the listener to endure. The careful observations of Messrs. Cornu and Mercadier (ibid., E, p. 787), confirming those of Prof. Helmholtz (p. 640 below, and p. 507 with especial reference to Joachim's practice), prove that, even now, musicians who have the command of unfixed tones never use equal temperament in harmony, unless forced to adopt it by a fixed tone accompaniment. Professor Helmholtz's own Appendix XVIII. bears witness to the importance which he attaches to the fact, that at least one popular system of teaching singing in England adopts just intonation as its basis without reserve. History shews us, indeed, that the music of the immediate past was a high development of the mean-tone temperament,—still the basis of most of existing musical theory and notation, though both retain traces of Greek influence. The demands of modulation, far outstripping the mechanical construction of instruments, made equal temperament a temporary necessity, and the music of the immediate future may possibly rest upon a high development of some corresponding theory, as yet unformulated. But no such theory can reconstruct the eternal laws of elastic vibration and the human sensations of tone. If then every instrument and voice is to execute its own melody, while all blend together in one harmony, certain acoustical conditions must be fulfilled, which neither of the above temperaments can satisfy. These conditions have been scientifically evolved, and clearly stated by Professor Helmholtz in the present work. When the inspired musician of the future comes to know the new mine of wealth thus placed at his command, he is not likely to let his genius be over-ridden by a temperament, required only for the mechanical convenience of a few instrumentalists. The composer

rules for writing sharps and flats in harmony, though the introduction of equal temperament has made them as utterly meaningless as the English change of pronunciation has rendered some of our own historical orthographies, and, like them, a mere source of difficulty and bewilderment to learners. I have indeed heard musicians state that the difficulties of musical notation were an absolute benefit to the learner, that musical teachers are rejoiced at them, and would not alter them for the world. I have heard similar observations made on the advantages of our spelling to children learning to read. But such opinions are utterly opposed to all I know of the most elementary principles of the science and art of education—for other subjects.

AUTHOR'S PREFACE

TO

THE FIRST GERMAN EDITION.

In laying before the Public the result of eight years' labour, I must first pay a debt of gratitude. The following investigations could not have been accomplished without the construction of new instruments, which did not enter into the inventory of a Physiological Institute, and which far exceeded in cost the usual resources of a German philosopher. The means for obtaining them have come to me from unusual sources. The apparatus for the artificial construction of vowels, described on pp. 176 to 188 and 604–7, I owe to the munificence of his Majesty King Maximilian of Bavaria, to whom German science is indebted, on so many of its fields, for ever-ready sympathy and assistance. For the construction of my Harmonium in perfectly natural intonation, described on p. 492, I was able to use the Soemmering prize which had been awarded me by the Senckenberg Physical Society (*die Senckenbergische naturforschende Gesellschaft*) at Frankfurt-on-the-Main. While publicly repeating the expression of my gratitude for this assistance in my investigations, I hope that the investigations themselves as set forth in this book will prove far better than mere words, how earnestly I have endeavoured to make a worthy use of the means thus placed at my command.

H. HELMHOLTZ.

HEIDELBERG:
October 1862.

AUTHOR'S PREFACE

TO

THE THIRD GERMAN EDITION.

THE PRESENT THIRD EDITION has been much more altered in some parts than the second. Thus in the sixth chapter I have been able to make use of the new physiological and anatomical researches on the ear. This has led to a modification of my view of the action of Corti's arches. Again, it appears that the peculiar articulation between the auditory ossicles called 'hammer' and 'anvil' might easily cause within the ear itself, the formation of harmonic upper partial tones for simple tones which are sounded loudly. By this means that peculiar series of upper partial tones, on the existence of which the present theory of music is essentially founded, receives a new subjective value, entirely independent of external alterations in the quality of tone. To illustrate the anatomical descriptions, I have been able to add a series of new woodcuts, principally from Henle's Manual of Anatomy, with the author's permission, for which I here take the opportunity of publicly thanking him.

I have made many changes in re-editing the section on the History of Music, and hope that I have improved its connection. I must, however, request the reader to regard this section as a mere compilation from secondary sources; I have neither time nor preliminary knowledge sufficient for original studies in this extremely difficult field. The older history of music to the commencement of Discant, is scarcely more than a confused heap

of secondary subjects, while we can only make hypotheses concerning the principal matters in quest. Of course, however, every theory of music must endeavour to bring some order into this chaos, and it cannot be denied that it contains many important facts.

For the representation of pitch in just or natural intonation, I have abandoned the method originally proposed by Hauptmann, which was not sufficiently clear in involved cases, and have adopted the system of Herr A. von Oettingen, as had already been done in M. G. Guéroult's French translation of this book.

[A comparison of the Third with the Second editions, shewing the changes and additions individually, is here omitted.]

If I may be allowed in conclusion to say a few words on the reception experienced by the Theory of Music here propounded, I should say that published objections almost exclusively relate to my Theory of Consonance, as if this were the pith of the matter. Those who prefer mechanical explanations express their regret at my having left any room in this field for the action of artistic invention and esthetic inclination, and they have endeavoured to complete my system by new numerical speculations. Other critics with more metaphysical proclivities have rejected my Theory of Consonance, and with it, as they imagine, my whole Theory of Music, as too coarsely mechanical.

I hope my critics will excuse me if I conclude from the opposite nature of their objections, that I have struck out nearly the right path. As to my Theory of Consonance, I must claim it to be a mere systematisation of *observed facts* (with the exception of the functions of the *cochlea* of the ear, which is moreover an hypothesis that may be entirely dispensed with). But I consider it a mistake to make the Theory of Consonance the essential foundation of the Theory of Music, and I had thought that this opinion was clearly enough expressed in my book. The essential basis of Music is *Melody*. Harmony has become to Western Europeans during the last three centuries an essential, and, to our present taste, indispensable means of strengthening melodic relations, but finely developed music existed for thousands of years and still

exists in ultra-European nations, without any harmony at all. And to my metaphysico-esthetical opponents I must reply, that I cannot think I have undervalued the artistic emotions of the human mind in the Theory of Melodic Construction, by endeavouring to establish the physiological facts on which esthetic feeling is based. But to those who think I have not gone far enough in my physical explanations, I answer, that in the first place a natural philosopher is never bound to erect systems about everything he knows and does not know; and secondly, that I should consider a theory which claimed to have shewn that all the laws of modern Thorough Bass were natural necessities, to stand condemned as having proved too much.

Musicians have found fault with the manner in which I have characterised the Minor Mode. I must refer in reply to those very accessible documents, the musical compositions of A.D. 1500 to A.D. 1750, during which the modern Minor was developed. These will shew how slow and fluctuating was its development, and that the last traces of its incomplete state are still visible in the works of Sebastian Bach and Handel.

H. HELMHOLTZ.

HEIDELBERG:
May 1870.

Addendum.

Since pp. 724–741 were printed off, the second and concluding part of Herr von Quanten's paper on Professor Helmholtz's 'Vowel Theory' has appeared (Pogg. Ann. vol. 154, pp. 522–552). Herr von Quanten concludes (ibid. p. 551) that 'the investigations into the nature of vowel sounds cannot be regarded as completed,' and that 'though Helmholtz has made important contributions towards the solution of this scientific question, he has not succeeded in answering it,' and he does not attempt to answer it himself.

It would be quite impracticable within the limits of an 'Addendum' to enter upon the new matters thus introduced. Many have been anticipated, and, although others might require much additional examination, it does not appear necessary to modify the remarks already made on pp. 724–741 below.

Errata.

p. 166, note 2, line 2, *for*: Sieler, *read*: Seiler.

p. 299, note 2, line 8 from bottom, *for*: but I do not find that he has raised it, *read*: as he has noted, p. xxxvi and elsewhere.

p. 413, table, line 5, Doric, for: *c—d—e*♭— &c., read: *c—d*♭*—e*♭— &c.

p. 704, line 3 from bottom, *for*: and when, *read*: as when.

CONTENTS.

[Notice by Translator, pp. iii—xi.]
Author's Preface to the First German Edition, p. xii.
Author's Preface to the Third German Edition, pp. xiii—xv.

INTRODUCTION, pp. 1–8.

Relation of Musical Science to Acoustics, 1–4.
Distinction between Physical and Physiological Acoustics, 5–6.
Plan of the Investigation, 6–8.

PART I. (pp. 9–226).

ON THE COMPOSITION OF VIBRATIONS.

Upper Partial Tones and Qualities of Tone.

CHAPTER I. On the Sensation of Sound in General, pp. 11–37.

Distinction between Noise and Musical Tone, 11
Musical Tone due to Periodic, Noise to non-Periodic Motions in the air, 12–13
General Property of Undulatory Motion: while Waves continually advance, the Particles of the Medium through which they pass execute Periodic Motions, 14–16.
Differences in Musical Tones due to Force, Pitch, and Quality, 16
Force of Tone depends on Amplitude of Oscillation, Pitch on the length of the Period of Oscillation, 17
Simple relations of Vibrational Numbers for the Consonant Intervals, 22
Vibrational Numbers of Consonant Intervals calculated for the whole Scale, 24
Quality of Tone must depend on Vibrational Form, 28
Conception of and Graphical Representation of Vibrational Form, 30
Harmonic Upper Partial Tones, 33
Terms explained: Tone, Musical Tone, Simple Tone, Partial Tone, Compound Tone, Pitch of Compound Tone, 35.

Reinforcement or Enfeeblement of Sound, due to difference of Phase, 246
Interference gives rise to Beats when the Pitch of the two Tones is slightly different, 247
Law for the Number of Beats, 248
Visible Beats on Bodies vibrating sympathetically, 249
Limits of Rapidity of Audible Beats, 251

CHAPTER IX. DEPTH AND DEEPEST TONES, pp. 264–271.

Former Investigations were insufficient, because there was a possibility of the ear being deceived by Upper Partial Tones, as is shewn by the number of Beats on the Siren, 264
Tones of less than thirty Vibrations in a second fall into a Drone, of which it is nearly or quite impossible to determine the Pitch, 266
Beats of the Higher Upper Partials of one and the same Deep Compound Tone, 269

CHAPTER X. BEATS OF THE UPPER PARTIAL TONES, pp. 272–299.

Any two Partial Tones of any two Compound Tones may beat if they are sufficiently near in pitch, but if they are of the same pitch there will be consonance, 272
Series of the different Consonances, in order of the Distinctness of their Delimitation, 277
Number of Beats which arise from Mistuning Consonances, and their effect in producing Roughness, 279
Disturbance of any Consonance by the adjacent Consonances, 283
Order of Consonances in respect to Harmoniousness, 284

CHAPTER XI. BEATS DUE TO COMBINATIONAL TONES, pp. 300–320.

The Differential Tones of the first order generated by two Partial Tones are capable of producing very distinct beats, 300
Differential Tones of higher orders produce weaker beats, even in the case of simple generating tones, 302
Influence of Quality of Tone on the Harshness of Dissonances and the Harmoniousness of Consonances, 311

CHAPTER XII. CHORDS, pp. 321–353.

Consonant Triads, 321
Major and Minor Triads distinguished by their Combinational Tones, 326
Relative Harmoniousness of Chords in different Inversions and Positions, 330
Retrospect on Preceding Investigations, 341

PART III. (pp. 355–578.)

THE RELATIONSHIP OF MUSICAL TONES.

Scales and Tonality.

CHAPTER XIII. GENERAL VIEW OF THE DIFFERENT PRINCIPLES OF MUSICAL STYLE IN THE DEVELOPMENT OF MUSIC, pp. 357–383.

Difference between the Physical and the Esthetical Method, 357
Scales, Keys, and Harmonic Tissues depend upon esthetic Principles of Style as well as Physical Causes, 358
Illustration from the Styles of Architecture, 359
Three periods of Music have to be distinguished, 361
1. Homophonic Music, 361
2. Polyphonic Music, 373
3. Harmonic Music, 377

CHAPTER XIV. THE TONALITY OF HOMOPHONIC MUSIC, pp. 384–446.

Esthetical Reason for Progression by Intervals, 384
Tonal Relationship in Melody depends on the identity of two partial tones, 389
The Octave, Fifth, and Fourth were thus first discovered, 390
Scales of Five Tones, used by Chinese and Gaels, 398
The Tetrachordal Systems of the Greeks, 404
The Greek Scales of Seven tones, 409
The Greek and Ecclesiastical Tonal Modes, 410
Early Ecclesiastical Modes, 416
The Rational Construction of the Diatonic Scales by the principle of Tonal Relationship in the first and second degrees gives the five Ancient Melodic Scales, 418
Introduction of a more Accurate Notation for Pitch, 423
The Arabic and Persian Tonal Systems, and the peculiar way in which they express the natural Thirds, 430
The leading note and consequent alterations in the Modern Scales, 437

CHAPTER XV. THE CONSONANT CHORDS OF THE TONAL MODES, pp. 447–480.

Chords as the Representatives of Musical Tones with peculiar qualities, 447
Reduction of all Tones to the closest relationship in the popular harmonies of the Major Mode, 449
Ambiguity of Minor Chords, 454
The Tonic Chord as the centre of the Sequence of Chords, 457
Relationship of Chords of the Scale, 459
The Major and Minor Modes are best suited for Harmonisation of all the Ancient Modes, 463

Harmonic Differences between Major and Minor, 464
Harmonisation of the Various Tonal Modes, 467
Major Mode and Mode of the Fourth, 467
Mode of the Minor Seventh, 469
Minor-Major Mode, 472
Mode of the Minor Sixth, 473
Remnants of the old Tonal Modes, 475

CHAPTER XVI. THE SYSTEM OF KEYS, pp. 481–513.

Relative and Absolute Character of the different Keys, 482
Modulation leads to Tempering the Intonation of the Intervals, 485
Hauptmann's System admits of a Simplification which makes its Realisation more Practicable, 491
Description of an Harmonium with Just Intonation, 492
Disadvantages of Tempered Intonation, 501
Modulation for Just Intonation, 511

CHAPTER XVII. OF DISCORDS, pp. 514–543.

Enumeration of Dissonances and Estimation of their Comparative Roughness, 515
Thirds and Sixths, 519
Fifths and Fourths, 521
Sevenths and Seconds, 522
Dissonant Triads, 525
Chords of the Seventh, 530
The Dissonant Note in a Discord, 538
The Compound Musical Tones represented by Chords of the Seventh, 538

CHAPTER XVIII. LAWS OF PROGRESSION OF PARTS, pp. 544–562.

The Musical Connection between two Consecutive Notes, 544
Consequent Rules for the Progression of Dissonant Notes, 548
Resolution of Discords, 549
Chordal Sequences and Resolution of Chords of the Seventh, 551
Prohibition of Consecutive Fifths and Octaves, 557
Hidden Fifths and Octaves, 561
False Relations, 561

CHAPTER XIX. ESTHETICAL RELATIONS, pp. 563–578.

Review of Results obtained, 563
Law of Unconscious Order in Works of Art, 569
The Law of Melodic Succession depends on Sensation, not on Consciousness, 572
And similarly for Consonance and Dissonance, 574
Conclusion, 577

APPENDICES, pp. 579—800.

I. On an Electro-Magnetic Driving Machine for the Siren, 579
II. On the Size and Construction of Resonators, 579
III. On the Motion of Plucked Strings, 582
IV. On the Production of Simple Tones by Resonance, 585
V. On the Vibrational Forms of Pianoforte Strings, 589
VI. Analysis of the Motion of Violin Strings, 595
VII. Influence of Resonance in Reed Pipes, 601
VIII. Practical Directions for Performing the Experiments on the Composition of Vowels, 604
IX. On the Phases of Waves caused by Resonance, 608
X. Relation between the Strength of Sympathetic Resonance and the Length of Time required for the Tone to die away, 612
XI. Vibrations of the Membrana Basilaris in the Cochlea, 613
XII. Theory of Combinational Tones, 621
XIII. Description of the Mechanism employed for opening the several Series of Holes in the Polyphonic Siren, 623
XIV. Variation in the Pitch of Simple Tones that Beat, 625
XV. Calculation of the Intensity of the Beats of Different Intervals, 626
XVI. On the Beats of Combinational Tones, and on Combinational Tones in the Siren and Harmonium, 629
XVII. Plan for Justly-Intoned Instruments with a single Manual, 633
XVIII. Just Intonation in Singing, 635
[XIX. Additions by the Translator, 641
[Notice, 641
[A. On the Notation and Calculation of Pitch, 642
[B. On the Notation of Intervals, 644
[C. On Temperaments, 647
[D. Scales of Quintal and Tertian Harmony, 660
[E. On Modulation, Decadation, and Duodenation, 665
[F. On Practical Tonal Systems, 673
[G. On the Construction of Experimental, Educational, and Practical Instruments with Fixed Tones, in Just, Skhismatic, and Practically Just Intonation, 679
[H. Miscellaneous Notes on Recent Acoustical and Musical Researches and Instruments, 699
[1. Professor Mayer's Acoustical Researches, 699
a. New Objective Analysis of Compound Sounds, 699
b. Limits of the Damping Powers of the Ear at different Pitches, or the Rapidity of Audibly Separable Impulses, 700
c. Observations on the Sensitive Hairs of the Antennæ of the Culex Mosquito, 703
d. Sharpening and Flattening by Motion, 703
e. Harmonic Curves, 703
[2. Mechanical Description of Combined Vibrations, 704
a. Mr. Donkin's Machine for Compounding Parallel Vibrations, 704
b. Mr. Tisley's Machine for compounding Rectangular Vibrations, 704

[3. Reference to Proofs of Fourier's Theorem, 705
[4. M. Terquem's Tonometer, 705
[5. Dr. W. H. Stone's Instrumental Researches, 705
a. New $C_{,}$ Contra-basso and Tension Bars for Viols, 705
b. New $C_{,}$ Contrafagotto, 706
c. Pressure on the Lungs in playing Wind Instruments, 707
[6. Mr. Hermann Smith and Herr H. Schneebeli on the Source of Tone in Flute Organ Pipes, 708
[7. On the Use of Striking and Free Reeds, 711
[8. The String Organ, 714
[9. New Musical Tones, 720
[10. On the Resolution of Compound Tones by Distance and Reflection, 721
[11. Observations on Herr Koenig's and Herr von Quanten's Remarks on Professor Helmholtz's Vowel Theory, 724

[Table I. Notation and Calculation of Pitch, 742
A. Notations, Just Intonation, 742
B. Logarithms for Musical Calculations, 743
C. Mutual Conversion of Logarithms and Numbers of Equal Semitones, 743
D. Fractions of a Comma and Skhisma, 745
[Table II. Value, Nomenclature, and Notation of Intervals, 746
i. Tertian Intervals, 746
ii. Septimal Intervals, 748
iii. Number of Intervals in an Octave, 748
[Table III. Scales of Quintal and Tertian Harmony, 749
A. Five-Fifths Scales, Quintal, Gaelic and Chinese, 749
B. Seven-Fifths Scales, Quintal, Greek, 750
C. Trichordal Scales, Tertian, Usual, 752
D. Tetrachordal Scales, Tertian, Modern, 757
[Table IV. Limits of Duodenation and Number of Tones, 759
[Table V. Skhistic Tones and their Synonyms, 760
[Table VI. Just and Tempered Intonation Compared, 761
A. Elements of the Calculation of the division B in this Table, 761
B. Just and Tempered Tones, expressed in Vibrational Numbers, Equal Semitones, and Logarithms, in Order of Pitch, 762
C. Absolute Pitch, 779
D. Examination of the two Consonant Triads in Just and Tempered Intonation, 781
E. Intervals actually played on Instruments with Variable Tones, 787
[Table VII. List of Trines and Duodenes in order of Fifths, 792
[Table VIII. Manuals for Duodenary Instruments, 793
[Table IX. Mr. Bosanquet's Generalised Keyboard, 794
[Table X. Chords Estimated as Artificial Qualities of Tone, 795
[Additional Note on Prof. Mayer's Researches, 800.]

INTRODUCTION.

In the present work an attempt will be made to form a connection between the boundaries of two sciences, which, although drawn together by many natural relations, have hitherto remained sufficiently distinct—the boundaries of *physical and physiological acoustics* on the one side, and of *musical science and esthetics* on the other. The class of readers addressed will, consequently, have had very different cultivation, and will be affected by very different interests. It will therefore be best for the author at the outset distinctly to state his intention in undertaking the work, and the aim he has sought to attain. The horizons of physics, philosophy, and art have of late been too widely separated, and, as a consequence, the language, the methods, and the aims of any one of these studies present a certain amount of difficulty for the student of any other of them; and possibly this is the principal cause why the problem here undertaken has not been long ago more thoroughly considered and advanced towards its solution.

It is true that acoustics constantly employs conceptions and names borrowed from the theory of harmony, and speaks of the 'scale,' 'intervals,' 'consonances,' and so forth; and similarly, manuals of Thorough Bass generally begin with a physical chapter which speaks of 'the numbers of vibrations,' and fixes their 'ratios' for the different intervals; but, up to the present time, this apparent *connection* of acoustics and music has been wholly external, and may be regarded rather as an expression given to the feeling that such a connection must exist, than its actual formulation. Physical knowledge may indeed have been useful for musical instrument makers, but for the development and foundation of the theory of harmony it has hitherto been totally barren. And yet the essential facts within the field here to be explained and turned to profit, have been known from the earliest times. Even Pytha-

goras (fl. circa B.C. 540-510) knew that when strings of different lengths but of the same make, and subjected to the same strain were used to give the perfect consonances of the Octave, Fifth, or Fourth, their lengths must be in the ratios of 1 to 2, 2 to 3, or 3 to 4 respectively, and if, as is probable, his knowledge was partly derived from the Egyptian priests, it is impossible to conjecture in what remote antiquity this law was first known. Later physics has extended the law of Pythagoras by passing from the lengths of strings to the number of vibrations, and thus making it applicable to the tones of all musical instruments, and the numerical relations 4 to 5 and 5 to 6 have been added to the above for the less perfect consonances of the major and minor Thirds, but I am not aware that any real step was ever made towards answering the question: *What have musical consonances to do with the ratios of the first six numbers?* Musicians, as well as philosophers and physicists, have generally contented themselves with saying in effect that human minds were in some unknown manner constituted so as to elicit the numerical relations of the vibrations of tones, and have a peculiar pleasure in contemplating simple and readily comprehensible ratios.

Meanwhile musical esthetics has made unmistakable advances in those points which depend for their solution rather on psychological feeling than on the action of the senses, by introducing the conception of *progression* in the examination of musical works of art. E. Hanslick, in his book 'on the Musically Beautiful' (*Ueber das musikalisch Schöne*), triumphantly attacked the false standpoint of exaggerated sentimentality, from which it was fashionable to theorise on music, and referred the critic to the simple elements of melodic progression. The esthetic relations for the structure of musical compositions, and the characteristic differences of individual forms of composition, are explained more fully in Vischer's 'Esthetics' (*Aesthetik*). In the inorganic world the kind of motion seen, reveals the kind of moving force in action and in the last resort the only method of recognising and measuring the elementary powers of nature consists in determining the motions they generate, and this is also the case for the motions of bodies or of voices which take place under the influence of human feelings. Hence the properties of the progressions or motions of tone which possess a graceful, dallying, heavy, forced, dull, powerful, quiet, exciting character, and so on, chiefly depend

on psychological action. In the same way questions relating to the equilibrium of the separate parts of a composition, to their development from one another and their connection as one clearly intelligible whole, bear a close analogy to similar questions in architecture. But all such investigations, however fertile they may have been, cannot have been otherwise than imperfect and uncertain, so long as they were without their proper origin and foundation, that is, so long as there was no scientific foundation for their elementary rules relating to the construction of scales, chords, keys and modes, in short, to all that is usually contained in works on 'Thorough Bass.' In this elementary region we have to deal not merely with unfettered artistic inventions, but with the natural power of immediate sensation. Music stands in a much closer connection with pure sensation than any of the other arts. The latter rather deal with what the senses apprehend, that is with the images of outward objects, collected by psychical processes from immediate sensation. *Poetry* aims most distinctly at merely exciting the formation of images, by addressing itself especially to imagination and memory, and it is only by subordinate auxiliaries of a more musical description, such as rhythm, and imitations of sounds, that it appeals to the immediate sensation of hearing. Hence its effects depend mainly on psychical action. The *plastic arts*, although they make use of the sensation of sight, address the eye almost in the same way as *poetry* addresses the ear. Their main purpose is to excite in us the image of an external object of determinate form and colour. The spectator is essentially intended to interest himself in this image, and enjoy its beauty; not to dwell upon the means by which it was created. It must at least be allowed that the pleasure of a connoisseur or virtuoso in the constructive art shewn in a statue or a picture, is not an essential portion of general artistic enjoyment.

It is only in painting that we find colour as an element which is directly appreciated by sensation, without any intervening act of the intellect. On the contrary, in *music*, the sensations of tone are the material of the art. So far as these sensations are excited in music, we do not create out of them any images of external objects or actions. Again, when in hearing a concert we recognise one tone as due to a violin and another to a clarinet, our artistic enjoyment does not depend upon our conception of this action of a

violin or clarinet, but solely on our hearing of the tones they produce, whereas the artistic enjoyment resulting from viewing a marble statue does not depend on the white light which it reflects into the eye, but upon the mental image of the beautiful human form which it calls up. In this sense it is clear that music has a more immediate connection with pure sensation than any other of the fine arts, and, consequently, that the theory of the sensations of hearing is destined to play a much more important part in musical esthetics, than, for example, the theory of *chiaroscuro* or of perspective in painting. Those theories are certainly useful to the artist, as means for attaining the most perfect representation of nature, but they have no part in the artistic effect of his work. But in music no such perfect representation of nature is aimed at; tones and the sensations of tone exist for themselves alone, and produce their effects independently of anything behind them.

This theory of the sensations of hearing belongs to natural science, and comes in the first place under *physiological acoustics.* Hitherto it is the *physical* part of the *theory of sound* that has been almost exclusively treated at length, that is, the investigations refer exclusively to the motions produced by solid, liquid, or gaseous bodies when they occasion the sounds which the ear appreciates. This *physical acoustics* is essentially nothing but a section of the theory of the motions of elastic bodies. It is physically indifferent whether observations are made on stretched strings, by means of spirals of brass wire, (which vibrate so slowly that the eye can easily follow their motions, and, consequently, do not excite any sensation of sound,) or by means of a violin string, (where the eye can scarcely perceive the vibrations which the ear readily appreciates). The laws of vibratory motion are precisely the same in both cases; its rapidity or slowness does not affect the laws themselves in the slightest degree, although it compels the observer to apply different methods of observation, the eye for one and the ear for the other. In physical acoustics, therefore, the phenomena of hearing are taken into consideration solely because the ear is the most convenient and handy means of observing the more rapid elastic vibrations, and the physicist is compelled to study the peculiarities of the natural instrument which he is employing, in order to control the correctness of its indications. In this way, although physical acoustics as hitherto pursued, has, undoubtedly, collected many observations and much knowledge con-

cerning the action of the ear, which, therefore, belong to *physiological acoustics*, these results were not the principal object of its investigations; they were merely secondary and isolated facts. The only justification for devoting a separate chapter to acoustics in the theory of the motions of elastic bodies, to which it essentially belongs, is that the application of the ear as an instrument of research, influenced the nature of the experiments and the methods of observation.

But in addition to a *physical* there is a *physiological theory of acoustics*, the aim of which is to investigate the processes that take place within the ear itself. The section of this science which treats of the conduction of the motions to which sound is due, from the entrance of the external ear to the expansions of the nerves in the labyrinth of the inner ear, has received much attention, especially in Germany, since ground was broken by Johannes Mueller. At the same time it must be confessed that not many results have as yet been established with certainty. But these attempts attacked only a portion of the problem, and left the rest untouched. Investigations into the processes of each of our organs of sense, have in general three different parts. First we have to discover how the agent reaches the nerves to be excited, as light for the eye and sound for the ear. This may be called the *physical* part of the corresponding physiological investigation. Secondly we have to investigate the various modes in which the nerves themselves are excited, giving rise to their various *sensations*, and finally the laws according to which these sensations result in mental images of determinate external objects, that is, in *perceptions*. Hence we have secondly a specially *physiological* investigation for sensations, and thirdly, a specially *psychological* investigation for perceptions. Now whilst the physical side of the theory of hearing has been already frequently attacked, the results obtained for its *physiological* and *psychological* sections are few, imperfect, and accidental. Yet it is precisely the specially physiological part—the theory of the sensations of hearing—to which the theory of music has to look for the foundations of its structure.

In the present work, then, I have endeavoured, in the first place, to collect and arrange such materials for a theory of the *sensations of hearing* as already existed, or as I was able to add from my own personal investigations. Of course such a first

attempt must necessarily be somewhat imperfect, and be limited to the elements and the most interesting divisions of the subject discussed. It is in this light that I wish these studies to be regarded. Although in the propositions thus collected there is little of entirely new discoveries, and although even such apparently new facts and observations as they contain are, for the most part, more properly to be termed immediate consequences of carrying already known theories and methods of investigation more completely to their legitimate consequences, and of more thoroughly exhausting their results than had been heretofore attempted, yet I cannot but think that the facts frequently receive new importance and new illustration, by being regarded from a fresh point of view and in a fresh connection.

The First Part of the following investigation is essentially physical and physiological. It contains a general investigation of the phenomenon of harmonic *upper partial tones*. The nature of this phenomenon is established, and its relation to *quality of tone* is proved. A series of qualities of tone are analysed in respect to their harmonic upper partial tones, and it results that these partial tones are not, as was hitherto thought, isolated phenomena of small importance, but that, with very few exceptions, they determine the qualities of tone of almost all instruments, and are of the greatest importance for those qualities of tone which are best adapted for musical purposes. The question how the ear is able to perceive these harmonic upper partial tones then leads to an hypothesis respecting the excitement of the auditory nerves, which is well fitted to reduce all the facts and laws in this department to a relatively simple mechanical conception.

The Second Part treats of the disturbances produced by the simultaneous production of two tones, namely the *combinational tones* and *beats*. The physiologico-physical investigation shews that the only case in which two tones can be simultaneously heard by the ear without mutual disturbance, is when they stand to each other in the perfectly determinate and well-known relations of intervals which form musical consonance. We are thus immediately introduced into the field of music proper, and are led to discover the physiological reason for that enigmatical numerical relation announced by Pythagoras. The magnitude of the consonant intervals is independent of the quality of tone, but the harmonious-

ness of the consonances, and the distinctness of their separation from dissonances, depends on the quality of tone. The conclusions of physiological theory here agree precisely with the musical rules for the formation of chords; they even go more into particulars than it was possible for the latter to do, and have, as I believe, the authority of the best composers in their favour.

In these first two Parts of the book, no attention is paid to esthetic considerations. Natural phenomena obeying a blind necessity, are alone treated. The Third Part treats of the construction of *musical scales* and *notes*. Here we come at once upon esthetic ground, and the differences of national and individual taste begin to appear. Modern music has especially developed the principle of *tonality*, and all the tones in a piece of music are strictly connected by their relationship to one chief tone, called the tonic. On admitting this principle, the results of the preceding investigations furnish a method of constructing our modern musical scales and modes, from which all arbitrary assumption is excluded.

I was unwilling to separate the physiological investigation from its musical consequences, because the correctness of these consequences must be to the physiologist a verification of the correctness of the physical and physiological views advanced, and the reader, who takes up my book for its musical conclusions alone, cannot form a perfectly clear view of the meaning and bearing of these consequences, unless he has endeavoured to get at least some conception of their foundations in natural science. But in order to facilitate the use of the book by readers who have no special knowledge of physics and mathematics, I have transferred to an appendix, at the end of the book, all special instructions for performing the more complicated experiments, and also all mathematical investigations. In this way I hope to have consulted the interests of both classes of readers.

It is of course impossible for anyone to understand the investigations thoroughly, who does not take the trouble of becoming acquainted by personal observation with at least the fundamental phenomena mentioned. Fortunately the usual musical instruments render it easy for anyone to become acquainted with harmonic upper partial tones, combinational tones, beats, and the like. Personal observation is better than the exactest description, especially when, as here, the subject of investigation is an analysis of

sensations themselves, which are always extremely difficult to describe to those who have not experienced them.

In my somewhat unusual attempt to pass from natural philosophy into the theory of the arts, I hope that I have kept the regions of physiology and esthetics sufficiently distinct. But I can scarcely disguise from myself, that although my researches are confined to the lowest grade of musical grammar, they may probably appear too mechanical and unworthy of the dignity of art, to those theoreticians who are accustomed to use even the enthusiastic feelings called forth by the highest works of art, for the scientific investigation of its basis. To these I would simply remark in conclusion, that the following investigation really deals only with the analysis of actually existing sensations—that the physical methods of observation employed are almost solely meant to facilitate and assure the work of this analysis and check its completeness—and that this analysis of the sensations would suffice to furnish the results required for musical theory, even independently of my physiological hypothesis concerning the mechanism of hearing, but that I was unwilling to omit that hypothesis because it is so well suited to furnish an extremely simple connection between all the very various and very complicated phenomena which present themselves in the course of this investigation.

CHAPTER I.

ON THE SENSATION OF SOUND IN GENERAL.

SENSATIONS result from the action of an external stimulus on the sensitive apparatus of our nerves. Kinds of sensation differ, partly with the organ of sense excited, and partly with the kind of stimulus employed. Each organ of sense produces peculiar sensations, which cannot be excited in any other; the eye gives sensations of light, the ear sensations of sound, the skin sensations of touch. Even when the same sunbeams which excite in the eye sensations of light, impinge on the skin and excite its nerves, they are felt only as heat, not as light. In the same way the vibration of elastic bodies heard by the ear, can also be felt by the skin, but in that case produce only a whirring fluttering sensation, not sound. The sensation of sound is therefore a species of reaction against external stimulus, peculiar to the ear, and excitable in no other organ of the body, and is completely distinct from the sensation of any other sense.

As our problem is to study the laws of the sensation of hearing, our first business will be to examine how many kinds of sensation the ear can generate, and what differences in the external means of excitement or sound, correspond to these differences of sensation.

The first and principal difference between various sounds experienced by our ear, is that between *noises* and *musical tones*. The soughing, howling, and whistling of the wind, the splashing of water, the rolling and rumbling of carriages, are examples of the first kind, and the tones of all musical instruments of the second. Noises and musical tones may certainly intermingle in very various degrees, and pass insensibly into one another, but their extremes are widely separated.

The nature of the difference between musical tones and noises, can generally be determined by attentive aural observation without

artificial assistance. We perceive that generally, a noise is accompanied by a rapid alternation of different kinds of sensations of sound. Think, for example, of the rattling of a carriage over granite paving stones, the splashing or seething of a waterfall or of the waves of the sea, the rustling of leaves in a wood. In all these cases we have rapid, irregular, but distinctly perceptible alternations of various kinds of sounds, which crop up fitfully. When the wind howls the alternation is slow, the sound slowly and gradually rises and then falls again. It is more or less possible to separate restlessly alternating sounds for the greater number of other noises. We shall hereafter become acquainted with an instrument, called a resonator, which will materially assist the ear in making this separation. On the other hand, a musical tone strikes the ear as a perfectly undisturbed, uniform sound which remains unaltered as long as it exists, and it presents no alternation of various kinds of constituents. To this then corresponds a simple, regular kind of sensation, whereas in a noise many various sensations of musical tone are irregularly mixed up and as it were tumbled about in confusion. We can easily compound noises out of musical tones, as, for example, by striking all the keys contained in one or two octaves of a pianoforte at once. This shews us that musical tones are the simpler and more regular elements of the sensations of hearing, and that we have consequently first to study the laws and peculiarities of this class of sensations.

Then comes the question: On what difference in the external means of excitement does the difference between noise and musical tone depend? The normal and usual means of excitement for the human ear is atmospheric vibration. The irregularly alternating sensation of the ear in the case of noises leads us to conclude that for these the vibration of the air must also change irregularly. For musical tones on the other hand we anticipate a regular motion of the air, continuing uniformly, and in its turn excited by an equally regular motion of the sonorous body, whose impulses were conducted to the ear by the air.

Those regular motions which produce musical tones have been exactly investigated by physicists. They are *oscillations*, *vibrations*, or swings, that is, up and down, or to and fro motions of sonorous bodies, and it is necessary that these oscillations should be regularly *periodic*. By a *periodic motion* we mean one which

constantly returns to the same condition after exactly equal intervals of time. The length of the equal intervals of time between one state of the motion and its next exact repetition, we call the *length of the oscillation* vibration or swing, or the *period* of the motion. The kind of motion of the moving body during one period, is perfectly indifferent. As illustrations of periodical motion, take the motion of a clock pendulum, of a stone attached to a string and whirled round in a circle with uniform velocity, of a hammer made to rise and fall uniformly by its connection with a water wheel. All these motions, however different be their form, are periodic in the sense here explained. The length of their periods, which in the cases adduced is generally from one to several seconds, is relatively long in comparison with the much shorter periods of the vibrations producing musical tones, the lowest or deepest of which makes at least 30 in a second, while in other cases their number may increase to several thousand in a second.

Our definition of periodic motion then enables us to answer the question proposed as follows:—*The sensation of a musical tone is due to a rapid periodic motion of the sonorous body; the sensation of a noise to non-periodic motions.*

The musical vibrations of solid bodies are often visible. Although they may be too rapid for the eye to follow them singly, we easily recognise that a sounding string, or tuning fork, or the tongue of a reed-pipe, is rapidly vibrating between two fixed limits, and the regular, apparently immovable image that we see, notwithstanding the real motion of the body, leads us to conclude that the backward and forward motions are quite regular. In other cases we can feel the swinging motions of sonorous solids. Thus, the player feels the trembling of the reed in the mouthpiece of a clarinet, oboe, or bassoon, or of his own lips in the mouthpieces of trumpets and trombones.

The motions proceeding from the sounding bodies are usually conducted to our ear by means of the atmosphere. The particles of air must also execute periodically recurrent vibrations, in order to excite the sensation of a musical tone in our ear. This is actually the case, although in daily experience sound at first seems to be some agent, which is constantly advancing through the air, and propagating itself further and further. We must, however, here distinguish between the motion of the individual particles of

air—which takes place periodically backwards and forwards within very narrow limits—and the propagation of the sonorous tremor. The latter is constantly advancing by the constant attraction of fresh particles into its sphere of tremor.

This is a peculiarity of all so-called *undulatory motions.* Suppose a stone to be thrown into a piece of calm water. Round the spot struck there forms a little ring of wave, which, advancing equally in all directions, expands to a constantly increasing circle. Corresponding to this ring of wave, sound also proceeds in the air from the excited point and advances in all directions as far as the limits of the mass of air extend. The process in the air is essentially identical with that on the surface of the water. The principal difference consists in the spherical propagation of sound in all directions through the atmosphere which fills all surrounding space, whereas the waves of the water can only advance in rings or circles on its surface. The crests of the waves of water correspond in the waves of sound to spherical shells where the air is condensed, and the troughs to shells of rarefaction. On the free surface of the water, the mass on compression can slip upwards and so form ridges, but in the interior of the sea of air, the mass must be condensed, as there is no unoccupied spot for its escape.

The waves of water, therefore, continually advance without returning. But we must not suppose that the particles of water of which the waves are composed advance in a similar manner to the waves themselves. The motion of the particles of water on the surface can easily be rendered visible by floating a chip of wood upon it. This will perfectly share the motion of the adjacent particles. Now, such a chip is not carried on by the rings of wave. It only bobs up and down and finally rests on its original spot. The adjacent particles of water move in the same manner. When the ring of wave reaches them they are set bobbing; when it has passed over them they are still in their old place, and remain there at rest, while the ring of wave continues to advance towards fresh spots on the surface of the water, and sets new particles of water in motion. Hence the waves which pass over the surface of the water, are constantly built up of fresh particles of water. What really advances as a wave is only the tremor, the altered form of the surface, while the individual particles of water themselves merely move up and down transiently, and never depart far from their original position.

The same relation is seen still more clearly in the waves of a rope or chain. Take a flexible string of several feet in length, or a thin metal chain, hold it at one end and let the other hang down, stretched by its own weight alone. Now, move the hand by which you hold it quickly to one side and back again. The excursion which we have caused in the upper end of the string by moving the hand, will run down it as a kind of wave, so that constantly lower parts of the string will make a sidewards excursion while the upper return again into the straight position of rest. But it is evident that while the wave runs down, each individual particle of the string can have moved only horizontal backwards and forwards, and can have taken no share at all in the advance of the wave.

The experiment succeeds still better with a long elastic line, such as a thick piece of india-rubber, or a brass-wire spiral spring, from eight to twelve feet in length, fastened at one end, and slightly stretched by being held with the hand at the other. The hand is then easily able to excite waves which will run very regularly to the other end of the line, be there reflected and return. In this case it is also evident that it can be no part of the line itself which runs backwards and forwards, but that the advancing wave is composed of continually fresh particles of the line. By these examples the reader will be able to form a mental image of the kind of motion to which sound belongs, where the material particles of the body merely make periodical oscillations, while the tremor itself is constantly propagated forwards.

Now let us return to the surface of the water. We have supposed that one of its points has been struck by a stone and set in motion. This motion has spread out in the form of a ring of wave over the surface of the water, and having reached the chip of wood has set it bobbing up and down. Hence by means of the wave, the motion which the stone first excited in one point of the surface of the water has been communicated to the chip which was at another point of the same surface. The process which goes on in the atmospheric ocean about us, is of a precisely similar nature. For the stone substitute a sounding body, which shakes the air; for the chip of wood substitute the human ear, on which impinge the waves of air excited by the shock, setting its movable parts in vibration. The waves of air proceeding from a sounding body, transport the tremor to the human ear exactly in the same way as

the water transports the tremor produced by the stone to the floating chip.

In this way also it is easy to see how a body which itself makes periodical oscillations, will necessarily set the particles of air in periodical motion. A falling stone gives the surface of the water a single shock. Now replace the stone by a regular series of drops falling from a vessel with a small orifice. Every separate drop will excite a ring of wave, each ring of wave will advance over the surface of the water precisely like its predecessor, and will be in the same way followed by its successors. In this manner a regular series of concentric rings will be formed and propagated over the surface of the water. The number of drops which fall into the water in a second will be the number of waves which reach our floating chip in a second, and the number of times that this chip will therefore bob up and down in a second, thus executing a periodical motion, the period of which is equal to the interval of time between the falling of consecutive drops. Similarly in the atmosphere, a periodically oscillating sonorous body produces a similar periodical motion, first in the mass of air, and then in the drum of our ear, and the period of these vibrations must be the same as that of the vibration in the sonorous body.

Having thus spoken of the principal division of sound into Noise and Musical Tones, and then described the general motion of the air for these tones, we pass on to the peculiarities which distinguish such tones one from the other. We are acquainted with three points of difference in musical tones, confining ourselves in the first place to such tones as are isolatedly produced by our usual musical instruments, and excluding the simultaneous sounding of the tones of different instruments. Musical tones are distinguished:—

1. By their *force* or *loudness*.
2. By their *pitch*, or *relative height*.
3. By their *quality*.

It is unnecessary to explain what we mean by the force and pitch of a tone. By the quality of a tone we mean that peculiarity which distinguishes the musical tone of a violin from that of a flute or that of a clarinet, or that of the human voice, when all these instruments produce the same note at the same pitch.

We have now to explain what peculiarities of the motion of

sound correspond to these three principal differences between musical tones.

First, We easily recognise that the *force* or loudness of a musical tone increases and diminishes with the extent or so-called *amplitude* of the oscillations of the particles of the sounding body. When we strike a string, its vibrations are at first sufficiently large for us to see them, and its corresponding tone is loudest. The visible vibrations become smaller and smaller, and at the same time the loudness diminishes. The same observation can be made on strings excited by a violin bow, and on the reeds of reed-pipes, and on many other sonorous bodies. The same conclusion results from the diminution of the loudness of a tone when we increase our distance from the sounding body in the open air, although the pitch and quality remain unaltered; for it is only the amplitude of the oscillations of the particles of air which diminishes as their distance from the sounding body increases. Hence loudness must depend on this amplitude, and none other of the properties of sound do so.[1]

The second essential difference between different musical tones consists in their *pitch* or relative height. Daily experience shews us that musical tones of the same pitch can be produced upon most diverse instruments by means of most diverse mechanical contrivances, and with most diverse degrees of loudness. All the motions of the air thus excited must be periodic, because they would not otherwise excite in us the sensation of a musical tone. But the motion within each single period may be of any kind whatever, and yet if the length of the periodic time of two musical tones is the same, they have the same pitch. Hence: *Pitch depends solely on the length of time in which each single vibration is executed, or*, which comes to the same thing, *on the vibrational number of the tone.* We are accustomed to take a second of time as the unit, and consequently mean by *vibrational number* the number of vibrations which the particles of a sounding body perform in one second of time. It is self-evident that we find the periodic time or *vibra-*

[1] Mechanically the force of the oscillations for tones of different pitch is measured by their *vis viva*, that is, by the square of the greatest velocity attained by the oscillating particles. But the ear has different degrees of sensibility for tones of different pitch, so that no measure can be found for the intensity of the sensation of sound, that is, for the loudness of sound at all pitches.

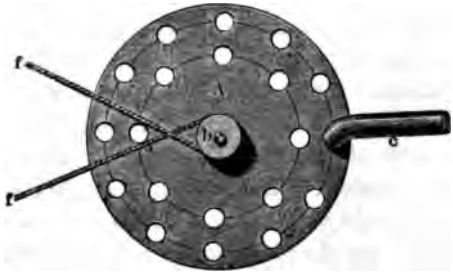
tional period, that is length of time which is occupied in performing a single vibration backwards and forwards, by dividing one second of time by the vibrational number.

Musical tones are higher, the greater their vibrational number, that is, the shorter is their vibrational period.

The exact determination of the vibrational number for such elastic bodies as produce audible tones, presents considerable difficulty, and physicists had to contrive many comparatively complicated processes in order to solve this problem for each particular case. Mathematical theory and numerous experiments had to render mutual asistance. It is consequently very convenient for the demonstration of the fundamental facts in this department of knowledge, to be able to apply a peculiar instrument for producing musical tones—the so-called *siren*—which is constructed in such a manner as to determine the vibrational number of the tone produced, by a direct observation. The principal parts of the simplest form of the siren are shewn in fig. 1, after Seebeck.

A is a thin disc of cardboard or tinplate, which can be set in rapid rotation about its axle b by means of a string f f, which passes over a larger wheel. On the margin of the disc there is punched a set of holes at equal intervals: of these there are twelve

FIG 1.



in the figure; one or more similar series of holes at equal distances are introduced on concentric circles, (there is one such of eight holes in the figure). c is a pipe which is directed over one of the holes. Now, on setting the disc in rotation and blowing through the pipe c, the air will pass freely whenever one of the holes passes the end of the pipe, but will be checked whenever an unpierced portion of the disc comes before it. Each hole of the disc, then, that passes the end of the pipe lets a single puff of air escape. Supposing the disc to make a single revolution and the

pipe to be directed to the outer circle of holes, we have twelve puffs corresponding to the twelve holes; but if the pipe is directed to the inner circle we have only eight puffs. If the disc is made to revolve ten times in one second, the outer circle would produce 120 puffs in one second, which would give rise to a weak and deep musical tone, and the inner circle eighty puffs. Generally, if we know the number of revolutions which the disc makes in a second, and the number of holes in the series to which the tube is directed, the product of these two numbers evidently gives the number of puffs in a second. This number is consequently far

FIG. 2.

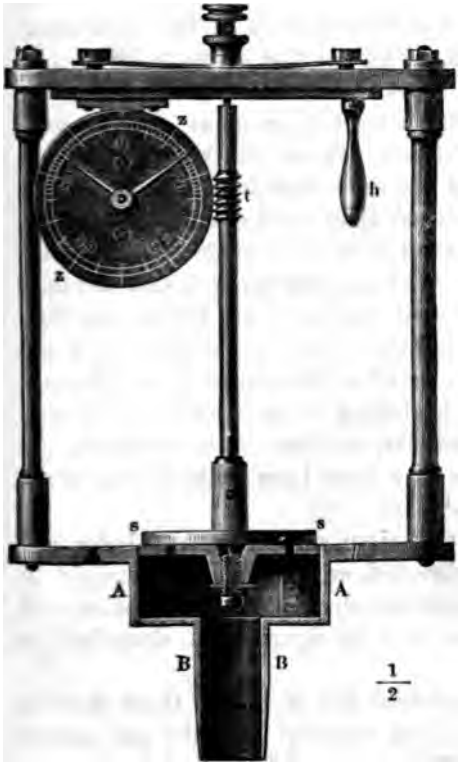


FIG. 3.

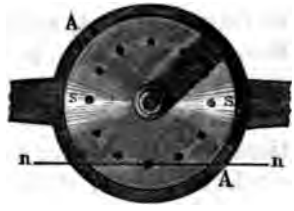
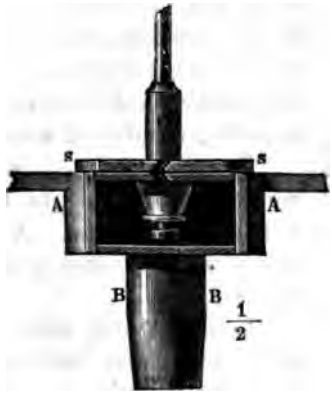


FIG. 4.



easier to determine exactly than in any other musical instrument, and sirens are accordingly extremely well adapted for studying all changes in musical tones resulting from the alterations and ratios of the vibrational numbers.

The form of siren here described gives only a weak tone. I have placed it first because its action can be most readily understood, and, by changing the disc, it can be easily applied to experiments of very different descriptions. A stronger tone is produced in the siren of Cagniard de la Tour, shewn in figures 2, 3, and 4, p. 19. Here s s is the rotating disc, of which the upper surface is shewn in fig. 3, and the side is seen in figs. 2 and 4. It is placed over a windchest A A, which is connected with a bellows by the pipe B B. The cover of the windchest A A, which lies immediately under the rotating disc s s, is pierced with precisely the same number of holes as the disc, and the direction of the holes pierced in the cover of the chest is oblique to that of the holes in the disc, as shewn in fig. 4, which is a vertical section of the instrument through the line n n in fig. 3. This position of the holes enables the wind escaping from A A to set the disc s s in rotation, and by increasing the pressure on the bellows, as much as 50 or 60 rotations in a second can be produced. Since all the holes of one circle are blown through at the same time in this siren, a much more powerful tone is produced than in Seebeck's, fig. 1. To record the revolutions, a counter z z is introduced, connected with a toothed wheel which works in the screw t, and advances one tooth for each revolution of the disc s s. By the handle h this counter may be moved slightly to one side, so that the wheelwork and screw may be connected or disconnected at pleasure. If they are connected at the beginning of one second, and disconnected at the beginning of another, the hand of the counter shews how many revolutions of the disc have been made in the corresponding number of seconds.[1]

Dove[2] introduced into this siren several rows of holes into which the wind might be directed, or from which it might be cut off, at pleasure. A polyphonic siren of this description with other peculiar arrangements will be figured and described in Chapter VIII.

It is clear that when the pierced disc of one of these sirens is made to revolve with a uniform velocity, and the air escapes through the holes in puffs, the motion of the air thus produced must be *periodic* in the sense already explained. The holes stand at equal intervals of space, and hence on rotation follow each

[1] See Appendix I. [2] [Pronounce *Do-ve*, in two syllables.—*Translator*.]

other at equal intervals of time. Through every hole there is poured, as it were, a drop of air into the external atmospheric ocean, exciting waves in it, which succeed each other at uniform intervals of time, just as was the case when regularly falling drops impinged upon a surface of water (p. 16). Within each separate period, each individual puff will have considerable variations of form in sirens of different construction, depending on the different diameters of the holes, their distance from each other, and the shape of the extremity of the pipe which conveys the air; but in every case, as long as the velocity of rotation and the position of the pipe remain unaltered, a regularly periodic motion of the air must result, and consequently the sensation of a musical tone must be excited in the ear, and this is actually the case.

It results immediately from experiments with the siren that two series of the same number of holes revolving with the same velocity, give musical tones of the same pitch, quite independently of the size and form of the holes, or of the pipe. We even obtain a musical tone of the same pitch if we allow a metal point to strike in the holes as they revolve instead of blowing. Hence it follows firstly that the musical height of a tone depends only on the *number* of puffs or swings, and not on their form, force, or method of production. Further it is very easily seen with this instrument that on increasing the velocity of rotation, and consequently the number of puffs produced in a second, the pitch becomes sharper or higher. The same result ensues if, maintaining a uniform velocity of rotation, we first blow into a series with a smaller and then into a series with a greater number of holes. The latter gives the sharper or higher pitch.

With the same instrument we also very easily find the remarkable relation which the vibrational numbers of two musical tones must possess in order to form a consonant interval. Make a series of 8 and another of 16 holes on a disc, and blow into both sets while the disc is kept at uniform velocity of rotation. Two tones will be heard which stand to one another in the exact ratio of an Octave. Increase the velocity of rotation; both tones will become sharper, but both will continue at the new pitch to form the interval of an Octave. Hence we conclude that *a musical tone which is an Octave higher than another, makes exactly twice as many vibrations in a given time as the latter.*

The disc shewn in fig. 1, p. 18, has two circles of 8 and 12 holes

respectively. Each, blown successively, gives two tones which form with each other a perfect Fifth, independently of the velocity of rotation of the disc. Hence, *two musical tones stand in the relation of a so-called Fifth when the higher tone makes three vibrations in the same time as the lower makes two.*

If we obtain a musical tone by blowing into a circle of 8 holes, we require a circle of 16 holes for its Octave, and 12 for its Fifth. Hence the ratio of the vibrational numbers of the Fifth and the Octave is 12 : 16 or 3 : 4. But the interval between the Fifth and the Octave is the Fourth, so that we see that *when two musical tones form a Fourth, the higher makes four vibrations while the lower makes three.*

The polyphonic siren of Dove has usually four circles of 8, 10, 12 and 16 holes respectively. The series of 16 holes gives the Octave of the series of 8 holes, and the Fourth of the series of 12 holes. The series of 12 holes gives the Fifth of the series of 8 holes, and the minor Third of the series of 10 holes. While the series of 10 holes gives the major Third of the series of 8 holes. The four series consequently give the constituent musical tones of a major chord.

By these and similar experiments we find the following relations of the vibrational numbers:—

1	:	2	Octave
2	:	3	Fifth
3	:	4	Fourth
4	:	5	major Third
5	:	6	minor Third.

When the fundamental tone of a given interval is taken an Octave higher, the interval is said to be *inverted.* Thus a Fourth is an inverted Fifth, a minor Sixth an inverted major Third, and a major Sixth an inverted minor Third. The corresponding ratios of the vibrational numbers are consequently obtained by doubling the smaller number in the original interval.

From 2 : 3 the Fifth, we thus have 3 : 4 the Fourth
„ 4 : 5 the major Third . . . . 5 : 8 the minor Sixth
„ 5 : 6 the minor Third, 6 : 10 = 3 : 5 the major Sixth.

These are all the consonant intervals which lie within the compass of an Octave. With the exception of the minor Sixth,

which is really the most imperfect of the above consonances, the ratios of their vibrational numbers are all expressed by means of the whole numbers, 1, 2, 3, 4, 5, 6.

Comparatively simple and easy experiments with the siren, therefore, corroborate that remarkable law mentioned in the introduction (p. 2), according to which the vibrational numbers of consonant musical tones bear to each other ratios expressible by small whole numbers. In the course of our investigation we shall employ the same instrument to verify more completely the strictness and exactness of this law.

Long before anything was known of vibrational numbers, or the means of counting them, Pythagoras had discovered that if a string be divided into two parts by a bridge, in such a way as to give two consonant musical tones when struck, the lengths of these parts must be in the ratio of these whole numbers. If the bridge is so placed that $\frac{2}{3}$ of the string lie to the right, and $\frac{1}{3}$ on the left, so that the two lengths are in the ratio of 2 : 1, they produce the interval of an Octave, the greater length giving the deeper tone. Placing the bridge so that $\frac{2}{3}$ of the string lie on the right and $\frac{1}{3}$ on the left, the ratio of the two lengths is 3 : 2, and the interval is a Fifth.

These measurements had been executed with great precision by the Greek musicians, and had given rise to a system of tones, contrived with considerable art. For these measurements they used a peculiar instrument, the *monochord*, consisting of a sounding board and box on which a single string was stretched with a scale below, so as to set the bridge correctly.

It was not till much later that, through the investigations of Galileo (1638), Newton, Euler (1729), and Daniel Bernouilli (1771), the law governing the motions of strings became known, and it was thus found that the simple ratios of the lengths of the strings existed also for the vibrational numbers of the tones they produced, and that they consequently belonged to the musical intervals of the tones of all instruments, and were not confined to the strings through which the law had been first discovered.

This relation of whole numbers to musical consonances was from all time looked upon as a wonderful mystery of deep significance. The Pythagoreans themselves made use of it in their speculations on the harmony of the spheres. From that time it

remained partly the goal and partly the starting point of the strangest and most venturesome, fantastic or philosophic combinations, till in modern times the majority of investigators adopted the notion accepted by Euler himself, that the human mind had a peculiar pleasure in simple ratios, because it could better understand them and comprehend their bearings. But it remained uninvestigated how the mind of a listener not versed in physics, who perhaps was not even aware that musical tones depended on periodical vibrations, contrived to recognise and compare these ratios of the vibrational numbers. To shew what processes take place in the ear to render sensible the difference between consonance and dissonance, will be one of the principal problems in the second part of this work.

Calculation of the Vibrational Numbers for all the Tones of the Musical Scale.

By means of the ratios of the vibrational numbers already assigned for the consonant intervals, it is easy, by pursuing these intervals throughout, to calculate the ratios for the whole extent of the musical scale.

The major triad or chord of three tones, consists of a major Third and a Fifth. Hence its ratios are:

$$\begin{array}{r} C : E : G \\ 1 : \tfrac{5}{4} : \tfrac{3}{2} \\ \text{or } 4 : 5 : 6 \end{array}$$

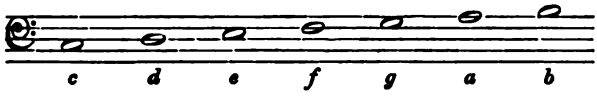
If we associate with this triad that of its dominant $G : B : D$, and that of its sub-dominant $F : A : C$, each of which has one tone in common with the triad of the tonic $C : E : G$, we obtain the complete series of tones for the major scale of C, with the following ratios of the vibrational numbers:

$$\begin{array}{r} C : D : E : F : G : A : B : c \\ 1 : \tfrac{9}{8} : \tfrac{5}{4} : \tfrac{4}{3} : \tfrac{3}{2} : \tfrac{5}{3} : \tfrac{15}{8} : 2. \\ [\text{or } 24 : 27 : 30 : 32 : 36 : 40 : 45 : 48] \end{array}$$

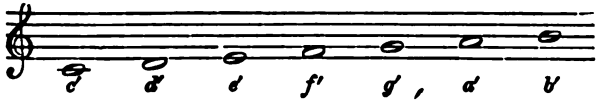
In order to extend the calculation to other octaves, we shall adopt the following notation of musical tones, marking the higher octaves by accents as is usual in Germany,[1] as follows:

[1] [English works use strokes above and below the letters, which is typographically inconvenient, and would interfere with a notation introduced in Chap. XIV. Hence the German notation, which is typographically more convenient, is retained. On notation of pitch generally see Appendix XIX.—*Translator.*]

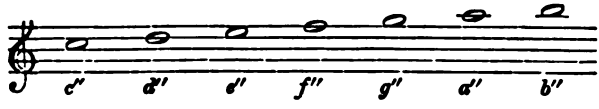
1. *The Unaccented or Small Octave* (the 4-foot Octave on the Organ):—



2. *The Once-accented Octave* (2-foot):—

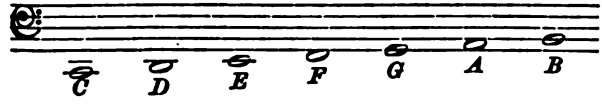


3. *The Twice-accented Octave* (1-foot):—



And so on for higher Octaves. Below the small Octave lies the great Octave, written with unaccented capital letters; its C requires an organ pipe of eight feet in length, and hence it is called the 8-foot Octave.

4. *Great or 8-foot Octave.*



Below this follows the 16-*foot* or *Contra-Octave*; the lowest on the pianoforte and most organs, the tones of which may be represented by $C_{,}$ $D_{,}$ $E_{,}$ $F_{,}$ $G_{,}$ $A_{,}$ $B_{,}$ with an inverted accent. On great organs there is a still deeper, 32-foot Octave, the tones of which may be written $C_{,,}$ $D_{,,}$ $E_{,,}$ $F_{,,}$ $G_{,,}$ $A_{,,}$ $B_{,,}$, with two inverted accents, but they scarcely retain the character of musical tones. (See Chap. IX.)

Since the vibrational numbers of any Octave are always twice as great as those for the next deeper, we find the vibrational numbers of the higher tones by *multiplying* those of the small or unaccented Octave as many times by 2 as its symbol has upper accents. And on the contrary the vibrational numbers for the deeper

Octaves are found by *dividing* those of the great Octave, as often as its symbol has lower accents.

$$\text{Thus } c'' = 2 \times 2 \times c = 2 \times 2 \times 2\, C$$
$$C_{,,} = \tfrac{1}{2} \times \tfrac{1}{2} \times C = \tfrac{1}{2} \times \tfrac{1}{2} \times \tfrac{1}{2}\, c.$$

For the pitch of the musical scale German physicists have generally adopted that proposed by Scheibler, and adopted subsequently by the German Association of Natural Philosophers (*die deutsche Naturforscherversammlung*) in 1834. This makes the once accented a' perform 440 vibrations in a second.[1] Hence results the following table for the scale of C major, which will serve to determine the pitch of all tones that are defined by their vibrational numbers in the following work.

Notes	Contra Octave $C_{,}$ to $B_{,}$	Great Octave C to B	Unaccented Octave c to b	Once accented Octave c' to b'	Twice accented Octave c'' to b''	Thrice accented Octave c''' to b'''	Four times accented Octave c'''' to b''''
C	33	66	132	264	528	1056	2112
D	37·125	74·25	148·5	297	594	1188	2376
E	41·25	82·5	165	330	660	1320	2640
F	44	88	176	352	704	1408	2816
G	49·5	99	198	396	792	1584	3168
A	55	110	220	440	880	1760	3520
B	61·875	123·75	247·5	495	990	1980	3960

[1] The Paris Academy has lately fixed the vibrational number of the same note at 435. This is called 870 by the Academy, because French physicists have adopted the inconvenient habit of counting the forward motion of a swinging body as one vibration and the backward as another, so that the whole vibration is counted as two. [This French method of counting is used everywhere for the seconds pendulum. In this work and in England generally, the backwards and forwards motion are counted musically as one vibration, hence sometimes called 'a double vibration.' In England the theoretical pitch gives 512 as the vibrational number of c'', and most tuning forks such as those issued by Mr. Hullah, and the Tonic Sol-faists, profess to give that pitch. The Society of Arts adopted 528 for c'', as in this table. Various other pitches are also in use. It should be observed that when 'equal temperament' is employed, and a' is taken as 440, c'' is only 523·25, and if c'' is taken as 528, a' in 'equal temperament' is 443·99 or nearly 444, that is, considerably sharper (nearly ¼ of a comma) than Scheibler's pitch. In the table in the text 'just intonation' is adopted. The difference between the 'just' and the 'equally tempered' scale, the latter of which is now almost universally employed for pianofortes and organs, will be explained in Chap. XVI. As many calculations are founded on the numbers given in this table, it was thought best not to reduce them to the English theoretical pitch in this translation. All the acoustical apparatus of Koenig (Appendix II.) is, however now constructed for $c'' = 512$, unless otherwise ordered; see the note prefixed to his catalogue of apparatus published in 1865. See also Appendix XIX.—*Translator.*]

The lowest tone on orchestral instruments is the $E_{,}$ of the double bass, making 41¼ vibrations in a second. Modern pianofortes and organs usually go down to $C_{,}$ with 33 vibrations, and the latest grand pianos even down to $A_{,,}$ with 27½ vibrations. On larger organs as already mentioned, there is also a deeper Octave reaching to $C_{,,}$ with 16½ vibrations. But the musical character of all these tones below $E_{,}$ is imperfect, because we are here near to the limit of the power of the ear to combine vibrations into musical tones. These lower tones cannot therefore be used musically except in connection with their higher Octaves to which they impart a character of greater depth without rendering the conception of the pitch indeterminate.

Upwards, pianofortes generally reach a'''' with 3520, or even c^{v} with 4224 vibrations. The highest tone in the orchestra is probably the five times accented d^{v} of the piccolo flute with 4752 vibrations. Despretz asserts that by means of small tuning-forks excited by a violin bow he has even reached the eight times accented $d^{v''''}$ with 38016 vibrations in a second. These high tones were very painfully unpleasant, and the pitch of those which exceed the boundaries of the musical scale was very imperfectly discriminated. More on these limits in Chap. IX.

The musical tones which can be used with advantage, and have clearly distinguishable pitch, have therefore between 40 and 4000 vibrations in a second, extending over 7 Octaves. Those which are audible at all have from 20 to 38000 vibrations, extending over 11 Octaves. This shews what a great variety of different vibrational numbers can be perceived and distinguished by the ear. In this respect the ear is far superior to the eye, which likewise distinguishes light of different periods of vibration by the sensation of different colours, for the compass of the vibrations of light distinguishable by the eye little exceeds an Octave.

Loudness and *pitch* were the two first differences which we found between musical tones; the third was *quality of tone*, which we now have to investigate. If the same note is sounded successively on a pianoforte, a violin, clarinet, oboe, or trumpet, or by the human voice, notwithstanding its having the same force and the same pitch, the musical tone of each of these instruments is different, and by means of this tone we recognise with the greatest ease which of these instruments was used. Varieties of quality of tone appear to be infinitely numerous. Not only do we know a

long series of musical instruments which could each produce the same note; not only do different individual instruments of the same form, and the voices of different individual singers shew certain more delicate shades of quality of tone, which our ear is able to distinguish; but the same note can sometimes be sounded on the same instrument with several qualitative varieties. In this respect the "bowed" instruments (i.e. those of the violin kind) are distinguished above all other. But the human voice is still richer, and human speech employs these very qualitative varieties of tone, in order to distinguish different letters. The different vowels, namely, belong to the class of sustained tones which can be used in music, while the character of consonants mainly depends upon brief and transient noises.

On inquiring to what external physical difference in the waves of sound the different qualities of tone correspond, we must remember that the amplitude of the vibration determines the force or loudness, and the period of vibration the pitch. Quality of tone can therefore depend upon neither of these. The only possible hypothesis, therefore, is that the quality of tone should depend upon the manner in which the motion is performed within the period of each single vibration. For the generation of a musical tone we have only required that the motion should be periodic, that is, that in any one single period of vibration exactly the same state should occur, in the same order of occurrence as it presents itself in any other single period. As to the kind of motion that should take place within any single period, no hypothesis was made. In this respect then an endless variety of motions might be possible for the production of sound.

Observe instances, taking first such periodic motions as are performed so slowly that we can follow them with the eye. Take a pendulum, which we can at any time construct by attaching a weight to a thread and setting it in motion. The pendulum swings from right to left with a uniform motion, uninterrupted by jerks. Near to either end of its path it moves slowly, and in the middle fast. Among sonorous bodies, which move in the same way, only very much faster, we may mention tuning forks. When a tuning fork is struck or is excited by a violin bow, and its motion is allowed to die away slowly, its two prongs oscillate backwards and forwards in the same way and after the same law

as a pendulum, only they make many hundred swings for each single swing of the pendulum.

As another example of a periodic motion, take a hammer moved by a water wheel. It is slowly raised by the millwork, then released, and falls down suddenly, is then again slowly raised, and so on. Here again we have a periodical backwards and forwards motion; but it is manifest that this kind of motion is totally different from that of the pendulum. Among motions which produce musical sounds, that of a violin string, excited by a bow, would most nearly correspond with this, as will be seen from the detailed description in Chap. V. The string clings for a time to the bow, and is carried along by it, then suddenly releases itself, like the hammer in the mill, and, like the latter, retreats somewhat with much greater velocity than it advanced, and is again caught by the bow and carried forward.

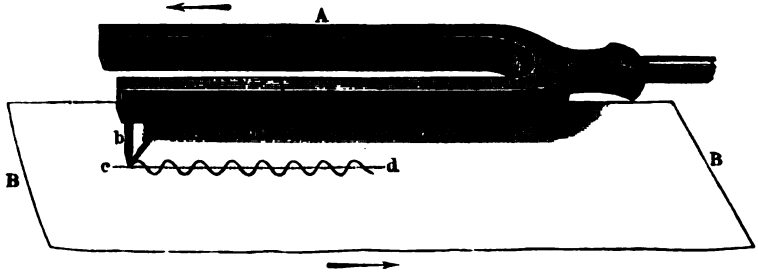
Again, imagine a ball thrown up vertically, and caught on its descent with a blow which sends it up again to the same height, and suppose this operation to be performed at equal intervals of time. Such a ball would occupy the same time in rising as in falling, but at the lowest point its motion would be suddenly interrupted, whereas above it would pass through gradually diminishing speed of ascent into a gradually increasing speed of descent. This then would be a third kind of alternating periodic motion, and would take place in a manner essentially different from the other two.

To render the law of such motions more comprehensible to the eye than is possible by lengthy verbal descriptions, mathematicians and physicists are in the habit of applying a graphical method, which must be frequently employed in this work, and should therefore be well understood.

To render this method intelligible suppose a drawing point b, fig. 5, p. 30, to be fastened to the prong A of a tuning fork in such a manner as to mark a surface of paper B B. Let the tuning fork be moved with a uniform velocity in the direction of the upper arrow, or else the paper be drawn under it in the opposite direction, as shewn by the lower arrow. When the fork is not sounding, the point will describe the dotted straight line d c. But if the prongs have been first set in vibration, the point will describe the undulating line d c, for as the prong vibrates, the attached point b will constantly move backwards and forwards,

and hence be sometimes on the right and sometimes on the left of the dotted straight line d c, as is shewn by the wavy line in the figure. This wavy line once drawn, remains as a permanent image of the kind of motion performed by the end of the fork during its musical vibrations. As the point b is moved in the direction of the straight line d c with a constant velocity, equal

FIG. 5.



sections of the straight line d c will correspond to equal sections of the time during which the motion lasts, and the distance of the wavy line on either side of the straight line will shew how far the point b has moved from its mean position to one side or the other during those sections of time.

In actually performing such an experiment as this, it is best to wrap the paper over a cylinder which is made to rotate uniformly by clockwork. The paper is wetted, and then passed over a turpentine flame which coats it with lampblack, on which a fine and somewhat smooth steel point will easily trace delicate lines. Fig. 6 is the copy of a drawing actually made in this way

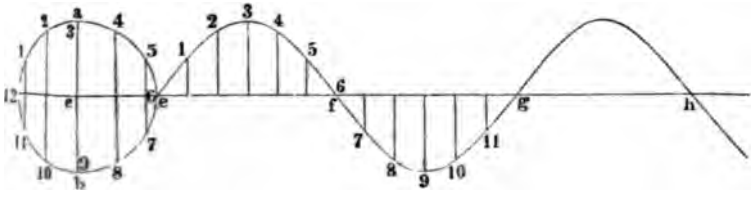
FIG. 6.

on the rotating cylinder of Messrs. Scott and Koenig's *Phonautograph.*

Fig. 7 shews a portion of this curve on a larger scale. It is easy to see the meaning of such a curve. The drawing point has passed with a uniform velocity in the direction c h. Suppose that it has described the section e g in $\frac{1}{10}$ of a second. Divide e g

into 12 eqnal parts, as in the figure, then the point has been $\frac{1}{120}$ of a second in describing the length of any such section horizontally, and the curve shews us on what side and at what distance from the position of rest the vibrating point will be at the end of $\frac{1}{120}$, $\frac{2}{120}$, and so on, of a second, or, generally, at any given short interval of time since it left the point e. We see, in the figure, that after $\frac{1}{120}$ of a second it had reached the height 1, and that it rose gradually till the end of $\frac{3}{120}$ of a second; then, however, it began to descend gradually till, at the end of $\frac{6}{120}=\frac{1}{20}$ second, it had reached its mean position f, and then it continued descending on the opposite side till the end of $\frac{9}{120}$ of a second and so on. We can also easily determine where the vibrating point was to be found at the end of any fraction of this hundred-and-twentieth of a second. A drawing of this kind consequently

FIG. 7.



shews immediately at what point of its path a vibrating particle is to be found at any given instant, and hence gives a complete image of its motion. If the reader wishes to reproduce the motion of the vibrating point, he has only to cut a narrow vertical slit in a piece of paper, and place it over fig. 6 or fig. 7, so as to shew a very small portion of the curve through the vertical slit, and draw the book slowly but uniformly under the slit, from right to left; the white or black point in the slit will then appear to move backwards and forwards in precisely the same manner as the original drawing point attached to the fork, only of course much more slowly.

We are not yet able to make all vibrating bodies describe their vibrations directly on paper, although much progress has recently been made in the methods required for this purpose. But we are able ourselves to draw such curves for all sounding bodies, when the law of their motion is known, that is, when we know how far the vibrating point will be from its mean position at any given

moment of time. We then set off on a horizontal line, such as e f, fig. 7, lengths corresponding to the interval of time, and let fall perpendiculars to it on either side, making their lengths equal or proportional to the distance of the vibrating point from its mean position, and then by joining the extremities of these perpendiculars we obtain a curve such as the vibrating body would have drawn if it had been possible to make it do so.

Thus fig. 8 represents the motion of the hammer raised by a water wheel, or of a point in a string excited by a violin bow. For the first 9 intervals it rises slowly and uniformly, and during the 10th it falls suddenly down.

FIG. 8.

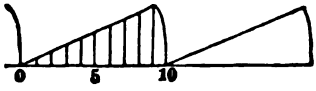


FIG. 9.

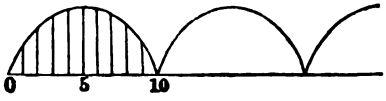


Fig. 9 represents the motion of the ball which is struck up again as soon as it comes down. Ascent and descent are performed with equal rapidity, whereas in fig. 8 the ascent takes much longer time. But at the lowest point the blow suddenly changes the kind of motion.

Physicists, then, having in their mind such curvilinear forms, representing the law of the motion of sounding bodies, speak briefly of the *form of vibration* of a sounding body, and assert that *the quality of tone depends on the form of vibration.* This assertion, which physicists hitherto based simply on the fact of their knowing that the quality of the tone could not possibly depend on the periodic time of a vibration, or on its amplitude, will be strictly examined hereafter. It will be shewn to be so far correct that every different quality of tone requires a different form of vibration, but on the other hand it will also appear that different forms of vibration may correspond to the same quality of tone.

On exactly and carefully examining the effect produced on the ear by different forms of vibration, as for example that in fig. 8, corresponding nearly to a violin string, we meet with a strange and unexpected phenomenon, long known indeed to individual musicians and physicists, but commonly regarded as a mere

curiosity, its generality and its great significance for all matters relating to musical tones not having been recognised. The ear when its attention has been properly directed to the effect of the vibrations which strike it, does not hear merely that one musical tone whose pitch is determined by the period of the vibrations in the manner already explained, but in addition to this it becomes aware of a whole series of higher musical tones, which we will call the *harmonic upper partial tones*, and sometimes simply the *upper partials* of that musical tone, in contradistinction to that first tone, the *fundamental* or *prime partial tone* or simply the *prime*, which is the lowest and generally the loudest of all, and by whose pitch we judge of the pitch of the whole *compound musical tone*, or simply the *compound*. The series of these upper partial tones is precisely the same for all compound musical tones which correspond to a uniformly periodical motion of the air. It is as follows:—

The first upper partial tone is the upper Octave of the prime tone, and makes double the number of vibrations in the same time. If we call the prime *c*, this upper octave will be *c′*.

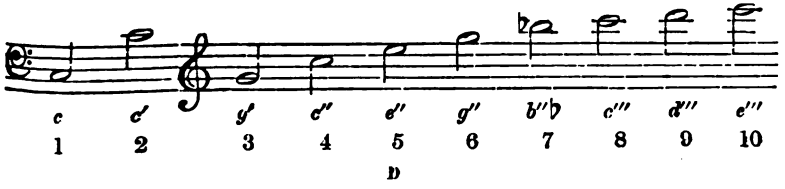
The second upper partial tone is the Fifth of this Octave, or *g′*, making three times as many vibrations in the same time as the prime.

The third upper partial tone is the second higher Octave or *c″*, making four times as many vibrations as the prime in the same time.

The fourth upper partial tone is the major Third of this second higher Octave, or *e″*, with five times as many vibrations as the prime in the same time.

The fifth upper partial tone is the Fifth of the second higher Octave, or *g″*, making six times as many vibrations as the prime in the same time.

And thus they go on, becoming continually fainter, to tones making 7, 8, 9, &c., times as many vibrations in the same time, as the prime tone. Or in musical notation



D

where the figures beneath shew how many times the corresponding vibrational number is greater than that of the prime tone.

The whole sensation excited in the ear by a periodic vibration of the air we have called a *musical tone*. We now find that this is *compound*, containing a series of different tones, which we distinguish as the *constituents* or *partial tones* of the *compound*. The first of these constituents is the *prime partial tone* of the compound, and the rest its *harmonic upper partial tones*. The *number* which shews the *order* of any partial tone in the series shews how many times its vibrational number exceeds that of the prime tone.[1] Thus, the second partial tone makes twice as many, the third three times as many vibrations in the same time as the prime tone, and so on.

G. S. Ohm was the first to declare that there is only one form of vibration which will give rise to no harmonic upper partial tones, and which will therefore consist solely of the prime tone. This is the form of vibration which we have described above as peculiar to the pendulum and tuning-forks, and drawn in figs. 6 and 7. We will call these *pendular vibrations*, or, since they cannot be analysed into a compound of different tones, *simple vibrations*. In what sense not merely other musical tones, but all other forms of vibration, may be considered as *compound*, will be shewn hereafter (Chap. IV.) The terms *simple* or *pendular vibration*,[2]

[1] [The ordinal number of a partial tone *generally*, must be distinguished from the ordinal number of an *upper* partial tone in particular. For the same tone the former number is always greater by unity than the latter, because the partials *generally* include the prime, which is reckoned as the first, and the *upper* partials exclude the prime, which being the *lowest* partial is of course not an *upper* partial at all. Thus the partials generally numbered 2 3 4 5 6 7 8 9 are the same as the upper partials numbered 1 2 3 4 5 6 7 8. As even the author has occasionally failed to note this distinction in the original German text, and other writers have constantly neglected it, too much weight cannot be here laid upon it. The presence or absence of the word *upper* before the word *partial* must always be carefully observed.—*Translator*.]

[2] The law of these vibrations may be popularly explained by means of the construction in fig. 10. Suppose a point to describe the circle of which c is the centre with a uniform velocity, and that an observer stands at a considerable distance in the prolongation of the line e h, so that he does not see the surface of the circle but only its edge, in which case the point will appear merely to move up and down along its diameter a b. This up and down motion would take place exactly according to the law of pendular vibration. To represent this motion graphically by means of a curve, divide the length e g, supposed to correspond to the time of a single period, into as many (here 12) equal parts as the circumference of the circle, and draw the perpendiculars 1, 2, 3, &c., on the dividing points of the line e g, in order, equal in length

be considered as in itself a chord or combination of various simple tones.[1]

[1] [The above paragraph relating to the English terms used in this translation, necessarily differs in many respects from the original, in which a justification is given of the use made by the author of certain *German* expressions. It has been my object to employ terms which should be thoroughly English, and should not in any way recall the German words. The word *tone* in English is extremely ambiguous. Prof. Tyndall (*Lectures on Sound*, 2nd ed. 1869, p. 117) has ventured to define a *tone* as a *simple tone*, in agreement with Prof. Helmholtz, who in the present passage limits the German word *Ton* in the same way. But I felt that an English reader could not be safely trusted to keep this very peculiar and important class of musical tones, which he has very rarely or never heard separately, invariably distinct from those musical tones with which he is familiar, unless the word *tone* were uniformly qualified by the epithet *simple*. The only exception I could make was in the case of a *partial tone*, which is received at once as a new conception. Even Prof. Helmholtz himself has not succeeded in using his word *Ton* consistently for a *simple* tone only, and this was an additional warning to me. English musicians have been also in the habit of using *tone* to signify a certain musical interval, and *semitone* for half of that interval, on the equally tempered scale. In this case I write *Tone* and *Semitone* with capital initials, a practice which I have found convenient for the names of all intervals, as *Thirds*, *Fifths*, &c. Prof. Helmholtz uses the word *Klang* for a *musical tone*, which generally, but not always, means a *compound tone*. Prof. Tyndall (*ibid.* p. 118) proposes to use the English word *clang* in the same sense. But *clang* has already a meaning in English, thus defined by Webster: 'a sharp shrill sound, made by striking together metallic substances, or sonorous bodies, as the *clang* of arms, or any like sound, as the *clang* of trumpets. This word implies a degree of harshness in the sound, or more harshness than *clink*.' Interpreted scientifically, then, *clang* according to this definition, is either *noise* or one of those *musical tones with inharmonic upper partials*, which will be subsequently explained. It is therefore totally unadapted to represent a *musical tone* in general, for which the simple word *tone* seems eminently suited, being of course originally the tone produced by a *stretched* string. Of course, if *clang* could not be used, Prof. Tyndall's suggestion to translate Prof. Helmholtz's *Klangfarbe* by *clangtint* (*ibid.*) fell to the ground. I can find no valid reason for supplanting the time-honoured expression *quality of tone*. Prof. Tyndall (*ibid.*) quotes Dr Young to the effect that 'this quality of sound is sometimes called its register, colour, or timbre.' *Register* has a distinct meaning in vocal music which must not be disturbed. *Timbre*, properly a kettle drum, then a helmet, then the coat of arms surmounted with a helmet, then the official stamp bearing that coat of arms (now used in France for a postage label), and then the mark which declared a thing to be what it pretends to be, Burns's 'guinea's stamp,' is a foreign word, often odiously mispronounced, and not worth preserving. *Colour* I have never met with, except at most as a passing metaphorical expression when applied to music. But the difference of tones in *quality* is familiar to our language. Then as to the Upper Partial Tones, Prof. Helmholtz uses *Theiltöne*, and *Partialtöne*, which are aptly Englished by *partial simple tones*. The word *simple*, however, may be omitted when *partial* is employed, as partial tones are necessarily simple. The *constituent tones* of a chord may be either *simple* or *compound*. The *Grundton* of a compound tone then becomes its *prime tone*, or briefly its *prime*. The *Grundton* of a *chord* will be further explained hereafter. *Upper* partial (simple) tones, that is, the partials exclusive of the prime, even when *harmonic*, that is, for the most part, belonging to the first six partial tones, must

CHAPTER II.

ON THE COMPOSITION OF VIBRATIONS.

AT the end of the last chapter we came upon the remarkable fact that the human ear is capable, under certain conditions, of separating the musical tone produced by a single musical instrument, into a series of simple tones, namely, the prime partial, and the various upper partial tones, each of which produces its own separate sensation. That the ear is capable of distinguishing from each other tones proceeding from different sources, that is, which do not arise from one and the same resonant body, we know from daily experience. There is no difficulty during a concert in following the melodic progression of each individual instrument or voice, if we direct our attention to it exclusively; and, after some practice, most persons can succeed in following the simultaneous progression of several united parts. This is true, indeed, not merely for musical tones, but also for noises, and for mixtures of music and noise. When several persons are speaking at once, we can generally listen at pleasure to the words of any single one of them, and even understand those words, provided that they are not too much overpowered by the mere loudness of the others. Hence it follows, first, that many different trains of waves of sound can be propagated at the same time through the same mass of air, without mutual disturbance; and, secondly, that the human ear is capable of again analysing into its constituent elements that composite motion of the air which is produced by the simultaneous action of several musical instruments. We will first investigate the nature of the motion of the air when it is produced by several simultaneous musical tones, and how such a compound motion is distinguished from that due to a single musical tone. We shall see that the ear has no decisive test by which it can in all cases distinguish between the effect of a motion of the air caused by several different musical tones arising from different sources, and that caused by the musical

tone of a single resonant body. Hence the ear has to analyse the composition of single musical tones, under proper conditions, by means of the same faculty which enabled it to analyse the composition of simultaneous musical tones. We shall thus obtain a clear conception of what is meant by analysing a single musical tone into a series of partial simple tones, and we shall perceive that this phenomenon depends upon one of the most essential and fundamental properties of the human ear.

We begin by examining the motion of the air which corresponds to several simple tones acting at the same time on the same mass of air. To illustrate this kind of motion it will be again convenient to refer to the waves formed on a calm surface of water. We have seen that if a point of the surface is agitated by a stone thrown upon it, the agitation is propagated in rings of waves over the surface to more and more distant points. Now, throw two stones at the same time on to different points of the surface, thus producing two centres of agitation. Each will give rise to a separate ring of waves, and the two rings gradually expanding, will finally meet. Where the waves thus come together, the water will be set in motion by both kinds of agitation at the same time, but this in no wise prevents both series of waves from advancing further over the surface, just as if each were alone present and the other had no existence at all. As they proceed, those parts of both rings which had just coincided, again appear separate and unaltered in form. These little waves, caused by throwing in stones, may be accompanied by other kinds of waves, such as those due to the wind or a passing steamboat. Our circles of waves will spread out over the water thus agitated, with the same quiet regularity as they did upon the calm surface. Neither will the greater waves be essentially disturbed by the less, nor the less by the greater, provided the waves never break; if that happened, their regular course would of course be impeded.

Indeed it is seldom possible to survey a large surface of water from a high point of sight, without perceiving a great multitude of different systems of waves, mutually overtopping and crossing each other. This is best seen on the surface of the sea, viewed from a lofty cliff, when there is a lull after a stiff breeze. We first see the great waves, advancing in far stretching ranks from the blue distance, here and there more clearly marked out by their white foaming crests, and following one another at regular

intervals towards the shore. From the shore they rebound, in different directions according to its sinuosities, and cut obliquely across the advancing waves. A passing steamboat forms its own wedge-shaped wake of waves, or a bird, darting on a fish, excites a small circular system. The eye of the spectator is easily able to pursue each one of these different trains of waves, great and small, wide and narrow, straight and curved, and observe how each passes over the surface, as undisturbedly as if the water over which it flits were not agitated at the same time by other motions and other forces. I must own that whenever I attentively observe this spectacle it awakens in me a peculiar kind of intellectual pleasure, because it bares to the bodily eye, what the mind's eye grasps only by the help of a long series of complicated conclusions for the waves of the invisible atmospheric ocean.

We have to imagine a perfectly similar spectacle proceeding in the interior of a ball-room, for instance. Here we have a number of musical instruments in action, speaking men and women, rustling garments, gliding feet, clinking glasses, and so on. All these causes give rise to systems of waves, which dart through the mass of air in the room, are reflected from its walls, return, strike the opposite wall, are again reflected, and so on till they die out. We have to imagine that from the mouths of men and from the deeper musical instruments there proceed waves of from 8 to 12 feet in length, from the lips of the women waves of 2 to 4 feet in length, from the rustling of the dresses a fine small crumple of wave, and so on; in short, a tumbled entanglement of the most different kinds of motion, complicated beyond conception.

And yet as the ear is able to distinguish all the separate constituent parts of this confused whole, we are forced to conclude that all these different systems of wave coexist in the mass of air, and leave one another mutually undisturbed. But how is it possible for them to coexist, since every individual train of waves has at any particular point in the mass of air its own particular degree of condensation and rarefaction, which determines the velocity of the particles of air to this side or that? It is evident that at each point in the mass of air, at each instant of time, there can be only one single degree of condensation, and that the particles of air can be moving with only one single determinate kind of motion, having only one single determinate amount of velocity, and passing in only one single determinate direction.

What happens under such circumstances is seen directly by the eye in the waves of water. If where the water shews large waves we throw a stone in, the waves thus caused will, so to speak, cut into the larger moving surface, and this surface will be partly raised, and partly depressed, by the new waves, in such a way that the fresh crests of the rings will rise just as much above, and the troughs sink just as much below the curved surfaces of the previous larger waves, as they would have risen above or sunk below the horizontal surface of calm water. Hence where a crest of the smaller system of rings of waves comes upon a crest of the greater system of waves, the surface of the water is raised by the sum of the two heights, and where a trough of the former coincides with a trough of the latter, the surface is depressed by the sum of the two depths. This may be expressed more briefly if we consider the heights of the crests above the level of the surface at rest, as positive magnitudes, and the depths of the troughs as negative magnitudes, and then form the so-called algebraical sum of these positive and negative magnitudes, in which case, as is well known, two positive magnitudes (heights of crests) must be added, and similarly for two negative magnitudes (depths of troughs); but when both negative and positive concur, one is to be subtracted from the other. Performing the addition then in this algebraical sense, we can express our description of the surface of the water on which two systems of waves concur, in the following simple manner: *The distance of the surface of the water at any point from its position of rest is at any moment equal to the sum of the distances at which it would have stood had each wave acted separately at the same place and at the same time.*

The eye most clearly and easily distinguishes the action in such a case as has been just adduced, where a smaller circular system of waves is produced on a large rectilinear system, because the two systems are then strongly distinguished from each other both by the height and shape of the waves. But with a little attention the eye recognises the same fact even when the two systems of waves have but slightly different forms, as when, for example, long rectilinear waves advancing towards the shore concur with those reflected from it in a slightly different direction. In this case we observe those well known comb-backed waves where the crest of one system of waves is heightened at some points by the crests of the other system, and at others depressed by its troughs. The

multiplicity of forms is here extremely great, and any attempt to describe them would lead us too far. The attentive observer will readily comprehend the result by examining any disturbed surface of water, without further description. It will suffice for our purpose if the first example has given the reader a clear conception of what is meant by *adding waves together*.[1]

Hence although the surface of the water at any instant of time can assume only one single form, while each of two different systems of waves simultaneously attempts to impress its own shape upon it, we are able to suppose in the above sense that the two systems coexist and are superimposed, by considering the actual elevations and depressions of the surface to be suitably separated into two parts, each of which belongs to one of the systems alone.

In the same sense, then, there is also a superimposition of different systems of sound in the air. By each train of waves of sound, the density of the air and the velocity and position of the particles of air, are temporarily altered. There are places in the wave of sound comparable with the crests of the waves of water, in which the quantity of the air is increased, and the air, not having free space to escape, is condensed; and other places in the mass of air, comparable to the troughs of the waves of water, having a diminished quantity of air, and hence diminished density. It is true that two different degrees of density, produced by two different systems of waves cannot coexist in the same place at the same time; nevertheless the condensations and rarefactions of the air can be (algebraically) added, exactly as the elevations and depressions of the surface of the water in the former case. Where two condensations are added we obtain increased condensation, where two rarefactions are added we have increased rarefaction; while a concurrence of condensation and rarefaction mutually, in whole or in part, destroy or neutralise each other.

The displacements of the particles of air are compounded in a similar manner. If the displacements of two different systems of waves are not in the same direction, they are compounded diagonally; for example, if one system would drive a particle of air

[1] The velocities and displacements of the particles of water are also to be added according to the law of the so-called parallelogram of forces. Strictly speaking, such a simple addition of waves as is spoken of in the text, is not perfectly correct, unless the heights of the waves are infinitely small in comparison with their lengths.

upwards, and another to the right, its real path will be obliquely upwards towards the right. For our present purpose there is no occasion to enter more particularly into such compositions of motion in different directions. We are only interested in the effect of the mass of air upon the ear, and for this we are only concerned with the motion of the air in the passages of the ear. Now the passages of our ear are so narrow in comparison with the length of the waves of sound, that we need only consider such motions of the air as are parallel to the axis of the passages, and hence have only to distinguish displacements of the particles of air outwards and inwards, that is towards the outer air and towards the interior of the ear. For the magnitude of these displacements as well as for their velocities with which the particles of air move outwards and inwards, the same (algebraical) addition holds good as for the crests and troughs of waves of water.

Hence, *when several resonant bodies in the surrounding atmosphere, simultaneously excite different systems of waves of sound, the changes of density of the air, and the displacements and velocities of the particles of the air within the passages of the ear, are each equal to the (algebraical) sum of the corresponding changes of density, displacements, and velocities, which each system of waves would have separately produced, if it had acted independently*;[1] and in this sense we can say that all the separate vibrations which separate waves of sound would have produced, coexist undisturbed at the same time within the passages of our ear.

After having thus in answer to the first question explained in what sense it is possible for several different systems of waves to coexist on the same surface of water or within the same mass of air, we proceed to determine the means possessed by our organs of sense, for analysing this composite whole into its original constituents.

I have already observed that an eye which surveys an extensive and disturbed surface of water, easily distinguishes the separate systems of waves from each other and follows their motions. The eye has a great advantage over the ear in being able to survey a large extent of surface at the same moment. Hence the eye readily sees whether the individual waves of water are rectilinear

[1] The same is true for the whole mass of external air, if only the addition of the displacements in different directions is made according to the law of the parallelogram of forces.

or curved, and whether they have the same centre of curvature, and in what direction they are advancing. All these observations assist it in determining whether two systems of waves are connected or not, and hence in discovering their corresponding parts. Moreover, on the surface of the water, waves of unequal length advance with unequal velocities, so that if they coincide at one moment to such a degree as to be difficult to distinguish, at the next instant one train pushes on and the other lags behind, so that they become again separately visible. In this way, then, the observer is greatly assisted in referring each system to its point of departure, and in keeping it distinctly visible during its further course. For the eye, then, two systems of waves having different points of departure can never coalesce; for example, such as arise from two stones thrown into the water at different points. If in any one place the rings of wave coincide so closely as not to be easily separable, they always remain separate during the greater part of their extent. Hence the eye could not be easily brought to confuse a compound with a simple undulatory motion. Yet this is precisely what the ear does under similar circumstances when it separates the musical tone which has proceeded from a single source of resonance, into a series of simple partial tones.

But the ear is much more unfavourably situated in relation to a system of waves of sound, than the eye for a system of waves of water. The ear is affected only by the motion of that mass of air which happens to be in the immediate neighbourhood of its tympanum within the aural passage. Since a transverse section of the aural passage is comparatively small in comparison with the length of waves of sound (which for serviceable musical tones varies from 6 inches to 32 feet), it corresponds to a single point of the mass of air in motion. It is so small that distinctly different degrees of density or velocity could scarcely occur upon it, because the positions of greatest and least density, of greatest positive and negative velocity, are always separated by half the length of a wave. The ear is therefore in nearly the same condition as the eye would be if it looked at one point of the surface of the water through a long narrow tube, which would permit of seeing its rising and falling, and were then required to undertake an analysis of the compound waves. It is easily seen that the eye would, in most cases, completely fail in the solution of such a problem. The ear is not in a condition to discover how the air is moving at dis-

tant spots, whether the waves which strike it are spherical or plane, whether they interlock in one or more circles, or in what direction they are advancing. The circumstances on which the eye chiefly depends for forming a judgment, are all absent for the ear.

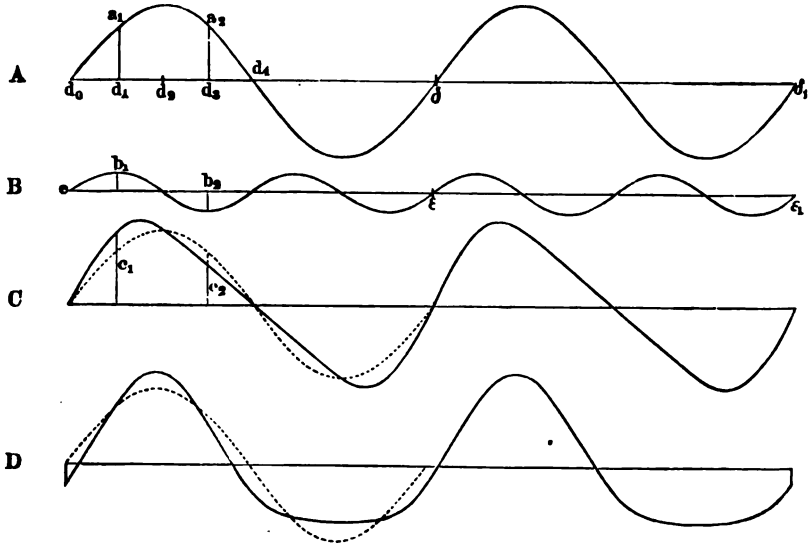
If, then, notwithstanding all these difficulties, the ear is capable of distinguishing musical tones arising from different sources—and it really shews a marvellous readiness in so doing—it must employ means and possess properties altogether different from those employed or possessed by the eye. But whatever these means may be—and we shall endeavour to determine them hereafter—it is clear that the analysis of a composite mass of musical tones must in the first place be closely connected with some determinate properties of the motion of the air, capable of impressing themselves even on such a very minute mass of air as that contained in the aural passage. If the motions of the particles of air in this passage are the same on two different occasions, the ear will receive the same sensation, whatever be the origin of those motions, whether they spring from one or several sources.

We have already explained that the mass of air which sets the tympanic membrane of the ear in motion, so far as the magnitudes here considered are concerned, must be looked upon as a single point in the surrounding atmosphere. Are there, then, any peculiarities in the motion of a single particle of air which would differ for a single musical tone, and for a combination of musical tones? We have seen that for each single musical tone there is a corresponding periodical motion of the air, and that its pitch is determined by the length of the periodic time, but that the kind of motion during any one single period is perfectly arbitrary, and may indeed be infinitely various. If then the motion of the air lying in the aural passage is not periodic, or if at least its periodic time is not as short as that of an audible musical tone, this fact will distinguish it from any motion which belongs to a musical tone; it must belong either to noises or to several simultaneous musical tones. Of this kind are really the greater number of cases where the different musical tones have been only accidentally combined, and are therefore not designedly framed into musical chords; nay, even where orchestral music is performed, the method of tempered tuning which at present prevails, prevents an accurate fulfilment of the conditions under which alone the resulting motion of the air can be exactly periodic. Hence in the greater

number of cases a want of periodicity in the motion might furnish a mark for distinguishing the presence of a composite mass of musical tones.

But a composite mass of musical tones may also give rise to a *purely periodic motion of the air*, namely, *when all the musical tones which intermingle, have vibrational numbers which are all multiples of one and the same whole number*, or, which comes to the same thing, when *all these musical tones, so far as their pitch is concerned, may be regarded as the upper partial tones of the same prime tone.* It was mentioned in Chapter I.

FIG. 11.



that the vibrational numbers of the upper partial tones are multiples of the vibrational nnmber of the prime tone. The meaning of this rule will be clear from a particular example. The curve A, fig. 11, represents a pendular motion in the manner explained in Chapter I., as produced in the air of the aural passage by a tuning fork in action. The horizontal lengths in the curves of fig. 11, consequently represent the passing time, and the vertical heights the corresponding displacements of the particles of air in the aural passage. Now suppose that with the first simple tone to which the curve A corresponds, there is

sounded a second simple tone, represented by the curve B, an octave higher than the first. This condition requires that two vibrations of the curve B should be made in the same time as one vibration of the curve A. In A, the sections of the curve $d_0\delta$ and $\delta\delta_1$ are perfectly equal and similar. The curve B is also divided into equal and similar sections $e\varepsilon$ and $\varepsilon\varepsilon_1$ by the points e, ε, ε_1. We could certainly halve each of the sections $e\varepsilon$ and $\varepsilon\varepsilon_1$, and also obtain equal and similar sections, each of which would then correspond to a single period of B. But by taking sections consisting of two periods of B, we divide B into larger sections, each of which is of the same horizontal length, and hence corresponds to the same duration of time, as the sections of A.

If then both simple tones are heard at once, and the times of the points e and d_0, ε and δ, ε_1 and δ_1 coincide, the heights of the portions of the section of curve $e\varepsilon$ have to be (algebraically) added to heights of the section of curve $d_0\delta$, and similarly for the sections $\varepsilon\varepsilon_1$ and $\delta\delta_1$. The result of this addition is shewn in the curve C. The dotted line is a duplicate of the section $d_0\delta$ in the curve A. Its object is to make the composition of the two sections immediately evident to the eye. It is easily seen that the curve C in every place rises as much above or sinks as much below the curve A, as the curve B respectively rises above or sinks beneath the medial straight line. The heights of the curve C are consequently, in accordance with the rule for compounding vibrations, equal to the (algebraical) sum of the corresponding heights of A and B. Thus the perpendicular c_1 in C is the sum of the perpendiculars a_1 and b_1 in A and B; the lower part of this perpendicular c_1, from the straight line up to the dotted curve, is equal to the perpendicular a_1, and the upper part, from the dotted to the continuous curve, is equal to the perpendicular b_1. On the other hand, the height of the perpendicular c_2 is equal to the height a_2 diminished by the depth of the fall b_2. And in the same way all other points in the curve C are found.[1]

It is evident that the motion represented by the curve C is also periodic, and that its periods have the same duration as those

[1] [Readers not used to geometrical constructions are strongly recommended to trace the two curves A and B, and to construct the curve C from them, by drawing a number of perpendiculars to a straight line, and then setting off upon them the lengths of the corresponding perpendiculars in A and B in proper directions, and joining the extremities of the lengths thus found by a curved line. In this way only can a clear conception of the composition of vibrations be rendered sufficiently familiar for subsequent use.—*Translator.*]

of A. Thus the addition of the section $d_0\delta$ of A and $e\varepsilon$ of B, must give the same result as the addition of the perfectly equal and similar sections $\delta\delta_1$ and $\varepsilon\varepsilon_1$, and, if we supposed both curves to be continued, the same would be the case for all the sections into which they would be divided. It is also evident that equal sections of both curves could not thus continually coincide after completing the addition, unless the curves thus added could be also separated into exactly equal and similar sections of the same length, as is the case in fig. 11, where two periods of B last as long or have the same horizontal length as one of A. Now the horizontal lengths of our figure represent time, and if we pass from the curves to the real motions, it results that the motion of air caused by the composition of the two simple tones, A and B, is also periodic, just because one of these simple tones makes exactly twice as many vibrations as the other in the same time.

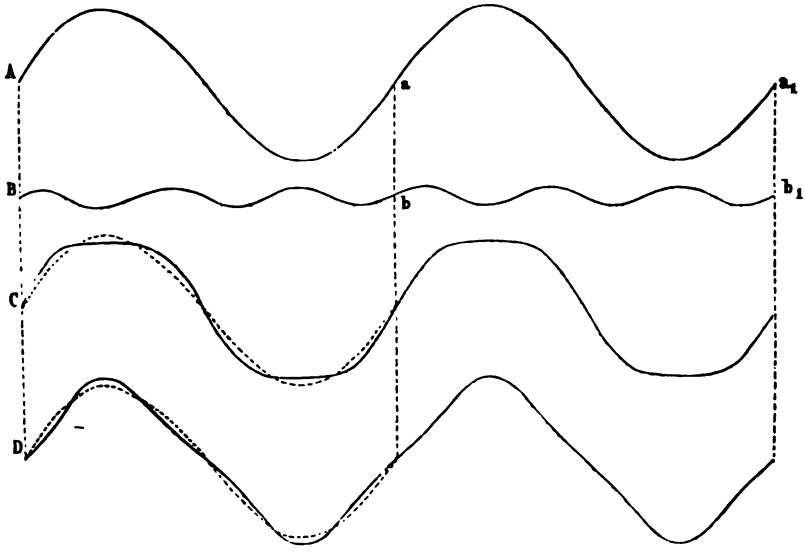
It is easily seen by this example that the peculiar form of the two curves A and B has nothing to do with the fact that their sum C is also a periodic curve. Whatever be the form of A and B, provided each can be separated into equal and similar sections which have the same horizontal lengths as the equal and similar sections of the other—no matter whether these sections correspond to one, or two, or three periods of the individual curves—then any one section of the curve A compounded with any one section of the curve B, will always give a section of the curve C, which will have the same length, and will be precisely equal and similar to any other section of the curve C obtained by compounding any other section of A with any other section of B.

When such a section embraces several periods of the corresponding curve (as in fig. 11, the sections $e\varepsilon$ and $\varepsilon\varepsilon_1$ consist each of two periods of the simple tone B,) then the pitch of this tone (as of B,) is that of an upper partial tone of that prime whose period has the length of that principal section (as the simple tone A in fig. 11), in accordance with the rule above cited.

In order to give a slight conception of the multiplicity of forms producible by comparatively simple compositions, I may remark that the compound curve would receive another form if the curves B, fig. 11, were displaced a little with respect to the curve A before the addition were commenced. Let B be displaced by being slid to the right until the point e falls under d_1 in A, and the composition will then give the curve D with narrow crests and

broad troughs, both sides of the crest being, however, equally steep; whereas in the curve C one side is steeper than the other. If we displace the curve B still more by sliding it to the right till e falls under d_1, the compound curve would resemble the reflection of C in a mirror: that is, it would have the same form as C reversed as to right and left; the steeper inclination which in C lies to the left would now lie to the right. Again, if we displace B till e falls under d_2 we obtain a curve similar to D, fig. 11, but reversed as to up and down, as may be seen by holding the book upside-down, the crests being broad and the troughs narrow.

FIG. 12.



All these curves with their various transitional forms are periodic curves. Other composite periodic curves are shewn at C, D, fig. 12 above, where they are compounded of the two curves A and B, having their periods in the ratio of 1 to 3. The dotted curves are as before copies of the first complete vibration or period of the curve A, in order that the reader may see at a glance that the compound curve is always as much higher or lower than A, as B is higher or lower than the horizontal line. In C, the curves A and B are added as they stand, but for D the curve B has been first slid half a wave's length to the right, and then the

addition has been effected. Both forms differ from each other and from all preceding ones. C has broad crests and broad troughs, D narrow crests and narrow troughs.

In these and similar cases we have seen that the compound motion is perfectly and regularly periodic, that is, it is exactly of the same kind as if it proceeded from a single musical tone. The curves compounded in these examples correspond to the motions of single simple tones. Thus, the motions shewn in fig. 11 (on p. 46) might have been produced by two tuning forks, of which one sounded an Octave higher than the other. But we shall hereafter see that a flute by itself when gently blown is sufficient to create a motion of the air corresponding to that shewn in C or D of fig. 11. The motions of fig. 12 might be produced by two tuning forks of which one sounded the twelfth of the other. Also a single closed organ pipe of the narrower kind (the stop called *Quintaten*[1]) would give nearly the same motion as that of C or D in fig. 12.

Here then the motion of the air in the aural passage has no property by which the composite[2] musical tone can be distinguished from the single musical tone. If the ear is not assisted by other accidental circumstances, as by one tuning fork beginning to sound before the other, so that we hear them struck, or, in the other case, the whistling of the wind against the mouthpiece of the flute or lip of the organ pipe, it has no means of deciding whether the musical tone is simple or composite.

Now, in what relation does the ear stand to such a motion of the air? Does it analyse it, or does it not? Experience shews us that when two tuning forks, an Octave or a Twelfth apart in pitch,

[1] [The names of the stops on German organs do not always agree with those on English organs. I find it best, therefore, not to translate them, but to give their explanations from E. J. Hopkins's 'The Organ, its History and Construction,' 1855, pp. 434–443. In this case Mr. Hopkins prints the word 'quinta*ton*,' clearly a misprint, and defines it, in 16 feet tone, as 'double stopped diapason, of rather small scale, producing the Twelfth of the fundamental sound, as well as the ground-tone itself, that is, sounding the 16 and 5⅓ ft. tones.' And similarly, in other cases, 'a pipe for sounding the Twelfth in addition to the fundamental tone.' It seems to be properly the English stop '*Twelfth—Duodecima*,' No. 562, p. 119 of Hopkins.—*Translator.*]

[2] [The reader must distinguish between *single* and *simple* musical tones. A *single* may be a *compound* tone inasmuch as it may be compounded of several simple musical tones, but it is *single* because it is produced by *one* resonant body. A *composite* musical tone is necessarily *compound*, but it is called *composite* because it is made up of tones (simple or compound) produced by *several* resonant bodies.—*Translator.*]

are sounded together, the ear is quite able to distinguish their simple tones, although the distinction is a little more difficult with these than with other intervals. But if the ear is able to analyse a composite musical tone produced by two tuning forks, it cannot but be in a condition to carry out a similar analysis, when the same motion of the air is produced by a single flute or organ pipe. And this is really the case. The single musical tone of such instruments, proceeding from a single source, is, as we have already mentioned, analysed into partial simple tones, consisting in each case of a prime tone, and one upper partial tone, the latter being different in the two cases.

The analysis of a single musical tone into a series of partial tones depends, then, upon the same property of the ear as that which enables it to distinguish different musical tones from each other, and it must necessarily affect both analyses by a rule which is independent of the fact that the waves of sound are produced by one or by several musical instruments.

The rule by which the ear proceeds in its analysis was first laid down as generally true by G. S. Ohm. Part of this rule has been already enunciated in the last chapter (p. 34), where it was stated that only that particular motion of the air which we have denominated a *simple vibration*, for which the vibrating particles swing backwards and forwards according to the law of pendular motion, is capable of exciting in the ear the sensation of a single simple tone. *Every motion of the air, then, which corresponds to a composite mass of musical tones, is, according to Ohm's law, capable of being analysed into a sum of simple pendular vibrations, and to each such single simple vibration corresponds a simple tone, sensible to the ear, and having a pitch determined by the periodic time of the corresponding motion of the air.*

The proofs of the correctness of this law, the reasons why, of all vibrational forms, only that one which we have called a simple vibration plays such an important part, must be left for Chapters IV. and VI. Our present business is only to gain a clear conception of what the rule means.

The simple vibrational form is inalterable and always the same. It is only its amplitude and its periodic time which are subject to change. But we have seen in figs. 11 and 12 (p. 46 and p. 49) what varied forms the composition of only two simple vibrations can produce. The number of these forms might be greatly in-

creased, even without introducing fresh simple vibrations of different periodic times, by merely changing the proportions which the heights of the two simple vibrational curves A and B bear to each other, or displacing the curve B by other lengths to the right or left, than those already selected in the figures. By these simplest possible examples of such compositions, the reader will be able to form some idea of the enormous variety of forms which would result from using more than two simple forms of vibration, each form giving an upper partial tone of the same prime, and hence, on addition, always producing other periodic curves. We should be able to make the heights of each single simple vibrational curve greater or smaller at pleasure, and displace each one separately by any amount in respect to the prime,—or, in physical language, we should be able to alter their amplitudes and the difference of their phases; and each such alteration of amplitude and difference of phase in each one of the simple vibrations would produce a fresh change in the resulting composite vibrational form.

The multiplicity of vibrational forms which can be thus produced by the composition of simple pendular vibrations is not merely extraordinarily great: it is so great that it cannot be greater. The French mathematician Fourier has proved the correctness of a mathematical law, which in reference to our present subject may be thus enunciated: *Any given regular periodic form of vibration can always be produced by the addition of simple vibrations, having vibrational numbers which are once, twice, thrice, four times, &c., as great as the vibrational number of the given motion.*

The *amplitudes* of the elementary simple vibrations to which the height of our wave-curves corresponds, and the *difference of phase*, that is, the relative amount of horizontal displacement of the wave-curves, can always be found in every given case, as Fourier has shewn, by peculiar methods of calculation, which, however, do not admit of any popular explanation, so that *any given regularly periodic motion can always be exhibited in one single way, and in no other way whatever, as the sum of a certain number of pendular vibrations.*

Since, according to the results already obtained, any regularly periodic motion corresponds to some musical tone, and any simple pendular vibration to a single musical tone, these propositions of Fourier may be thus expressed for the purpose of their application to the theory of sound:

Any vibrational motion of the air in the aural passages, corresponding to a musical tone, may be always, but for each case only in one single way, exhibited as the sum of a number of simple vibrational motions, corresponding to the partial tones of this musical tone.

Since, according to these propositions, any form of vibration, no matter what shape it may take, can be expressed as the sum of simple vibrations, its analysis into such a sum is quite independent of the power of the eye to perceive by looking at its representative curve, whether it contains simple vibrations or not, and if it does, what they are. I am obliged to lay stress upon this point, because I have by no means unfrequently found even physicists start on the false hypothesis, that the vibrational form must exhibit little waves corresponding to the several audible upper partial tones. A mere inspection of the figs. 11 and 12 (p. 46 and p. 49) will suffice to shew that although the composition can be easily traced in the parts where the curve of the prime tone is dotted in, this is quite impossible in those parts of the curves C and D in each figure, where no such assistance has been provided. Or, if we suppose that an observer who had rendered himself thoroughly familiar with the curves of simple vibrations imagined that he could trace the composition in these easy cases, he would certainly utterly fail on attempting to discover by his eye alone the composition of such curves as are shewn in figs. 8 and 9 (p. 32). In these will be found straight lines and acute angles. Perhaps it will be asked how it is possible by compounding such smooth and uniformly rounded curves as those of our simple vibrational forms A and B in figs. 11 and 12, to generate at one time straight lines, and at another acute angles. The answer is, that an infinite number of simple vibrations are required to generate curves with such discontinuities as are there shewn. But when a great many such curves are combined, and are so chosen that in certain places they all bend in the same direction, and in others in opposite directions, the curvatures mutually strengthen each other in the first case, finally producing an infinitely great curvature, that is, an acute angle, and in the second case they mutually weaken each other, so that ultimately a straight line results. Hence we can generally lay it down as a rule that the force or loudness of the upper partial tones is the greater, the sharper the discontinuities of the atmospheric motion. When the motion alters uniformly and gradually, answering to a vibra-

tional curve proceeding in smoothly curved forms, only the deeper partial tones, which lie nearest to the prime tone, have any perceptible intensity. But where the motion alters by jumps, and hence the vibrational curves shew angles or sudden changes of curvature, the upper partial tones will also have sensible force, although in all these cases the amplitudes decrease as the pitch of the upper partial tones becomes higher.[1]

We shall become acquainted with examples of the analysis of given vibrational forms into separate partial tones in Chapter V.

The theorem of Fourier here adduced shews first that it is mathematically possible to consider a musical tone as a sum of simple tones, in the meaning we have attached to the words, and mathematicians have indeed always found it convenient to base their acoustic investigations on this mode of analysing vibrations. But it by no means follows that we are obliged to consider the subject in this way. We have rather to inquire, do these partial constituents of a musical tone, such as the mathematical theory distinguishes and the ear perceives, really exist in the mass of air external to the ear? Is this means of analysing forms of vibration which Fourier's theorem prescribes and renders possible, not merely a mathematical fiction, permissible for facilitating calculation, but not necessarily having any corresponding actual meaning in things themselves? What makes us hit upon pendular vibrations, and none other, as the simplest element of all motions producing sound? We can conceive a whole to be split into parts in very different and arbitrary ways. Thus we may find it convenient for a certain calculation to consider the number 12 as the sum $8+4$, because the 8 may have to be cancelled, but it does not follow that 12 must always and necessarily be considered as merely the sum of 8 and 4. In another case it might be more convenient to consider 12 as the sum of 7 and 5. Just as little does the mathematical possibility, proved by Fourier, of compounding all periodic vibra-

[1] Supposing n to be the number of the order of a partial tone, and n to be very large, then the amplitude of the upper partial tones decreases: 1) when the amplitude of the vibrations themselves makes a sudden jump, as $\frac{1}{n}$; 2) when their differential quotient makes a sudden jump, and hence the curve has an acute angle, as $\frac{1}{n.n}$; 3) when the curvature alters suddenly, as $\frac{1}{n.n.n}$; 4) when none of the differential quotients are discontinuous, they must decrease at least as fast as e^{-n}.

CHAPTER III.

ANALYSIS OF MUSICAL TONES BY SYMPATHETIC RESONANCE.

WE proceed to shew that the simple partial tones contained in a composite mass of musical tones, produce peculiar mechanical effects in nature, altogether independent of the human ear and its sensations, and also altogether independent of merely theoretical considerations. These effects consequently give a peculiar objective significance to this peculiar method of analysing vibrational forms.

Such an effect occurs in the phenomenon of *sympathetic resonance.* This phenomenon is always found in those bodies which when once set in motion by any impulse, continue to perform a long series of vibrations before they come to rest. When these bodies are struck gently, but periodically, although each blow may be separately quite insufficient to produce a sensible motion in the vibratory body, yet, provided the periodic time of the gentle blows is precisely the same as the periodic time of the body's own vibrations, very large and powerful oscillations may result. But if the periodic time of the regular blows is different from the periodic time of the oscillations, the resulting motion will be weak or quite insensible.

Periodic impulses of this kind generally proceed from another body which is already vibrating regularly, and in this case the swings of the latter in the course of a little time, call into action the swings of the former. Under these circumstances we have the process called *sympathetic oscillation* or *sympathetic resonance.* The essence of the mechanical effect is independent of the rate of motion, which may be fast enough to excite the sensation of sound, or slow enough not to produce anything of the kind. Musicians are well acquainted with *sympathetic resonance.* When for example the strings of two violins are in exact unison, and one string is bowed, the other will begin to vibrate. But the

nature of the process is best seen in instances where the vibrations are slow enough for the eye to follow the whole of their successive phases.

Thus for example it is known that the largest church-bells may be set in motion by a man or even a boy who pulls the ropes attached to them at proper and regular intervals, even when their weight of metal is so great that the strongest man would scarcely be able to move them sensibly, if he did not apply his strength in definite periodical intervals. When such a bell is once set in motion, it continues, like a struck pendulum, to oscillate for some time, until it gradually returns to rest, even if it is left quite by itself, and no force is employed to arrest its motion. The motion diminishes gradually, as we know, because the friction on the axis and the resistance of the air at every swing destroys a portion of the existing moving force.

As the bell swings backwards and forwards, the lever and rope fixed to its axis rise and fall. If when the lever falls a boy clings to the lower end of the bellrope, his weight will act so as to increase the rapidity of the existing motion. This increase of velocity may be very small, and yet it will produce a corresponding increase in the extent of the bell's swings, which again will continue for a while, until destroyed by the friction and resistance of the air. But if the boy clung to the bellrope at a wrong time, while it was ascending, for instance, the weight of his body would act in opposition to the motion of the bell, and the extent of swing would decrease. Now, if the boy continued to cling to the rope at each swing so long as it was falling, and then let it ascend freely, at every swing the motion of the bell would be only increased in speed, and its swings would gradually become greater and greater, until by their increase the motion imparted on every oscillation of the bell to the walls of the belfry, and the external air would become so great as exactly to be covered by the power exerted by the boy at each swing.

The success of this process depends therefore essentially on the boy's applying his force only at those moments when it will increase the motion of the bell. That is, he must employ his strength periodically, and the periodic time must be equal to that of the bell's swing, or he will not be successful. He would just as easily bring the swinging bell to rest, if he clung to the rope only during its ascent, and thus let his weight be raised by the bell.

A similar experiment which can be tried at any instant is the following. Construct a pendulum by hanging a heavy body (such as a ring) to the lower end of a thread, holding the upper end in the hand. On setting the ring into pendular vibration, it will be found that this motion can be gradually and considerably increased by watching the moment when the pendulum has reached its greatest departure from the vertical, and then giving the hand a very small motion in the opposite direction. Thus, when the pendulum is furthest to the right, move the hand very slightly to the left; and when the pendulum is furthest to the left, move the hand to the right. The pendulum may be similarly set in motion from a state of rest by giving the hand similar very slight motions having the same periodic time as the pendulum's own swings. The displacements of the hand may be so small under these circumstances, that they can scarcely be perceived with the closest attention, a circumstance to which is due the superstitious application of this little apparatus as a divining rod.[1] If namely the observer, without thinking of his hand, follows the swings of the pendulum with his eye, the hand readily follows the eye, and involuntarily moves a little backwards or forwards, precisely in the same time as the pendulum, after this has accidentally begun to move. These involuntary motions of the hand are usually overlooked, at least when the observer is not accustomed to exact observations on such unobtrusive influences. By this means any existing vibration of the pendulum is increased and kept up, and any accidental motion of the ring is readily converted into pendular vibrations, which seem to arise spontaneously without any co-operation of the observer, and are hence attributed to the influence of hidden metals, running streams, and so on.

If on the other hand the motion of the hand is intentionally made in the contrary direction, the pendulum soon comes to rest.

The explanation of the process is very simple. When the upper end of the thread is fastened to an immovable support, the pendulum, once struck, continues to swing for a long time, and the extent of its swings diminishes very slowly. We can suppose the extent of the swings to be measured by the angle which the

[1] [See many details of the motion of this so-called 'Odometer or Divining Ring' in Letter XII. of Dr. Herbert Mayo's 'On the Truths contained in Popular Superstitions, with an account of Mesmerism,' 8vo, pp. 248, London, 3rd ed. 1851.—*Translator.*]

thread makes with the vertical on its greatest deflection from it. If the attached body at the point of greatest deflection lies to the right, and we move the hand to the left, we manifestly increase the angle between the string and the vertical, and consequently also augment the extent of the swing. By moving the upper end of the string in the opposite direction we should decrease the extent of the swing.

In this case there is no necessity for moving the hand in the same periodic time as the pendulum swings. We might move the hand backwards and forwards only at every third or fifth or other swing of the pendulum, and we should still produce large swings. Thus, when the pendulum is to the right, move the hand to the left, and keep it still, till the pendulum has swung to the left, then again to the right, and then once more to the left, and then return the hand to its first position, afterwards wait till the pendulum has swung to the right, then to the left, and again to the right, and then recommence the first motion of the hand. In this way three complete vibrations or double excursions of the pendulum, will correspond to one left and right motion of the hand. In the same way one left and right motion of the hand may be made to correspond with seven or more swings of the pendulum. The meaning of this process is always that the motion of the hand must in each case be made at such a time and in such a direction as to be opposed to the deflection of the pendulum and consequently to increase it.

By a slight alteration of the process we can easily make two, four, six, &c., swings of the pendulum correspond to one left and right motion of the hand; for a sudden motion of the hand at the instant of the pendulum's passage through the vertical has no influence on the size of the swings. Hence when the pendulum lies to the right move the hand to the left, and so increase its velocity, let it swing to the left, watch for the moment of its passing the vertical line, and at that instant return the hand to its original position, allow it to reach the right, and then again the left and once more the right extremity of its arc, and then recommence the first motion of the hand.

We are able then to communicate violent motion to the pendulum by very small periodical vibrations of the hand, having their periodic time exactly as great, or else two, three, four, &c. times as great as that of the pendular oscillation. We have here con-

sidered that the motion of the hand is backwards. This is not necessary. It may take place continuously in any other way we please. When it moves continuously there will be generally portions of time during which it will increase the pendulum's motion, and others perhaps in which it will diminish the same. In order to create strong vibrations in the pendulum, then, it will be necessary that the increments of motion should be permanently predominant, and should not be neutralised by the sum of the decrements.

Now if a determinate periodic motion were assigned to the hand, and we wished to discover whether it would produce considerable vibrations in the pendulum, we could not always predict the result without calculation. Theoretical mechanics would, however, prescribe the following process to be pursued: *Analyse the periodic motion of the hand into a sum of simple pendular vibrations of the hand*—exactly in the same way as was laid down in the last chapter for the periodic motions of the particles of air, —*then, if the periodic time of one of these vibrations is equal to the periodic time of the pendulum's own oscillations, the pendulum will be set into violent motion, but not otherwise.* We might compound small pendular motions of the hand out of vibrations of other periodic times, as much as we liked, but we should fail to produce any lasting strong swings of the pendulum. Hence the analysis of the motion of the hand into pendular swings has a real meaning in nature, producing determinate mechanical effects, and for the present purpose no other analysis of the motion of the hand into any other partial motions can be substituted for it.

In the above examples the pendulum could be set into sympathetic vibration, when the hand moved periodically at the same rate as the pendulum; in this case the longest partial vibration of the hand, corresponding to the prime tone of a resonant vibration, was, so to speak, in unison with the pendulum. When three swings of the pendulum went to one backwards and forwards motion of the hand, it was the third partial swing of the hand, answering as it were to the Twelfth of its prime tone, which set the pendulum in motion. And so on.

The same process that we have thus become acquainted with for swings of long periodic time, holds precisely for swings of so short a period as resonant vibrations. Any elastic body which is so fastened as to admit of continuing its vibrations for some length

of time when once set in motion, can also be made to vibrate sympathetically, when it receives periodic agitations of comparatively small amounts, having a periodic time corresponding to that of its own tone.

Gently touch one of the keys of a pianoforte without striking the string, so as to raise the damper only, and then sing a note of the corresponding pitch forcibly directing the voice against the strings of the instrument. On ceasing to sing, the note will be echoed back from the piano. It is easy to discover that this echo is caused by the string which is in unison with the note, for directly the hand is removed from the key, and the damper is allowed to fall, the echo ceases. The sympathetic vibration of the string is still better shewn by putting little paper riders upon it, which are jerked off as soon as the string vibrates. The more exactly the singer hits the pitch of the string, the more strongly it vibrates. A very little deviation from the exact pitch fails in exciting sympathetic vibration.

In this experiment the sounding board of the instrument is first struck by the vibrations of the air excited by the human voice. The sounding board is well-known to consist of a broad flexible wooden plate, which, owing to its extensive surface, is better adapted to convey the agitation of the strings to the air, and of the air to the strings, than the small surface over which string and air are themselves directly in contact. The sounding board first communicates the agitations which it receives from the air excited by the singer, to the points where the string is fastened. The magnitude of any single such agitation is of course infinitesimally small. A very large number of such effects must necessarily be aggregated, before any sensible motion of the string can be caused. And such a continuous addition of effects really takes place, if, as in the preceding experiments with the bell and the pendulum, the periodic time of the small agitations which are communicated to the extremities of the string by the air through the intervention of the sounding board, exactly correspond to the periodic time of the string's own vibrations. When this is the case, the string will really be set by a long series of such vibrations into motion which is very violent in comparison with the exciting cause.

In place of the human voice we might of course use any other musical instrument. Provided only that it can produce the tone

of the pianoforte string accurately and sustain it powerfully, it will bring the latter into sympathetic vibration. In place of a pianoforte, again, we can employ any other stringed instrument having a sounding board, as a violin, guitar, harp, &c., and also stretched membranes, bells, elastic tongues or plates &c., provided only that the latter are so fastened as to admit of their giving a tone of sensible duration when once made to sound.

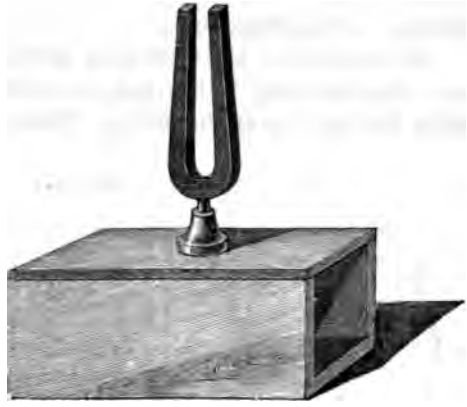
When the pitch of the original resonant body is not exactly that of the sympathising body, or that which is meant to vibrate in sympathy with it, the latter will nevertheless often make sensible sympathetic vibrations, which will diminish in amplitude as the difference of pitch increases. But in this respect different resonant bodies shew great differences, according to the length of time for which they continue to sound after having been set in action before communicating their whole motion to the air.

Bodies of small mass, which readily communicate their motion to the air, and quickly cease to sound, as, for example, stretched membranes or violin strings, are readily set in sympathetic vibration, because the motion of the air is conversely readily transferred to them, and they are also sensibly moved by sufficiently strong agitations of the air, even when the latter have not precisely the same periodic time as the natural tone of the sympathising bodies. The limits of pitch capable of exciting sympathetic vibration are consequently a little broader in this case. By the comparatively greater influence of the motion of the air upon light elastic bodies of this kind which offer but little resistance, their natural periodic time can be slightly altered, and adapted to that of the exciting tone. Massive elastic bodies, on the other hand, which are not readily movable, and are slow in communicating their resonant vibrations to the air, such as bells and plates, and continue to sound for a long time, are also more difficult to move by the air. A much longer addition of effects is required for this purpose, and consequently it is also necessary to hit the pitch of their own tone with much greater nicety, in order to make them vibrate sympathetically. Still it is well known that bell-shaped glasses can be put into violent motion by singing their proper tone into them; indeed it is related that singers with very powerful and pure voices, have sometimes been able to crack them by the agitation thus caused. The principal difficulty in this experiment is in hitting the pitch with sufficient precision,

and retaining the tone at that exact pitch for a sufficient length of time.

Tuning forks are the most difficult to set in sympathetic vibration. To effect this they must be fastened on sounding boxes which have been exactly tuned to their tone, as shewn in fig. 13. If we have two such forks of exactly the same pitch, and excite one by a violin bow, the other will begin to vibrate in sympathy, even if placed at the further end of the same room, and it will continue to sound, when the first is damped. The astonishing nature of such a case of sympathetic vibration will appear, if we merely compare the heavy and powerful mass of steel set in motion, with the light yielding mass of air which produces the effect by such small motive powers that they could not stir the lightest spring which was not in tune with the fork. With such forks the time required to set them in full swing by sympathetic action, is also of sensible duration, and the slightest disagreement in pitch is sufficient to produce a sensible diminution in the sympathetic effect. By sticking a piece of wax to one prong of the second fork, sufficient to make it vibrate once in a second less than the first—a difference of pitch scarcely sensible to the finest ear—the sympathetic vibration will be wholly destroyed.

FIG. 13.



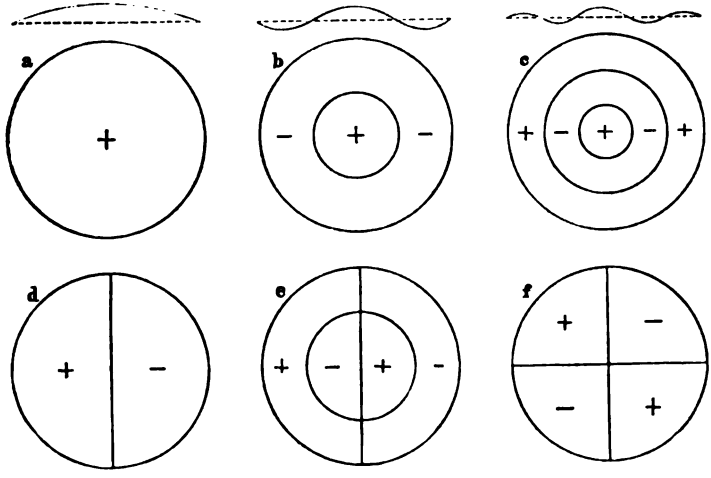
After having thus described the phenomenon of sympathetic vibration in general, we proceed to investigate the influence exerted in sympathetic resonance by the different forms of wave of a musical tone.

First, it must be observed that most elastic bodies which have been set into sustained vibration by a gentle force acting periodically, are (with a few exceptions to be considered hereafter) always made to swing in pendular vibrations. But they are in general capable of executing several kinds of such vibration, with different periodic times and with a different distribution over the

various parts of the vibrating body. Hence to the different lengths of the periodic times correspond different simple tones producible on such an elastic body. These are its so-called *proper tones.* It is, however, only exceptionally, as in strings and the narrower kinds of organ pipes, that these proper tones correspond in pitch with the harmonic upper partial tones of a musical tone already mentioned. They are for the most part inharmonic in relation to the prime tone.

In many cases the vibrations and their mode of distribution over the vibrating bodies can be rendered visible by strewing a little fine sand over the latter. Take, for example, a membrane

FIG. 14.



(as a bladder or piece of thin india-rubber) stretched over a circular ring. In fig. 14 are shewn the various forms which a membrane can assume when it vibrates. The diameters and circles on the surface of the membrane, mark those points which remain at rest during the vibration, and are known as *nodal lines.* By these the surface is divided into a number of compartments which bend alternately up and down, in such a way that while those marked (+) rise, those marked (—) fall. Over the figures a, b, c, are shewn the forms of a section of the membrane during vibration. Only those forms of motion are drawn which correspond with the deepest and the most easily producible

tones of the membrane. The number of circles and diameters can be increased at pleasure by taking a sufficiently thin membrane, and stretching it with sufficient regularity, and in this case the tones would continually sharpen in pitch. By strewing sand on the membrane the figures are easily rendered visible, for as soon as it begins to vibrate the particles of sand collect on the nodal lines.

In the same way it is possible to render visible the nodal lines and forms of vibration of oval and square membranes, and of differently-shaped plane elastic plates, bars, and so on. These form a series of very interesting phenomena discovered by Chladni, but to pursue them would lead us too far from our proper subject. It will suffice to give a few details respecting the simplest case, that of a circular membrane.

For the time required by the membrane to execute 100 vibrations of the form a, fig. 14, (p. 64), the number of vibrations executed by the other forms is as follows:—

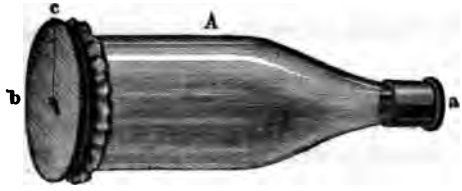
Form of Vibration	Vibrational Number	Pitch
a without nodal lines	100	*c*
b with one circle	229·6	*d'* +
c with two circles	359·9	*b'*♭ +
d with one diameter	159	*a*♭
e with one diameter and one circle	292	*g'*
f with two diameters	214	*c'*♯

The prime tone has been here arbitrarily assumed as *c*, in order to note the intervals of the higher tones. Those simple tones produced by the membrane which are slightly higher than those of the note written, are marked (+); those lower, by (−). In this case there is no commensurable ratio between the prime tone and the other tones, that is, none expressible in whole numbers.

Strew a very thin membrane of this kind with sand, and sound its prime tone strongly in its neighbourhood; the sand will be driven by the vibrations towards the edge, where it collects. On producing another of the tones of the membrane, the sand collects in the corresponding nodal lines, and we are thus easily able to determine to which of its tones the membrane has responded. A singer who knows how to hit the tones of the membrane correctly,

can thus easily make the sand arrange itself at pleasure in one order or the other, by singing the corresponding tones powerfully at a distance. But in general the simpler figures of the deeper tones are more easily generated than the complicated figures of the upper tones. It is easiest of all to set the membrane in general motion by sounding its prime tone, and hence such membranes have been much used in acoustics to prove the existence of some determinate tone in some determinate spot of the surrounding air. It is most suitable for this purpose to connect the membrane with an inclosed mass of air. A, fig. 15, is a glass bottle, having an open mouth a, and in place of its bottom b, a stretched membrane, consisting of wet pig's bladder, allowed to dry after it has been stretched and fastened. At c is attached a single fibre of a silk cocoon, bearing a drop of sealing wax, and hanging down like a pendulum against the membrane. As soon as the membrane swings the little pendulum is violently agitated. Such a pendulum is very convenient as long as we have no reason to apprehend any confusion of the prime tone of the membrane with any other of its proper tones. There is no scattering of sand, and the apparatus is therefore always in order. But to decide with certainty what tones are really agitating the membrane, we must after all place the bottle with its mouth downwards and strew sand on the membrane. However, when the bottle is of the right size, and the membrane uniformly stretched and fastened, it is only the prime tone of the membrane (slightly altered by that of the sympathetically vibrating mass of air in the bottle) which is easily excited. This prime tone can be made deeper by increasing the size of the membrane, or the volume of the bottle, or by diminishing the tension of the membrane or size of the orifice of the bottle.

FIG. 15.



A stretched membrane of this kind, whether it is or is not attached to the bottom of a bottle, will not only be set in vibration by musical tones of the same pitch as the proper tone of the bottle, but also by such musical tones as contain the proper tone of the membrane among its upper partial tones. Generally, given a

number of interlacing waves, to discover whether the membrane will vibrate sympathetically, suppose the motion of the air at the given place to be mathematically analysed into a sum of pendular vibrations. If there is one among them whose periodic time is the same as that of any one of the proper tones of the membrane, the corresponding vibrational form of the membrane will be superinduced. But if there are none such, or none sufficiently powerful, the membrane will remain at rest.

In this case also, then, we find that the analysis of the motion of the air into pendular vibrations, and the existence of certain vibrations of this kind, are decisive for the sympathetic vibration of the membrane, and in this case no other similar analysis of the motion of the air can be substituted for its analysis into pendular vibrations. The pendular vibrations into which the composite motion of the air can be analysed, here shew themselves capable of producing mechanical effects in external nature, independently of the ear, and independently of mathematical theory. Hence the statement is confirmed, that the theoretical view which first led mathematicians to this method of analysing compound vibrations, is founded in the nature of the thing itself.

As an example take the following description of a single experiment:—

A bottle of the shape shewn in fig. 15 (p. 66) was covered with a thin vulcanised india-rubber membrane, of which the vibrating surface was 49 millimètres (1·93 inches) in diameter, the bottle being 140 millimètres (6·51 inches) high, and having an opening at the brass mouth of 13 millimètres (·41 inches) in diameter. When blown it gave $f'\sharp$, and the sand heaped itself in a circle near the edge of the membrane. The same circle resulted from my giving the same tone $f'\sharp$ on a physharmonica,[1] or its deeper Octave $f\sharp$, or the deeper Twelfth B. Both $F\sharp$ and D gave the same circle, but more weakly. Now the $f'\sharp$ of the membrane is the prime tone of the physharmonica tone $f'\sharp$, the first upper partial tone of $f\sharp$, the second of B, the third of $F\sharp$ and fourth of D.[2] All these notes on being sounded set the membrane in the

[1] [Any such instrument as an accordion, mouth-harmonica, pitch-reed, formed of harmonium vibrators, &c.—*Translator.*]

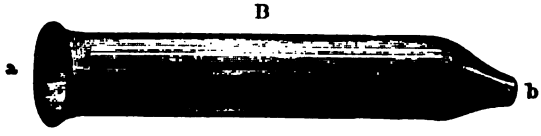
[2] [For the partials of f are $f\sharp$, $f'\sharp$, &c.; of B are B, b, $f'\sharp$, &c.; of $F\sharp$ are $F\sharp$, $f\sharp$, $d\sharp$, $f'\sharp$, &c. and of D are D, d, $\dagger a$, d', $f'\sharp$, &c., where $\dagger a$ has the vibrational number $222\frac{2}{3}$ instead of the 220 of a; see Appendix XIX, Section A.—*Translator.*]

motion due to its deepest tone. A second smaller circle, 19 millimètres (·75 inches) in diameter was produced on the membrane by b' and the same more faintly by b, and there was a trace of it for the deeper Twelfth e, that is, for simple tones of which vibrational numbers were $\frac{1}{2}$ and $\frac{1}{3}$ that of b'.

Such stretched membranes are very convenient for these and similar experiments on compound tones. They have the great advantage of being independent of the ear, but they are not very sensitive for the fainter simple tones. Their sensitiveness is far inferior to that of the *res'onātors* which I have introduced. These are hollow spheres of glass or metal, or tubes, with two openings as shewn in figs. 16 a and 16 b. One opening (a) has sharp edges, the other (b) is funnel shaped, and adapted for insertion into the ear. This smaller end I usually surround with melted sealing wax, and when it has cooled down enough not to hurt the finger on being touched, but is still soft, I press the opening into the entrance of my ear. The sealing wax thus moulds itself to the shape of the inner surface of this opening, and when I subsequently

FIG. 16a.

FIG. 16b.



use the resonator, it fits easily and is air-tight. Such an instrument is very like the resonance bottle already described, fig. 15 (p. 66), for which the observer's own tympanic membrane has been made to replace the former artificial membrane.

The mass of air in a resonator, together with that in the aural passage, and with the tympanic membrane or drum-skin itself, forms an elastic system which is capable of vibrating in a peculiar manner, and, in especial, the prime tone of the sphere, which is

much deeper than any other of its proper tones, can be set into very powerful sympathetic vibration, and then the ear, which is in immediate connection with the air inside the sphere perceives this augmented tone by direct action. If we stop one ear (which is best done by a plug of sealing wax moulded into the form of the entrance of the ear), and apply a resonator to the other, most of the tones produced in the surrounding air will be considerably damped; but if the proper tone of the resonator is sounded, it brays into the ear most powerfully. Hence any one, even if he has no ear for music or is quite unpractised in detecting musical sounds, is put in a condition to pick the required simple tone, even if comparatively faint, from out of a great number of others. The proper tone of the resonator may even be sometimes heard cropping up in the whistling of the wind, the rattling of carriage wheels, the splashing of water. For these purposes such resonators are incomparably more sensitive than tuned membranes. When the simple tone to be observed is faint in comparison with those which accompany it, it is of advantage to apply the resonator and withdraw it a little alternately. We thus easily feel whether the proper tone of the resonator begins to sound when the instrument is applied, whereas a uniform continuous tone is not so readily perceived.

A properly tuned series of such resonators is therefore an important instrument for experiments in which individual faint tones have to be distinctly heard, although accompanied by others which are strong, as in observations on the combinational and upper partial tones, and a series of other phenomena to be hereafter described relating to chords. By their means such researches can be carried out even by ears quite untrained in musical observation, whereas it had been previously impossible to conduct them except by trained musical ears, and much strained attention properly assisted. These tones were consequently accessible to the observation of only a very few individuals; and a large number of physicists and even musicians had indeed never succeeded in distinguishing them. And again even the trained ear is now able, with the assistance of resonators, to carry the analysis of a mass of musical tones much further than before. Without their help, indeed, I should scarcely have succeeded in making the observations hereafter described, with so much precision and certainty, as at present.[1]

[1] See Appendix II. for the measures and construction of these Resonators.

It must be carefully noted that the ear does not hear the required tone with augmented force, unless that tone attains a considerable intensity within the mass of air inclosed in the resonator. Now the mathematical theory of the motion of the air shews that, so long as the amplitude of the vibrations is sufficiently small, the inclosed air will execute pendular oscillations of the same periodic time as those in the external air, and none other, and that only those pendular oscillations whose periodic time corresponds with that of the proper tone of the resonator, have any considerable strength; the intensity of the rest diminishing as the difference of their pitch from that of the proper tone increases. All this has nothing to do with the connection of the ear and resonator, except in so far as the tympanic membrane forms one of the inclosing walls of the mass of air. Theoretically this apparatus does not differ from the bottle with an elastic membrane, in fig. 15 (p. 66), but its sensitiveness is amazingly increased by using the drumskin of the ear for the closing membrane of the bottle, and thus bringing it in direct connection with the auditory nerves themselves. Hence we cannot obtain a powerful tone in the resonator except when an analysis of the motion of the external air into pendular vibrations, would shew that one of them has the same periodic time as the proper tone of the resonator. Here again no other analysis but that into pendular vibrations would give a correct result.

It is easy for an observer to convince himself of the above-named properties of resonators. Apply one to the ear, and let a piece of harmonised music, in which the proper tone of the resonator frequently occurs, be executed by any instruments. As often as this tone is struck, the ear to which the instrument is held, will hear it violently contrast with all the other tones of the chord.

This proper tone will also often be heard, but more weakly, when deeper musical tones occur, and on investigation we find that in such cases tones have been struck which include the proper tone of the resonator among their upper partial tones. Such deeper musical tones are called the *harmonic subtones* of the resonator. They are musical tones whose periodic time is exactly 2, 3, 4, 5, and so on, times greater than that of the resonator. Thus if the proper tone of the resonator is c'', it will be heard when a musical instrument sounds c', f, c, $A\flat$, F, D, C, and so on.

In this case the resonator is made to sound in sympathy with one of the harmonic upper partial tones of the compound musical tone which is vibrating in the external air. It must, bowever, be noted that by no means all the harmonic upper partial tones occur in the compound tones of every instrument, and that they have very different degrees of intensity in different instruments. In the musical tones of violins, pianofortes, and physharmonicas, the first five or six are generally very distinctly present. A more detailed account of the upper partial tones of strings will be given in the next chapter. On the physharmonica the uneven partial tones (1, 3, 5, &c.) are generally stronger than the even ones (2, 4, 6, &c.) In the same way, the upper partial tones are clearly heard by means of the resonators in the singing tones of the human voice, but differ in strength for the different vowels, as will be shewn hereafter.

Among the bodies capable of strong sympathetic vibration must be reckoned stretched strings, which are connected with a sounding board, as on the pianoforte. The principal mark of distinction between strings and the other bodies which vibrate sympathetically, is that different forms of vibrating strings give simple tones corresponding to the *harmonic* upper partial tones of the prime tone, whereas the secondary simple tones of membranes, bells, rods, &c., have a different form of vibration, *in*harmonic with the prime tone, and the masses of air in resonators have generally only very high upper partial tones, also chiefly *in*harmonic with the prime tone, and not capable of being much reinforced by the resonator.

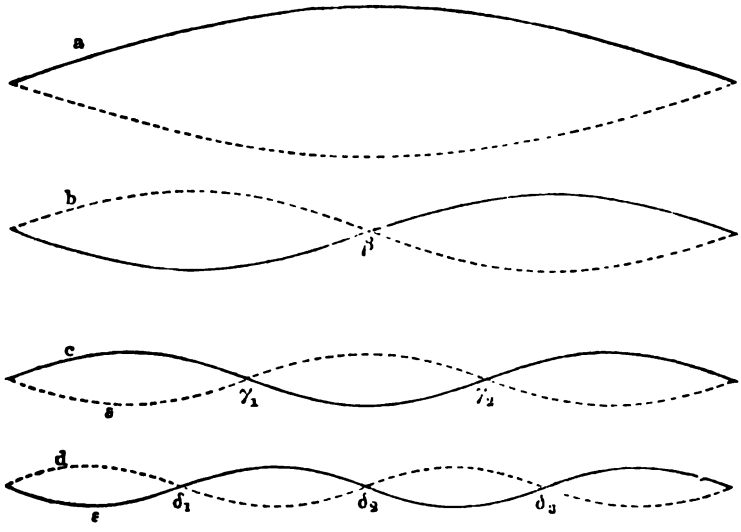
The vibrations of strings may be studied either on elastic chords loosely stretched, and not sonorous, but swinging so slowly that their motion may be followed with the hand and eye, or else on sonorous strings, as those of the pianoforte, guitar, monochord, or violin. Strings of the first kind are best made of thin spirals of brass wire, six to ten feet in length. They should be gently stretched, and both ends should be fastened. A string of this construction makes ample excursions with great regularity, which are easily seen by a large audience. The swings are excited by moving the string regularly backwards and forwards by the finger near to one of its extremities.

A string may be first made to vibrate as in fig. 17, a (p. 72), so

that its appearance when displaced from its position of rest is always that of a simple half wave. The string in this case gives a single simple tone, the deepest it can produce, and no other harmonic secondary tones are audible.

But the string may also during its motion assume the forms fig. 17, b, c, d. In this case the form of the string is that of two, three, or four half waves of a simple wave curve. In the vibrational form b the string produces only the upper Octave of its prime tone, in the form c the Twelfth, and in the form d the second Octave. The dotted lines shew the position of the string

FIG. 17.



at the end of half its periodic time. In b the point β remains at rest, in c two points γ and γ_2 remain at rest, in d three points δ_1, δ_2, δ_3. These points are called *nodes*. In a swinging spiral wire the nodes are readily seen, and for a resonant string they are shewn by little paper riders, which are jerked off from the vibrating parts and remain sitting on the nodes. When, then, the string is divided by a node into two swinging sections, it produces a simple tone having a vibrational number double that of the prime tone. For three sections the vibrational number is tripled, for four sections quadrupled, and so on.

To bring a spiral wire into these different forms of vibration, we either move it periodically with the finger near one extremity, adopting the period of its slowest swings for a, twice that rate for b, three times for c, and four times for d. Or we just gently touch one of the nodes nearest the extremity with the finger, and pluck the string half-way between this node and the nearest end. Hence when γ_1 in c, or δ_1 in d, is kept at rest by the finger, we pluck the string at ε. The other nodes then appear when the vibration commences.

For a resonant string the vibrational forms of fig. 17 (p. 72) are most purely produced by applying to its sounding board the handle of a tuning fork which has been struck and gives the simple tone corresponding to the form required. If only a determinate number of nodes are desired, and it is indifferent whether the individual points of the string do or do not execute simple vibrations, it is sufficient to touch the string very gently at one of the nodes and either pluck the string or rub it with a violin bow. By touching the string with the finger all those simple vibrations are damped which have no node at that point, and only those remain which allow the string to be at rest in that place.

The number of nodes in long thin strings may be considerable. They cease to be formed when the sections which lie between the nodes are too short and stiff to be capable of resonant vibration. Very fine strings consequently give a greater number of higher tones than thicker ones. On the violin and the lower pianoforte strings it is not very difficult to produce tones with ten sections; but with extremely fine wires tones with sixteen or twenty sections can be made to sound.

The forms of vibration here spoken of are those in which each point of the string performs pendular oscillations. Hence these motions excite in the ear the sensation of only a single simple tone. In all other vibrational forms of the strings, the oscillations are not simply pendular, but take place according to a different and more complicated law. This is always the case when the string is plucked in the usual way with the finger (as for guitar, harp, zither) or is struck with a hammer (as on the pianoforte), or is rubbed with a violin bow. The resulting motions may then be regarded as compounded of many simple vibrations, which taken separately correspond to those in fig. 17 (p. 72). The multiplicity of such composite forms of motion is

infinitely great, the string may indeed be considered as capable of assuming any given form (provided we confine ourselves in all cases to very small deviations from the position of rest), because, according to what was said in Chapter II., any given form of wave can be compounded out of a number of simple waves such as are shewn in fig. 17, a, b, c, d (p. 72). A plucked, struck, or bowed string therefore allows a great number of harmonic upper partial tones to be heard at the same time as the prime tone, and generally the number increases with the fineness of the string. The peculiar clink or chink of very fine metallic strings, is clearly due to these very high secondary tones. It is easy to distinguish the upper simple tones up to the sixteenth by means of resonators. Beyond the sixteenth they are too close to each other to be distinctly separable by this means.

Hence when a string is sympathetically excited by a musical tone in its neighbourhood, answering to the pitch of the prime tone of the string, a whole series of different simple vibrational forms will generally be at the same time generated in the string. For when the prime tone of the musical tone corresponds to the prime tone of the string all the harmonic upper partial tones of the first correspond to those of the second, and are hence capable of exciting the corresponding vibrational forms in the string. Generally the string will be brought into as many forms of sympathetic vibration by the motion of the air as the analysis of that motion shews that it possesses simple vibrational forms with a periodic time equal to that of a vibrational form, which the string is capable of assuming. But as a general rule when there is one such simple vibrational form in the air, there are several such, and it will often be difficult to determine by which out of the many possible simple tones which would produce the effect, the string has been excited. Consequently the usual unweighted strings are not so convenient for the determination of the pitch of such simple tones existent in a composite mass of air as the membranes or the inclosed air of resonators.

To make experiments with the pianoforte on the sympathetic vibrations of strings, select a flat instrument, raise its lid so as to expose the strings, then press down the key of the string (for *c′* suppose) which you wish to put into sympathetic vibration, but so slowly that the hammer does not strike, and place a little chip of wood across this *c′* string. You will find the chip put in motion,

or even thrown off when certain other strings are struck. The motion of the chip is greatest when one of the *subtones* of c' (p. 70) is struck, as c, F, C, $A\flat$, $F_{,}$, $D_{,}$ or $C_{,}$. Some but much less motion also occurs when one of the upper partial tones of c' is struck, as c'', g'' or c''', but in this last case the chip will not move if it has been placed over one of the corresponding nodes of the string. Thus if it is laid across the middle of the string it will be still for c'' and c''', but will move for g''. Placed at one third the length of the string from its extremity, it will not stir for g'', but will move for c'' or c'''. Finally the string c' will also be put in motion when a subtone of one of its upper partial tones is struck; for example, the note f, of which the third partial tone c'' is identical with the second partial tone of c'. In this case also the chip remains at rest when put on to the middle of the string c', which is its node for c''. In the same way the string c' will move, with the formation of two nodes, for g', g, or $e\flat$, all which notes have g'' as an upper partial tone, which is also the third partial of c'.

Observe that on the pianoforte, where one end of the strings is commonly concealed, the position of the nodes is easily found by pressing the string gently on both sides and striking the key. If the finger is at a node the corresponding upper partial tone will be heard purely and distinctly, otherwise the tone of the string is dull and bad.

As long as only one upper partial tone of the string c' is excited, the corresponding nodes can be discovered, and hence the particular form of its vibration determined. But this is no longer possible by the above mechanical method when two upper partial tones are excited, such as c'' and g'', as would be the case if both these notes were struck at once on the pianoforte, because the whole string of c' would then be in motion.

Although the relations for strings appear more complicated to the eye, their sympathetic vibration is subject to the same law as that which holds for resonators, membranes, and other elastic bodies. The sympathetic vibration is always determined by the analysis of whatever resonant motions exist, into simple pendular vibrations. If the periodic time of one of these simple vibrations corresponds to the periodic time of one of the proper tones of the elastic body, that body, whether it be a string, a membrane, or a mass of air, will be put into strong sympathetic vibration.

These facts give a real value to the analysis of resonant motion into simple pendular vibration, and no such value would attach to any other analysis. Every individual single system of waves formed by pendular vibrations exists as an independent mechanical unit, expands, and sets in motion other elastic bodies having the corresponding proper tone, perfectly undisturbed by any other simple tones of other pitches which may be expanding at the same time, and which may proceed either from the same or another source of sound. Each single simple tone, then, can, as we have seen, be separated from the composite mass of tones, by mechanical means, namely by bodies which will vibrate sympathetically with it. Hence every individual partial tone exists in the compound musical tone produced by a single musical instrument, just as truly, and in the same sense, as the different colours of the rainbow exist in the white light proceeding from the sun or any other luminiferous body. Light is also only a vibrational motion of a peculiar elastic medium, the luminiferous ether, just as sound is a vibrational motion of the air. In a beam of white light there is a species of motion which *may* be represented as the sum of many oscillatory motions of various periodic times, each of which corresponds to one particular colour of the solar spectrum. But of course each particle of ether at any particular moment has only *one* determinate velocity, and only *one* determinate departure from its mean position, just like each particle of air in a space traversed by many systems of resonant waves. The really existing motion of any particle of ether is of course only one and individual; and our theoretical treatment of it as compound, is in a certain sense arbitrary. But the undulatory motion of light can also be analysed into the waves corresponding to the separate colours by external mechanical means, such as by refraction in a prism, or by transmission through fine gratings, and each individual simple wave of light corresponding to a simple colour, exists mechanically by itself, independently of any other colour.

We must therefore not hold it to be an illusion of the ear, or mere imagination, when in the musical tone of a single note emanating from a musical instrument, we distinguish many partial tones, as I have found musicians inclined to think even when they have heard those partial tones quite distinctly with their own ears. If we admitted this, we should have also to look upon the colours of the spectrum which are separated from white light, as a

CHAPTER IV.

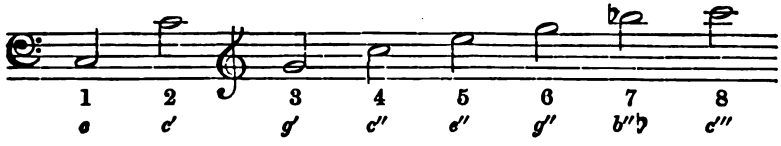
ON THE ANALYSIS OF MUSICAL TONES BY THE EAR.

It was frequently mentioned in the preceding chapter that musical tones could be resolved by the ear alone, unassisted by any peculiar apparatus, into a series of partial tones corresponding to the simple pendular vibrations in a mass of air, that is, into the same constituents as those into which the motion of the air is resolved by the sympathetic vibration of elastic bodies. We proceed to shew the correctness of this assertion.

Any one who endeavours for the first time to distinguish the upper partial tones of a musical tone, generally finds considerable difficulty in merely hearing them.

The analysis of our sensations when it cannot be attached to corresponding differences in external objects, meets with peculiar difficulties, the nature and significance of which will occupy our attention hereafter. The attention of the observer has generally to be drawn to the phenomenon he has to observe, by peculiar aids properly selected, until he knows precisely what to look for; after he has once succeeded, he will be able to throw aside such crutches. Similar difficulties meet us in the observation of the upper partial tones of a musical tone. I shall first give a description of such processes as will most easily put an untrained observer into a position to recognise upper partial tones, and I will remark in passing that a musically trained ear will not necessarily hear upper partial tones with greater ease and certainty than an untrained ear. Success depends rather upon a peculiar power of mental abstraction or a peculiar mastery over attention, than upon musical training. But a musically trained observer has an essential advantage over one not so trained in his power of figuring to himself how the simple sounds sought for ought to sound, whereas the untrained observer has continually to hear these tones sounded by other means in order to keep their effect fresh in his mind.

First we must note, that the uneven partial tones, as the Fifths, Thirds, Sevenths, &c., of the prime tones are usually easier to hear than the even ones, which are Octaves either of the prime tone or of some of the upper partials which lie near it, just as in a chord we more readily distinguish whether it contains Fifths and Thirds than whether it has Octaves. The second, fourth, and eighth partials are higher Octaves of the prime, the sixth partial an Octave above the third partial, or the Twelfth of the prime; and some practice is required for distinguishing these. Among the uneven partials which are more easily distinguished, the first place must be assigned, from its usual loudness, to the third partial, the Twelfth of the prime, or the Fifth of its first higher Octave. Then follows the fifth partial as the major Third of the prime, and, generally very faint, the seventh partial as the minor Seventh [1] of the second higher Octave of the prime, as will be seen by their following expression in musical notation, for the compound tone *c*.



In commencing to observe upper partial tones, it is advisable just before producing the musical tone itself which you wish to analyse, to sound the note you wish to distinguish in it, very gently, and if possible in the same quality of tone as the compound itself. The pianoforte and harmonium are well adapted for these experiments, because they both have upper partial tones of considerable power.

First gently strike on a piano the note *g′*, as marked above, and after letting the digital rise so as to damp the string, strike the note *c*, of which *g′* is the third partial, with great force, and keep your attention directed to the pitch of the *g′* which you had just heard, and you will hear it again in the compound tone of *c*. Similarly, first strike the fifth partial *e″* gently, and then *c* strongly. These upper partial tones are often more distinct as the sound dies away, because they appear to lose force more slowly than the prime. The seventh and ninth partials *b″♭* and *d‴* are

[1] [Or more correctly *sub*-minor Seventh; see the table of intervals in Chapter X. —*Translator*.]

most weak, or quite absent on modern pianos. If the same experiments are tried with an harmonium in one of its louder stops, the seventh partial will generally be well heard, and sometimes even the ninth.

To the objection which is sometimes made that the observer only imagines he hears the partial tone in the compound, because he had just heard it by itself, I need only remark at present that if *e''* is first heard as a partial tone of *c* on a good piano, tuned in equal temperament, and then *e''* is struck on the instrument itself, it is quite easy to perceive that the latter is a little sharper. This follows from the method of tuning. But if there is a difference in pitch between the two tones, one is certainly not a continuation of the mental effect produced by the other. Other facts which completely refute the above conception, will be subsequently adduced.

A still more suitable process than that just described for the piano, can be adopted on any stringed instrument, as the piano, monochord, or violin. It consists in first producing the tone we wish to hear, as an harmonic, [p. 37, note] by touching the corresponding node of the string when it is struck or rubbed. The resemblance of the tone first heard to the corresponding partial of the compound is then much greater, and the ear discovers it more readily. It is usual to place a divided scale by the string of a monochord, to facilitate the discovery of the nodes. Those for the third partial, as shewn in the last chapter, divide the string into three equal parts, those for the fifth into five, and so on. On the piano and violin the position of these points is easily found experimentally, by touching the string gently with the finger in the neighbourhood of the node, which has been approximatively determined by the eye, then striking or bowing the string, and moving the finger about till the required harmonic comes out strongly and purely. By then sounding the string at one time with the finger on the node and at another without, we obtain the required upper partial at one time as an harmonic, and at another in the compound tone of the whole string, and thus learn to recognise the existence of the first as part of the second, with comparative ease. Using thin strings which have loud upper partials, I have thus been able to recognise the partials separately, up to the sixteenth. Those which lie still higher are too near to each other in pitch for the ear to separate them readily.

In such experiments I recommend the following process. Touch the node of the string on the pianoforte or monochord with a camel's hair pencil, strike the note, and immediately remove the pencil from the string. If the pencil has been pressed tightly on the string, we either continue to hear the required partial as an harmonic, or else in addition hear the prime tone gently sounding with it. On repeating the excitement of the string, and continuing to press more and more lightly with the camel's hair pencil, and at last removing the pencil entirely, the prime tone of the string will be heard more and more distinctly with the harmonic, till at last we have the full natural musical tone of the string. By this means we obtain a series of gradual transitional stages between the isolated partial and the compound tone, in which the first is readily retained by the ear. By means of this last process I have generally succeeded in making quite untrained ears recognise the existence of upper partial tones.

It is at first more difficult to hear the upper partials on most wind instruments and in the human voice, than on stringed instruments, harmoniums, and the more penetrating stops of an organ, because it is then not so easy first to produce the upper partial softly in the same quality of tone. But still a little practice suffices to lead the ear to the required partial tone, by previously touching it on the piano. The partial tones of the human voice are comparatively most difficult to distinguish for reasons which will be given subsequently. Nevertheless they were distinguished even by Rameau[1] without the assistance of any apparatus. The process is as follows:—

Get a powerful bass voice to sing *e*♭ to the vowel O, in *more* (more like *aw* in *maw* than *ow* in *mow*), gently touch *b′*♭ on the piano, which is the Twelfth, or third partial tone of the note *e*♭, and let its sound die away while you are listening to it attentively. The note *b′*♭ on the piano will appear really not to die away, but to keep on sounding, even when its string is damped by removing the finger from the digital, because the ear unconsciously passes from the tone of the piano to the partial tone of the same pitch produced by the singer, and takes the latter for a continuation of the former. But when the finger is removed from the key, and the damper has fallen, it is of course impossible that the tone of

[1] Nouveau Système de Musique théorique. Paris: 1726. Préface.

the string should have continued sounding. To make the experiment for g'' the fifth partial, or major Third of the second Octave above $e\flat$, the voice should sing to the vowel A in *father*.

The resonators described in the last chapter furnish an excellent means for this purpose, and can be used for the tones of any musical instrument. On applying to the ear the resonator corresponding to any given upper partial of the compound c, such as g', this g' is rendered much more powerful when c is sounded. Now hearing and distinguishing g' in this case by no means proves that the ear alone and without this apparatus would hear g' as part of the compound c. But the increase of the loudness of g' caused by the resonator may be used to direct the attention of the ear to the tone it is required to distinguish. On gradually removing the resonator from the ear, the force of g' will decrease. But the attention once directed to it by this means, remains more readily fixed upon it, and the observer continues to hear this tone in the natural and unchanged compound tone of the given note, even with his unassisted ear. The sole office of the resonators in this case is to direct the attention of the ear to the required tone.

By frequently instituting similar experiments for perceiving the upper partial tones, the observer comes to discover them more and more easily, till he is finally able to dispense with any aids. But a certain amount of undisturbed concentration is always necessary for analysing musical tones by the ear alone, and hence the use of resonators is quite indispensable for an accurate comparison of different qualities of tones, especially in respect to the weaker upper partials. At least, I must confess, that my own attempts to discover the upper partial tones in the human voice, and to determine their differences for different vowels, were most unsatisfactory until I applied the resonators.

We now proceed to prove that the human ear really does analyse musical tones according to the law of simple vibrations. Since it is not possible to institute an exact comparison of the strength of our sensations for different simple tones, we must confine ourselves to proving that when an analysis of a composite tone into simple vibrations, effected by theoretic calculation or by sympathetic resonance, shews that certain upper partial tones are absent, the ear also does not perceive them.

The tones of strings are again best adapted for conducting this

proof, because they admit of many alterations in their quality of tone, according to the manner and the spot in which they are excited, and also because the theoretic or experimental analysis is most easily and completely performed for this case. Thomas Young[1] first shewed that when a string is plucked or struck, or, as we may add, bowed at any point in its length which is the node of any of its so-called harmonics, those simple vibrational forms of the string which have a node in that point are not contained in the compound vibrational form. Hence, if we attack the string at its middle point, all the simple vibrations due to the even partial tones, each of which has a node at that point, will be absent. This gives the sound of the string a peculiar hollow or nasal twang. If we excite the string at $\frac{1}{3}$ of its length, the vibrations corresponding to the third, sixth, and ninth partial tones will be absent; if at $\frac{1}{4}$, then those corresponding to the fourth, eighth, twelfth partial tones will fail; and so on.[2]

This result of mathematical theory is confirmed, in the first place, by analysing the compound tone of the string by sympathetic resonance, either by the resonators or by other strings. The experiments may be easily made on the pianoforte. Press down the digitals for the notes *c* and *c′*, without allowing the hammer to strike, so as merely to free them from their dampers, and then pluck the string *c* with the nail till it sounds. On damping the string *c* the higher *c′* will echo the sound, except in the particular case when the *c* string has been plucked exactly at its middle point, which is the point where it would have to be touched in order to give its first harmonic when struck by the hammer.

If we touch the *c* string at $\frac{1}{3}$ or $\frac{2}{3}$ its length, and strike it with the hammer, we obtain the harmonic *g′*; and if the damper of the *g′* is raised, this string echoes the sound. But if we pluck the *c* string with the nail, at either $\frac{1}{3}$ or $\frac{2}{3}$ its length, *g′* is not echoed, as it will be if the *c* string is plucked at any other spot.

In the same way observations with the resonators shew that when the *c* string is plucked at its middle the Octave *c′* is missing, and when at $\frac{1}{3}$ or $\frac{2}{3}$ its length the Twelfth *g′* is absent. The analysis of the sound of a string by the sympathetic resonance of strings or resonators, consequently fully confirms Thomas Young's law.

[1] London. Philosophical Transactions, 1800, vol. i. p. 137.

[2] See Appendix, No. III.

But for the vibration of strings we have a more direct means of analysis than that furnished by sympathetic resonance. If we, namely, touch a vibrating string gently for a moment with the finger or a camel's-hair pencil, we damp all those simple vibrations which have no node at the point touched. Those vibrations, however, which have a node there are not damped, and hence will continue to sound without the others. Consequently, if a string has been made to speak in any way whatever, and we wish to know whether there exists among its simple vibrations one corresponding to the Twelfth of the prime tone, we need only touch one of the nodes of this vibrational form at $\frac{1}{3}$ or $\frac{2}{3}$ the length of the string, in order to reduce to silence all simple tones which have no such node, and leave the Twelfth sounding, if it were there. If neither it, nor any of the sixth, ninth, twelfth, &c., of the partial tones were present, giving corresponding harmonics, the string will be reduced to absolute silence by this contact of the finger.

Press down one of the digitals of a piano, in order to free a string from its damper. Pluck the string at its middle point, and immediately touch it there. The string will be completely silenced, shewing that plucking it in its middle excited none of the even partial tones of its compound tone. Pluck it at $\frac{1}{3}$ or $\frac{2}{3}$ its length, and immediately touch it in the same place; the string will be silent, proving the absence of the third partial tone. Pluck the string anywhere else than in the points named, and the second partial will be heard when the middle is touched, the third when the string is touched at $\frac{1}{3}$ or $\frac{2}{3}$ of its length.

The agreement of this kind of proof with the results from sympathetic resonance, is well adapted for the experimental establishment of the proposition based in the last chapter solely upon the results of mathematical theory, namely, that sympathetic vibration occurs or not, according as the corresponding simple vibrations are or are not contained in the compound motion. In the last described method of analysing the tone of a string, we are quite independent of the theory of sympathetic vibration, and the simple vibrations of strings are exactly characterised and recognisable by their nodes. If the compound tones admitted of being analysed by sympathetic resonance according to any other vibrational forms except those of simple vibration, this agreement could not exist.

If, after having thus experimentally proved the correctness of

Thomas Young's law, we try to analyse the tones of strings by the unassisted ear, we shall continue to find complete agreement.[1] If we pluck or strike a string in one of its nodes, all those upper partial tones of the compound tone of the string to which the node belongs, disappear for the ear also, but they are heard if the string is plucked at any other place. Thus, if the string *c* be plucked at $\frac{1}{3}$ its length, the partial tone *g'* cannot be heard, but if the string be plucked at only a little distance from this point the partial tone *g'* is distinctly audible. Hence the ear analyses the sound of a string into precisely the same constituents as are found by sympathetic resonance, that is, into simple tones, according to Ohm's definition of this conception. These experiments are also well adapted to shew that it is no mere play of imagination when we hear upper partial tones, as some people believe on hearing them for the first time, for those tones are not heard when they do not exist.

The following modification of this process is also very well adapted to make the upper partial tones of strings audible. First, strike alternately in rhythmical sequence, the third and fourth partial tone of the string alone, by damping it in the corresponding nodes, and request the listener to observe the simple melody thus produced. Then strike the undamped string alternately and in the same rhythmical sequence, in these nodes, and thus reproduce the same melody in the upper partials, which the listener will then easily recognise. Of course, in order to hear the third partial, we must strike the string in the node of the fourth, and conversely.

The compound tone of a plucked string is also a remarkably striking example of the power of the ear to analyse into a long series of partial tones, a motion which the eye and the imagination are able to conceive in a much simpler manner. A string, which is pulled aside by a sharp point, or the finger nail, assumes the form fig. 18, A (p. 86) before it is released. It then passes through the series of forms, fig. 18, B, C, D, E, F, till it reaches G, which is the inversion of A, and then returns through the same to A again. Hence it alternates between the forms A and G. All these forms, as is clear, are composed of three straight lines, and on expressing the velocity of the individual points of the strings by vibrational curves, these would have the same form. Now the string scarcely imparts any perceptible portion of its

[1] See Brandt in 'Poggendorff's Annalen der Physik,' vol. cxii., p. 324, where this fact is proved.

own motion directly to the air. Scarcely any audible tone results when both ends of a string are fastened to immovable supports, as metal bridges, which are again fastened to the walls of a room. The sound of the string reaches the air through that one of its extremities which rests upon a bridge standing on an elastic sounding board. Hence the sound of the string essentially depends on the motion of this extremity, through the pressure which it exerts on the sounding board. The magnitude of this pressure, as it alters periodically with the time, is shewn in fig. 19, where the height of the line h h corresponds to the amount of pressure exerted on the bridge by that extremity of the string when the string is at rest. Along h h suppose lengths to be set off corresponding to consecutive intervals of time, the vertical heights of the broken line above or below h h

FIG. 18.

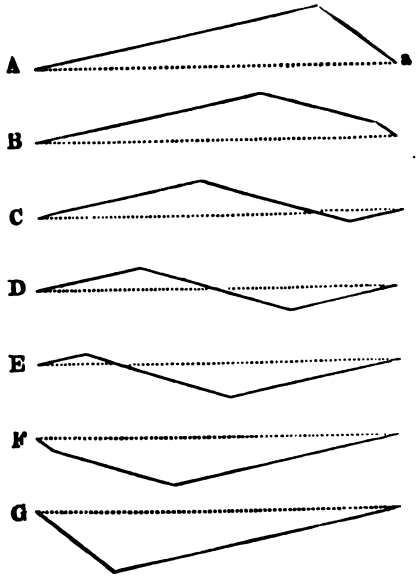
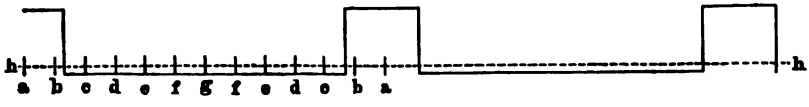


FIG. 19.



represent the corresponding augmentations or diminutions of pressure at those times. The pressure of the string on the sounding board consequently alternates, as the figure shews, between a higher and a lower value. For some time the greater pressure remains unaltered; then the lower suddenly ensues, and likewise remains for a time unaltered. The letters a to g in fig. 19 correspond to the times at which the string assumes the forms A to G in fig. 18. It is this alternation between a greater and a smaller pressure which produces the sound in the air. We cannot but feel astonished that a motion produced by

means so simple and so easy to comprehend, should be analysed by the ear into such a complicated sum of simple tones. For the eye and the understanding the action of the string on the sounding board can be figured with extreme simplicity. What has the simple broken line of fig. 19 (p. 86) to do with wave-curves, which, in the course of one of their periods, shew 3, 4, 5, up to 16, and more, crests and troughs? This is one of the most striking examples of the different ways in which eye and ear comprehend a periodic motion.

There is no resonant body whose motions under varied conditions can be so completely calculated theoretically and contrasted with observation as a string. The following are examples in which theory can be compared with analysis by ear:—

I have discovered a means of exciting simple pendular vibrations in the air. A tuning-fork when struck gives no harmonic upper partial tones, or, at most, traces of them when it is brought into such excessively strong vibration that it no longer exactly follows the law of the pendulum. On the other hand, tuning-forks have some very high inharmonic secondary tones, which produce that peculiar sharp tinkling of the fork at the moment of being struck, and generally become rapidly inaudible. If the tuning-fork is held in the fingers, it imparts very little of its tone to the air, and cannot be heard unless it is held close to the ear. Instead of holding it in the fingers, we may screw it into a thick board, on the under side of which some pieces of india-rubber tubing have been fastened. When this is laid upon a table, the india-rubber tubes on which it is supported convey no sound to the table, and the tone of the tuning-fork is so weak that it may be considered inaudible. Now if the prongs of the fork be brought near a resonance chamber[1] of a bottle-form of such a size and shape that, when we blow over its mouth, the air it contains gives a tone of the same pitch as the fork's, the air within this chamber vibrates sympathetically, and the tone of the fork is thus conducted with great strength to the outer air. Now the higher secondary tones of such resonance chambers are also inharmonic to the prime tone, and in general the secondary tones of the chambers corre-

[1] Either a bottle of a proper size, which can readily be more accurately tuned by pouring oil or water into it, or a tube of pasteboard quite closed at one end, and having a small round opening at the other. See the proper sizes of such resonance chambers in Appendix IV.

spond neither with the harmonic nor the inharmonic secondary tones of the forks; this can be determined in each particular case by producing the secondary tones of the bottle by stronger blowing, and discovering those of the forks with the help of strings set into sympathetic vibration, as will be presently described. If, then, only one of the tones of the fork, namely the prime tone, corresponds with one of the tones of the chamber, this alone will be reinforced by sympathetic vibration, and this alone will be communicated to the external air, and thus conducted to the observer's ear. The examination of the motion of the air by resonators shews that in this case, provided the tuning-fork be not set into too violent motion, no tone but the prime is present, and in such case the unassisted ear hears only a single simple tone, namely the common prime of the tuning-fork and of the chamber, without any accompanying upper partial tones.

The tone of a tuning-fork can also be purified from secondary tones by placing its handle upon a string and moving it so near to the bridge that one of the proper tones of the section of string lying between the fork and the bridge is the same as that of the tuning-fork. The string then begins to vibrate strongly, and conducts the tone of the tuning-fork with great power to the sounding board and surrounding air, whereas the tone is scarcely, if at all, heard as long as the above-named section is not in unison with the tone of the fork. In this way it is easy to find the lengths of string which correspond to the prime and upper partial tones of the fork, and accurately determine the pitch of the latter. If this experiment is conducted with ordinary strings which are uniform throughout their length, we shield the ear from the inharmonic secondary tones of the fork, but not from the harmonic upper partials, which are sometimes faintly present when the fork is made to vibrate strongly. Hence to conduct this experiment in such a way as to create purely pendular vibrations of the air, it is best to weight one point of the string, if only so much as by letting a drop of melted sealing-wax fall upon it. This causes the upper proper tones of the string itself to be inharmonic to the prime tone, and hence there is a distinct interval between the points where the fork must be placed to bring out the prime tone and its audible octave, if it exists.

In most other cases the mathematical analysis of the motions of sound is not nearly far enough advanced to determine with certainty what upper partials will be present and what intensity they

will possess. In circular plates and stretched membranes which are struck, it is theoretically possible to do so, but their inharmonic secondary tones are so numerous and so nearly of the same pitch that most observers would probably fail to separate them satisfactorily. On elastic rods, however, the secondary tones are very distant from each other, and are inharmonic, so that they can be readily distinguished from each other by the ear. The following are the proper tones of a rod which is free at both ends; the vibrational number of the prime tone taken to be *c*, is reckoned as 1:—

	Vibrational Number	Notation
Prime tone	1·0000	*c*
Second proper tone	2·7576	*f'* −0·2
Third proper tone	5·4041	*f''* +0·1
Fourth proper tone	13·3444	*a'''* −0·1

The notation is adapted to the equal temperament, and the appended fractions are parts of the interval of a complete tone.

Where we are unable to execute the theoretical analysis of the motion, we can, at any rate, by means of resonators and other sympathetically vibrating bodies, resolve any individual musical tone that is produced, and then compare this resolution, which is determined by the laws of sympathetic vibration, with that effected by the unassisted ear. The latter is naturally much less sensitive than one armed with a resonator; so that it is frequently impossible for the unarmed ear to recognise amongst a number of other stronger simple tones those which the resonator itself can only faintly indicate. On the other hand, so far as my experience goes, there is complete agreement to this extent: the ear recognises without resonators the simple tones which the resonators greatly reinforce, and perceives no upper partial tone which the resonator does not indicate. To verify this conclusion, I performed numerous experiments, both with the human voice and the harmonium, and they all confirmed it.

By the above experiments the proposition enunciated and defended by G. S. Ohm must be regarded as proved, viz., that *the human ear perceives pendular vibrations alone as simple tones, and resolves all other periodic motions of the air into a series of pendular vibrations, hearing the series of simple tones which correspond with these simple vibrations.*

Calling, then, as already defined (in pp. 34–6 and note), the sensation excited in the ear by any periodical motion of the air a *musical tone*, and the sensation excited by a simple pendular vibration a *simple tone*, the rule asserts that *the sensation of a musical tone is compounded out of the sensations of several simple tones.* In particular, we shall henceforth call the sound produced by a single resonant body its (simple or compound) *tone*, and the sound produced by several musical instruments acting at the same time a *composite tone*, consisting generally of several (simple or compound) tones. If, then, a single note is sounded on a musical instrument, as a violin, trumpet, organ, or by a singing voice, it must be called in exact language a *tone* of the instrument in question. This is also the ordinary language, but it did not then imply that the tone might be *compound.* When the tone is, as usual, a compound tone, it will be distinguished by this term, or the abridgement, a *compound*; while *tone* is a general term which includes both simple and compound tones.[1] The prime tone is generally louder than any of the upper partial tones, and hence it alone generally determines the *pitch* of the compound. The tone produced by any resonant body reduces to a *single* simple tone in very few cases indeed, as the tone of tuning-forks imparted to the air by resonance chambers in the manner already described. The tones of wide-stopped organ-pipes when gently blown are almost free from upper partials, and are accompanied only by a rush of wind.

It is well known that this union of several simple tones into one compound tone, which is naturally effected in the tones produced by most musical instruments, is artificially imitated on the organ by peculiar mechanical contrivances. The tones of organ-pipes are comparatively poor in upper partials. When it is desir-

[1] [Here, again, as on pp. 34–6, I have, in the translation, been necessarily obliged to deviate slightly from the original. *Klang*, as here defined, embraces *Ton* as a particular case. I use *tone* for the general term, and *compound tone* and *simple tone* for the two particular cases. Thus, as presently mentioned in the text, the *tone* produced by a tuning-fork held over a proper resonance chamber we know, on analysis, to be *simple*, but before analysis it is to us only a (musical) *tone* like any other, and hence in this case the author's *Klang* becomes the author's *Ton*. I believe that the language used in my translation is best adapted for the constant accurate distinction between compound and simple tones by English readers, as I leave nothing which runs counter to old habits, and by the use of the words simple and compound, constantly recall attention to this newly discovered and extremely important relation.—*Translator.*]

able to use a stop of incisive penetrating quality of tone and great power, the wide pipes (*principal register* and *weitgedackt*[1]) are not sufficient; their tone is too soft, too defective in upper partials; and the narrow pipes (*geigen-register* and *quintaten*[2]) are also unsuitable, because, although more incisive, their tone is weak. For such occasions, then, as in accompanying congregational singing, recourse is had to the *compound stops*.[3] In these stops every key is connected with a larger or smaller series of pipes, which it opens simultaneously, and which give the prime tone and a certain number of the first upper partials of the compound tone of the note in question. It is very usual to connect the upper Octave with the prime tone, and after that the Twelfth. The more complex compounds (*cornet*) give the first six partial tones, that is, in addition to the two Octaves of the prime tone and its Twelfth, the higher major Third, and the Octave of the Twelfth. This is as much of the series of upper partials as belongs to the tones of a major chord. But to prevent these compound stops from being insupportably noisy, it is necessary to reinforce the deeper tones of each note by other rows of pipes, for in all natural tones which are suited for musical purposes the higher partials decrease in force as they rise in pitch. This has to be regarded in their imitation by compound stops. These compound stops were a monster in the path of the old musical theory, which was acquainted only with the prime tones of compounds; but the practice of organ builders and organists necessitated their retention, and when they are suitably arranged and properly applied, they form a very effective musical apparatus. The nature of the case at the same time fully

[1] [*Principal*—double open diapason. *Grossgedackt*—double stopped diapason.' Hopkins, *Organ*, p. 435.—*Translator.*]

[2] [' *Geigen Principal*—violin or crisp-toned diapason, eight feet.' Hopkins, *Organ*, p. 435. 'A manual stop of eight feet, producing a pungent tone very like that of the Gamba, except that the pipes, being of larger scale, speak quicker and produce a fuller tone. The stop is not much known in England; two specimens, of eight and four feet, however, occurring on the choir manual of the Exchange Organ at Northampton.' Ibid. p. 117. For *quintaten*, see *suprà*, p. 50, note.—*Translator.*]

[3] [As described in Hopkins, *Organ*, pp. 120, 122, these are the *sesquialtera* ' of five, four, three, or two ranks of open metal pipes, tuned in Thirds, Fifths, and Octaves to the Diapason.' The *mixture*, consisting of five to two ranks of open metal pipes smaller than the last, is in England the second, in Germany the first, compound stop. The *Furniture* of five to two sets of small open pipes, is variable. The *Cornet*, 1) *mounted* has five ranks of very large and loudly voiced pipes, 2) *echo*, is similar, but light and delicate, and is enclosed in a box. In German organs the *cornet* is also a pedal reed stop of four and two feet.—*Translator.*]

[illegible] The musician is bound to regard the tones of [illegible] instruments as compounded in the same way as the [illegible] stops of organs, and the important part this method of [illegible] plays in the construction of musical scales and chords [illegible] be made evident in subsequent chapters.

We have thus been led to an appreciation of upper partial tones, which, as it differs considerably from that previously entertained by musicians, and even physicists, must meet with considerable opposition. The upper partial tones were indeed known, but almost only in such compound tones as those of strings, where there was a favourable opportunity for observing them; but they appear in previous physical and musical works as an isolated accidental phenomenon of small intensity, a kind of curiosity, which was certainly occasionally adduced, in order to give some support to the opinion that nature had prefigured the construction of our major chord, but which on the whole remained almost entirely disregarded. In opposition to this we have to assert, and we shall prove the assertion in the next chapter, that upper partial tones are, with a few exceptions already named, a general constituent of all musical tones, and that a certain stock of upper partials is an essential condition for a good musical quality of tone. Finally, these upper partials have been erroneously considered as weak, because they are difficult to observe, while, in point of fact, for some of the best musical qualities of tone, the loudness of the first upper partials is not far inferior to that of the prime tone itself.

There is no difficulty in verifying this last fact by experiments on the tones of strings. Strike the string of a piano or monochord, and immediately touch one of its nodes for an instant with the finger; the constituent partial tones having this node will remain with unaltered loudness, and the rest will disappear. We might just as well touch the node at the instant of striking, and thus obtain the corresponding constituent partial tones from the first, in place of the complete compound tone of the note. In both ways we can readily convince ourselves that the first upper partials, as the Octave and Twelfth, are by no means weak and difficult to hear, but have a very appreciable strength. In some cases we are able to assign numerical values for the intensity of the upper partial tones, as will be shewn in the next chapter. For tones not produced on strings this *à posteriori* proof is not so easy to conduct, because we are not able to make the upper partials speak

separately. But even then by means of the resonator we can appreciate the intensity of these upper partials by producing the corresponding note on the same or some other instrument until its loudness, when heard through the resonator, agrees with that of the former.

The difficulty we experience in hearing upper partial tones is no reason for considering them to be weak; for this difficulty does not depend on their intensity, but upon entirely different circumstances, which could not be properly estimated until the advances recently made in the physiology of the senses. On this difficulty of observing the upper partial tones have been founded the objections which A. Seebeck[1] has advanced against Ohm's law of the decomposition of a musical tone; and perhaps many of my readers who are unacquainted with the physiology of the other senses, especially with that of the eye, might be inclined to adopt Seebeck's opinions. I am therefore obliged to enter into some details concerning this difference of opinion, and the peculiarities of the perceptions of our senses, on which the solution of the difficulty depends.

Seebeck, although extremely accomplished in acoustical experiments and observations, was not always able to recognise upper partial tones, where Ohm's law required them to exist. But we are also bound to add that he did not apply the methods already indicated for directing the attention of his ear to the upper partials in question. In other cases when he did hear the theoretical upper partials, they were weaker than the theory required. He concluded that the definition of a simple tone as given by Ohm was too limited, and that not only pendular vibrations, but other vibrational forms, provided they were not too widely separated from the pendular, were capable of exciting in the ear the sensation of a single simple tone, which, however, had a variable quality. He consequently asserted that when a musical tone was compounded of several simple tones, part of the intensity of the upper constituent tones went to increase the intensity of the prime tone, with which it fused, and that at most a small remainder excited in the ear the sensation of an upper partial tone. He did not formulate any determinate law, assigning the vibrational forms which would give the impression of a simple and those which would give the impression of a compound tone. The experiments of Seebeck, on which

[1] In Poggendorff's *Annalen der Physik*, vol. lx. p. 449, vol. lxiii. pp. 353 and 368. —*Ohm*, ibid. vol. lix. p. 513, and vol. lxii. p. 1.

he founded his assertions, need not be here described in detail. Their object was only to produce musical tones for which either the intensity of the simple vibrations corresponding to the upper partials could be theoretically calculated, or in which these upper partials could be rendered separately audible. For the latter purpose the siren was used. We have just described how the same object can be attained by means of strings. Seebeck shews in each case that the simple vibrations corresponding to the upper partials have considerable strength, but that the upper partials are either not heard at all, or heard with difficulty in the compound tone itself. This fact has been already mentioned in the present chapter. It may be perfectly true for an observer who has not applied the proper means for observing upper partials, while another, or even the first observer himself when properly assisted, can hear them perfectly well.

Upper partial tones, namely, are a phenomenon belonging to pure auditory *sensation*. The aggregation of a series of partial tones into a compound tone, belonging to any determinate musical instrument, is not a process of sensation, but of *perception*. I had to draw attention to this difference in the Introduction to this work (p. 5). *Impressions on our senses*, in so far as we become conscious of them only as conditions of our body, and, in particular, of our nervous apparatus, are called *sensations*; but in so far as we form from them a mental image of external objects, they are termed *perceptions*. When we apprehend a certain sound, as the tone of a violin, we have a perception; we conclude that a certain instrument exists which usually produces tones of that kind. But when we try to resolve this musical tone into its partial tones, this is a matter of pure sensation. No individual partial tone corresponds to any particular resonant body or part of a resonant body. Separated from the accompanying partial tones, any one in particular is nothing but a part of our sensation. Hence, although when for a scientific purpose we institute investigations into our sensations, as in the present book, we may have a great interest in hunting up these partial tones, yet, in the every-day use of our ears, we have no interest of the kind, for then the only value of our sensations is to assist us in comprehending what is going on in the world without us. For this purpose it is enough to apprehend the musical tones correctly. Their resolution into partial

tones, even if we were conscious of it, would not only be of no use, but would be a source of extreme disturbance.

Now practice and experience play a far greater part in the use of our senses than we are usually inclined to assume, and since, as just remarked, our sensations derived from the senses are primarily of importance only for enabling us to form a correct conception of the world without us, our practice in the observation of these sensations usually does not extend in the slightest degree beyond what is necessary for this purpose. We are certainly only far too much disposed to believe that we must be immediately conscious of all that we feel and of all that enters into our sensations. This natural belief, however, is founded only on the fact that we are always immediately conscious, without taking any special trouble, of everything necessary for the practical purpose of forming a correct acquaintance with external nature, because during our whole life we have been daily and hourly using our organs of sense and collecting results of experience for this precise object. And even if we do not go beyond the circle of sensations corresponding to external objects, the influence of practice is apparent. It is well known how much more delicately and rapidly a painter distinguishes colour and *chiaroscuro* than an eye unused to such distinctions; how a musician or a musical-instrument maker easily and surely apprehends distinctions of pitch and quality of tone which have no existence for the layman; how, even in the lower regions of cookery and wine-tasting, it requires much training and frequent comparisons to make a master. The value of practice, however, is much more striking when we proceed to sensations solely conditioned by the internal relations of our organs of sense and of our whole nervous system, and not corresponding to external objects or the impressions they produce, and hence of no value to us for the apprehension of external nature. Recent physiological inquiries into the action of the organs of sense have made us acquainted with many such cases, which have been discovered partly by mere chance, partly by theoretical questions and speculations, partly even by a peculiar talent for observation in gifted individuals, as Goethe and Purkinje. These so-called *subjective* phenomena are extremely difficult to discover, and when discovered they almost always require peculiar means for assisting our attention and directing it to the observation of the matter in question,

insomuch that it is generally very difficult to re-discover the phenomena, even when the descriptions given by the first observer are known. We are, indeed, not only unpractised in observing these subjective phenomena of the senses, but we are even extremely practised in disregarding them, just because a consciousness of their existence would disturb us in the observation of external nature. It is only when they are sufficiently powerful to impede that observation that we begin to remark them; and sometimes in dreams or delirium they form the points of attachment for various delusions.

As examples, I will only refer to some tolerably well-known instances in physiological optics.[1] So-called *mouches volantes* probably exist in all eyes; these are little fibres, lumps, or drops, which floating about in the vitreous humour of the eye, and casting their shadow on the retina, appear as little dark moving images in the field of view. They are most easily visible when we look attentively at an extensive bright surface without other marks upon it, as, for example, at the blue sky. Most persons whose attention has not been specially directed to them are not aware of their existence until diseased eyes lead them to pay more attention to subjective symptoms. The common complaint, then, is that the *mouches volantes* came with the disease; and the patients are thus led to disquiet themselves greatly over these harmless objects, and to follow them attentively. They then refuse to believe the assurance that these same images have existed in their eyes all their life, and, in fact, do exist in every healthy eye. I even knew an old gentleman who had to bandage up an eye which was accidentally diseased, and then observed for the first time, to his own great dismay, that he was totally blind with the other eye; and, on examination, it appeared to be a blindness of such a nature that it must have existed for years without having been observed.

Who, moreover, would believe, without having performed the proper experiments, that when one eye is closed, there is a small gap not far from the middle of the field of view of the open eye in which we can see nothing at all, and which we fill up by imagination only? This is the so-called *punctum caecum.*

[1] [Prof. Helmholtz is the author of a Manual of Physiological Optics (*Handbuch der physiologischen Optik*, Leipzig, 1867, pp. 874), forming vol. ix. of G. Karsten's *Allgemeine Encyklopädie der Physik.—Translator.*]

Mariotte, who was led to the discovery of this phenomenon by theoretical speculations, excited considerable astonishment by exhibiting it at the court of Charles II. of England (1666); and the experiment was then repeated with many variations and used for amusement. In reality, this gap is so large that seven full-moons could be placed on its diameter, and that a human face six or seven feet off can be made to disappear entirely within it. In ordinary unconstrained vision, however, this gap in the field of vision is not observed, because we are perpetually sweeping it over with our eye, which we immediately direct to any point of interest. Hence it is that the objects which excite our attention at any moment, and, consequently, the blind point of our field of vision, never usually become objects of attention. We have to direct our eye purposely towards one object, and then to move a second small object into the neighbourhood of the blind spot, while we force ourselves to notice the second object without moving the axis of our eye directed to the first, and this feat is so opposed to our usual habits that many persons cannot succeed in performing it; but unless we can do so we cannot see the second object disappear, and thus become convinced of the existence of the gap in our eye.

Finally I may recall the double images present on vision with two eyes. Whenever we direct both eyes to a single point all objects which are much nearer or much further off than that point appear double. A little attentive observation shews us this immediately. Hence we can conclude that during the whole of our life we have been constantly seeing by far the majority of external objects doubled, and yet there are many persons who are not aware of this circumstance, and are immensely astonished when their attention is for the first time directed to it. But at the same time we have also constantly seen those particular objects single, to which our attention was directed at the moment; for to this we had always simultaneously directed the axes of both eyes.[1]

[1] [When objects pass rapidly before the eye, as when we look at near buildings from the window of a railway carriage in rapid motion, or look from the platform at a passing train, we become conscious of this simultaneous direction of the eyes by the pain caused by the rapid changes necessary to see these passing objects. If we cover one eye with the hand all pain disappears, owing to the absence of any necessity for converging the axes of the eyes, and we thus become aware of the usual habit of vision. The relief which is thus afforded, and which I have frequently experienced, is often extremely useful, as it enables us, when necessary, to see without inconvenience even the lettering upon a near train in rather rapid motion, or upon a station which an express train shoots past.—*Translator.*]

Hence our attention in the ordinary use of our eyes was constantly directed away from all those objects which appeared double, and of course we knew nothing about them. Before we could discover the phenomenon, we had first to direct our attention to a new and unusual object; we had to commence examining the outskirts of our field of vision, not for the purpose of becoming acquainted with the objects within it, but to analyse our sensations.

The same difficulty which besets observations on such subjective sensations as do not correspond to any external object, besets also the analysis of compound sensations which correspond to a single and not to a compound object. Of this kind are the sensations of musical tones. After the sound of a violin has, every time we have heard it, constantly reproduced the same sum of partial tones in our sensorium, this sum of partial tones comes to be regarded as the compound sign for the musical tone of a violin; and similarly another combination of partial tones as the sensible symbol for the musical tone of a clarinet, and so on. The oftener such a combination has been heard, the more accustomed are we to apprehend it as a connected whole, and the more difficult it is to analyse it by direct observation. I believe this to be one of the principal causes of the comparative difficulty felt in analysing the singing tones of the human voice. Similar fusions of several sensations into a single whole of conscious perception are of frequent occurrence in the action of our organs of sense.

We find again interesting examples of this in physiological optics. The perception of solidity in any near object is always produced by the union of two different images formed by the object in the two eyes, their difference depending on the slightly different points from which the two eyes view it, and the consequent slightly different perspective images thrown on the retinas. That this is really the case could only be conjectured before the invention of the stereoscope, but is now easy to prove at any moment by the use of that instrument. In the stereoscope are combined two flat drawings, representing the perspective views as formed in the two eyes, in such a manner that each eye views the image belonging to it from a proper point. The result is the perception of a solid body as complete and as natural as if the real body were before us. By first shutting one eye and then the other we can direct our attention to the difference between the images, provided it is not very slight; but for the stereoscopic

perception of depth, images suffice which are so slightly different, that it is difficult to discover their differences even on close examination. And certainly in our usual unconstrained observation of solid objects, we have no notion that our perception arises from the combination of two perspective images blended together, because this perception of solidity is of a totally different nature from that of either flat perspective image separately viewed. In this case then two different sensations, one in each eye, fuse into a third mental image which is totally different from either, just as the partial tones fuse into the mental image of the musical tone of some determinate instrument. And just as we learn to separate the partial tones of a string, by first making them sound separately on touching a node, so we learn to separate the images of our two eyes, by shutting them separately and alternately.

There are several much more complicated cases in which many sensations must concur to furnish the foundation for a very simple perception. For example, to perceive that an object seen lies in a certain direction, we have first to distinguish that a given part of our optic nerves has been excited by its light, and that other parts have not; this determines the position of the object with respect to the eye. Then we have to form a correct judgment of the position of the axis of the eyes in respect to the head, by means of the muscular sensation of the muscles directing our eyeballs, and finally we have to form a correct judgment of the position assumed by the head with respect to the body, by means of the muscular sensation of the muscles at the back of the neck. If one of these processes is faulty we obtain an incorrect mental image of the position of the object seen. By altering the refraction of the light by means of an interposed prism, and thus causing other parts of the nerve to be excited by the light of the object viewed, or by pressing the ball of the eye on one side, and thus impeding the free action of the muscles of the eye, and so on, we are able to shew that the sensations of all these different organs must concur to obtain the simple perception of the object's position; but it would certainly be quite impossible to see all this immediately in the sensible impression made on us by the object. And even when we have instituted such experiments and have convinced ourselves in every possible way that this must be the case, it still remains concealed for our own immediate observation of ourselves.

These examples must suffice to shew the important part which directed attention and training play in the perceptions of our senses. Now let us apply this conclusion to observations made by the ear. The common problem which our ear has to solve on the concurrence of several musical sounds, is to distinguish from each other the different musical tones belonging to single resonant bodies or musical instruments; it is only thus far that we have an objective interest in the analysis made by the ear. We are desirous of knowing, when several people are speaking at once, what each is saying, or when several instruments are playing or voices singing together, what melody is performed by each. On the other hand, the further analysis, by which these individual musical tones are reduced to their partial tones, although it can be executed by the same means and the same properties of the ear as the first, would teach us nothing new concerning the sources of the musical tones, but would rather tend to make us confused over their number. Hence we usually limit the direction of our attention to the analysis of a mass of musical tones into the tones of individual instruments, and as it were withdraw it from any further analysis of the compound into simple tones. We are then as much untrained in the latter, as we are trained in the former analysis.

Now there are many circumstances which assist us first in separating the musical tones arising from different sources, and secondly, in keeping together the partial tones of each separate source. Thus when one musical tone is heard for some time before being joined by the second, and then the second continues after the first has ceased, the separation in sound is facilitated by the succession of time. We have already heard the first musical tone by itself, and hence know immediately what we have to deduct from the compound effect for the effect of this first tone. Even when several parts proceed in the same rhythm in polyphonic music, the mode in which the tones of different instruments and voices commence, the nature of their increase in force, the certainty with which they are held, and the manner in which they die off, are generally slightly different for each. Thus the tones of a pianoforte commence suddenly with a blow, and are consequently strongest at the first moment, and then rapidly decrease in power; the tones of brass instruments, on the other hand, commence sluggishly, and require a small but sensible time to develope their full

strength; the tones of bowed instruments are distinguished by their extreme mobility, but when either the player or the instrument is not unusually perfect they are interrupted by little, very short, pauses, producing in the ear the sensation of scraping, as will be described more in detail when we come to analyse the musical tone of a violin. When then such instruments are sounded together there are generally points of time when one or the other is predominant, and it is consequently easily distinguished by the ear. But besides all this, in good part music, especial care is taken to facilitate the separation of the parts by the ear. In polyphonic music proper, where each part has its own distinct melody, a principal means of clearly separating the progression of each part has always consisted in making them proceed in different rhythms and on different divisions of the bars; or where this could not be done, or was at any rate only partly possible, as in four-part chorales, it is an old rule, contrived for this purpose, to let three parts, if possible, move by single degrees of the scale, and let the fourth leap over several. The small amount of alteration in the pitch makes it easier for the listener to keep the identity of the several voices distinctly in mind.

All these helps fail in the resolution of musical tones into their constituent partials. When a compound tone commences to sound, all its partial tones commence with the same comparative strength; when it swells, all of them generally swell uniformly; when it ceases, all cease simultaneously. Hence no opportunity is generally given for hearing them separately and independently. In precisely the same manner as the naturally connected partial tones form a single source of sound, the partial tones in a compound stop on the organ fuse into one, being all struck with the same finger key, and all moving in the same melodic progression as their prime tone.

Moreover, the tones of most instruments are usually accompanied by characteristic irregular noises, as the scratching and rubbing of the violin bow, the rush of wind in flutes and organ pipes, the grating of reeds, &c. These noises, with which we are already familiar as characterising the instruments, materially facilitate our power of distinguishing them in a composite mass of sounds. The partial tones in a compound have, of course, no such characteristic marks.

Hence we have no reason to be surprised that the resolution of

a compound tone into its partials is not quite so easy for the ear to accomplish, as the resolution of composite masses of the musical sounds of many instruments into their proximate constituents, and that even a trained musical ear requires the application of a considerable amount of attention when it undertakes the former problem.

It is easy to see that the auxiliary circumstances already named do not always suffice for a correct separation of musical tones. In uniformly-sustained musical tones, where one might be considered as an upper partial of another, our judgment might readily make default. This is really the case. G. S. Ohm proposed a very instructive experiment to shew this, using the tones of a violin. But it is more suitable for such an experiment to use simple tones, as those of a stopped organ pipe. The best instrument, however, is a glass bottle of the form shewn in fig. 20, which is easily procured and prepared for the experiment. A little rod c supports a gutta-percha tube a in a proper position. The end of the tube, which is directed towards the bottle, is softened in warm water and pressed flat, forming a narrow chink, through which air can be made to rush over the mouth of the bottle. When the tube is fastened by an india-rubber pipe to the nozzle of a bellows, and wind is driven over the bottle, it produces a hollow obscure sound, like the vowel *oo* in *too*, which is freer from upper partial tones than even the tone of a stopped pipe, and is only accompanied by a slight noise of wind. I find that it is easier to keep the pitch unaltered in this instrument while the pressure of the wind is slightly changed, than in stopped pipes. We deepen the tone by partially shading the orifice of the bottle with a little wooden plate; and we sharpen it by pouring in oil or melted wax. We are thus able to make any required little alterations in pitch. I tuned a large bottle to *b*♭ and a smaller one to *b′*♭, and united them with the

FIG. 20.

same bellows, so that when used both began to speak at the same instant. When thus united they gave a musical tone of the pitch of the deeper *b♭*, but having the quality of tone of the vowel *oa* in *toad*, instead of *oo* in *too*. When, then, I compressed first one of the india-rubber tubes and then the other, so as to produce the tones alternately, separately, and in connection, I was at last able to hear them separately when sounded together, but I could not continue to hear them separately for long, for the upper tone gradually fused with the lower. This fusion takes place even when the upper tone is somewhat stronger than the lower. The alteration in the quality of tone which takes place during this fusion is characteristic. On producing the upper tone first and then letting the lower sound with it, I found that I at first continued to hear the upper tone with its full force, and the under tone sounding below it in its natural quality of *oo* in *too*. But by degrees, as my recollection of the sound of the isolated upper tone died away, it seemed to become more and more indistinct and weak, while the lower tone appeared to become stronger, and sounded like *oa* in *toad*. This weakening of the upper and strengthening of the lower tone was also observed by Ohm on the violin. As Seebeck remarks, it certainly does not always occur, and probably depends on the liveliness of our recollections of the tones as heard separately, and the greater or less uniformity in the simultaneous production of the tones. But where the experiment succeeds, it gives the best proof of the essential dependence of the result on varying activity of attention. With the tones produced by bottles, in addition to the reinforcement of the lower tone, the alteration in its quality is very evident and is characteristic of the nature of the process. This alteration is less striking for the penetrating tones of the violin.

This experiment has been appealed to both by Ohm and by Seebeck as a corroboration of their different opinions. When Ohm stated that it was an "illusion of the ear" to apprehend the upper partial tones wholly or partly as a reinforcement of the prime tone (or rather of the compound tone whose pitch is determined by that of its prime), he certainly used a somewhat incorrect expression, although he meant what was correct, and Seebeck was justified in replying that the ear was the sole judge of auditory sensations, and that the mode in which it apprehended tones ought not to be called an "illusion." However, our experiments just described

shew that the judgment of the ear differs according to the liveliness of its recollection of the separate auditory impressions here fused into one whole, and according to the intensity of its attention. Hence we can certainly appeal from the sensations of an ear directed without assistance to external objects, whose interests Seebeck represents, to the ear which is attentively observing itself and is suitably assisted in its observation. Such an ear really proceeds according to the law laid down by Ohm.

Another experiment should be adduced. Raise the dampers of a pianoforte so that all the strings can vibrate freely, then sing the vowel *a* in *father*, *art*, loudly to any note of the piano, directing the voice to the sounding-board of the piano; the sympathetic resonance of the strings distinctly re-echoes the same *a*. On singing *oe* in *toe*, the same *oe* is re-echoed. On singing *a* in *fare*, this *a* is re-echoed. For *ee* in *see* the echo is not quite so good. The experiment does not succeed so well if the damper is removed only from the note on which the vowels are sung. The vowel character of the echo arises from the re-echoing of those upper partial tones which characterise the vowels. These, however, will echo better and more clearly when their corresponding higher strings are free and can vibrate sympathetically. In this case, then, in the last resort, the musical effect of the resonance is compounded of the tones of several strings, and several separate partial tones combine to produce a musical tone of a peculiar quality. In addition to the vowels of the human voice, the piano will also quite distinctly imitate the quality of tone produced by a clarinet, when strongly blown on to the sounding board.

Finally, we must remark, that although the pitch of a compound tone is, for musical purposes, determined by that of its prime, the influence of the upper partial tones is by no means unfelt. They give the compound tone a brighter and higher effect. Simple tones are dull. When they are compared with compound tones of the same pitch, we are inclined to estimate the compound as belonging to a higher Octave than the simple tones. The difference is of the same kind as that heard when first the vowel *oo* in *too* and then *a* in *tar* are sung to the same note. It is often extremely difficult to compare the pitches of compound tones of different qualities. It is very easy to make a mistake of an Octave. This has happened to the most celebrated musicians and

CHAPTER V.

ON THE DIFFERENCES IN THE QUALITY OF MUSICAL TONES.

TOWARDS the close of Chapter I. (p. 32), we found that differences in the quality of musical tones must depend on the form of the vibration of the air. The reasons for this assertion were only negative. We had seen that loudness depended on amplitude, and pitch on rapidity of vibration: nothing else was left to distinguish quality but vibrational form. We then proceeded to shew that the existence and loudness of the upper partial tones which accompanied the prime depend also on the vibrational form, and hence we could not but conclude that musical tones of the same quality would always exhibit the same combination of partials, seeing that the peculiar vibrational form which excites in the ear the sensation of a certain quality of tone, must always evoke the sensation of its corresponding upper partials. The question then arises, can, and if so, to what extent can the differences of musical quality be reduced to the combination of different partial tones with different intensities in different musical tones? At the conclusion of last chapter (p. 103), we saw that even artificially combined simple tones were capable of fusing into a musical tone of a quality distinctly different from that of either of its constituents, and that consequently the existence of a new upper partial really altered the quality of a tone. By this means we gained a clue to the hitherto enigmatical nature of quality of tone, and to the cause of its varieties.

There has been a general inclination to credit quality with all possible peculiarities of musical tones that were not evidently due to loudness and pitch. This was correct to the extent that quality of tone was merely a negative conception. But very slight consideration will suffice to shew that many of these peculiarities of musical tones depend upon the way in which they begin and end. The methods of attacking and releasing tones are sometimes so

characteristic that for the human voice they have been noted by a series of different letters. To these belong the explosive consonants B, D, G, and P, T, K. The effects of these letters are produced by opening the closed, or closing the open passage through the mouth. For B and P the closure is made by the lips, for D and T by the tongue and upper teeth,[1] for G and K by the back of the tongue and soft palate. The series of the *mediae* B, D, G is distinguished from that of the *tenues* P, T, K, by the glottis being sufficiently narrowed when the closure of the former is released to produce voice, or at least the rustle of whisper, whereas for the latter or *tenues* the glottis is wide open,[2] and cannot sound. The *mediae* are therefore accompanied by voice, which is capable of commencing at the beginning of a syllable an instant before the opening of the mouth, and of lasting at the end of a syllable a moment after the closure of the mouth, because some air can be still driven into the closed cavity of the mouth and the vibration of the vocal chords in the larynx can be still maintained. On account of the narrowing of the glottis the influx of air is more moderate, and the noise of the wind less sharp for the *mediae* than the *tenues*, which, being spoken with open glottis, allow of a great deal of wind being forced at once from the chest.[3] But although

[1] [This is true for German, and most Continental languages, and for some dialectal English, especially in Cumberland, Westmoreland, Yorkshire, Lancashire, the Peak of Derbyshire and Ireland, but even then only in connection with the trilled R. Throughout England generally, the tip of the tongue is quite free from the teeth, except for TH in *thin* and *then*, and for T and D it only touches the hard palate, seldom advancing so far as the root of the gums. In the Peak of Derbyshire two dialects are distinguished by one possessing the *dental* and the other this latter *coronal*, T D. Some native Indians recognise the English T, D as identical with their own *cerebrals*, and the French T, D as identical with their own *dentals*.—*Translator*.]

[2] [This again is true for German, but not for English, French, or Italian, and not even for the adjacent Slavonic languages. In these the glottis is quite closed for both the mediae and the tenues in ordinary speech, but the voice begins for the mediae *before* releasing the closure of the lips or tongue and palate, and for the tenues *at the moment of release*. Although in giving vowel sounds, &c. I have generally contented myself with translating the same into English symbols and examples, it seemed better in the present case, where the author was speaking especially of the phenomena of speech to which he was personally accustomed, to leave the text unaltered and draw attention to English peculiarities in foot-notes.—*Translator*.]

[3] [Observe again that this description of the rush of wind accompanying P, T, K, although true for German habits of speech, is not true for the usual English habits, which require the windrush between the opening of the mouth and sounding of the vowel to be entirely suppressed. The English result is a gliding vowel sound preceding the true vowel on commencing, and following the vowel on ending a syllable.

we are able to shew how these letters are produced, and to hear differences in the commencements of the tone of the voice, we are not yet in a position to determine with exactness what differences are actually caused in the motion of the air.

As with consonants, the differences in the quality of tone of struck strings, also partly depends on the rapidity with which the tone dies away. When the strings have little mass (strings of gut), and are fastened to a very mobile sounding board (as for a violin, guitar, or *zither*), or when the parts on which they rest or which they touch are but slightly elastic (as when the violin strings, for example, are pressed by the soft point of the finger on the finger-board), their vibrations rapidly disappear after striking, and the tone is dry, short, and without ring, as in the *pizzicato* of a violin. But if the strings are of metal wire, and hence of greater weight and tension, and if they are attached to strong heavy bridges which cannot be much shaken, they give out their vibrations slowly to the air and the sounding board; their vibrations continue longer, their tone is more durable and fuller, as in the pianoforte, but is comparatively less powerful and penetrating than that of strings which give up their tone more readily and are struck with the same force. Hence the *pizzicato* of bowed instruments when well executed is much more piercing than the tone of a pianoforte. Pianofortes with strong and heavy understays for the strings have, for the same strength of string, a less penetrating but a much more lasting tone than those with lighter understays.

It is very characteristic of brass instruments, as trumpets and trombones, that their tones commence abruptly and heavily. The various tones in these instruments are produced by exciting different upper partials through different styles of blowing, which serve to throw the column of air into vibrating portions of different numbers and lengths similar to those on a string. It always

The difference between English P and German P is precisely the same (as I have verified by actual observation) as that between the simple Sanscrit tenuis P, and the postaspirated Sanscrit Ph, as now actually pronounced by cultivated Bengalese. For Indian T, Th there is the dental difference only, and for K, Kh there are other peculiarities which cannot be here mentioned. For the aspirated B, D, G again the case is entirely different. These sounds are exceedingly common in the Irish pronunciation of English, and Indians repudiate altogether their ordinary pronunciation by German professors of Sanscrit. See my 'Early English Pronunciation,' p. 1136, col. 1.—*Translator.*]

requires a certain amount of effort to excite the new condition of vibration in place of the old, but when once established it is maintained with less exertion. On the other hand, the transition from one tone to another is easy for wooden wind instruments, as the flute, oboe, and clarinet, where the length of the column of air is readily changed by application of the fingers to the side holes and keys, and where the style of blowing has not to be materially altered.

These examples will suffice to shew how certain characteristic peculiarities in the tones of several instruments depend on the mode in which they begin and end. When we speak in what follows of *musical quality of tone*, we shall disregard these peculiarities of beginning and ending, and confine our attention to the peculiarities of the musical tone which continues uniformly.

But even when a musical tone continues with uniform or variable intensity, it is mixed up in the general methods of excitement with certain noises, which express greater or less irregularities in the motion of the air. In wind instruments where the tones are maintained by a stream of air, we generally hear more or less whizzing and hissing of the air which breaks against the sharp edges of the mouthpiece. In strings, rods, or plates excited by a violin bow, we usually hear a good deal of noise from the rubbing. The hairs of the bow are naturally full of many minute irregularities, the resinous coating is not spread over it with absolute evenness, and there are also little inequalities in the motion of the arm which holds the bow and in the amount of pressure, all of which influence the motion of the string, and make the tone of a bad instrument or an unskilful performer rough, scraping, and variable. We shall not be able to explain the nature of the motions of the air and sensations of the ear which correspond to these noises till we have investigated the conception of *beats*. Those who listen to music make themselves deaf to these noises by purposely withdrawing attention from them, but a slight amount of attention generally makes them very evident for all tones produced by blowing or rubbing. It is well known that most consonants in human speech are characterised by the maintenance of similar noises, as F, V; S, Z; TH in *thin* and in *then*; the Scotch and German guttural CH, and Dutch G. For some the tone is made still more irregular by trilling parts of the mouth, as for R and L. In the case of R the stream of air is

periodically entirely interrupted by trilling the uvula[1] or the tip of the tongue; and we thus obtain an intermitting sound to which these interruptions give a peculiar jarring character. In the case of L the soft side edges of the tongue are moved by the stream of air, and, without completely interrupting the tone, produce inequalities in its strength.

Even the vowels themselves are not free from such noises, although they are kept more in the background by the musical character of the tones of the voice. Donders first drew attention to these noises, which are partly identical with those which are produced when the corresponding vowels are indicated in low voiceless speech. They are strongest for *ee* in *see*, the French *u* in *vu* (which is nearly the same as the Norfolk and Devon *oo* in *too*), and for *oo* in *too*. For these vowels they can be made audible even when speaking aloud.[2] By simply increasing their force the vowel *ee* in *see* becomes the consonant *y* in *yon*, and the vowel *oo* in *too* the consonant *w* in *wan*.[3] For *a* in *art*, *a* in *at*, *e* in *met*, *there*, and *o* in *more*, the noises appear to me to be produced in the glottis alone when speaking gently, and to be absorbed into the voice when speaking aloud.[4] It is remarkable that in speaking, the vowels *a* in *art*, *a* in *at*, and *e* in *met*, *there*, are produced with less musical tone than in singing. It seems as if a feeling of greater compression in the larynx caused the tuneful tone of the voice to give way to one of a more jarring character which admits of more evident articulation. The greater intensity thus given to the noises appears in this case to facilitate the characterisation of the peculiar vowel quality. In singing, on the contrary, we try to favour the musical part of its quality and hence often render the articulation somewhat obscure.[5]

[1] [In many parts of Germany and France, never in England or Italy, except as an organic defect. There are also many other trills, into which, as into other phonetic details, it is not necessary to enter.—*Translator.*]

[2] [I have heard M. Got, of the Comédie Française, pronounce the word *oui* entirely without voice tones, and yet make it audible throughout the theatre.—*Translator.*]

[3] [That this is not the whole of the phenomenon is shewn by the words *ye*, *woo*. The whole subject is discussed at length in my 'Early English Pronunciation,' pp. 1092–1094, and 1149–1151.—*Translator.*]

[4] [By 'speaking gently' (*leise*) seems to be meant either speaking absolutely without voice, that is with an open glottis, or in a whisper, with the glottis nearly closed. For voice the glottis is quite closed, and this is indicated by speaking aloud (*beim lauten Sprechen*). It would lead too far to discuss the important phonetic observations in the text.—*Translator.*]

[5] [These observations must not be considered as exhausting the subject of the

Such accompanying noises and little inequalities in the motion of the air, furnish much that is characteristic in the tones of musical instruments, and in the vocal tones of speech which correspond to the different positions of the mouth; but besides these there are numerous peculiarities of quality belonging to the musical tone proper, that is, to the perfectly regular portion of the motion of the air. The importance of these can be better appreciated by listening to musical instruments or human voices, from such a distance that the comparatively weaker noises are no longer audible. Notwithstanding the absence of these noises, it is generally possible to discriminate the different musical instruments, although it must be acknowledged that under such circumstances the tone of a horn may be occasionally mistaken for that of the singing voice, or a violoncello may be confused with an harmonium. For the human voice, consonants first disappear at a distance, because they are characterised by noises, but M, N, and the vowels can be distinguished at a greater distance.[1] The formation of M and N in so far resembles that of vowels, that no noise of wind is generated in any part of the cavity of the mouth, which is perfectly closed, and the sound of the voice escapes through the nose. The nose merely forms a resonance chamber which alters the quality of tone. It is interesting in calm weather to listen to the voices of men who are descending from high hills to the plain. Words can no longer be recognised, or at most only such as are composed of M, N, and vowels, as *Mamma*, *No*. But the vowels contained in the spoken words are easily distinguished. They form a strange series of alternations of quality and singular inflections of tone, wanting the thread which connects them into words and sentences.[2]

difference between the singing and the speaking voice, which requires a peculiar study here merely indicated. The difference between English and German habits of speaking and singing must also be borne in mind, and allowed for by the reader. The English vowels given in the text are not the perfect equivalents of Prof. Helmholtz's German sounds. The noises which accompany the vowels are not nearly so marked in English as in German, but they differ very much locally, even in England.—*Translator.*]

[1] [The nasals M and N are distinguished from B and D, for example, as being nasals, but they are readily confused with each other even at small distances. The consonants, especially P, T, K in English, are also distinguished by peculiar gliding vowel sounds, which, however, are too delicate to be heard at great distances, and are easily overwhelmed by shouting on the continuous vowel sounds.—*Translator.*]

[2] [The three vowels *ee* in *see*, *a* in *art*, and *oo* in *too* are most easily distinguished at

In the present chapter we shall at first disregard all irregular portions of the motion of the air, and the mode in which sounds commence or terminate, directing our attention solely to the musical part of the tone, properly so called, which corresponds to a uniformly sustained and regularly periodic motion of the air, and we shall endeavour to discover the relations between the quality of the sound and its composition out of individual simple tones. The peculiarities of quality of sound belonging to this division, we shall briefly call its *musical quality*.

The object of the present chapter is, therefore, to describe the different composition of musical tones as produced by different instruments, for the purpose of shewing how different modes of combining the upper partial tones correspond to characteristic varieties of musical quality. Certain general rules will result for the arrangement of the upper partial tones which answer to such species of musical quality as are called, *soft*, *piercing*, *braying*, *hollow* or *poor*, *full* or *rich*, *dull*, *bright*, *crisp*, *pungent*, and so on. Independently of our immediate object (the determination of the physiological action of the ear in the discrimination of musical quality, which is reserved for the following chapter), the results of this investigation are important for the resolution of purely musical questions in later chapters, because they shew us how rich in upper partial tones, good musical qualities of tone are found to be, and also point out the peculiarities of musical quality favoured on those musical instruments, for which the quality of tone has been to some extent abandoned to the caprice of the maker.

Since physicists have worked comparatively little at this subject, I shall be forced to enter somewhat more minutely into the mechanism by which the tones of several instruments are produced, than will be, perhaps, agreeable to many of my readers. For such the principal results collected at the end of this chapter will suffice. On the other hand, I must ask indulgence for leaving many large gaps in this almost unexplored region, and for confining myself principally to instruments sufficiently well known for us to obtain a tolerably satisfactory view of the source of their tones. In this

a distance. This suggests the construction of a voice telegraph for night purposes, where the three signals might answer in the above order to the *point*, *dash*, and *blank* of the ordinary telegraph alphabet, and the necessarily inaudible and consequently useless consonants might be introduced at pleasure to facilitate articulation.—*Translator.*]

inquiry lie rich materials for interesting acoustical work. But I have felt bound to confine myself to what was necessary for the continuation of the present investigation.

1. *Musical Tones without Upper Partials.*

We begin with such musical tones as are not decomposable, but consist of a single simple tone. These are most readily and purely produced by holding a struck tuning-fork over the mouth of a resonance tube, as has been described in the last chapter (p. 87). These tones are uncommonly soft and free from all shrillness and roughness. As already remarked, they appear to lie comparatively deep, so that such as correspond to the deep tones of a bass voice produce the impression of a most remarkable and unusual depth. The musical quality of such deep simple tones is also rather dull. The simple tones of the soprano pitch sound bright, but even those corresponding to the highest tones of a soprano voice are very soft, without a trace of that cutting, rasping shrillness which is displayed by most intruments at such pitches, with the exception, perhaps, of the flute, for which the tones are very nearly simple, being accompanied with very few and faint upper partials. Among vowels the *oo* in *too* comes nearest to a simple tone, but even this vowel is not entirely free from upper partials. On comparing the musical quality of a simple tone thus produced with that of a compound tone in which the first harmonic upper partial tones are developed, the latter will be found to be more tuneful, metallic, and brilliant. Even the vowel *oo* in *too*, although the dullest and least tuneful of all vowels, is sensibly more brilliant and less dull than a simple tone of the same pitch. The series of the first six partial tones of a compound tone may be regarded musically as a major chord with a very predominant fundamental tone, and in fact the musical quality of a compound tone possessing these partials, as, for example, a fine singing voice, when heard beside a simple tone, very distinctly produces the agreeable effect of a consonant chord.

Since the form of simple waves of known periodic time is completely given when their amplitude is given, simple tones of the same pitch can only differ in loudness and not in musical quality. In fact, the difference of quality remains perfectly indistinguishable, whether the simple tone is conducted to the external air in

the preceding methods by a tuning fork and a resonance tube of any given material, glass, metal, or pasteboard, or by a string, provided only that we guard against any chattering in the apparatus.

Simple tones accompanied only by the noise of rushing wind can also be produced, as already mentioned, by blowing over the mouth of bottles with necks (p. 102). If we disregard the friction of the air, the proper musical quality of such tones is really the same as that produced by tuning forks.

2. *Musical Tones with Inharmonic Upper Partials.*

Nearest to musical tones without any upper partials are those with secondary tones which are inharmonic to the prime, and such tones, therefore, in strictness, should not be reckoned as musical tones at all. They are exceptionally used in artistic music, but only when it is contrived that the prime time should be so much more powerful than the secondary tones, that the existence of the latter may be ignored. Hence they are placed here next to the simple tones, because musically they are available only for the more or less good simple tones which they represent. The first of these are *tuning forks* themselves, when they are struck and applied to a sounding board, or brought very near the ear. The upper partial tones of tuning forks lie very high. In those which I have examined, the first made from 5·8 to 6·6 as many vibrations in the same time as the prime tone, and hence lay between its third diminished Fifth and major Sixth. The vibrational numbers of these high upper partial tones were to one another as the squares of the odd numbers. In the time that the first upper partial would execute $3.3 = 9$ vibrations, the next would execute $5.5 = 25$, and the next $7.7 = 49$, and so on. Their pitch, therefore, increases with extraordinary rapidity, and they are usually all inharmonic with the prime, though some of them may exceptionally become harmonic. If we call the prime tone of the fork *c*, the next succeeding tones are nearly $a''\flat$, $d'^{\vee}$, $c^{\vee}\sharp$. These high secondary tones produce a bright inharmonic clink, which is easily heard at a considerable distance when the fork is first struck, whereas when it is brought close to the ear, the prime tone alone is heard. The ear readily separates the prime from the upper tones and has no inclination to fuse them. The high simple tones usually die off rapidly, while the prime tone remains audible for a long time. It

should be remarked, however, that the mutual relations of the proper tones of tuning forks differ somewhat according to the form of the fork, and hence the above indications must be looked upon as merely approximate. In theoretical determinations of the upper partial tones, each prong of the fork may be regarded as a rod fixed at one end.

The same relations hold for straight elastic rods, which also, when struck, give rather high inharmonic upper partial tones. When such a rod is firmly supported at the two nodal lines of its prime tone, the continuance of that tone is favoured in preference to the other higher tones, and hence the latter disturb the effect very slightly, more especially as they rapidly die away after the rod has been struck. Such rods, however, are not suitable for real artistic music, although they have lately been introduced for military and dance music on account of their penetrating qualities of tone. Glass rods or plates, and wooden rods, were formerly used in this way for the *glass harmonica* and *wood harmonica*. The rods were inserted between two pairs of intertwisted strings, which grasped them at their two nodal lines. The wooden rods in the German *straw-fiddle* were simply laid on straw cylinders. They were struck with hammers of wood or cork.

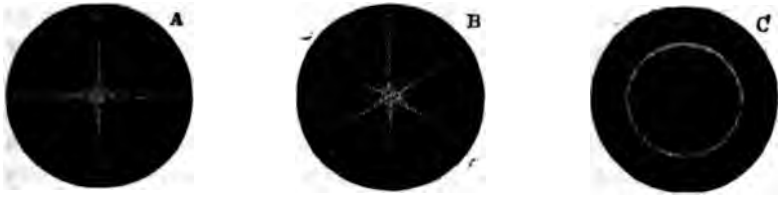
The only effect of the material of the rods on the quality of tone in these cases, consists in the greater or less length of time that it allows the proper tones at different pitches to continue. These secondary tones, including the higher ones, usually continue to sound longest in elastic metal of fine uniform consistency, because its greater mass gives it a greater tendency to continue in any state of motion which it has once assumed, and among metals the most perfect elasticity is found in steel, and the better alloys of copper and zinc, or copper and tin. In slightly alloyed precious metals, their greater specific gravity lengthens the duration of the tone, notwithstanding their inferior elasticity. Superior elasticity appears to favour the continuance of the higher proper tones, because imperfect elasticity and friction generally seem to damp rapid more quickly than slow vibrations. Hence I think that I may describe the general characteristic of what is usually called a *metallic* quality of tone, as the comparatively continuous and uniform maintenance of higher upper partial tones. The quality of tone for glass is similar;

but as it breaks when violently agitated, the tone is always weak and soft, and it is also comparatively high, and dies rapidly away, on account of the smaller mass of the vibrating body. In wood the mass is small, the internal structure comparatively rough, being full of countless interstices, and the elasticity also comparatively imperfect, so that the proper tones, especially the higher ones, rapidly die away. And for this reason the *straw-fiddle* or wood-harmonica is perhaps more satisfactory to a musical ear, than harmonicas formed of steel or glass rods or plates, with their piercing inharmonic upper partial tones,—at least so far as simple tones are suitable for music at all, of which I shall have to speak later on.

For all of these instruments which have to be struck, the hammers are made of wood or cork, and covered with leather. This renders the highest upper partials much weaker than if only hard metal hammers were employed. Greater hardness of the striking mass produces greater discontinuities in the original motion of the plate. The influence exerted by the manner of striking will be considered more in detail, in reference to strings, where it is also of much importance.

According to Chladni's discoveries, *elastic plates*, cut in circular, oval, square, oblong, triangular, or hexagonal forms, will sound in a great number of different vibrational forms, usually producing simple tones which are mutually inharmonic. Fig. 21

FIG. 21.



gives the more simple vibrational forms of a circular plate. Much more complicated forms occur when several circles or additional diameters appear as nodal lines, or where both circles and diameters occur. Supposing the vibrational form A to give the tone *c*, the others give the following proper tones:—

Number of Nodal Circles	Number of Diameters					
	0	1	2	3	4	5
0 1 2	$g\sharp$ $g''\sharp+$	$b'\flat$	c g''	d'	c''	g''—$g''\sharp$

This shews that many proper tones of nearly the same pitch are produced by a plate of this kind. When a plate is struck, those proper tones which have no node at the point struck, will all sound together. To obtain a particular determinate tone it is of advantage to support the plate in points which lie in the nodal lines of that tone; because those proper tones which have no node in those points will then die off more rapidly. For example, if a circular plate is supported at 3 points in the nodal circle of fig. 21, C (p. 116), and is struck exactly in its middle, the simple tone called $g\sharp$ in the table, which belongs to that form, will be heard, and all those other proper tones which have diameters as some of their nodal lines will be very weak, for example $c, d', c'', g'', b'\flat$ in the table. In the same way the tone $g''\sharp$ with two nodal circles, dies off immediately, because the points of support fall on one of its ventral segments, and the first proper tone which can sound loudly at the same time is that corresponding to three nodal circles, one of its nodal lines being near to that of No. 2. But this is 3 Octaves and more than a whole Tone higher than the proper tone of No. 2, and on account of this great interval does not disturb the latter. Hence a disc thus struck gives a tolerably good musical tone, whereas plates in general produce sounds composed of many inharmonic proper tones of nearly the same pitch, giving an empty tin-kettle sort of quality, which cannot be used in music. But even when the disc is properly supported the tone dies away rapidly, at least in the case of glass plates, because contact at many points, even when nodal, sensibly impedes the freedom of vibration.

The sound of *bells* is also accompanied by inharmonic secondary tones, which, however, do not lie so close to one another as those of flat plates. The vibrations which usually arise have 4, 6, 8, 10, &c. nodal lines extending from the vertex of the bell to its margin, at equal intervals from each other. The corre-

sponding proper tones for glass bells which have approximatively the same thickness throughout, are nearly as the squares of the numbers 2, 3, 4, 5, so that if we call the lowest tone *c*, we have for the

Number of nodal lines . . .	4	6	8	10	12
Tones	*c*	*d'*	*c''*	*g''*♯ –	*d'''* –

The tones, however, vary with the greater or less thickness of the wall of the bell towards the margin, and it appears to be an essential point in the art of casting bells, to make the deeper proper tones mutually harmonic by giving the bell a certain empirical form. According to the observations of the organist Gleitz,[1] the bell cast for the cathedral at Erfurt in 1477 has the following proper tones: *E*, *e*, *g*♯, *b*, *e'*, *g'*♯, *b'*, *c'*♯. The bell of St. Paul's, London, gives *a* and *c'*♯. Hemony of Zütphen, a master in the seventeenth century, required a good bell to have three Octaves, two Fifths, one major and one minor Third. The deepest tone is not the strongest. The body of the bell when struck gives a deeper tone than the 'sound bow,' but the latter gives the loudest tone. Probably other vibrational forms of bells are also possible in which nodal circles are formed parallel to the margin. But these seem to be produced with difficulty and have not yet been examined.

If a bell is not perfectly symmetrical in respect to its axis, if, for example, the wall is a little thicker at one point of its circumference than at another, it will give, on being struck, two different tones of very nearly the same pitch, which will 'beat' together. Four points on the margin will be found, separated from each other by quarter-circles, in which only one of these tones can be heard without accompanying beats, and four others, half-way between the pairs of the others, where the second tone only sounds. If the bell is struck elsewhere both tones are heard, producing beats, and such beats may be perceived in most bells as their tone dies gradually away.

Stretched *membranes* have also inharmonic proper tones of

[1] 'Historical Notes on the Great Bell and the other Bells in Erfurt Cathedral' (*Geschiehtliches über die grosse Glocke und die übrigen Glocken des Domes zu Erfurt*). Erfurt, 1867. – See also Schafhäutl in the 'Kunst- und Gewerbeblatt für das Königreich Bayern,' 1868, LIV. 325 to 350; 385 to 427.

nearly the same pitch. For a circular membrane, of which the deepest tone is *c*, these are, in order of pitch:

Number of Nodal Lines		Tone
Diameters	Circles	
0	0	*e*
1	0	*a*b
2	0	*c*♯ +0·1
0	1	*d* +0·2
1	1	*g* −0·2
0	2	*b*♭ +0·1

These tones rapidly die out. If the membranes are associated with an air chamber, as in the *kettledrum*, the relation of the proper tones may be altered, and the prime tone seems to gain in strength with respect to the others. No detailed investigations have yet been made on the secondary tones of the kettledrum. The drum is used in artistic music, but only to mark certain accents. It is tuned, indeed, not for the purpose of filling up chords, but only to prevent injury to the harmony.

The common character of the instruments hitherto described is that when struck they produce inharmonic upper partial tones. If these are of nearly the same pitch as the prime tone, their quality of sound is in the highest degree unmusical, bad, and tinkettly. If the secondary tones are of very different pitch from the prime, and weak in force, the quality of sound is more musical, as for example in tuning-forks, harmonicas of rods, and bells; and such tones are applicable for marches and other boisterous music, principally intended to mark time. But for really artistic music, such instruments as these have always been rejected, as they ought to be, for the inharmonic secondary tones, although they rapidly die out, always disturb the harmony most unpleasantly, renewed as they are at every fresh blow. A very striking example of this was furnished by a company of bell-ringers, said to be Scotch, that lately travelled about Germany, and performed all kinds of musical pieces, some of which had an artistic character. The accuracy and skill of the performance was undeniable, but the musical effect was detestable, on account of the heap of false secondary tones which accompanied the music, although care was taken to damp each bell as soon as the proper duration of its note had expired, by placing it on a table covered with cloth.

Resonant bodies with inharmonic partials, may be also set in action by violin bows, and then by properly damping them in a nodal line of the desired tone, the secondary tones which lie near it can be prevented from interfering. One simple tone then predominates distinctly, and it might consequently be used for musical purposes. But when the violin bow is applied to any bodies with inharmonic upper partial tones, as tuning-forks, plates, bells, we hear a strong scratching sound, which on investigation with resonators, is found to consist mainly of these same inharmonic secondary tones of such bodies, not sounding continuously but only in short irregular fits and starts. Intermittent tones, as I have already noted, produce the effect of grating or scratching. It is only when the body excited by the violin bow has harmonic upper partials, that it can perfectly accommodate itself to every impulse of the bow, and give a really musical quality of tone. The reason of this is that any required periodic motion such as the bow aims at producing, can be compounded of motions corresponding to harmonic upper partial tones, but not of other, inharmonic vibrations.

3. *Musical Tones of Strings.*

We now proceed to the analysis of musical tones proper, which are characterised by harmonic upper partials. These may be best classified according to their mode of excitement: 1. By striking. 2. By bowing. 3. By blowing against a sharp edge. 4. By blowing against elastic tongues or vibrators. The two first classes comprehend stringed instruments alone, as longitudinally vibrating rods, the only other instruments producing harmonic upper partial tones, are not used for musical purposes. The third class embraces flutes and the flute or flue stops of organs; the fourth all other wind instruments, including the human voice.

Strings excited by Striking.—Among musical instruments at present in use, this section embraces the *pianoforte*, *harp*, *guitar*, and *zither*; among physical, the *monochord*, arranged for an accurate examination of the laws controlling the vibrations of strings; the *pizzicato* of bowed instruments must also be placed in this category. We have already mentioned that the musical tones produced by strings which are struck or plucked, produce numerous upper partial tones. We have the advantage of possessing a complete theory for the motion of plucked strings, by

which the force of their upper partial tones may be determined. In the last chapter we compared some of the conclusions of this theory with the results of experiment, and found them agree. A similarly complete theory may be formed for the case of a string which has been struck in one of its points by a hard sharp edge. The problem is not so simple when soft elastic hammers are used, such as those of the pianoforte, but even in this case it is possible to assign a theory for the motion of the string which embraces at least the most essential features of the process, and indicates the force of the upper partial tones.[1]

The loudness or force of the upper partial tones in a struck string, depends in general on:—

1. The *nature of the stroke.*
2. The *place struck.*
3. The *density*, *rigidity*, and *elasticity* of the *string.*

First, as to the *nature of the stroke.* The string may be *plucked*, by drawing it on one side with the finger or a point (the plectrum, or the ring of the zither-player), and then letting it go. This is a usual mode of exciting a string in a great number of ancient and modern stringed instruments. Among the modern, I need only mention the *harp*, *guitar*, and *zither*. Or else the string may be struck with a hammer-shaped body, as in the pianoforte, and its ancient congeners, the *spinet*, &c. I have already remarked that the strength and number of the upper partial tones increases with the abruptness of the discontinuities in the motion excited. This fact determines the various modes of exciting a string. When a string is plucked, the finger, before quitting it, removes it from its position of rest throughout its whole length. A discontinuity in the string arises only by its forming a more or less acute angle at the place where it wraps itself about the finger or point. The angle is more acute for a sharp point than for the finger. Hence the sharp point produces a shriller tone with a greater number of high tinkling upper partials, than the finger. But in each case the intensity of the prime tone exceeds that of any upper partial. If the string is struck with a sharp-edged metallic hammer which rebounds instantly, only the one single point struck is set in motion directly. Immediately after the blow the remainder of the string is at rest. It does not move until a wave of deflection arises, and runs back-

[1] See Appendix, No. V.

wards and forwards over the string. This limitation of the original motion to a single point produces the most abrupt discontinuities, and a corresponding long series of upper partial tones, having intensities[1] in most cases equalling or even surpassing that of the prime. When the hammer is soft and elastic, the motion has time to spread before the hammer rebounds. When thus struck the point of the string in contact with such a hammer is not set in motion with a jerk, but increases gradually and continuously in velocity during the contact. The discontinuity of the motion is consequently much less, diminishing as the softness of the hammer increases, and the force of the higher upper partial tones is correspondingly decreased.

We can easily convince ourselves of the correctness of these statements by opening the lid of any pianoforte, and, keeping one of the digitals down with a weight, so as to free the string from the damper, plucking the string at pleasure with a finger or a point, and striking it with a metallic edge or the pianoforte hammer itself. The qualities of tone thus obtained will be entirely different. When the string is struck or plucked with hard metal, the tone is piercing and tinkling, and a little attention suffices to make us hear a multitude of very high partial tones. These disappear, and the tone of the string becomes duller, softer, and more harmonious, when we pluck the string with the soft finger or strike it with the proper soft hammer. We also readily recognise the different loudness of the prime tone. When we strike with metal, the prime tone is scarcely heard and the quality of tone is correspondingly *poor*. The peculiar quality of tone commonly termed *poverty*, as opposed to *richness*, arises from the upper partials being comparatively too strong for the prime tone. The prime tone is heard best when the string is plucked with a soft finger, which produces a rich and yet harmonious quality of tone. The prime tone is not so strong, at least in the middle and deeper Octaves of the instrument, when the strings are struck with the pianoforte hammer, as when they are plucked with the finger.

This is the reason why it has been found advantageous to cover pianoforte hammers with thick layers of felt, rendered elastic by much compression. The outer layers are the softest and most yielding, the lower are firmer. The surface of the hammer comes

[1] When *intensity* is here mentioned, it is always measured objectively, by the *vis viva*, or *mechanical equivalent of work* of the corresponding motion.

in contact with the string without any audible impact; the lower layers give the elasticity which throws the hammer back from the string. If you remove a pianoforte hammer and strike it strongly on a wooden table or against a wall, it rebounds from them like an india-rubber ball. The heavier the hammer and the thicker the layers of felt—as in the hammers for the lower Octaves—the longer must it be before it rebounds from the string. The hammers for the upper Octaves are lighter and have thinner layers of felt. The makers of these instruments have been clearly led by practice to discover certain relations of the elasticity of the hammer to the best tones of the string. The make of the hammer has an immense influence on the quality of tone. Theory shews that those upper partial tones are especially favoured whose periodic time is nearly equal to twice the time during which the hammer lies on the string, and that, on the other hand, those disappear whose periodic time is 6, 10, 14, &c. times as great.

My experiments with very good and new grand pianofortes made by Kaim and Günther, shew that in the middle and deeper Octaves, the first weak or vanishing partial is generally the seventh, but it is also sometimes the sixth or fifth. Differences often occur for closely adjacent notes. Hence it follows that the time of the hammer's contact is about half the periodic time of the second partial tone of the string. In the upper Octaves, on the other hand, this time of contact appears to approach half the periodic time of the prime, or even to exceed it. The intensity of the individual upper partials as calculated from these data will be given hereafter.

Secondly as to the *place struck*. In the last chapter, when verifying Ohm's law for the analysis of musical tones by the ear, we remarked that whether strings are plucked or struck, those upper partials disappear which have a node at the point excited. Conversely; those partials are comparatively strongest which have a maximum displacement at that point. Generally, when the same method of striking is successively applied to different points of a string, the individual upper partials increase or decrease with the intensity of motion, at the point of excitement, for the corresponding simple vibrations of the string. The composition of the musical tone of a string can be consequently greatly varied by merely changing the point of excitement.

Thus if a string be struck in its middle, the second partial tone

disappears, because it has a node at that point. But the third partial tone comes out forcibly, because as its nodes lie at $\frac{1}{3}$ and $\frac{2}{3}$ the length of the string from its extremities, the string is struck half-way between these two nodes. The fourth partial has its nodes at $\frac{1}{4}$, $\frac{3}{4}$ $(=\frac{1}{2})$, and $\frac{3}{4}$ the length of the string from its extremity. It is not heard, because the point of excitement corresponds to its second node. The sixth, eighth, and generally the even partials disappear in the same way, but the fifth, seventh, ninth, and the other odd partials are heard. By this disappearance of the even partial tones when a string is struck at its middle, its tone obtains a peculiar quality, essentially different from that usually heard from strings. It sounds somewhat hollow or nasal. The experiment is easily made on any piano when it is opened and the dampers are raised. The middle of the string is easily found by trying where the finger must be laid to bring out the first upper partial clearly and purely on striking the finger-key.

If the string is struck at $\frac{1}{3}$ its length, the third, sixth, ninth, &c. partials vanish. This also gives a certain amount of hollowness, but less that when the string is struck in its middle. When the point of excitement approaches the end of the string, the prominence of the higher upper partials is favoured at the expense of the prime and lower upper partial tones, and the sound of the string is empty and tinkling.

In pianofortes, the point struck is about $\frac{1}{7}$ to $\frac{1}{9}$ the length of the string from its extremity, for the middle part of the instrument. We must therefore assume that this place has been chosen because experience has shewn it to give the finest musical tone, which is most suitable for harmonies. The selection is not due to theory. It results from attempts to meet the requirements of artistically trained ears, and from the technical experience of two centuries. This gives particular interest to the investigation of the composition of musical tones for this point of excitement. An essential advantage in the choice of this position seems to be that the seventh and ninth partial tones disappear or at least become very weak. These are the first in the series of partial tones which do not belong to the major chord of the prime tone. Up to the sixth partial we have only Octaves, Fifths, and major Thirds of the prime tone; the seventh is nearly a minor Seventh, the ninth a major Second of the prime. Hence these will not fit into the major chord. Experiments on pianofortes shew that when the

string is struck by the hammer and touched at its nodes, it is easy to bring out the six first partial tones (at least on the strings of the middle and lower Octaves), but that it is either not possible to bring out the seventh, eighth, and ninth at all, or that we obtain at best very weak and imperfect results. The difficulty here is not occasioned by the incapacity of the string to form such short vibrating sections, for if instead of striking the digital we pluck the string nearer to its end, and damp the corresponding nodes, the seventh, eighth, ninth, nay even the tenth and eleventh partial may be clearly and brightly produced. It is only in the upper Octaves that the strings are too short and stiff to form the high upper partial tones. For these, several instrument-makers place the striking point nearer to the extremity, and thus obtain a brighter and more penetrating tone. The upper partials of these strings, which their stiffness renders it difficult to bring out, are thus favoured as against the prime tone. A similarly brighter tone, but at the same time a thinner and emptier one, can be obtained from the lower strings by placing a bridge nearer the striking point, so that the hammer falls at a point less than $\frac{1}{7}$ of the effective length of the string from its extremity.

While on the one hand the tone can be rendered more tinkling, shrill, and acute, by striking the string with hard bodies, on the other hand it can be rendered duller or more obtuse, that is, the prime tone may be made to outweigh the upper partials, by striking it with a soft and heavy hammer, as, for example, a little iron hammer covered with a thick sheet of india-rubber. The strings of the lower Octaves then produce a much fuller but duller tone. To compare the different qualities of tone thus produced by using hammers of different constructions, care must be taken always to strike the string at the same distance from the end as it is struck by the proper hammer of the instrument, as otherwise the results would be mixed up with the effects depending on altering the striking point. These circumstances are of course well known to the instrument-makers, because they have themselves selected heavier and softer hammers for the lower, and lighter and harder for the upper Octaves. But when we see that they have not given more than a certain weight to the hammers and have not increased it sufficiently to reduce the intensity of the upper partial tones still further, we feel convinced that a musically

trained ear prefers that an instrument to be used for rich combinations of harmony should possess a quality of tone which contains upper partials with a certain amount of strength. In this respect the composition of the tones of pianoforte strings is of great interest for the whole theory of music. In no other instrument is there so wide a field for alteration of quality of tone; in no other, then, was a musical ear so unfettered in the choice of a tone that would meet its wishes.

As I have already observed, the middle and lower Octaves of pianoforte strings generally allow the six first partial tones to be clearly produced by striking the digital, and the three first of them are strong, the fifth and sixth distinct, but much weaker. The seventh, eighth, and ninth are eliminated by the position of the striking point. Those higher than the ninth are always very weak. For closer comparison I subjoin a table in which the intensities of the partial tones of a string for different methods of striking have been theoretically calculated from the formulæ developed in the Appendix V. The effect of the stroke of a hammer depends on the time for which it touches the string. This time is given in the table in fractions of the periodic time of the prime tone. To this is added a calculation for strings plucked by the finger. The striking point is always assumed at $\frac{1}{7}$ of the length of the string from its extremity.

Theoretical Intensity of the Partial Tones of Strings.

Striking point at $\frac{1}{7}$ of the length						
Number of the Partial Tone	Excited by Plucking	Struck by a hammer which touches the string for $\frac{3}{7}$ of the periodic time of the prime tone c''	$\frac{3}{10}$ g'	$\frac{3}{14}$ $C_{,}-c'$	$\frac{3}{20}$	Struck by a perfect hard Hammer
1	100	100	100	100	100	100
2	81·2	99·7	189·4	249	285·7	324·7
3	56·1	8·9	107·9	242·9	357·0	504·9
4	31·6	2·3	17·3	118·9	259·8	504·9
5	13	1·2	0	26·1	108·4	324·7
6	2·8	0·01	0·5	1·3	18·8	100·0
7	0	0	0	0	0	0

For easier comparison the intensity of the prime tone has been throughout assumed as 100. I have compared the calculated intensity of the upper partials with their loudness on the grand

pianoforte already mentioned, and found that the first series, under $\frac{3}{7}$, suits for about the region of c''. In higher parts of the instrument the upper partials were much weaker than in this column. On striking the digital for c'', I obtained a powerful second partial and an almost inaudible third. The second column marked $\frac{3}{10}$ corresponded nearly to the region of g', the second and third partials were very strong, the fourth partial was weak. The third column, inscribed $\frac{3}{14}$, corresponds with the deeper tones from c' downwards; here the four first partials are strong, and the fifth weaker. In the next column, under $\frac{3}{20}$, the third partial tone is stronger than the second; there was no corresponding note on the pianoforte which I examined. With a perfectly hard hammer the third and fourth partials have the same strength, and are stronger than all the others. It results from the calculations in the above table, that pianoforte tones in the middle and lower octaves have their fundamental tone weaker than the first, or even than the two first upper partials. This can also be confirmed by a comparison with the effects of plucked strings. For the latter the second partial is weaker than the first; and it will be found that the prime tone is much more distinct in the tones of the strings when plucked by the finger, than when struck by the hammer.

Although, as is shewn by the mechanism of the upper Octaves on pianofortes, it is possible to produce a compound tone in which the prime is predominant, makers have preferred arranging the method of striking the lower strings in such a way as to preserve the five or six first partials distinctly, and to give the second and third greater intensity than the prime.

Thirdly, as regards the *thickness and material of the strings.* Very rigid strings will not form any very high upper partials, because they cannot readily assume inflections in opposite directions within very short sections. This is easily observed by stretching two strings of different thicknesses on a monochord and endeavouring to produce their high upper partial tones. We always succeed much better with the thinner than with the thicker string. To produce very high upper partial tones, it is preferable to use strings of extremely fine wire, such as gold lace makers employ, and when they are excited in a suitable manner, as for example by plucking or striking with a metal point, these high upper partials may be heard in the compound itself. The numerous high upper partials which lie close to each other in the scale,

give that peculiar high inharmonious noise which we are accustomed to call 'tinkling.' From the eighth partial tone upwards these simple tones are less than a whole Tone apart, and from the fifteenth upwards less than a Semitone. They consequently form a series of dissonant tones. On a string of the finest iron wire, such as is used in the manufacture of artificial flowers, 700 centimetres (22·97 feet) long, I was able to isolate the eighteenth partial tone. The peculiarity of the tones of the zither depends on the presence of these tinkling upper partials, but the series does not extend so far as that just mentioned, because the strings are shorter.

Strings of gut are much lighter than metal strings of the same compactness, and hence produce higher partial tones. The difference of their musical quality depends partly on this circumstance and partly on the inferior elasticity of the gut, which damps their partials, especially their higher partials, much more rapidly. The tone of plucked cat-gut strings (*guitar, harp*) is consequently much less tinkling than that of metal strings.

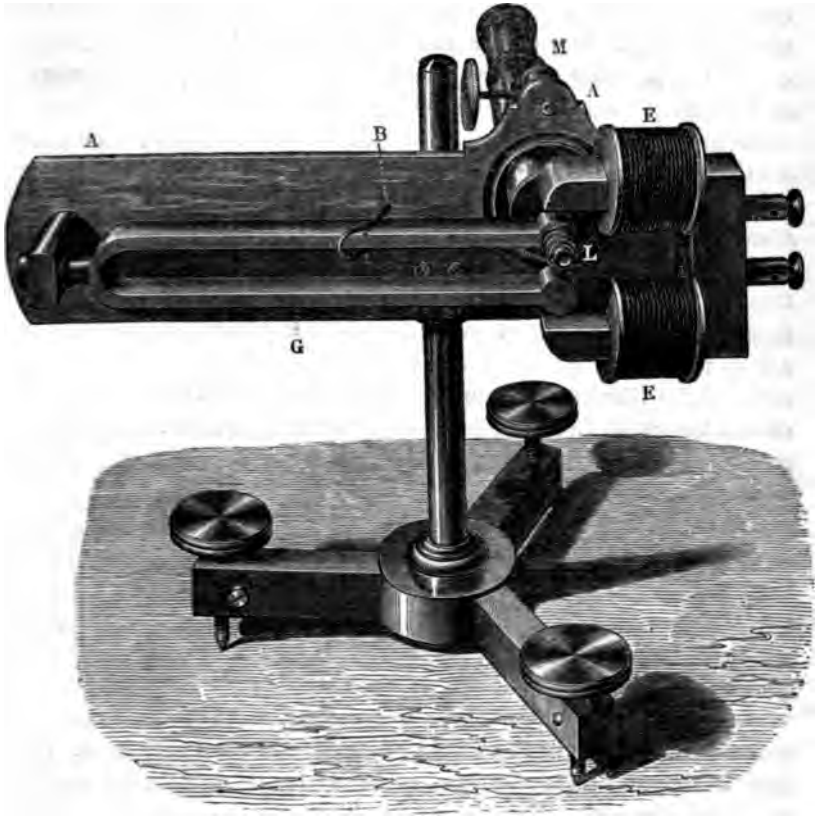
4. *Musical Tones of Bowed Instruments.*

No complete mechanical theory can yet be given for the motion of strings excited by the violin-bow, because the mode in which the bow affects the motion of the string is unknown. But by applying a peculiar method of observation, proposed in its essential features by the French physicist *Lissajous*, I have found it possible to observe the vibrational form of individual points in a violin string, and from this observed form, which is comparatively very simple, to calculate the whole motion of the string and the intensity of the upper partial tones.

Look through a hand magnifying glass consisting of a strong convex lens, at any small bright object, as a grain of starch reflecting a flame, and appearing as a fine point of light. Move the lens about while the point of light remains at rest, and the point itself will appear to move. In the apparatus I have employed, which is shewn in fig. 22 (p. 129), this lens is fastened to the end of one prong of the tuning-fork G, and marked L. It is in fact a combination of two achromatic lenses, like those used for the object glasses of microscopes. These two lenses may be used alone as a doublet, or be combined with others. When more magnifying power is required, we can introduce behind the metal

plate A A, which carries the fork, the tube and eye-piece of a microscope, of which the doublet then forms the object-glass. This instrument may be called a *vibration microscope*. When it is so arranged that a fixed luminous point may be clearly seen through it, and the fork is set in vibration, the doublet L moves periodically up and down in pendular vibrations. The observer, however,

FIG. 22.



appears to see the luminous point itself vibrate, and, since the separate vibrations succeed each other so rapidly that the impression on the eye cannot die away during the time of a whole vibration, the path of the luminous point appears as a fixed straight line, increasing in length with the excursions of the fork.[1]

[1] The end of the other prong of the fork is thickened to counterbalance the weight of the doublet. The iron loop B which is clamped on to one prong serves to alter

The grain of starch which reflects the light to be seen, is then fastened to the resonant body whose vibrations we intend to observe, in such a way that the grain moves backwards and forwards horizontally, while the doublet moves up and down vertically. When both motions take place at once, the observer sees the real horizontal motion of the luminous point combined with its apparent vertical motion, and the combination results in an apparent curvilinear motion. The field of vision in the microscope then shews an apparently steady and unchangeable bright curve, when either the periodic times of the vibrations of the grain of starch and of the tuning fork are exactly equal, or one is exactly two or three or four times as great as the other, because in this case the luminous point passes over exactly the same path every one or every two, three, or four vibrations. If these ratios of the vibrational numbers are not exactly perfect, the curves alter slowly, and the effect to the eye is as if they were drawn on the surface of a transparent cylinder which slowly revolved on its axis. This slow displacement of the apparent curves is not disadvantageous, as it allows the observer to see them in different positions. But if the ratio of the vibrational numbers of the observed body and the fork differs too much from one expressible by small whole numbers, the motion of the curve is too rapid for the eye to follow it, and all becomes confusion.

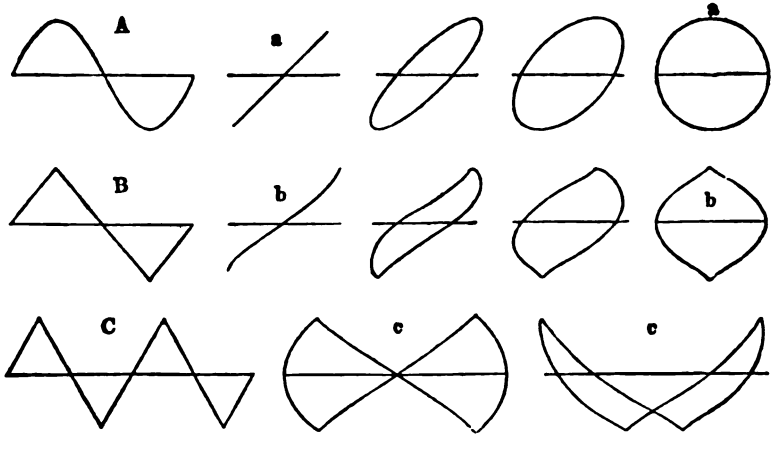
If the vibration microscope has to be used for observing the motion of a violin string, the luminous point must be attached to that string. This is done by first marking the required spot on the string with ink, and, when it is dry, rubbing it over with wax, and powdering this with starch so that two or three grains remain sticking. The violin is then fixed with its strings in a vertical direction opposite the microscope, so that the luminous reflection from one of the grains of starch can be clearly seen. The bow is drawn across the strings in a direction parallel to the prongs of the fork. Every point in the string then moves horizontally, and on setting the fork in motion at the same time, the observer sees the peculiar vibrational curves already mentioned. For the purposes of observation I used the a' string, which I tuned

the pitch of the fork slighty; we flatten the pitch by moving it towards the end of the prong. E is an electro-magnet by which the fork is kept in constant uniform vibration on passing intermittent electrical currents through its wire coils, as will be described more in detail in Chapter VI.

a little higher, as *b'*♭, so that it was exactly two Octaves higher than the tuning fork of the microscope, which sounded *B*♭.

In fig. 23 are shewn the resulting vibrational curves as seen in the vibration microscope. The straight horizontal lines in the figures, a to a, b to b, c to c, shew the apparent path of the observed luminous point, before it had itself been set in vibration; the curves and zigzags in the same figures, shew the path of the luminous point when it also was made to move. By their side, in A, B, C, the same vibrational forms are exhibited according to the methods used in Chapters I. and II., the lengths of the horizontal line being directly proportional to the corresponding lengths of *time*, whereas in figures a to a, b to b, c to c, the horizontal lengths are proportional to the *excursions* of the

FIG. 23.



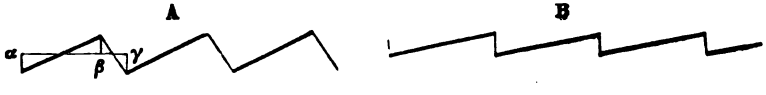
vibrating microscope. A, and a to a, shew the vibrational curves for a tuning fork, that is for a simple pendular vibration; B and b to b those of the middle of a violin string in unison with the fork of the vibration microscope; C and c, c, those for a string which was tuned an octave higher. We may imagine the figures a to a, b to b, and c to c, to be formed from the figures A, B, C, by supposing the surface on which these are drawn to be wrapped round a transparent cylinder whose circumference is of the same length as the horizontal line. The curve drawn upon the surface of the cylinder must then be observed from such a point, that the

horizontal line which when wrapped round the cylinder forms a circle, appears perspectively as a single straight line. The vibrational curve A will then appear in the forms a to a, B in the forms b to b, C in the forms c to c, according to the angular state of the cylinder. When the pitch of the two vibrating bodies is not in an exact harmonic ratio, this imaginary cylinder on which the vibrational curves are drawn, appears to revolve so that the forms a to a, &c., are assumed in succession.

It is now easy to rediscover the forms A, B, C, from the forms a to a, b to b, and c to c, and as the former give a more intelligible image of the motion of the string than the latter, the curves, which are seen as if they were traced on the surface of a cylinder, will be drawn as if their trace had been unrolled from the cylinder into a plane figure like A, B, C. The meaning of our vibrational curves will then precisely correspond to the similar curves in preceding chapters. When four vibrations of the violin string correspond to one vibration of the forte (as in our experiments, where the fork gave *B*♭ and the string *b′*♭, p. 131 top), so that four waves seem to be traced on the surface of the imaginary cylinder, and when moreover they are made to rotate slowly and are thus viewed in different positions, it is not at all difficult to draw them from immediate inspection as if they had been rolled off on to a plane, for the middle jags have then nearly the same appearance on the cylinder as if they were traced on a plane.

The figures 23 B and 23 C (p. 131), give the direct vibrational forms for the middle of a violin string, when the bow bites well, and the prime tone of the string is fully and powerfully produced. It is easily seen that these vibrational forms are essentially different from that of a simple vibration. When the point is taken nearer the ends of the string the vibrational figure is shewn in fig. 24, A, and the two sections $\alpha\beta$, $\beta\gamma$, of any wave, are to one another as the two sections of the string which lie on either side of the observed point. In the figure this ratio is 3 : 1, the point being

FIG. 24.



at $\frac{1}{4}$ the length of the string from its extremity. Close to the end of the string the form is as in fig. 24, B. The short lengths of

line in the figure have been made faint because the corresponding motion of the luminous point is so rapid that it often becomes invisible, and the thicker lengths are alone seen.

These figures shew that every point of the string between its two extremities vibrates with a constant velocity. For the middle point, the velocity of ascent is equal to that of descent. If the violin bow is used near the right end of the string descending, the velocity of descent on the right half of the string is less than that of ascent, and the more so the nearer to the end. On the left half of the string the converse takes place. At the place of bowing the velocity of descent appears to be equal to that of the violin bow. During the greater part of each vibration the string here clings to the bow, and is carried on by it; then it suddenly detaches itself and rebounds, whereupon it is seized by other points in the bow and again carried forward.[1]

Our present purpose is chiefly to determine the upper partial tones. The vibrational forms of the individual points of the string being known, the intensity of each of the partial tones can be completely calculated. The necessary mathematical formulæ are developed in Appendix VI. The following is the result of the calculation. When a string excited by a violin bow speaks well, all the upper partial tones which can be formed by a string of its degree of rigidity, are present, and their intensity diminishes as their pitch increases. The amplitude and the intensity of the second partial is one-fourth of that of the prime tone, that of the third partial a ninth, that of the fourth a sixteenth, and so on. This is the same scale of intensity as for the partial tones of a string plucked in its middle, with this exception, that in the latter case the even partials all disappear, whereas they are now all present. The upper partials in the compound tone of a violin are heard easily and will be found to be strong in sound if they have been first produced as so-called harmonics on the string, by bowing lightly while gently touching a node of the required partial tone. The strings of a violin will allow the harmonics to be produced as high as the sixth partial tone with ease, and with some difficulty even up to the tenth. The lower tones speak best when the string is bowed at from one-tenth to one-twelfth the length of the vibrating portion of the string from its extremity.

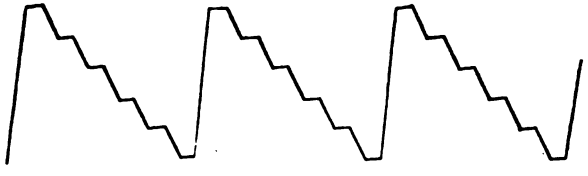
[1] These facts suffice to determine the complete motion of bowed strings. See Appendix No. VI.

For the higher harmonics where the sections are smaller, the strings must be bowed at about one-fourth or one-sixth of their vibrating length from the end.

The prime in the compound tones of bowed instruments is comparatively more powerful than in those produced on a pianoforte or guitar by striking or plucking the strings near to their extremities; the first upper partials are comparatively weaker; but the higher upper partials from the sixth to about the tenth, are much more distinct, and give these tones their cutting character.

The fundamental form of the vibrations of a violin string just described, is, when the string speaks well, tolerably independent of the place of bowing, at least in all essential features. It does not in any respect alter like the vibrational form of struck or plucked strings according to the position of the point excited. Yet there are certain observable differences of the vibrational figure which depend upon the bowing point. Little crumples are usually perceived on the lines of the vibrational figure, as in fig. 25,

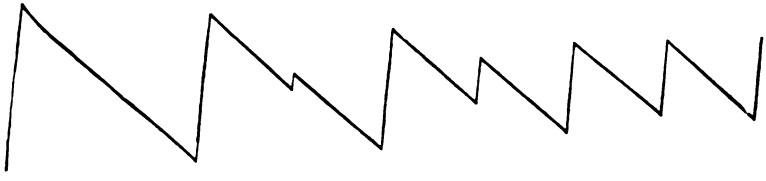
FIG. 25.



which increase in breadth and height the further the bow is removed from the extremity of the string. When we bow at a node of one of the higher upper partials which is near the bridge, these crumples are simply reduced to the absence of that part of the normal motion of the string which depends on the partial tones having a node at that place. When the observation on the vibrational form is made at one of the other nodes belonging to the deepest tone produced, none of these crumples are seen. Thus if the string is bowed at one-seventh, or six-sevenths, or five-sevenths, or four-sevenths, &c., its length from the bridge, the vibrational figure is simple, as in fig. 24 (p. 132). But if we observe between two nodes, the crumples appear as in fig. 25. Variations in the quality of tone partly depend upon this condition. When the violin bow is brought too near the finger board, the end of

which is one-fifth the length of the string from the bridge, the fifth or sixth partial tone, which is generally distinctly audible, will be absent. The tone is thus rendered duller. The usual place of bowing is at about one-tenth of the length of the string, for piano passages it is somewhat further from the bridge and for forte somewhat nearer to it. If the bow is brought near the bridge, and at the same time but lightly pressed, another alteration of quality occurs, which is readily seen on the vibrational figure. It consists of a mixture of the prime tone and first harmonic of the string. By light and rapid bowing, namely at about one-twentieth of the length of the string from the bridge, we sometimes obtain the upper Octave of the prime tone by itself, a node being formed in the middle of the string. On bowing more firmly the prime tone immediately sounds. Intermediately the higher Octave may mix with it in any proportion. This is immediately recognised in the vibrational figure. Fig. 26 gives the corresponding series of

FIG. 26.



forms. It is seen how a fresh crest appears on the longer side of the front of a wave, jutting out at first slightly, then more strongly, till at length the crest of the new waves are as high as those of the old, and then the vibrational number has doubled, and the pitch has passed into the Octave above. The quality of the lowest tone of the string is rendered softer and brighter, but less full and powerful when the intermixture commences. It is interesting to observe the vibrational figure while little changes are made in the style of bowing, and note how the resulting slight changes of quality are immediately rendered evident by very distinct changes in the vibrational figure itself.

The vibrational forms just described may be maintained in a uniformly steady and unchanged condition by carefully uniform bowing. The instrument has then an uninterrupted and pure musical quality of tone. Any scratching of the bow is immediately shewn by sudden jumps, or discontinuous displacements and

changes in the vibrational figure. If the scratching continues, the eye has no longer time to perceive a regular figure. The scratching noises of a violin bow must therefore be regarded as irregular interruptions of the normal vibrations of the string, making them to recommence from a new starting point. Sudden jumps in the vibrational figure betray every little stumble of the bow which the ear alone would scarcely observe. Inferior bowed instruments seem to be distinguished from good ones by the frequency of such greater or smaller irregularities of vibration. On the string of a monochord, which was only used for the occasion as a bowed instrument, great neatness of bowing was required to preserve a steady vibrational figure for sufficient time for the eye to apprehend it; and the tone was rough in quality, accompanied by much scratching. With a very good modern violin made by Bausch it was easier to maintain the steadiness of the vibrational figure for some time; but I succeeded much better with an old Italian violin of Guadanini, which was the first one on which I could keep the vibrational figure steady enough to count the crumples. This great uniformity of vibration is evidently the reason of the purer tone of these old instruments, since every little irregularity is immediately felt by the ear as a roughness or scratchiness in the quality of tone.

An appropriate structure of the instrument, and wood of the most perfect elasticity procurable, are probably the important conditions for regular vibrations of the string, and when these are present, the bow can be easily made to work uniformly. This allows of a pure flow of tone, undisfigured by any roughness. On the other hand, when the vibrations are so uniform the string can be more vigorously attacked with the bow. Good instruments consequently allow of a much more powerful motion of the string, and the whole intensity of their tone can be communicated to the air without diminution, whereas the friction caused by any imperfection in the elasticity of the wood destroys part of the motion. Much of the advantages of old violins may, however, also depend upon their age, and especially their long use, both of which cannot but act favourably on the elasticity of the wood. But the art of bowing is evidently the most important condition of all. How delicately this must be cultivated to obtain certainty in producing a very perfect quality of tone and its different varieties, cannot be more clearly demonstrated than by the observation of vibrational

figures. It is also well known that great players can bring out full tones from even indifferent instruments.

The preceding observations and conclusions refer to the vibrations of the strings of the instrument and the intensity of their upper partial tones solely in so far as they are contained in the compound vibrational movement of the string. But partial tones of different pitches are not equally well communicated to the air, and hence do not strike the ear of the listener with precisely the same degrees of intensity as those they possess on the string itself. They are communicated to the air by means of the sonorous body of the instrument. As we have had already occasion to remark, vibrating strings do not directly communicate any sensible portion of their motion to the air. The vibrating strings of the violin, in the first place, agitate the bridge over which they are stretched. This stands on two feet over the most mobile part of the 'belly' between the two '*f* holes.' One foot of the bridge rests upon a comparatively firm support, namely, the 'sound post,' which is a solid rod inserted between the two plates, back and belly, of the instrument. It is only the other leg which agitates the elastic wooden plates, and through them the included mass of air.

An inclosed mass of air, like that of the violin, viola, and violoncello, bounded by elastic plates, has certain proper tones which may be evoked by blowing across the openings, or '*f* holes.' The violin thus treated gives *c′* according to Savart, who examined instruments made by Stradivario (Straduarius). Zamminer found the same tone constant on even imperfect instruments. For the violoncello Savart found on blowing over the holes, F, and Zamminer G. According to Zamminer the sound box of the viola (tenor) is tuned to be a Tone deeper than that of the violin. On placing the ear against the back of a violin and playing a scale on the pianoforte, some tones will be found to penetrate the ear with more force than others, owing to the resonance of the instrument. On a violin made by Bausch two tones of greatest resonance were thus discovered, one between *c′* and *c′*♯, and the other between *a′* and *b′*♭. For a viola (tenor) I found the two tones about a Tone deeper, which agrees with Zamminer's calculation.

The consequence of this peculiar relation of resonance is that those tones of the strings which lie near the proper tones of the inclosed body of air, must be proportionably more reinforced. This is clearly perceived on both the violin and violoncello, at

least for the lowest proper tone, when the corresponding notes are produced on the strings. They sound particularly full, and the prime tone of these compounds is especially prominent. I think that I heard this also for a' on the violin, which corresponds to its higher proper tone.

Since the lowest tone on the violin is g, the only upper partials of its musical tones which can be somewhat reinforced by the resonance of the higher proper tone of its inclosed body of air, are the higher Octaves of its three deepest notes. But the prime tones of its higher notes will be reinforced more than their upper partials, because these prime tones are more nearly of the same pitch as the proper tones of the body of air. This produces an effect similar to that of the construction of the hammer of a piano, which favours the upper partials of the deep notes, and weakens those of the higher notes. For the violoncello, where the lowest string gives C, the stronger proper tone of the body of air is, as on the violin, a Fourth or a Fifth higher than the pitch of the lowest string. There is consequently a similar relation between the favoured and unfavoured partial tones, but all of them are a Twelfth lower than on the violin. On the other hand, the most favoured partial tones of the viola (tenor) corresponding nearly with b', do not lie between the first and second strings, but between the second and third; and this seems to be connected with the altered quality of tone on this instrument. Unfortunately this influence cannot be expressed numerically. The maximum of resonance for the proper tones of the body of air is not very marked; were it otherwise there would be much more inequality in the scale as played on these bowed instruments, immediately on passing the pitch of the proper tones of their bodies of air. We must consequently conjecture that their influence upon the relative intensity of the individual partials in the musical tones of these instruments is not very prominent.

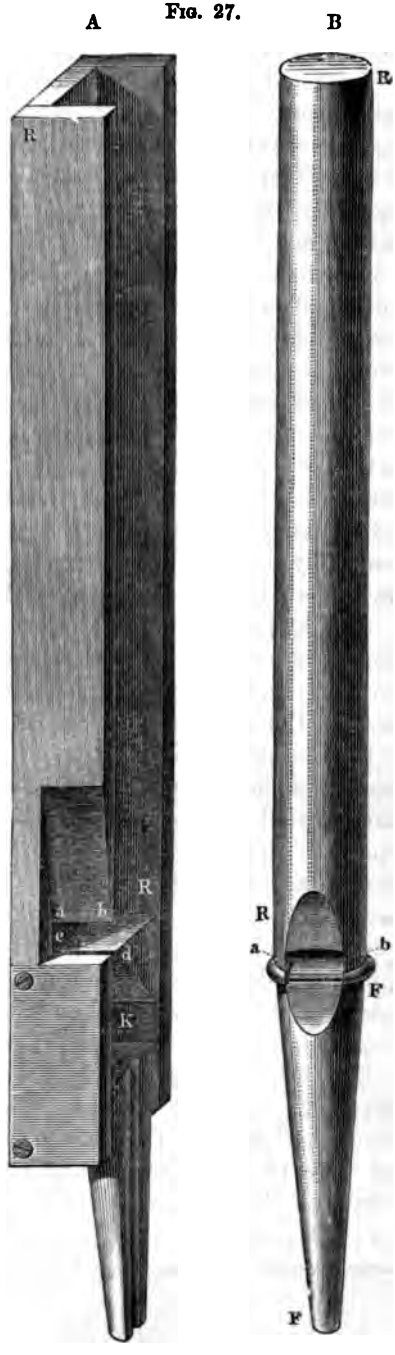
5. *Musical Tones of Flute or Flue Pipes.*

In these instruments the tone is produced by driving a stream of air against an opening, generally furnished with sharp edges, in some hollow space filled with air. To this class belong the bottles described in the last chapter, and shown in fig. 20 (p. 102), and especially flutes and the majority of organ pipes. For flutes, the

resonant body of air is included in its own cylindrical bore. It is blown with the mouth, which directs the breath against the somewhat sharpened edges of its mouth hole. The construction of organ pipes will be seen from the two adjacent figures. Fig. 27, A, shews a square wooden pipe, cut open longwise, and B the external appearance of a round tin pipe. R R in each limits the tube which incloses the resonant body of air, a b the *mouth hole* where it is blown, terminating in a sharp *lip*. In fig. 27, A, we see the air chamber or *throat* K into which the air is first driven from the bellows, and whence it can only escape through the narrow slit c d, which directs it against the edge of the lip. The wooden pipe A as here drawn is *open*, that is its extremity is uncovered, and it produces a tone with a wave of air *twice* as long as the tube R R. The other pipe, B, is *stopped*, that is, its upper extremity is closed. Its tone has a wave *four times* the length of the tube R R, and hence an Octave deeper than an open pipe of the same length.

Any air chamber can be made to give a musical tone, just like organ pipes, flutes, the bottles previously described, the wind-chests of violins, &c., provided they have a sufficiently narrow opening,

FIG. 27.



furnished with somewhat projecting sharp edges, by directing a thin flat stream of air across the opening and against its edges.

These edges are the source of the musical tone of all such instruments. The directed stream of air breaking against the edges, generates a peculiar hissing or rushing noise, which is all we hear when the pipe does not speak, or when we blow against the edges of a hole in a flat plate instead of a pipe. The narrower the opening and the stronger the blast, the higher will be this noise of the wind. Such a noise, as we have already found, may be considered as a mixture of several inharmonic tones of nearly the same pitch. When the air chamber of the pipe is brought to bear upon these tones its resonance strengthens such as correspond with the proper tones of that chamber, and makes them predominate over the rest, which this predominance conceals. Hence in all such pipes we always hear the tone accompanied more or less distinctly by a rush of wind, and this gives a peculiar character to its quality. Precisely in the same way as the tones of the noise created by the wind are strengthened by resonance, the tone of a tuning fork can be also reinforced by bringing it near the mouth of the pipe, when the pitch of the fork corresponds to one of the proper tones of the inclosed mass of air. By a series of different tuning forks, then, we are enabled to find and determine the proper tones of the air chamber with ease and certainty.[1] The character of the musical quality of tone in such pipes of course essentially depends upon whether or not the harmonic upper partials of the tone created by blowing, correspond to the proper tones of the pipe with sufficient accuracy, to admit of being reinforced at the same time as the prime tone. It is only in narrow cylindrical open pipes, as flutes, and the fiddle stop of organs, that the higher upper partials of the tube exactly correspond with the harmonic upper partials of the prime tone. By blowing more strongly, and thus raising the pitch of the exciting wind-rush itself, the higher proper tones of the tube can be made to speak without the lower. A flute which when gently blown, with all its holes stopped, gives d', will on stronger blowing give d'', and on still stronger a'' and d''', that is, the first second and third upper partial tone of d'. Hence for narrow cylindrical pipes, not only the prime tone but also a series of its harmonic upper partial tones are

[1] In my mathematical investigations, I have consequently called these the *tones of strongest resonance*. Crelle, 'Journal für Mathematik,' vol. 57.

reinforced by the resonance of the tube, when it is blown with sufficient force for the wind-rush itself to contain several higher partial tones. And in reality on forcibly blowing the narrow cylindrical pipes of an organ (in the *geigenprincipal*, *violoncell*, *violonbass*, *viola di gamba* stops)[1] we hear a series of upper partials distinctly and powerfully accompany the prime tone, giving them a more cutting quality, resembling a violin. By using resonators I find that on narrow pipes of this kind the partial tones may be clearly heard up to the sixth. For wide open pipes, on the other hand, the adjacent proper tones of the tube are all somewhat sharper than the corresponding harmonic tones of the prime, and hence these tones will be much less reinforced by the resonance of the tube. Wide pipes, having larger masses of vibrating air and admitting of being much more strongly blown without jumping up into an harmonic, are used for the great body of sound on the organ, and are hence called *principalstimmen*.[2] For the above reasons they produce the prime tone alone strongly and fully, with a much weaker retinue of secondary tones. For wooden 'principal' pipes, I find the prime tone and its Octave or first upper partial very distinct; the Twelfth or second upper partial is but weak, and the higher upper partials no longer distinctly perceptible. The quality of tone in these pipes is fuller and softer than that of the *geigenprincipal*. When flute or flue stops of the organ, and

[1] [*Geigenprincipal*—violin or crisp-toned diapason, 8 feet,—violin principal, 4 feet. See suprà, p. 91, note. *Violoncello*—'crisp-toned open stop, of small scale, the Octave to the violone, 8 feet.' *Violonbass*—this fails in Hopkins, but it is probably his '*violone*—double bass, a unison open wood stop, of much smaller scale than the Diapason, and formed of pipes that are a little wider at the top than at the bottom, and furnished with ears and beard at the mouth; the tone of the Violone is crisp and resonant, like that of the orchestral Double Bass; and its speech being a little slow, it has the Stopped Bass always drawn with it, 16 feet.' *Gamba* or *viol da gamba*—'bass viol. unison stop, of smaller scale, and thinner but more pungent tone than the violin diapason, 8 feet,' one of the most highly esteemed and most frequently disposed stops in Continental organs; the German gamba is usually composed of cylindrical pipes.' In England till very recently it was made exclusively conical with a bell top. From Hopkins on the *Organ*, pp. 116, 435, 441, 442—*Translator*.]

[2] [Literally 'principal voices or parts;' may probably be best translated 'principal work' or 'diapason-work,' including 'all the open cylindrical stops of Open Diapason measure, or which have their scale deduced from that of the Open Diapason; such stops are the chief, most important or "*principal*," as they are also most numerous in an organ. The Unison and Double Open Diapasons, Principal, Fifteenth and Octave Fifteenth; the Fifth, Twelfth, and Larigot; the Tenth and Tierce; and the Mixture Stops, when of full or proportional scale, belong to the Diapason-work.' From Hopkins on the *Organ*, p. 109.—*Translator*.]

the German flute are blown softly, the upper partials lose strength at a greater rate than the prime tone, and hence the musical quality becomes weak and soft.

Another variety is observed on the pipes which are conically narrowed at their upper end, in the *salicional*, *gemshorn*, and *spitzflöte* stops.[1] Their upper opening has generally half the diameter of the lower section; for the same length the *salicional* pipe has the narrowest, and the *spitzflöte* the widest section. These pipes have, I find, the property of rendering some higher partial tones, from the fifth to the seventh, comparatively stronger than the lower. The quality of tone is consequently poor, but peculiarly bright.

The *narrower stopped cylindrical pipes* have proper tones corresponding to the uneven partials of the prime, that is, the third partial or Twelfth, the fifth partial or higher major Third, and so on. For the *wider* stopped pipes, as for the wide open pipes, the next adjacent proper tones of the mass of air are distinctly higher than the corresponding upper partials of the prime, and consequently these upper partials are very slightly, if at all, reinforced. Hence wide stopped pipes, especially when gently blown, give the prime tone almost alone, and they were therefore previously adduced as examples of simple tones (p. 102). Narrow stopped pipes, on the other hand, let the Twelfth be very distinctly heard at the same time with the prime tone; and have hence been called *quintaten* (*quintam tenentes*).[2] When these pipes are strongly blown they also give the fifth upper partial, or higher major Third, very distinctly. Another variety of quality is produced by the *rohrflöte*.[3] Here a tube, open at both ends, is inserted in the cover of a

[1] [*Salicional*—'reedy Double Dulciana, 16 feet and 8 feet, octave salicional, 4 feet.' The Double Dulciana is described as 'an open metal cylindrical stop, belonging to the Flute-work, of small scale and delicate tone, and somewhat slow of speech' (Hopkins, p. 113). *Gemshorn*, literally 'chamois horn;' in Hopkins, 'Goat-horn, a unison open metal stop, more conical than the Spitz-Flöte, 8 feet.' 'A member of the Flute-work, and met with of 8, 4, or 2 feet length in Continental organs. The pipes of this stop are only ⅓ the diameter at the top that they are at the mouth; and the tone is consequently light, but very clear and travelling' (*Ibid.* p. 118). *Spitzflöte*—'Spire or taper flute, a unison open metal stop formed of pipes with conical bodies, 8 feet.' 'This stop is found of 8, 4, and 2 feet length in German organs. In England it has hitherto been made chiefly as a 4-feet stop; i.e. of principal pitch. The pipes of the Spitz-flute are slightly conical, being about ¼ narrower at top than at the mouth, and the tone is therefore rather softer than that of the cylindrical stop, but of very pleasing quality' (*Ibid.* p. 118).—*Translator.*]

[2] [See suprà, p. 50, note.—*Translator.*]

[3] [*Rohrflöte*—'Double Stopped Diapason of metal pipes with chimneys, 16 feet,

stopped pipe, and in the examples I examined, its length was that of an open pipe giving the fifth partial tone of the prime tone of the stopped pipe. The fifth partial tone is thus proportionably stronger than the rather weak third partial on these pipes, and the quality of tone becomes peculiarly bright. Compared with open pipes the quality of tone in stopped pipes, where the even partial tones are absent, is somewhat hollow; the wider stopped pipes have a dull quality of tone, especially when deep, and are soft and powerless. But their softness offers a very effective contrast to the more cutting qualities of the narrower open pipes and the noisy *compound stops*, of which I have already spoken (p. 91), and which, as is well known, form a compound tone by uniting many pipes corresponding to a prime and its upper partial tones.

Wooden pipes do not produce such a cutting wind rush as metal pipes. Wooden sides also do not resist the agitation of the waves of sound so well as metal ones, and hence the vibrations of higher pitch seem to be destroyed by friction. For these reasons wood gives a softer, but duller, less penetrating quality of tone than metal.

It is characteristic of all pipes of this kind that they speak readily, and hence admit of great rapidity in musical divisions and figures, but, as a little increase of force in blowing distinctly alters the pitch, their loudness of tone can scarcely be changed. Hence on the organ *forte* and *piano* have to be produced by stops, which regulate the introduction of pipes with various qualities of tone, sometimes more, sometimes fewer, now the loud and cutting, now the weak and soft. The means of expression on this instrument are therefore somewhat limited, but, on the other hand, it clearly owes part of its magnificent properties to its power of sustaining tones with unaltered force, undisturbed by subjective excitement.

6. *Musical Tones of Reed Pipes.*

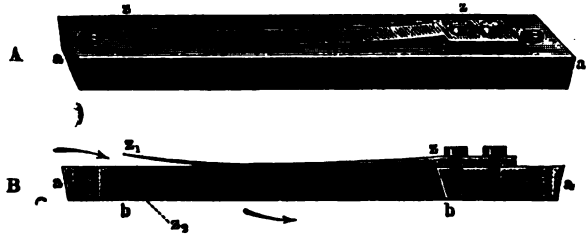
The mode of producing the tones on these instruments resembles that used for the siren: the passage for the air being alternately closed and opened, its stream is separated into a series of individual pulses. This is effected on the siren, as we have

Reed-flute, Metal Stopped Diapason, with reeds, tubes or chimneys, 8 feet. Stopped Metal Flute, with reeds, tubes or chimneys, 4 feet' (Hopkins, *ibid.*)—*Translator.*]

already seen, by means of a rotating disc pierced with holes. In reed instruments, elastic plates or tongues are employed which are set in vibration and thus alternately close and open the aperture in which they are fastened. To these belong—

1. *The reed pipes of organs and the vibrators of harmoniums.* Their tongues, shewn in perspective in fig. 28, A, and in section in fig. 28, B, are thin oblong metal plates, z z, fastened to a brass block, a a, in which there is a hole, b b, behind the tongue and of the same shape. When the tongue is in its

FIG. 28.

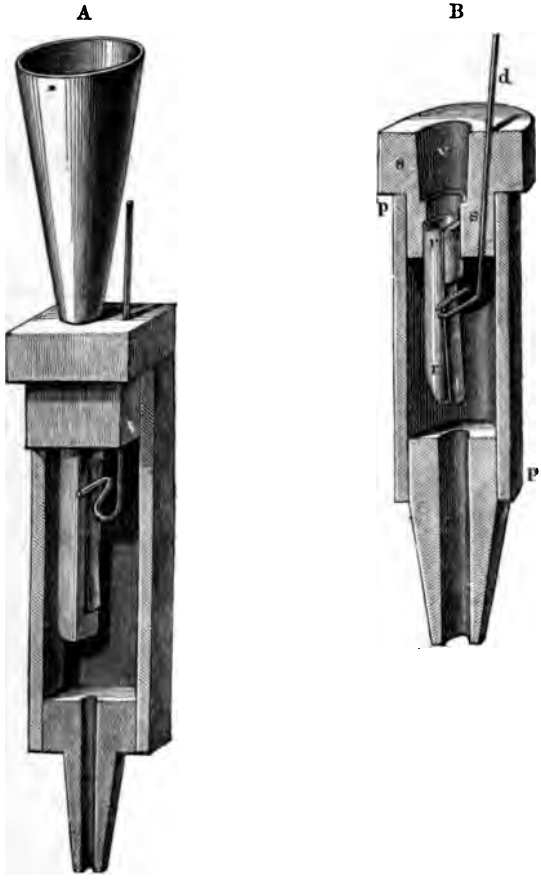


position of rest, it closes the hole completely, with the exception of a very fine chink all round its margin. When in motion it oscillates between the positions marked z_1 and z_2 in fig. 28, B. In the position z_1 there is an aperture for the stream of air to enter, in the direction shewn by the arrow, and this is closed when the tongue has reached the other extreme position z_2. The tongue shewn is a *free vibrator* or *anche libre*, such as is now universally employed. These tongues are slightly smaller than the corresponding opening, so that they can bend inwards without touching the edges of the hole. Formerly, *striking vibrators* or *reeds* were employed, which on each oscillation struck against their frame. But as these produced a harsh quality of tone they have gone out of use.

The mode in which tongues are fastened in the reed stops of organs is shewn in fig. 29, A and B (p. 145). A bears a resonant cup above; B is a longitudinal section; p p is the air chamber into which the wind is driven; the tongue l is fastened in the guttur r, which fits into the block s; d is the tuning wire, which presses against the tongue, and being pushed down shortens it and hence sharpens its pitch, and, conversely, flattens the pitch when pulled up. Slight variations of pitch are thus easily produced.

2. In similar manner are constructed the tongues of *clarinets*, *oboes*, and *bassoons*, which are cut out of elastic reed plates. The clarinet has a single wide tongue which is fastened before the corresponding opening of the mouthpiece like the metal tongues previously described, and would strike the frame if its excursions were long enough. But its excursions are small, and the pressure

FIG. 29.



of the lips brings it just near enough to make the chink sufficiently small without allowing it to strike. For the oboe and bassoon two reeds or tongues of the same kind are placed opposite each other at the end of the mouthpiece. They are separated by

a narrow chink, and by blowing are pressed near enough to close the chink whenever they swing inwards.

3. *Membranous tongues.*—The peculiarities of these tongues are best studied on those artificially constructed. Cut the end of a wooden or gutta-percha tube obliquely on both sides, as shewn in fig. 30, leaving two nearly rectangular points standing between the two edges which are cut obliquely. Then gently stretch strips of vulcanised india-rubber over the two oblique edges, so as to leave a small slit between them, and fasten them with a thread. A reed mouthpiece is thus constructed which may be connected in any way with tubes or other air chambers. When the membranes bend inwards the slit is closed; when outwards, it is open. Membranes which are fastened in this oblique manner speak much better than those which are laid at right angles to the axis of the tube, as Johannes Müller proposed, for in the latter case they require to be bent outwards by the air before they can begin to open and shut alternately. Membranous tongues of the kind proposed may be blown either in the direction of the arrows or in the opposite direction. In the first case the slit is opened when they move against the air-chamber, that is, towards the further end of the conducting tube. Tongues of this kind I distinguish as *striking inwards.* When blown they always give deeper tones than they would do if allowed to vibrate freely, that is, without being connected with an air-chamber. The tongues of organ pipes, harmoniums, and wooden wind instruments are always arranged to strike inwards. But both membranous and metal tongues may be arranged so as to act against the stream of air, and hence to open when they move towards the outer opening of the instrument. I then say that they *strike outwards.* The tones of tongues which strike outwards are always sharper than those of isolated tongues.

FIG. 30.

Only two kinds of membranous tongues have to be considered as musical instruments: the human *lips* in *brass instruments,* and the human *larynx* in *singing.*

The lips must be considered as very slightly elastic membranous tongues, loaded with much inelastic tissue containing water, and they would consequently vibrate very slowly, if they could be brought to vibrate by themselves.[1] In brass instruments they form membranous tongues which strike outwards, and consequently by the above rule produce tones sharper than their proper tones. But as they offer very slight resistance, they are readily set in motion, by the alternate pressure of the vibrating column of air, when used with brass instruments.

In the larynx, the elastic vocal chords act as membranous tongues. They are stretched across from the windpipe, front to back, like the india-rubber strips in fig. 30 (p. 146), and leave a small slit, the glottis, between them. They have the advantage over all artificially constructed tongues of allowing the width of their slit, their tension, and even their form to be altered at plea sure with extraordinary rapidity and certainty, at the same time that the resonant tube formed by the opening of the mouth admits of much variety of form, so that many more qualities of tone can be thus produced than on any instrument of artificial construction. If the vocal chords are examined from above with a laryngoscope, while producing a tone, they will be seen to make very large vibrations for the deeper breast voice, shutting the glottis tightly whenever they strike inwards.

The pitch of the various reeds or tongues just mentioned is altered in very different manners. The metal tongues of the *organ* and *harmonium* are always intended to produce one single tone apiece. On the motion of these comparatively heavy and stiff tongues, the pressure of the vibrating air has very small influence, and their pitch within the instrument is consequently not much different from that of the isolated tongues. There must be at least one tongue for each note on such instruments.

In *wooden wind instruments* a single tongue has to serve for the whole series of notes. But the tongues of these instruments are made of light elastic wood, which is easily set in motion by the alternate pressure of the vibrating column of air, and swings sympathetically with it. Such instruments, therefore, in addition

[1] [Babies may be often heard to 'trill the lips,' and German coachmen use a very audible lip trill to stop their horses, but in each case the real tone is produced in the larynx, and the vibration of the lips only serves to interrupt it; these examples, however, allow us to judge of the rate of vibration natural to the lips.—*Translator.*]

to those very high tones, which nearly correspond to the proper tones of their tongues, can, as theory and experience alike shew, also produce deep tones of a very different pitch,[1] because the waves of air which arise in the tube of the instrument excite an alternation in the pressure of air adjacent to the tongue itself sufficiently powerful to make it vibrate sensibly. Now in a vibrating column of air the alternation of pressure is greatest where the velocity of the particles of air is smallest; and since the velocity is always null, that is a minimum, at the end of a closed tube, such as a stopped organ pipe, and the alternation of pressure in that place is consequently a maximum, the tones of these reed pipes must be the same as those which the resonant tube alone would produce, if it were stopped at the place where the tongue is placed, and were blown as a stopped pipe. In musical practice, then, such tones of the instrument as correspond to the proper tones of the tongues are not used at all, because they are very high and screaming, and their pitch cannot be preserved with sufficient steadiness when the tongue is wet. The only tones produced are considerably deeper than the proper tone of the tongue, and have their pitches determined by the length of the column of air, which corresponds to the proper tones of the stopped pipe.

The *clarinet* has a cylindrical tube, the proper tones of which correspond to the third, fifth, seventh, &c., partial tone of the prime. By altering the style of blowing, it is possible to pass from the prime to the Twelfth or the higher major Third. The acoustic length of the tube may also be altered by opening the side holes of the clarinet, in which case the vibrating column of air is principally that between the mouthpiece and the uppermost open side hole.

The *oboe* (hautbois) and *bassoon* (fagot) have conical tubes which are closed up to the vertex of their cone, and have proper tones that are the same as those of open tubes of the same length. Hence the tones of both of these instruments nearly correspond to those of open pipes. By overblowing they give the Octave, Twelfth, second Octave, and so on, of the prime tone. Intermediate tones are produced by opening side holes.

[1] See Helmholtz, 'Verhandlungen des naturhistorischen medicinischen Vereins zu Heidelberg,' July 26, 1861, in the 'Heidelberger Jahrbücher.' Poggendorff's Annalen, 1861.

The older *French horns* and *trumpets* consist of long conical twisted tubes, without keys or side holes. They can produce such tones only as correspond to the proper tones of the tube, and these again are the natural harmonic upper partials of the prime. But as the prime tone of such a long tube is very deep, the upper partial tones in the middle parts of the scale lie very close together, especially in the very long tubes of the French horn,[1] so that they give most of the degrees of the scale. The trumpet is restricted to these natural tones. But by introducing the hand into the bell of the French horn and thus partly closing it, and by lengthening the tube of the trombrone, it was possible in some degree to supply the missing tones and improve the faulty ones. In later times trumpets and horns have been frequently supplied with keys to supply the missing tones, but at some expense of power in the tone and the brilliancy in its quality. The vibrations of the air in these instruments are unusually powerful, and require the resistance of firm, smooth, unbroken tubes to preserve their strength. In the use of brass instruments, the different form and tension of the lips of the player act only to determine which of the proper tones of the tube shall speak; the pitch of the individual tones is almost entirely independent of the tension of the lips.

On the other hand, in the *larynx* the tension of the vocal chords, which here form the membranous tongues, is itself variable, and determines the pitch of the tone. The air chambers connected with the larynx are not adapted for materially altering the tone of the vocal chords. Their walls are so yielding that they cannot allow the formation of vibrations of the air within them sufficiently powerful to force the vocal chords to oscillate with a period which is different from that required by their own elasticity. The cavity of the mouth is also far too short, and generally too widely open to serve as a resonance chamber which could have material influence on the pitch.

[1] The tube of the French horn [the *Waldhorn* or foresthorn, hunting horn of the Germans] is, according to Zamminer, 27 feet long. Its proper prime tone is $E_{,}$♭. This and the next E♭ are not used, but only the other tones, B♭, e♭, g, b♭, d'♭ —, e'♭, f', g', a'♭ +, b'♭, &c. [The tones d'♭ —, a'♭ +, corresponding to the 7th and 11th partial tone, are respectively flatter and sharper than the tones in the justly intoned scale of e♭ in the ratios of 63 : 64 and 33 : 32, as indicated by the — and + signs.—*Translator.*]

In addition to the tension of the vocal chords (which can be increased not only by separating the points of their insertion in the cartilages of the larynx, but also by voluntarily stretching the muscular fibres within them), their density seems also to be variable. Much soft watery inelastic tissue lies underneath the proper elastic fibrils and muscular fibres of the vocal chords, and in the breast voice this probably acts to weight them and retard their vibrations. The head voice is probably produced by drawing aside the mucous coat below the chords, thus rendering the edge of the chords sharper, and the weight of the vibrating part less, while the elasticity is unaltered.[1]

We now proceed to investigate the *quality of tone* produced on reed pipes, which is our proper subject. The sound in these pipes is excited by intermittent pulses of air, which at each swing break through the opening that is closed by the tongue. A freely vibrating tongue has far too small a surface to communicate any appreciable quantity of resonant motion to the surrounding air; and it is as little able to excite the air enclosed in pipes. The sound seems to be really produced by pulses of air, as in the siren, where the metal plate that opens and closes the orifice does not vibrate at all. By the alternate opening and closing of a passage, a continuous influx of air is changed into a periodic motion, capable of affecting the air. Like any other periodic motion of the air, the one thus produced can also be resolved into a series of simple vibrations. We have already remarked that the number of terms in such a series will increase with the discontinuity of the motion to be thus resolved. Now the motion of the air which passes through a siren, or past a vibrating tongue, is discontinuous in a very high degree, since the individual pulses of air must be generally separated by complete pauses during the closures of the opening. Free tongues without a resonance tube, in which all the individual simple tones of the vibration which they excite in the air are given off freely to the surrounding atmosphere, have consequently always a very sharp, cutting, jarring quality of tone, and we can really hear with either armed or unarmed ears a long series of strong and clear partial tones up to the 16th or 20th, and there are evidently still higher partials present, although it is difficult or impossible to distinguish them from each other, because they do not lie so much as a

[1] [The mode in which the different *registers* of the voice are produced is still under inquiry.—*Translator.*]

Semitone apart.[1] This whirring of dissonant partial tones makes the musical quality of free tongues very disagreeable. A tone thus produced also shews that it is really due to puffs of air. I have examined the vibrating tongue of a reed pipe, like that in fig. 28 (p. 144), when in action with the vibration microscope of Lissajous, in order to determine the vibrational form of the tongue, and I found that the tongue performed perfectly regular simple vibrations. Hence it would communicate to the air merely a simple tone and not a compound tone, if the sound were directly produced by its own vibrations.

The intensity of the upper partial tones of a free tongue, unconnected with a resonance tube, and their relation to the prime, are greatly dependent on the nature of the tongue, its position with respect to its frame, the tightness with which it closes, &c. Striking tongues which produce the most discontinuous pulses of air, also produce the most cutting quality of tone. The shorter the puff of air, the more sudden its action, the greater number of high upper partials should we expect, exactly as we find in the siren, according to Seebeck's investigations. Hard, unyielding material, such as that used for brass tongues, will produce pulses of air which are much more disconnected than those formed by soft and yielding substances.

This is probably the reason why the singing tones of the human voice are softer than all others which are produced by reed pipes. Nevertheless the number of upper partial tones in the human voice, when used in emphatic *forte*, is very great, and they reach

[1] [The difficulties felt by the unarmed ear in observing such upper partials, or recognising the effects heard as due to their presence, is well shewn by such assertions as these: 'The springs of harmoniums emit no harmonics,' p. xxxiii.; 'pure fundamental tones are always detached in harmoniums, because they have *no* audible harmonics,' p. 233; 'the springs [of symphonions, which are of the same construction as harmonium vibrators] do not yield harmonics, therefore there is no confusion of sounds; it is the deficiency as to harmonics that makes the harmonium an unsatisfactory substitute for the organ,' p. 246 of 'The History of Music, Art, and Science, vol. i., from the Earliest Records to the fall of the Roman Empire, with explanations of Antient Systems of Music, Musical Instruments, and of the True Physiological Basis for the Science of Music, whether Ancient or Modern,' by W. Chappell, F.S.A. author of 'A History of the Ballad Literature and Popular Music of the Olden Time.' Introduction dated June 1, 1874. The title and date are worth preserving, to shew to what extent practical and antiquarian musicians at the present day (among whom Mr. Chappell has already earned an honourable position) are acquainted with the physical constitution of the materials with which and on which they work.—*Translator.*]

distinctly and powerfully up to the four times accented Octave (p. 26). To this we shall have to return.

The tones of tongues are essentially changed by the addition of resonance tubes, because they reinforce and hence give prominence to those upper partial tones which correspond to the proper tones of these tubes, precisely as was the case for the rush of wind in the flute or flue pipes of organs. In this case the resonance tubes must be considered as closed at the point where the tongue is inserted.[1]

A brass tongue such as is used in organs, and tuned to *b*♭, was applied to one of my larger spherical resonators, also tuned to *b*♭, instead of its usual resonance tube. After considerably increasing the pressure of wind in the bellows, the tongue spoke somewhat flatter than usual, but with an extraordinarily full, beautiful, soft tone, from which almost all upper partials were absent. Very little wind was used, but it was under high pressure. In this case the prime tone of the compound was in unison with the resonator, which gave a powerful resonance, and consequently the prime tone had also great power. None of the higher partial tones could be reinforced. The theory of the vibrations of air in the sphere further shews that the greatest pressure must occur in the sphere at the moment that the tongue opened. Hence the necessity of strong pressure in the bellows to overcome the increased pressure in the sphere, and yet not much wind really passed.

If instead of a glass sphere, resonant tubes are employed, which admit of a greater number of proper tones, the resulting musical tones are more complex. In the clarinet we have a cylindrical tube which by its resonance reinforces the uneven partial tones. The conical tubes of the oboe, bassoon, trumpet, and French horn, on the other hand, reinforce all the harmonic upper partial tones of the compound up to a certain height, determined by the incapacity of the tubes to resound for waves of sound that are not much longer than the width of the opening. By actual trial I found only uneven partial tones, distinct to the seventh inclusive, in the compound tones of the clarinet, whereas on the other instruments, which have conical tubes, I found the even partials also. I have not yet had an opportunity of making observations on the further differences of quality in the tones of

[1] See Appendix VII.

individual instruments with conical tubes. This opens rather a wide field for research, since the quality of tone is altered in many ways by the style of blowing, and even on the same instrument the different parts of the scale, when they require the opening of side holes, shew considerable differences in quality. On wooden wind instruments these differences are striking. The opening of side holes is by no means a complete substitute for shortening the tube, and the reflection of the waves of sound at the points of opening is not the same as at the free open end of the tube. The upper partials of compound tones produced by a tube limited by an open side hole, must certainly be in general materially deficient in harmonic purity, and this will also have a marked influence on their resonance.

7. *Vowel Qualities of Tone.*

We have hitherto discussed those cases of resonance, generated in such air chambers as were capable of reinforcing the prime tone principally, but also a certain number of the harmonic upper partial tones of the compound tone produced. The case, however, may also occur in which the lowest tone of the resonance chamber applied does not correspond with the prime, but only with some one of the upper partials of the compound tone itself, and in these cases we find, in accordance with the principles hitherto developed, that the corresponding upper partial tone is really more reinforced than the prime or other partials by the resonance of the chamber, and consequently predominates extremely over all the other partials in the series. The quality of tone thus produced has consequently a peculiar character, and more or less resembles one of the vowels of the human voice. For the vowels of speech are in reality tones produced by membranous tongues (the vocal chords), with a resonance chamber (the mouth) capable of altering in length, width, and resonant pitch, and hence capable also of reinforcing at different times different partials of the compound tone to which it is applied.[1]

[1] The theory of vowel tones was first enunciated by Wheatstone in a criticism, unfortunately little known, on Willis's experiments. The latter are described in the 'Transactions of the Cambridge Philosophical Society,' vol. iii., p. 231, and Poggendorff's 'Annalen der Physik,' vol. xxiv., p. 397. Wheatstone's report upon them is contained in the 'London and Westminster Review' for October 1837.

In order to understand the composition of vowel tones, we must in the first place bear in mind that the source of their sound lies in the vocal chords, and that when the voice is heard, these chords act as membranous tongues, and like all tongues produce a series of decidedly discontinuous and sharply separated pulses of air, which, on being represented as a sum of simple vibrations, must consist of a very large number of them, and hence be received by the ear as a very long series of partials belonging to a compound musical tone. With the assistance of resonators it is possible to recognise very high partials, up to the sixteenth, when one of the brighter vowels is sung by a powerful bass voice at a low pitch, and, in the case of a strained forte in the upper notes of any human voice, we can hear, more clearly than on any other musical instrument, those high upper partials that belong to the middle of the four times accented Octave (the highest on modern pianofortes), and these high tones have a peculiar relation to the ear, to be subsequently considered. The loudness of such upper partials, especially those of highest pitch, differs considerably in different individuals. For cutting bright voices it is greater than for soft and dull ones. The quality of tone in cutting screaming voices may perhaps be referred to a want of sufficient smoothness or straightness in the edges of the vocal chords, to enable them to close in a straight narrow slit without striking one another. This circumstance would give the larynx more the character of striking tongues, and the latter have a much more cutting quality than the free tongues of the normal vocal chords. Hoarseness in voices may arise from the glottis not entirely closing during the vibrations of the vocal chords. At any rate, when alterations of this kind are made in artificial membranous tongues, similar results ensue. For a strong and yet soft quality of voice it is necessary that the vocal chords should, even when most strongly vibrating, join rectilinearly at the moment of approach with perfect tightness, effectually closing the glottis for the moment, but without overlapping or striking against each other. If they do not close perfectly, the stream of air will not be completely interrupted, and the tone cannot be powerful. If they overlap, the tone must be cutting, as before remarked, because arising from striking tongues. On examining the vocal chords in action by means of a laryngoscope, it is marvellous to observe the accuracy with which they close even

when making vibrations occupying nearly the entire breadth of the chords themselves.

There is also a certain difference in the way of putting on the voice in speaking and in singing, which gives the speaking voice a much more cutting quality of tone, especially in the open vowels, and occasions a sensation of much greater pressure in the larynx. I suspect that in speaking the vocal chords act as striking tongues.

When the mucous membrane of the larynx is affected with catarrh, the laryngoscope sometimes shews little flakes of mucus in the glottis. When these are too great they disturb the motion of the vibrating chords and make them irregular, causing the tone to become unequal, jarring, or hoarse. It is, however, remarkable what comparatively large flakes of mucus may lie in the glottis without producing a very striking deterioration in the quality of tone.

It has already been mentioned that it is generally more difficult for the unassisted ear to recognise the upper partials in the human voice, than in the tones of musical instruments. Resonators are more necessary for this examination than for the analysis of any other kind of musical tone. They have nevertheless been heard at times by attentive observers. Rameau had heard them at the beginning of last century. And at a later period Seiler of Leipzig relates that while listening to the chant of the watchman during a sleepless night, he occasionally heard at first, when the watchman was at a distance, the Twelfth of the melody, and afterwards the prime tone. The reason of this difficulty is most probably that we have all our lives remarked and observed the tones of the human voice more than any other, and always with the object of grasping it as a whole and obtaining a clear knowledge and perception of its manifold changes of quality.

We may certainly assume that in the tones of the human larynx, as in all other tongued instruments, the upper partial tones would decrease in force as they increase in pitch, if they could be observed without the resonance of the cavity of the mouth. In reality they satisfy this assumption tolerably well, for those vowels which are spoken with a wide funnel-shaped cavity of the mouth, as *a* in *art*, or *a* in *bat* lengthened, which is nearly the same as *a* in *bare*. But this relation is materially altered by the resonance which takes place in the cavity of the

mouth. The more this cavity is narrowed, either by the lips or the tongue, the more distinctly marked is its resonance for tones of determinate pitch, and the more therefore does this resonance reinforce those partial tones in the compound tone produced by the vocal chords, which approach the favoured pitch, and the more, on the contrary, will the others be damped. Hence on investigating the compound tones of the human voice by means of resonators we find pretty uniformly that the first six to eight partial tones are clearly perceptible, but with very different degrees of force according to the different forms of the cavity of the mouth, sometimes screaming loudly into the ear, at others scarcely audible.

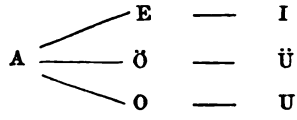
Under these circumstances the investigation of the resonance of the cavity of the mouth is of great importance. The easiest and surest method of finding the tones to which the air in the oral cavity is tuned for the different shapes it assumes in the production of vowels, is that which is used for glass bottles and other spaces filled with air. Tuning forks of different pitches are struck and held before the opening of the air chamber—in the present case the open mouth—and the louder the proper tone of the fork is heard, the nearer does it correspond with one of the proper tones of the included mass of air. Since the shape of the oral cavity can be altered at pleasure, it can always be made to suit the tone of any given tuning fork, and we thus easily discover what shape the mouth must assume for its included mass of air to be tuned to a determinate pitch.

Having a series of tuning forks at command, I was thus able to obtain the following results :—

The pitch of strongest resonance of the oral cavity depends solely upon the vowel for pronouncing which the mouth has been arranged, and alters considerably for even slight alterations in the vowel quality, such, for example, as occur in the different dialects of the same language. On the other hand, the proper tones of the cavity of the mouth are nearly independent of age and sex. I have in general found the same resonances in men, women, and children. The want of space in the oral cavity of women and children can be easily replaced by a great closure of its opening, which will make the resonance as deep as in the larger oral cavities of men.

The vowels can be arranged in three series, according to the

position of the parts of the mouth, which may be written thus, in accordance with Du Bois-Reymond the elder [1]:—



The vowel A (*a* in *father*, or Scotch *a* in *man*) forms the common origin of all three series. With this vowel corresponds a funnel-shaped resonance cavity, enlarging with tolerable uniformity from the larynx to the lips. For the vowels of the lower series, O (*o* in *more*) and U (*oo* in *poor*), the opening of the mouth is contracted by means of the lips, more for U than for O, while the cavity is enlarged as much as possible by depression of the tongue, so that on the whole it becomes like a bottle without a neck, with rather a narrow mouth, and a cavity undi-

[1] 'Norddeutsche Zeitschrift,' edited by de la Motte Fouqué, 1812. 'Kadmus oder allgemeine Alphabetik, von F. H. du Bois-Reymond,' Berlin, 1862, p. 152. [This is the arrangement usually adopted. But in 1867 Mr. Melville Bell, an orthoepical teacher of many years' standing, who had been led professionally to pay great attention to the shapes of the mouth necessary to produce certain sounds, in his 'Visible Speech; the Science of Universal Alphabetics' (London: Simpkin, Marshall and Co., 4to., pp. x., 126, with sixteen lithographic tables) proposed a more elaborate method of classifying vowels by the shape of the mouth. He commenced with 9 positions of the tongue, consisting of 3 in which the middle, or as he terms it, 'front' of the tongue was raised, highest for *ea* in *seat*, not so high for *a* in *sate*, and lowest for *a* in *sat*; 3 others in which the *back*, instead of the middle, of the tongue was raised, highest for *oo* in *snood*, lower for *o* in *node*, and lowest for *aw* in *gnawed* (none of which three are determined by the position of the tongue alone), and 3 intermediate positions, where the whole tongue is raised almost evenly at three different elevations. These 9 lingual positions might be accompanied with the ordinary or with increased distension of the pharynx, giving 9 primary and 9 'wide' vowels. And each of the 18 vowels thus produced, could be 'rounded,' that is modified by shading the mouth in various degrees with the lips. He thus obtains 36 distinct vowel cavities, among which almost all those used for vowel qualities in different nations may be placed. Subsequent research has shown how to extend this arrangement materially. See my 'Early English Pronunciation,' p. 1279. In following Prof. Helmholtz it must be remembered that the German vowels differ materially in quality from the English, and that consequently complete agreement between his observations and those of any Englishman, who repeats his experiments, must not be expected. I have consequently thought it better in this place to leave his German notation untranslated, and merely subjoin in parentheses the nearest English sounds. For the table in the text we may assume A to = *a* in *father*, or else Scotch *a* in *man* (different sounds), E to = *e* in *there*, I to = *i* in *machine*, O to = *o* in *more*, U to = *u* in *sure*; and Ö to = *eu* in French *peu* or else in *peuple* (different sounds), and Ü to = *u* in French *pu*.—*Translator*.]

vided in any direction.[1] The pitch of such bottles is lower the larger the cavity and the narrower its mouth. Usually only one upper partial with strong resonance can be clearly recognised; when other proper tones exist they are comparatively very high, or have only weak resonance. In conformity with these results, which may be obtained with glass bottles, we find the resonance deepest for U (*oo* in *poor* nearly), where the oral cavity is widest and the mouth narrowest; it answers to the unaccented *f*. On passing from U to O (*o* in *more* nearly) the resonance gradually rises; and for a full, ringing, pure O the pitch is *b'*♭. The position of the mouth for O is peculiarly favourable for resonance, the opening of the mouth being neither too large nor too small, and the internal cavity sufficiently spacious. Hence if a *b'*♭ tuning-fork be struck and held before the mouth while O is gently uttered, or the O-position merely assumed without really speaking, the tone of the fork will resound so fully and loudly that a large audience can hear it. The usual *a'* tuning fork of musicians may also be used for this purpose, but then it will be necessary to make a somewhat duller O, if we wish to bring out the full resonance.

On gradually bringing the shape of the mouth from the position proper to O, through those due to O[a] (nearly *o* in *cot*, with rather more of the O sound), and A[o] (nearly *au* in *caught*, with rather more of the A sound) into that for A (Scotch *a* in *man*, with rather more of an O quality in it than English *a* in *father*), the resonance gradually rises an Octave, and reaches *b''*♭. This tone corresponds with the North German A; the somewhat brighter A (*a* in *father*) of the English and Italians rises up to *d'''*, or a Third higher. It is particularly remarkable what little differences in pitch correspond to very sensible varieties of vowel quality in the neighbourhood of A; and I should therefore recom-

[1] [This depressed position of the tongue answers better for English *aw* in *saw* than for either *o* in *more* or *oo* in *poor*. For the *o* the tongue is slightly more raised, especially at the back, while for English *oo* the back of the tongue is almost as high as for *k*, and greatly impedes the oral cavity. If, however, the tongue be kept in the position for *aw* by sounding this vowel, and, while sounding it steadily, the lips be gradually contracted, the sound will be found to pass through certain obscure qualities of tone till it suddenly comes out clearly as a sound a little more like *aw* than *o* in more (really the Danish *aa*), and then again passing through other obscure phases, comes out again clearly as a deep sound, not so bright as our *oo* in *poor*, but more resembling the Swedish *o* to which it will reach if the tongue be slightly raised into the A position. It is necessary to bear these facts in mind when following the text, where U is only almost, not quite = *oo* in *poor*, which is the long sound of *u* in *pull*, and is duller than *oo* in *pool* or French *ou* in *poule*.—*Translator*.]

mend philologists who wish to define the vowels of different languages to fix them by the pitch of loudest resonance.[1]

For the vowels already mentioned I have not been able to detect any second proper tone, and the analogy of the phenomena presented by artificial resonance chambers of similar shapes would hardly lead us to expect any of sensible loudness. Experiments hereafter described shew that the resonance of this single tone is sufficient to characterise the vowels above mentioned.

The second series of vowels consists of A, Ä, E, I. The lips are drawn so far apart that they no longer contract the issuing stream of air, but a fresh contraction is produced between the front (middle) parts of the tongue and the hard palate, the space immediately above the larynx being widened by depressing the root of the tongue, while the larynx simultaneously rises. The form of the oral cavity consequently resembles a bottle with a narrow neck. The belly of the bottle is behind, in the pharynx, and its neck is the narrow passage between the upper surface of the tongue and the hard palate. In the above series of letters, Ä, E, I, these changes increase until for I the internal cavity of the bottle is greatest and the neck narrowest. For Ä (the broadest French *é*, broader than *e* in *there*, and nearly as broad as *a* in *bat* lengthened, with which the name of their city is pronounced by the natives of Bath), the whole channel is, however, tolerably wide, so that it is quite easy to see down to the larynx when the laryngoscope is used. Indeed this vowel gives the very best position of the mouth for the application of this instrument, because the root of the tongue, which impedes the view when A is uttered, is depressed, and the observer can see over and past it.

[1] [Great difficulties lie in the way of carrying out this recommendation. The ear of philologists and even of those who are readily able to discriminate vowel sounds, is frequently not acute for differences of pitch. The determination of the pitch even under favourable circumstances is not easy, especially, as it will be seen, for the higher pitches. Without mechanical appliances even good ears are deceived in the Octave. The differences of pitch noted by Helmholtz, Donders, and Merkel, as given further on, probably point to fundamental differences of pronunciation, and shew the desirability of a very extensive series of experiments being carried out with special apparatus, by an operator with an extremely acute musical ear, on speakers of various nationalities and also on various speakers of the same nationality. Great difficulty will even then be experienced on account of the variability of the same speaker in his vowel quality for differences of pitch and expression, the want of habit to maintain the position of the mouth unmoved for a sufficient length of time to complete an observation satisfactorily, and, worst of all, the involuntary tendency of the organs to accommodate themselves to the pitch of the fork presented.—*Translator.*]

When a bottle with a long narrow neck is used as a resonance chamber, two simple tones are readily discovered, of which one can be regarded as the proper tone of the belly, and the other as that of the neck of the bottle. Of course the air in the belly cannot vibrate quite independently of that in the neck, and both proper tones in question must consequently be different, and indeed somewhat deeper than they would be if belly and neck were separate and had their resonance examined independently. The neck is approximately a short pipe open at both ends. To be sure its inner end debouches into the cavity of the bottle instead of the open air, but if the neck is very narrow, and the belly of the bottle very wide, the latter may be in some respect regarded as an open space in respect of the vibrations of the air inclosed in the neck. These conditions are best satisfied for I, in which the length of the channel between tongue and palate, measured from the upper teeth to the back edge of the bony palate, is about six centimètres (2·36 or about $2\frac{1}{3}$ inches). An open pipe of this length when blown would give e'''', while the observations made for determining the tone of loudest resonance for I gives nearly d'''', which is as close an agreement as could possibly have been expected in such an irregularly shaped pipe as that formed by the tongue and palate.

In accordance with these experiments the vowels Ä, E, I have each a higher and a deeper resonance tone. The higher tones continue the ascending series of the proper tones of the vowels U, O, A. By means of tuning forks I found for Ä a tone between g''' and $a'''\flat$, and for E the tone $b'''\flat$. I had no fork suitable for I, but by means of the whistling noise of the air, to be considered presently, its proper tone was determined with tolerable exactness to be d''''.

The deeper proper tones which are due to the back part of the oral cavity are rather more difficult to discover. Tuning forks may be used, but the resonance is comparatively weak, because it must be conducted through the long narrow neck of the air chamber. It must further be remembered that this resonance only occurs while the corresponding vowel is gently whispered, and disappears as soon as the whisper ceases, because the form of the chamber on which the resonance depends then immediately changes. The tuning forks after being struck must be brought as close as possible to the opening of the air chamber behind the upper teeth. By this means I found d'' for Ä and f' for E. For

I, direct observation with tuning-forks was not possible: but from the upper partial tones, I conclude that its proper tone is as deep as that of U, or near *f*. Hence, when we pass from A to I, these deeper proper tones of the oral cavity sink, and the higher ones rise in pitch.[1]

For the third series of vowels from A through Ö (French *eu* in *peu*, or the deeper *eu* in *peuple*), towards Ü (French *u* in *pu*, which is rather deeper than the German sound), we have the same internal positions of the mouth as in the last-named. For Ü the mouth is placed in nearly the same position as for a vowel lying between E and I, and for Ö as for an E which inclines towards Ä. In addition to the contraction between the tongue and palate as in the second series, we have also a contraction of the lips, which are made into some sort of a tube, forming a front prolongation of that made by the tongue and palate. The air-chamber of the mouth, therefore, in this case also resembles a bottle with a neck, but the neck is longer than before. For I the neck was six centimètres (rather more than $2\frac{1}{3}$ inches) long, for Ü, measured from the front edge of the upper teeth to the commencement of the soft palate, it is eight centimètres (3·15, or rather more than $3\frac{1}{10}$ inches). The pitch of the higher proper tone corresponding to the resonance of the neck must be therefore about a Fourth deeper than for I. If both ends were free, a pipe of this length would give *b'''*, according to the usual calculation. In reality it resounded for a fork lying between *g'''* and *a'''*♭, a difference similar to that found for I, and also probably attributable to the back end of the tube debouching into a wider but not quite open space. The resonance of the back space has to be observed in the same way as for the I series. For Ö it is *f'*, the same as for E, and for Ü it is *f*, the same as for I.

The fact that the cavity of the mouth for different vowels is

[1] [Mr. Graham Bell, son of Mr. Melville Bell, already mentioned (p. 157, note), was in the habit of bringing out this fact by placing his mouth in the required positions and then tapping against a finger placed just in front of the upper teeth, for the higher resonance, and placed against the neck, just above the larynx, for the lower. He obligingly performed the experiment several times privately before me, and the successive alterations and differences in their direction were striking. The tone was dull and like a wood harmonica. Considerable dexterity seemed necessary to produce the effect, and I could not succeed in doing so. He carried out the experiment much further than is suggested in the text, embracing the whole nine positions of the tongue in his father's vowel scheme, and obtaining a double resonance in each case.—*Translator*.]

tuned to different pitches was first discovered by Donders,[1] not with the help of tuning-forks, but by the whistling noise produced in the mouth by whispering. The cavity of the mouth is thus blown like an organ pipe, and by its resonance reinforces the corresponding tones of the wind-rush, which are produced partly in the contracted glottis,[2] and partly in the forward contracted passages of the mouth. In this way it is not usual to obtain a complete musical tone; this only happens for Ü and U, when a real whistle is produced. This, however, would be a fault in speaking. We have rather only such a degree of reinforcement of the noise of the air as occurs in an organ pipe, which does not speak well, either from a badly-constructed lip or an insufficient pressure of wind. But a noise of this kind, although not brought up to being a complete musical tone, has nevertheless a tolerably determinate pitch, which can be estimated by a practised ear. But, as in all cases where tones of very different qualities have to be compared, it is easy to make a mistake in the Octave. However, after some of the important pitches have been determined by tuning-forks, and others, as Ü and Ö, by allowing the whisper to pass into a regular whistle, the rest are easily determined by arranging them in a melodic progression with the first. Thus the series:—

Clear A	Ä	E	I
(*a* in *father*) *d'''*	(*a* in *man*) *g'''*	(*e* in *there*) *b'''*♭	(*i* in *machine*) *d''''*

[1] 'Archiv für die Holländischen Beiträge für Natur- und Heilkunde von *Donders* und *Berlin*,' vol. i. p. 157. Older incomplete observations of the same circumstance in Samuel Reyher's 'Mathesis Mosaica,' Kiel 1619.—Chr. Hellwag, 'De Formatione Loquelae Diss., Tubingae,' 1710.—Flörcke, 'Neue Berliner Monatsschrift,' Sept. 1803, Feb. 1804.—Olivier, 'Ortho-epo-graphisches Elementar-Werk,' 1804, part iii. p. 21.

[2] In whispering, the vocal chords are kept close, but the air passes through a small triangular opening at the back part of the glottis between the arytenoid cartilages. [According to Czermak ('Sitzungsberichte, Wiener Akad., Math.-Naturw. Cl.' April 29, 1858, p. 576) the vocal chords as seen through the laryngoscope are not quite close for whisper, but are nicked in the middle. Merkel ('Die Funktionen des menschlichen Schlund- und Kehlkopfes. . . . nach eigenen pharyngo- und laryngoskopischen Untersuchungen,' Leipzig, 1862, p. 77) distinguishes two kinds of whispering: (1) the loud, in which the opening between the chords is from $\frac{1}{8}$ to $\frac{3}{4}$ of a line wide, producing no resonant vibrations, and that between the arytenoids is somewhat wider; (2) the gentle, in which the vowel is commenced as in loud speaking, with closed glottis, and, after it has begun, the back part of the glottis is opened, while the chords remain close and motionless.—*Translator.*]

forms an ascending minor chord of *g* in the position of the Fourth and Sixth, and can be readily compared with the same melodic progression on the pianoforte. I was able to determine the pitch for clear A, Ä, and E by tuning-forks, and hence fix that for I also.[1]

[1] The statements of Donders differ slightly from mine, partly because they have reference to Dutch pronunciation, while mine refer to the North German vowels; and partly because Donders, not having been assisted by tuning-forks, was not always able to determine with certainty to what Octave the noises he heard should be assigned.

Vowel	Pitch according to Donders	Pitch according to Helmholtz
U	*f′*	*f*
O	*d′*	*b′*♭
A	*b′*♭	*b″*♭
Ö	*g* ?	*c‴*
Ü	*a″*	*g‴* – *a‴*♭
E	*c‴*♯	*b‴*♭
I	*f‴*	*d⁗*

[The following passage and table translated from C. L. Merkel ('Physiologie der menschlichen Sprache (physiologische Laletik),' Leipzig, 1866, p. 47) will shew what great uncertainty still prevails in the determination of these pitches. The pitch according to Donders, given in this table, differs materially from the foregoing. Merkel, who is a Middle German, says:—'On attempting to determine the vibrational numbers of isolated vowels, I obtained very various results. I was often doubtful as to the Octave; as to whether the pitch of U which seemed to be the most fixed, was *d* or *d′*, and still more as to the pitch of I, whether, assuming that of U to be *d*, that of I belonged to the once-accented or twice-accented Octave. The case was still worse for the other vowels, especially for the intermediate or mixed vowels, Ä, Ö, Ü. The reader will obtain an idea of the difficulties by the following table of my own various results and those of my predecessors:—

Observer.	U	O	A	Ö	Ü	Ä	E	I
Merkel	*d* – *d*♯	*f* – *f*♯ – *g*	*a* – *b* – *c′*	*g g′ d″* *f′*♯ – *g′* + *d″* – *d″*♯	*a″ d‴* *a′* + *c″* – *d″* – *c″*	*d′ g′ d″* (*g*♯ – *a′*) + *d″*	*f′* + *d″* + *f″*	*g″ a″ b″*
Beyher	*c*	*d*♯	*a c′*	—	—	*d*♯	*f′*	*c″*
Hellwag	*c*	*c*♯	*f*♯	*g*♯	*b*♭	*a*	*b*	*c′*
Flörcke	*c*	*g*	*c′*	*c′*	*g′*	*g′*	*a′*	*c″*
Donders	*e f*	*e*	*b*	*g*	*a′*	—	*c′*	*f″*
Helmholtz	*f*	*b′*♭	*b″*♭ – *a‴*	[1]) *c″*♯ [2]) *f′*	[1]) *g″* [2]) *f*	[1]) *g″* – *a‴*♭ [2]) *a″*	[1]) *b″*♭ [2]) *f′*	[1]) *d⁗* [2]) *f*

where [1]) refers to the pitch of the mouth, and [2]) to that of the pharynx.'

And on p. 109 Merkel gives the following as the results of his most recent experi-

partial tones in the vowels, and I was not acquainted with them when my previous accounts were published.[1] They are best observed in high notes of women's voices, or the falsetto of men's voices. The upper partials of high notes in that part of the scale are not so nearly of the same pitch as those of deeper notes, and hence they are more readily distinguished. On *b'*♭, for example, women's voices could easily bring out all the vowels, with a full quality of tone, but at higher pitches the choice is more limited. When *b'*♭ is sung, then, the Twelfth *f'''* is heard for the broad Ä, the double Octave *b'''*♭ for E, the high Third *d''''* for I, all clearly, the last even piercingly. [See table on p. 181, note.]

In these experiments it must be remembered that certain vowels speak better in certain parts of the scale than in others.[2] So far as my own observations go, which, however, have not been extensive in this division of my subject, vowels always speak best when their characteristic tone is a little higher than that of the note sung, and those speak next best which have a characteristic tone that is the second or third partial of the note sung. Thus, for men's voices, U, having the characteristic pitch *f*, speaks best on *d*, *e*, and *f*, and then, an Octave deeper, or *F*. E, with the characteristic tone *f'*, speaks best on the upper notes of the bass *d'*, *e'*, and *f'*, and next in order on the harmonic subtones of *f'*, namely, *f* and B♭. At *b'*♭, which is the limit of my own falsetto, I can only sing O, Ä, or A^5, for which *b'*♭ is characteristic. The influence of the vowels is most striking on those notes which are difficult to reach because they lie at the limits of the voice. Below *c'* all female voices incline to an obscure O or O^u, the proper tones of these vowels lying in this neighbourhood. In women's high notes, above *e''* or *f''*, the vowel A speaks best, its characteristic tone being *b''*♭. Above *b''*♭ the vowel I sounds best; its proper tone lies an Octave higher, and it is more agreeable to the ear than Ä, which lies at the same height.[3]

[1] 'Gelehrte Anzeigen der Bayerischen Akademie der Wissenschaften,' June 18, 1859.

[2] These differences, which are clearly of great importance for singers, have been further considered in E. Sieler, 'Altes und Neues über die Ausbildung des Gesangorgans,' Leipzig, 1861, p. 52.—G. Engel, 'Die Vocal-Theorie,' Berlin, 1867, and the same author's paper in Reichert's 'Archiv für Anatomie,' 1869, p. 309. [Madame E. Seiler's work has been translated into English, under the title: 'The Voice in Singing,' Philadelphia, J. B. Lippincott and Co., 1871. The passage referred to is p. 102 of this translation —*Translator.*]

[3] [The vowel actually best suited to the pitch sung, varies also with the *register* in

When, for the purpose of observing the partial tones, a note is chosen for singing, and it happens to be one for which certain vowels speak with remarkable power, the upper partials are heard proportionably too loud. This is not of much consequence in the deeper male voices, because U and I are the only vowels which have reinforcements at a low pitch; and that pitch lies in the most convenient middle region of the voice, where the strength of the different vowels can be easily equalised. For female voices, on the other hand, the influence is much greater. Thus the high soprano notes, which fall into the reinforcing region of the vowel A, speak so much more powerfully on A than on any other vowel, that even the upper partials of such an A, lying in the upper half of the thrice-accented Octave, come out much stronger than the partial tones of E and I, which are here reinforced. Hence we must always select from among the harmonic subtones of the resonator applied, one on which the singer can give the vowels under observation for comparison, with equal power, or at least warn him to moderate his tone in the favourable cases, so as to bring them down to the level of the unfavourable. In observations with resonators as well as with tuning-forks, I have found the pitch of the reinforced partial tones the same in several female voices as in male voices, with the exception that in women the reinforced tones of U and I were too deep to be sensible.

A peculiar circumstance must also be mentioned which distinguishes the human voice from all other instruments and has a peculiar relation to the human ear. Above the higher reinforced partial tones of I, in the neighbourhood of *e''''* up to *g''''*, the notes of a pianoforte have a peculiar cutting effect, and we might be easily led to believe that the hammers were too hard, or that their mechanism somewhat differed from that of the adjacent notes. But the phenomenon is the same on all pianofortes, and if a very small glass tube or sphere is applied to the ear, the cutting effect ceases, and these notes become as soft and weak as the rest, but another and deeper series of notes now becomes stronger and more cutting. Hence it follows that the human ear by its own resonance favours the tones between *e''''* and *g''''*, or, in other words,

which it is sung. On the boundaries of the registers in any voice there are generally several notes which can be sung in either register. The scale of vowels which seems to ascend from U towards I with the ascending voice, retrogrades when the register is changed. Observations on this subject are still much needed.—*Translator.*]

that it is tuned to one of these pitches.[1] These notes produce a feeling of pain in sensitive ears. Hence the upper partial tones which have nearly this pitch, if any such exist, are extremely prominent and affect the ear powerfully. This is generally the case for the human voice when it is strained, and will help to give it a screaming effect. In powerful male voices singing forte, these partial tones sound like a clear tinkling of little bells, accompanying the voice, and are most audible in choruses, when the singers shout a little. Every individual male voice at such pitches produces dissonant upper partials. When basses sing their high *e'*, the 7th[2] partial tone is *d''''*, the 8th *e''''*, the 9th *f''''*♯, and the 10th *g''''*♯. Now, if *e''''* and *f''''*♯ are loud, and *d''''* and *g''''*♯, though weaker, are audible, there is of course a sharp dissonance. If many voices are sounding together, producing these upper partials with small differences of pitch, the result is a very peculiar kind of tinkling, which is readily recognised a second time when attention has been once drawn to it. I have not noticed any difference of effect for different vowels in this case, but the tinkling ceases as soon as the voices are taken *piano*; although the tone produced by a chorus will of course still have considerable power. This kind of tinkling is peculiar to human voices; orchestral instruments do not produce it in the same way either so sensibly or so powerfully. I have never heard it from any other musical instrument so clearly as from human voices.

The same upper partials are heard also in soprano voices when they sing forte; in harsh, uncertain voices they are tremulous, and hence shew some resemblance to the tinkling heard in the tones of male voices. But I have heard them brought out with exact purity, and continue to sound on perfectly and quietly, in some steady and harmonious female voices, and also in some excellent tenor voices. In the melodic progression of a voice part, I then hear these high upper partials of the four-times accented Octave, falling and rising at different times within the compass of a minor Third, according as different upper partials of the notes sung enter

[1] I have lately found that my right ear is most sensitive for *f''''*, and my left for *c''''*. When I drive air into the passage leading to the tympanum, the resonance descends to *c''''*♯ and *g''''*♯. The chirp of the cricket corresponds precisely to the higher resonance, and on merely applying a short paper tube to the entrance of my ear, this chirp is rendered extraordinarily weak.

[2] [The first six partial tones are *e'*, *e''*, *b''*, *e'''*, *g'''*♯, *b'''*, the seventh is slightly flatter than *d''''*.—*Translator*.]

the region for which our ear is so sensitive. It is certainly remarkable that it should be precisely the human voice which is so rich in those upper partials for which the human ear is so sensitive. Madame E. Seiler, however, remarks that dogs are also very sensitive for the high *e*'''' of the violin.

This reinforcement of the upper partial tones belonging to the middle of the four-times accented Octave, has, however, nothing to do with the characterisation of vowels: I have only mentioned it because these high tones are readily remarked in investigations into the vowel qualities of tone, and the observer must not be misled to consider them as peculiar characteristics of individual vowels. They are simply a characteristic of strained voices.

The humming tone heard when singing with closed mouth, lies nearest to U. This hum is used in uttering the consonants M, N and N^{g}. The size of the exit of the air (the nostrils) is in this case much smaller in comparison with the resonant chamber (the internal nasal cavity) than the opening of the lips for U in comparison with the corresponding resonant chamber in the mouth. Hence, in humming, the peculiarities of the U tone are much enhanced. Thus although upper partials are present, even up to a considerably high pitch, yet they decrease in strength as they rise in pitch much faster than for U. The upper Octave is tolerably strong in humming, but all the higher partial tones are weak. Humming in the N-position differs a little from that in the M-position, by having its upper partials less damped than for M. But it is only at the instant when the cavity of the mouth is opened or closed that a clear difference exists between these consonants. We cannot enter into the details of the composition of the sound of the other consonants, because they produce noises which have no constant pitch, and are not musical tones, to which we have here to confine our attention.

The theory of vowel sounds here explained may be confirmed by experiments with artificial reed pipes, to which proper resonant chambers are attached. This was first done by Willis, who attached reed pipes to cylindrical chambers of variable length, and produced different tones by increasing the length of the resonant tube. The shortest tubes gave him I, and then E, A, O, up to U, until the tube exceeded the length of a quarter of a wave. On further increasing the length the vowels returned in converse order. His determination of the pitch of the re-

sonant pipes agrees well with mine for the deeper vowels. The pitch found by Willis for the higher vowels was relatively too high, because in this case the length of the waves was smaller than the diameter of the tubes, and consequently the usual calculation of pitch from the length of the tubes alone was no longer applicable.

The vowels E and I were also far from accurately resembling those of the voice, because the second resonance was absent, and hence, as Willis himself states, they could not be well distinguished.

Vowel	In the Word[1]	Pitch, Willis	Pitch, Helmholtz	Length of Tube in Inches
O	No	c''	c''	4·7
A°	Nought	$e''\flat$	$e''\flat$	3·8
	Paw	g''	g''	3·05
A	Part	$a'''\flat$	$a'''\flat$	2·2
	Pad	f'''		1·8
E	Pay	a''''	$b''\flat$	1·0
	Pet	c''''	c''''	0·6
I	See	g''''	d''''	0·38 (?)

The vowels are obtained much more clearly and distinctly with properly tuned resonators, than with cylindrical resonance chambers. On applying to a reed pipe which gave $b\flat$, a glass resonator tuned to $b\flat$, I obtained the vowel U; changing the resonator to one tuned for $b'\flat$, I obtained O; the $b''\flat$ resonator gave a rather close A, and the d''' resonator a clear A. Hence by tuning the applied chambers in the same way we obtain the same vowels quite independently of the form of the chamber and nature of its walls. I also succeeded in producing various grades of Ä, Ö, E, and I with

[1] [Probably the first treatise on phonology in which Willis's experiments were given at length, and the above table cited, with Wheatstone's article from the 'London and Westminster Review,' which was kindly brought under my notice by Sir Charles Wheatstone himself, was my *Alphabet of Nature*, London, 1845. The table includes U exemplified by *but*, *boot*, with an indefinite length of pipe. The word *pad* is misprinted *paa* in all the German editions of Helmholtz, and as he therefore could not separate its A from that in *part*, he gives no pitch. It is really the nearest English representative of the German Ä. The sounds in *nought*, *paw*, which Sir John Herschell, when citing Willis (Art. 'Sound,' in 'Encyc. Metropol.,' par. 375), could not distinguish, were probably meant for the broad Italian open O, or English *o* in *more*, and the English *aw* in *maw* respectively. The length of the pipe in inches is here added from Willis's paper. I have heard Willis's experiments repeated by Wheatstone.—*Translator*.]

the same reed pipe, by applying glass spheres into whose external opening glass tubes were inserted from 6 to 10 centimètres (2·36 to 3·93, say $2\frac{1}{2}$ to 4 inches) in length, in order to imitate the double resonance of the oral cavity for these vowels.

Willis has also given another interesting method for producing vowels. If a toothed wheel, with many teeth, is turned round rapidly, and a spring be applied to its teeth, the spring will be raised by each tooth as it passes, and a tone will be produced having its vibrational number equal to the number of teeth by which it has been struck in a second. Now if one end of the spring is well fastened, and the spring be set in vibration, it will itself produce a tone which will increase in pitch as the spring diminishes in length. If then we turn the wheel with a constant velocity, and allow a watch spring of variable length to strike against its teeth, we shall obtain for a long spring a quality of tone resembling U, and as we shorten the spring other qualities in succession like O, A, E, I, the tone of the spring here playing the part of the reinforced tone which determines the vowel. But this imitation of the vowels is certainly much less complete than that obtained by reed pipes. The reason of this process also evidently depends upon our producing compound tones in which certain upper partials (which in this case correspond with the proper tones of the spring itself) are more reinforced than others.

Willis himself advanced a theory concerning the nature of vowel tones which differs from that I have laid down in agreement with the whole connection of all other acoustical phenomena. Willis imagines that the pulses of air which produce the vowel qualities, are themselves tones which rapidly die away, corresponding to the proper tone of the spring in his last experiment, or the short echo produced by a pulse or a little explosion of air in the mouth, or in the resonance chamber of a reed pipe. In fact something like the sound of a vowel will be heard if we only tap against the teeth with a little rod, and set the cavity of the mouth in the position required for the different vowels. Willis's description of the motion of sound for vowels is certainly not a great way from the truth; but it only assigns the mode in which the motion of the air ensues, and not the corresponding reaction which this produces in the ear. That this kind of motion as well as all others is actually resolved by the ear into a series of partial tones, according to the laws of sympathetic resonance, is shewn by the

agreement of the analysis of vowel qualities of tone made by the unarmed ear and by the resonators. This will appear still more clearly in the next chapter, where experiments will be described shewing the direct composition of vowel qualities from their partial tones.

Vowel qualities of tone consequently are essentially distinguished from the tones of most other musical instruments by the fact that the loudness of their partial tones does not depend upon the numerical order but upon the absolute pitch of those partials; thus when I sing the vowel A to the note *E*♭, the reinforced tone *b*″♭ is the 12th partial tone of the compound; and when I sing the same vowel A to the note *b*′♭, the reinforced tone is still *b*″♭, but is now the 2nd partial of the compound tone sung.

From the examples adduced to show the dependence of quality of tone from the mode in which a musical tone is compounded, we may deduce the following general rules:—

1. *Simple Tones*, like those of tuning-forks applied to resonance chambers and wide stopped organ pipes, have a very soft, pleasant sound, free from all roughness, but wanting in power, and dull at low pitches.

2. *Musical Tones*, which are accompanied by a moderately loud series of the lower upper partial tones, up to about the sixth partial,[1] are more harmonious and musical. Compared with simple tones they are rich and splendid, while they are at the same time perfectly sweet and soft if the higher upper partials are absent. To these belong the musical tones produced by the pianoforte, open organ pipes, the softer piano tones of the human voice and of the French horn. The last-named tones form the transition to musical tones with high upper partials; while the tones of flutes, and of pipes on the flute-stops of organs with a low pressure of wind, approach to simple tones.

3. If only the uneven partials are present (as in narrow stopped organ pipes, pianoforte strings struck in their middle points, and

[1] [That is the fifth *upper* partial, excluding the prime tone. By an accident the German text says the 'sixth *upper* partial,' which would be the *seventh* partial, and produce some degree of roughness.—*Translator.*]

clarinets), the quality of tone is *hollow*, and, when a large number of such upper partials are present, *nasal*.[1] When the prime tone predominates the quality of tone is *rich* or *full*; but when the prime tone is not sufficiently superior in strength to the upper partials, the quality of tone is *poor* or *empty*. Thus the quality of tone in the wider open organ pipes is fuller than that in the narrower; strings struck with pianoforte hammers give tones of a fuller quality than when struck by a stick or pulled by the finger; the tones of reed pipes with suitable resonance chambers have a fuller quality than those without resonance chambers.

4. When partial tones higher than the sixth or seventh are very distinct, the quality of tone is *cutting* and *rough*. The reason for this will be seen hereafter to lie in the dissonances which they form with one another. The degree of harshness may be very different. When their force is inconsiderable the higher upper partials do not essentially detract from the musical applicability of the compound tones; on the contrary, they are useful in giving character and expression to the music. The most important musical tones of this description are those of bowed instruments and of most reed pipes, oboe (hautbois), bassoon (fagot), physharmonica (harmonium, concertina, accordion), and the human voice. The rough, braying tones of brass instruments are extremely penetrating, and hence are better adapted to give the impression of great power than similar tones of a softer quality. They are consequently little suitable for artistic music when used alone, but produce great effect in an orchestra. Why high dissonant upper partials should make a musical tone more penetrating will appear hereafter.

[1] ['When the bagpipe sings i' the nose.'—'Merch. of V.' Act IV., sc. i., v. 49.—*Translator.*]

CHAPTER VI.

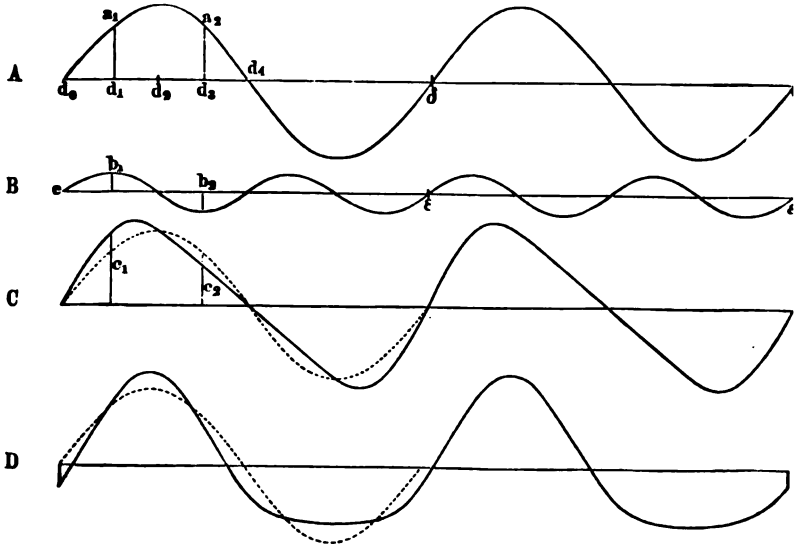
ON THE APPREHENSION OF QUALITIES OF TONE.

UP to this point we have not endeavoured to analyse given musical tones further than to determine the differences in the number and loudness of their partial tones. Before we can determine the function of the ear in apprehending qualities of tone, we must inquire whether a determinate relative strength of the upper partial tones suffices to give us the impression of a determinate musical quality, or whether there are not also other perceptible differences in quality which are independent of such a relation. Since we deal only with *musical tones*, that is, with such as are produced by exactly periodic motions of the air, and exclude all irregular motions of the air which appear as noises, we can give this question a more definite form. If we suppose the motion of the air corresponding to the given musical tone to be resolved into a sum of pendular vibrations of air, such individual pendular vibrations will not only differ from each other in force or amplitude for different forms of the compound motion, but also in their relative position, or, according to physical terminology, in their *difference of phase*. For example, if we superimpose the two pendular vibrational curves A and B, fig. 31 (p. 175), first with the point e of B on the point d_0 of A, and next with the point e of B on the point d^1 of A, we obtain the two entirely distinct vibrational curves C and D. By further displacement of the initial point e so as to place it on d_2 or d_3 we obtain other forms, which are the inversions of the forms C and D, as has been already shewn (supra, p. 49). If, then, musical quality of tone depends solely on the relative force of the partial tones, all the various motions C, D, &c. must make the same impression on the ear. But if the relative position of the two waves, that is the difference of phase, produces any effect, they must make different impressions on the ear.

Now to determine this point it was necessary to compound

various musical tones out of simple tones artificially, and to see whether an alteration of quality ensued when force was constant but phase varied. Simple tones of great purity, which can have both their force and phase exactly regulated, are best obtained from tuning-forks having the lowest proper tone reinforced by a resonance chamber, as has been already described (p. 87), and thus communicated to the air. To set the tuning-forks in very uniform motion, they were placed between the limbs of a little electro-magnet, as shewn in fig. 32 (p. 176). Each tuning-fork was screwed

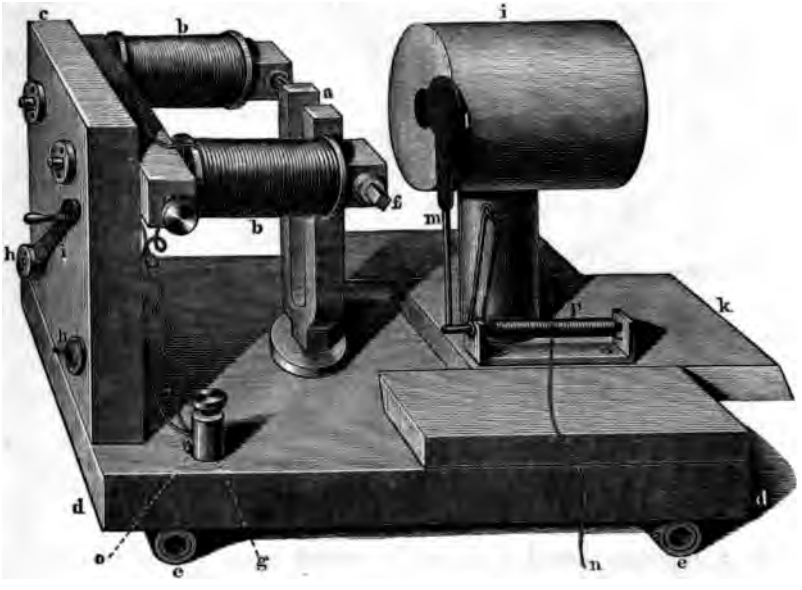
FIG. 31.



into a separate board d d, which rested upon pieces of india-rubber tubing e e that were cemented below it, to prevent the vibrations of the fork from being directly communicated to the table and hence becoming audible. The limbs b b of the electro-magnet are surrounded with wire, and its pole f is directed to the fork. There are two clamp screws g on the board d d which are in conductive connection with the coils of the electro-magnet, and serve to introduce other wires which conduct the electric current. To set the forks in strong vibration the strength of these streams must alternate periodically. These are generated by a separate apparatus, described below (fig. 33, p. 178).

When forks thus arranged are set in vibration, very little indeed of their tone is heard, because they have so little means of communicating their vibrations to the surrounding air or adjacent solids. To make the tone strongly audible, the resonance chamber i which has been previously tuned to the pitch of the fork, must be brought near it. This resonance chamber is fastened to another board k, which slides in a proper groove made in the board d d, and thus allows its opening to be brought very near to the fork. In the figure the resonance chamber it shewn at a distance from the fork in order to exhibit the separate parts distinctly; when in

FIG. 32.



use, it is brought as closely as possible to the fork. The mouth of the resonance chamber can be closed by a lid l attached to a lever m. By pulling the string n the lid is withdrawn from the opening and the tone of the fork is communicated to the air with great force. When the thread is let loose, the lid is brought over the mouth of the chamber by the spring p, and the tone of the fork is no longer heard. By partial opening of the mouth of the chamber, the tone of the fork can be made to receive any desired intermediate degree of strength. The whole of the strings which open the

various resonance chambers belonging to a series of such forks are attached to a keyboard in such a way that by pressing a key the corresponding chamber is opened.

At first I had eight forks of this kind, giving the tones B♭ and its first seven harmonic upper partials, namely *b*♭, *f′*, *b′*♭, *d″*, *f″*, *a″*♭,[1] and *b″*♭. The prime tone B♭ corresponds to the pitch in which bass voices naturally speak. Afterwards I had forks made of the pitches *d‴*, *f‴*, *a‴*♭[1] and *b‴*♭, and assumed *b*♭ for the prime tone of the compound.

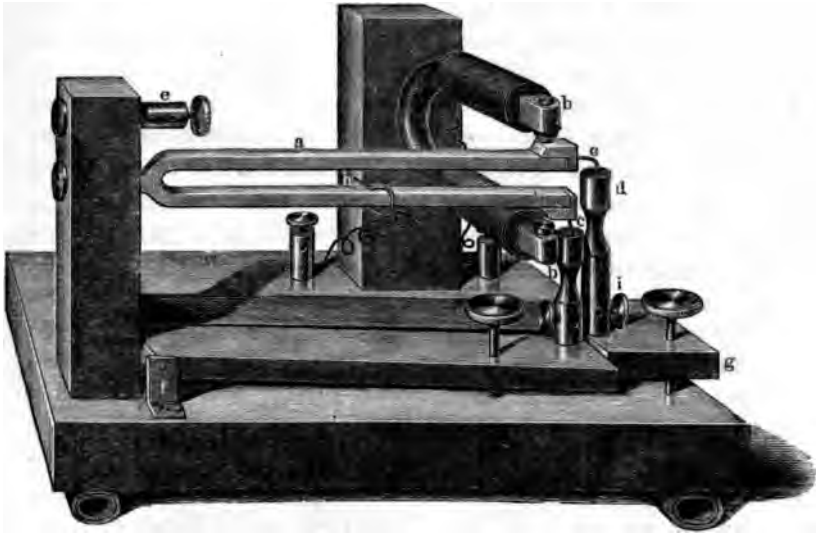
To set the forks in motion, intermittent electrical currents had to be conducted through the coils of the electro-magnet, giving as many electrical shocks as the lowest forks made vibrations in a second, namely 120. Every shock makes the iron of the electro-magnet b b momentarily magnetic, and hence enables it to attract the prongs of the fork, which are themselves rendered permanently magnetic. The prongs of the lowest fork B♭ are thus attracted by the poles of the electro-magnet, for a very short time, once in *every* vibration; the prongs of the second for *b*♭, which moves twice as fast, once every *second* vibration, and so on. The vibrations of the forks are therefore both excited and kept up as long as the electric currents pass through the apparatus. The vibrations of the lower forks are very powerful, those of the higher proportionally weaker.

The apparatus shewn in fig. 33 (p. 178) serves to produce intermittent currents of exactly determinate periodicity. A tuning-fork a is fixed horizontally between the limbs b b of an electro-magnet; at its extremities are fastened two platinum wires c c, which dip into two little cups d filled half with mercury and half with alcohol, forming the upper extremities of brass columns. These columns have clamping screws i to receive the wires, and stand on two boards f, g, which revolve about an axis, as at f, and which can each be somewhat raised or lowered by a thumb-screw, as at g, so as to make the points of the platinum wires c c exactly touch the mercury below the alcohol in the cups d. A third clamping screw e is in conductive connection with the handle of the tuning-fork. When the fork vibrates, and an electric current passes through it from i to e, it will be broken every time that the end of the fork a rises above the surface of the mercury in the cup d, and re-made

[1] [A little flatter than the *a″*b and *a‴*b, in the justly intoned scale of *e*b.—*Translator.*]

every time the platinum wire dips again into the mercury. This intermittent current being at the same time conducted through the electro-magnet b b, fig. 33, the latter becomes magnetic every time it passes, and thus keeps up the vibrations of the fork a, which is itself magnetic. Generally only one of the cups d is used for conducting the current inwards. Alcohol is poured over the mercury to prevent the latter from being burned by the electrical sparks which arise when the stream is interrupted. This method of interrupting the current was invented by Neef, who used a simple vibrating spring in place of the tuning-fork, as may be

FIG. 33.



generally seen in the induction apparatus so much used for medical purposes. But the vibrations of a spring communicate themselves to all adjacent bodies and are for our purposes both too audible and too irregular. Hence the necessity of substituting a tuning-fork for the spring. The handle of a well worked symmetrical tuning-fork is extremely little agitated by the vibrations of the fork and hence does not itself agitate the bodies connected with it, so powerfully as the fixed end of a straight spring. The tuning-fork of the apparatus in fig. 33, must be in exact unison with the prime tone B♭. To effect this I employ a little clamp of steel

wire h, placed on one of the prongs. By slipping this towards the free end of the prong the tone is deepened, and by slipping it towards the handle of the fork, the tone is raised.[1]

When the whole apparatus is in action, but the resonance chambers are closed, all the forks are maintained in a state of uniform motion, but no sound is heard, beyond a gentle humming caused by the direct action of the forks on the air. But on opening one or more resonance chambers, the corresponding tones are heard with sufficient loudness, and are louder as the lid is more widely opened. By this means it is possible to form in rapid succession, different combinations of the prime tone with one or more harmonic upper partials having different degrees of loudness, and thus produce tones of different qualities.

Among the natural musical tones which appear suitable for imitation with forks, the vowels of the human voice hold the first rank, because they are accompanied by comparatively little extraneous noise and shew distinct differences of quality which are easy to seize. Most vowels also are characterised by comparatively low upper partials, which can be reached by our forks; E and I alone somewhat exceed these limits. The motion of the very high forks is too weak for this purpose when influenced only by such electrical currents as I was able to use without disturbance from the noise of the electric sparks.

The first series of experiments was made with the eight forks $B\flat$ to $b''\flat$. With these U, O, Ö, and even A could be imitated; the last not very well because of my not possessing the upper partials c''' and d''', which lie immediately above its characteristic tone $b''\flat$, and are sensibly reinforced in the natural sound of this vowel. The prime tone $B\flat$ of this series, when sounded alone, gave a very dull U, much duller than could be produced in speech. The sound became more like U when the second and third partial tones $b\flat$ and f' were allowed to sound feebly at the same time. A very fine O was produced by taking $b'\flat$ strong, and $b\flat$, f', d'' more feebly; the prime tone $B\flat$ had, however, to be somewhat damped. On suddenly changing the pressure on the keys and hence the

[1] This apparatus was made by Fessel in Cologne. More detailed descriptions of its separate parts, and instructions for the experiments to be made by its means, are given in Appendix No. VIII. [This apparatus was exhibited by R. Koenig (see Appendix No. II.) in the International Exhibition of 1872 in London, and may also be procured through Mr. Ladd, Beak Street, Regent Street, London.—*Translator.*]

position of the lids before the resonance chambers, so as to give *B*♭ strong, and all the upper partials weak, the apparatus uttered a good clear U after the O.

A or rather A° (nearly *o* in *not*) was produced by making the fifth to the eighth partial tones as loud as possible, and keeping the rest under.

The vowels of the second and third series, which have higher characteristic tones, could be only imperfectly imitated by bringing out their reinforced tones of the lower pitch. Though not very clear in themselves they became so by contrast on alternation with U and O. Thus a passably clear Ä was obtained by giving loudness chiefly to the fourth and fifth tones, and keeping down the lower ones, and a sort of E by reinforcing the third, and letting the rest sound feebly. The difference between O and these two vowels lay principally in keeping the prime tone *B*♭ and its octave *b*♭ much weaker for Ä and E than for O.[1]

To extend my experiments to the clearer vowels, I afterwards added the forks *d'''*, *f'''*, *a'''*♭, *b'''*♭, the two upper ones of which, however, gave a very faint tone, and I chose *b*♭ as the prime tone in place of *B*♭. With these I got a very good Ä and A, and at least a much more distinct E than before. But I could not get up to the high characteristic tone of I.

In this higher series of forks, the prime tone *b*♭, when sounded alone, reproduced U. The same prime *b*♭ with moderate force, accompanied with a strong octave *b'*♭, and a weaker Twelfth *f''*, gave O, with the characteristic tone *b'*♭. A was obtained by taking *b*♭, *b'*♭, and *f''* moderately strong, and the characteristic tones *b''*♭ and *d'''* very strong. To change A into Ä it was necessary to increase somewhat the force of *b'*♭ and *f''* which were adjacent to the characteristic tone *d''*, to damp *b''*♭, and bring out *d'''* and *f'''* as strongly as possible. For E the two deepest tones of the series, *b*♭ and *b'*♭, had to be kept moderately loud, as being adjacent to the deeper characteristic tone *f'*, while the highest *f'''*, *a'''*♭, *b'''*♭ had to be made as prominent as possible. But I have hitherto not succeeded so well with this as with the other vowels, because the high forks were too weak, and because perhaps the upper partials which lie above the highest characteristic tone *b'''*♭ could not be entirely dispensed with.[2]

[1] The statements in the *Münchener gelehrte Anzeigen* for June 20, 1859, must be corrected accordingly. At that time I did not know the higher upper partials of E and I, and hence made the O too dull to distinguish it from the imperfect E.

[2] [The following tabular statement of the above results will serve to shew their rela-

In precisely the same way as the vowels of the human voice, it is possible to imitate the quality of tone produced by organ pipes of different stops, but of course the whizzing noise, formed at the lip of the pipes as it cuts the stream of air impinging on it, is wanting in these imitations. The tuning forks are necessarily limited to the imitation of the purely musical part of the tone. The piercing high upper partials, required for imitating reed instruments were absent, but the nasality of the clarinet was given by using a series of uneven partials, and the softer tones of the French horn by the full chorus of all the forks.

But though it was not possible to imitate every kind of quality of tone by the present apparatus, it sufficed to decide the important question as to the effect of altered difference of phase upon quality of tone. As I particularly observed at the beginning of this chapter, this question is of fundamental importance for the theory of auditory sensation. The reader who is unused to physical investigations must excuse some apparently difficult and dry details in the explanation of the experiments necessary for its decision.

tions more clearly. In the first line are placed the notes of the forks and the numbers of the corresponding partials. The letters *pp, p, mf, f, ff* below them are the usual musical indications of force, *pianissimo, piano, mezzo forte, forte, fortissimo.* Where no such mark is added the partial is not mentioned in the text. For the second series of experiments the forks of corresponding pitches are kept under the old ones, but the whole are now numbered as partials of *b'*♭.

First Forks		1 *B*♭	2 *b*♭	3 *f'*	4 *b'*♭	5 *a''*	6 *f''*	7 *a''*♭	8 *b''*♭	10	12	14	16
Vowels	U	*f*	*pp*	*pp*									
	O	*mf*	*p*	*p*	*f*	*p*							
	A°	*p*	*p*	*p*	*p*	*ff*	*ff*	*ff*	*ff*				
	Ä	*p*	*pp*	*p*	*f*	*f*							
	E	*p*	*pp*	*f*	*p*	*p*	*p*	*p*	*p*				
Second Forks			1 *b*♭		2 *b'*♭		3 *f''*		4 *b'*♭	5 *d'''*	6 *f''''*	7 *a'''*♭	8 *b'''*♭
Vowels	U		*f*										
	O		*mf*		*f*		*p*						
	A		*mf*		*mf*		*mf*		*f*	*f*			
	Ä				*f*		*f*		*p*	*ff*	*ff*		
	E		*mf*		*mf*						*ff*	*ff*	*ff*

Translator.]

The simple means of altering the phases of the secondary tones consists in bringing the resonance chambers somewhat out of tune by narrowing their apertures, which weakens the resonance, and at the same time alters the phase. If the resonance chamber is tuned so that the simple tone which excites its strongest resonance coincides with the simple tone of the corresponding fork, then, as the mathematical theory shews,[1] the greatest velocity of the air at the mouth of the chamber in an outward direction, coincides with the greatest velocity of the ends of the fork in an inward direction. On the other hand, if the chamber is tuned to be slightly deeper than the fork, the greatest velocity of the air slightly precedes, and if it is tuned slightly higher, that greatest velocity slightly lags behind the greatest velocity of the fork. The more the tuning is altered, the greater will be the difference of phase, till at last it reaches the duration of a quarter of a vibration. The magnitude of the difference of phase agrees during this change precisely with the strength of the resonance, so that to a certain degree we are able to measure the former by the latter. If we represent the strength of the sound in the resonance chamber when in unison with the fork by 10, and divide the periodic time of a vibration, like the circumference of a circle, into 360 degrees, the relation between the strength of the resonance and the difference of phase is shewn by the following table:—

Strength of Resonance	Difference of Phase in angular degrees
10	0°
9	35° 54′
8	50° 12′
7	60° 40′
6	68° 54′
5	75° 31′
4	80° 48′
3	84° 50′
2	87° 42′
1	89° 26′

This table shews that a comparatively slight weakening of resonance by altering the tuning of the chamber occasions considerable differences of phase, but that considerable alterations of strength occasion relatively small changes of phase. We can take

[1] See Appendix No. IX.

advantage of this circumstance when compounding the vowel sounds by means of the tuning-forks to produce every possible alteration of phase. It is only necessary to let the lid shade the mouth of the resonance chamber till the strength of the tone is perceptibly diminished. As soon as we have learned how to estimate roughly the amount of diminution of loudness, the above table gives us the corresponding alteration of phase. We are thus able to alter the vibrations of the tones in question to any amount, up to a quarter of the periodic time of a vibration. Alterations of phase to the amount of half the periodic time are produced by sending the electric current through the electro-magnets of the corresponding fork in an opposite direction, which causes the ends of the fork to be repelled instead of attracted by the electro-magnets on the passage of the current, and thus sets the fork vibrating in the contrary direction. This counter-excitement of the fork, however, by repelling currents, must not be continued too long, as the magnetism of the fork itself would otherwise gradually diminish, whereas attracting currents strengthen it or maintain it at a maximum. It is well known that the magnetism of masses of iron that are violently agitated is easily altered.

After a tone has been compounded, in which some of the partials have been weakened and at the same time altered in phase by the half-shading of the apertures of their corresponding resonance chambers, we can re-compound the same tone by an equal amount of weakening in the same partials, but without shading the aperture, and therefore without change of phase, by simply leaving the mouths of the chambers wide open, and increasing their distances from the exciting forks, until the required amount of enfeeblement of sound is attained.

For example, let us first sound the forks *B*♭ and *b*♭, with fully opened resonance chambers, and perfect unison. They will vibrate as shewn by the vibrational forms fig. 31, A and B (p. 175), with the points e and d_0 coincident, and produce at a distance the compound vibration represented by the vibrational curve C. But by closing the resonance chamber of the fork *B*♭ we can make the point e on the curve B coincide with the points between d_0 and d_1 on the curve A. To make e coincide with d_1, the loudness of *B*♭ must be made about three-quarters of what it would be if the mouth of the chamber were unshaded. The point e can be made to coincide with d_4 by reversing the current in the electro-magnets

and fully opening the mouth of the resonance chamber; and then by imperfectly opening the chamber of $B\flat$ the point e can be made to move towards δ. On the other hand, an imperfect opening of the chamber $b\flat$ will make e recede, from coincidence with δ (which is the same thing as coincidence with d_0) or with d_4, towards d_4 or d_3 respectively. The proportions of loudness may be made the same in all these cases, without any alteration of phase, by removing the corresponding chambers to the proper distance from its forks without shading its mouth.

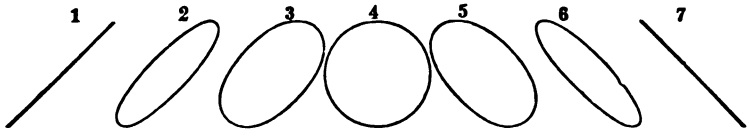
In this manner every possible difference of phase in the tones of two chambers can be produced. The same process can of course be applied to any required number of forks. I have thus experimented upon numerous combinations of tone with varied differences of phase, and I have never experienced the slightest difference in the quality of tone. So far as the quality of tone was concerned, I found that it was entirely indifferent whether I weakened the separate partial tones by shading the mouths of their resonance chambers, or by moving the chamber itself to a sufficient distance from the fork. Hence the answer to the proposed question is: *the quality of the musical portion of a compound tone depends solely on the number and relative strength of its partial simple tones, and in no respect on their differences of phase.*

The preceding proof that quality of tone is independent of difference of phase, is the easiest to carry out experimentally, but its force lies solely in the theoretical proposition that phases alter contemporaneously with strength of tone when the mouths of the resonance chambers are shaded, and this proposition is the result of mathematical theory alone. We cannot make vibrations of air directly visible. But by a slight change in the experiment it may be so conducted as to make the alteration of phase immediately visible. It is only necessary to put the tuning-forks themselves out of tune with their resonance chambers, by attaching little lumps of wax to the prongs. The same law holds for the phases of a tuning-fork kept in vibration by an electric current, as for the resonance chambers themselves. The phase gradually alters by a quarter period, while the strength of the tone of the fork is reduced from a maximum to nothing at all, by putting it out of tune. The phase of the motion of the air retains the same relation to the phase of the vibration of the fork, because the pitch, which is determined by the number of interruptions of the electrical current

in a second, is not altered by the alteration of the fork. The change of phase in the fork can be observed directly by means of Lissajou's vibration microscope, already described and shewn in fig. 22 (p. 129). Place the prongs of the fork and the microscope of this instrument horizontally, and the fork to be examined vertically; powder the upper end of one of its prongs with a little starch, direct the microscope to one of the grains of starch, and excite both forks by means of the electrical currents of the interrupting fork (fig. 33, p. 178.) The fork of Lissajou's instrument is in unison with the interrupting fork. The grain of starch vibrates horizontally, the object glass of the microscope vertically, and thus, by the composition of these two motions, curves are generated, just as in the observations on violin strings previously described.

When the observed fork is in unison with the interrupting fork, the curve becomes an oblique straight line (fig. 34, 1),

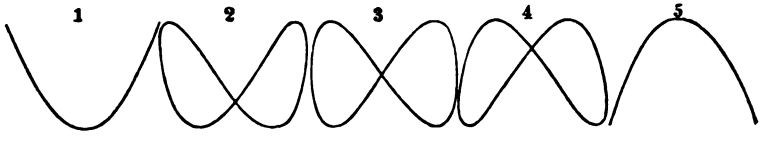
FIG. 34.



when both forks pass through their position of rest at the same moment. As the phase alters, the straight line passes through a long oblique ellipse (2, 3), till on the difference of phase becoming a quarter of a period, it develops into a circle (4); and then as the difference of phase increases, it passes through oblique ellipses (5, 6) in another direction, till it reaches another straight line (7), on the difference becoming half a period.

If the second fork is the upper Octave of the interrupting fork, the curves 1, 2, 3, 4, 5, in fig. 35, shew the series of forms.

FIG. 35.



Here 3 answers to the case when both forks pass through their position of rest at the same time; 2 and 4 differ from that position by $\frac{1}{12}$, and 1 and 5 by $\frac{1}{4}$ of a wave of the higher fork.

If we now bring the forks into the most perfect possible unison with the interrupting fork, so that both vibrate as strongly as possible, and then alter their tuning a little by putting on or removing pieces of wax, we also see one figure of the microscopic image gradually passing into another, and can thus easily assure ourselves of the correctness of the law already cited. Experiments on quality of tone are now conducted by first bringing all the forks as exactly as possible to the pitches of the harmonic upper partial tones of the interrupting fork, then removing the resonance chambers to such distances from the forks as will give the required relations of strength, and finally putting the forks out of tune as much as we please by sticking on lumps of wax. The size of these lumps should be previously so regulated by microscopical observation as to produce the required difference of phase. This, however, at the same time weakens the vibrations of the forks, and hence the strength of the tones must be restored to its former state by bringing the resonant chambers nearer to the forks.

The result in these experiments, where the forks are put out of tune, is the same as in those where the resonance chambers were put out of tune. There is no perceptible alteration of quality of tone. At least there is no alteration so marked as to be recognisable after the expiration of the few seconds necessary for resetting the apparatus, and hence certainly no such change of quality as would change one vowel into another.

An apparent exception to this rule must here be mentioned. If the forks $B\flat$ and $b\flat$ are not perfectly tuned, and are brought into vibration by rubbing or striking, an attentive ear will observe very weak beats which appear like small changes in the strength of the tone and its quality. These beats are certainly connected with the successive entrance of the vibrating forks on varying difference of phase. Their explanation will be given when combinational tones are considered, and it will then be shewn that these slight variations of quality are referable to changes in the strength of one of the simple tones.

Hence we are able to lay down the important law that *differences in musical quality of tone depend solely on the presence and strength of partial tones, and in no respect on the differences in phase under which these partial tones enter into composition.* It must be here observed that we are speaking only of musical quality as previously defined. When the musical tone is accom-

panied by unmusical noises, such as jarring, scratching, soughing, whizzing, hissing, these motions are either not to be considered as periodic at all, or else correspond to high upper partials, of nearly the same pitch, which consequently form strident dissonances. We were not able to embrace these in our experiments, and hence we must leave it for the present doubtful whether in such dissonating tones difference of phase is an element of importance. Subsequent theoretic considerations will lead us to suppose that it really is.

If we wish only to imitate vowels by compound tones without being able to distinguish the differences of phase in the individual constituent simple tones, we can effect our purpose tolerably well with organ pipes. But we must have at least two series of them, loud open and soft stopped pipes, because the strength of tone cannot be increased by additional pressure of wind without at the same time changing the pitch. I have had a double row of pipes of this kind made by Herr Appun in Hanau, giving the first sixteen partial tones of *B*♭. All these pipes stand on a common windchest, which also contains the valves by which they can be opened or shut. Two larger valves cut off the passage from the windchest to the bellows. While these valves are closed, the pipe valves are arranged for the required combination of tones, and then one of the main valves of the windchest is opened, allowing all the pipes to sound at once. The character of the vowel is better produced in this way by short jerks of sound, than by a long continued sound. It is best to produce the prime tone and the predominant upper partial tones of the required vowels on both the open and stopped pipes at once, and to open only the weak stopped pipes for the next adjacent tones, so that the strong tone may not be too isolated.

The imitation of the vowels by this means is not very perfect, because, among other reasons, it is impossible to graduate the strength of tone on the different pipes so delicately as on the tuning forks, and the higher tones especially are too screaming. But the vowel sounds thus composed are perfectly recognisable.

We proceed now to consider the part played by the ear in the apprehension of quality of tone. The assumption previously made respecting the function of the ear, was that it was capable of distinguishing both the vibrational number of a musical tone (which gives the pitch), and also the *form* of the vibrations (on which the

difference of quality depends). This last assertion was based simply on the exclusion of all other possible assumptions. As it could be proved that equality of pitch always required equal vibrational numbers, and as loudness visibly depended upon the magnitude of the vibrations, the quality of tone must necessarily depend on something which was neither the number nor the magnitude of the vibrations. There was nothing left us but form. We can now make this view more definite. The experiments just described shew that waves of very different forms (as fig. 31, C, D, p. 175, and fig. 12, C, D, p. 49), may have the same quality of tone, and indeed, for every case, except the simple tone, there is an infinite number of forms of wave of this kind, because any alteration of the difference of phase alters the form of wave without changing the quality of tone. The only decisive character of a quality of tone, is that the motion of the air which strikes the ear when resolved into a sum of pendular vibrations gives the same degree of strength to the same simple vibration.

Hence the ear does not distinguish the different forms of waves in themselves, as the eye distinguishes the different vibrational curves. The ear must be said rather to decompose every wave form into simpler elements according to a definite law. It then receives a sensation from each of these simpler elements as from an harmonious tone. By trained attention the ear is able to become conscious of each of these simpler tones separately. And what the ear distinguishes as different qualities of tone are only different combinations of these simpler sensations.

The comparison between ear and eye is here very instructive. When the vibrational motion is rendered visible, as in the vibration microscope, the eye is capable of distinguishing every possible different form of vibration one from another, even such as the ear cannot distinguish. But the eye is not capable of directly resolving the vibrations into simple vibrations, as the ear is. Hence the eye, assisted by the above-named instrument, really distinguishes the *form of vibration*, as such, and in so doing distinguishes *every different* form of vibration. The ear, on the other hand, does *not* distinguish *every* different form of vibration, but only such as when resolved into pendular vibrations, give different constituents. But on the other hand, by its capability of distinguishing and feeling these very constituents, it is again superior to the eye, which is quite incapable of so doing.

This analysis of compound into simple pendular vibrations is an astounding property of the ear. The reader must bear in mind that when we apply the term 'compound' to the vibrations produced by a single musical instrument, the 'composition' has no existence except for our auditory perceptions, or for mathematical theory. In reality, the motion of the particles of the air is not at all compound, it is quite simple, flowing from a single source. When we turn to external nature for an analogue of such an analysis of periodical motions into simple motions, we find none but the phenomena of sympathetic vibration. In reality if we suppose the dampers of a pianoforte to be raised, and allow any musical tone to impinge powerfully on its sounding board, we bring a set of strings into sympathetic vibration, namely *all* those strings, and *only* those, which correspond with the simple tones contained in the given musical tone. Here, then, we have a purely mechanical repetition of the resolution of air waves by the ear. The air wave, quite simple in itself, brings a certain number of strings into sympathetic vibration, and the sympathetic vibration of these strings depends on the same law as the sensation of harmonic upper partial tones in the ear.[1]

There is necessarily a certain difference between the two kinds of apparatus, because the pianoforte strings readily vibrate with their upper partials in sympathy, and hence separate into several vibrating sections. We will disregard this peculiarity in making our comparison. It would besides be easy to make an instrument in which the strings would not vibrate sensibly or powerfully for

[1] [On a piano, tune the notes *B*♭, *b*♭, *f'*, *b'*♭, *d''*, *f''*, *a''*♭, *b''*♭, *c'''*, *d'''*, *e'''*♭, *f'''*, *g'''*, *a'''*♭, *a''''*, *b'''*♭ accurately to the 16 partial tones of *B*♭. Observe that the 7th and 14th, a''♭ and *a'''*♭, are flatter than the *a''*♭ and *a'''*♭ in the key of *e*♭ (that is, than the Fourths above the *e''*♭ and *e'''*♭, which are the Fourths above *b'*♭ and *b''*♭), in the proportion of 63 : 64, also that the 11th *e'''*♭ is sharper than the Fourth above *b''*♭ in the proportion of 33 : 32; and that, on the other hand, the 13th *g'''* is *flatter* than the major Sixth above *b''*♭ in the proportion of 39 : 40. In tuning, at first omit these 7th, 11th, and 13th partial tones, and then insert them afterwards by estimation. The other 13 tones are easy to tune correctly. After these notes have been all correctly tuned, place weights on the finger keys and thus raise the dampers of these 16 partial tones, keeping the other strings damped. Expose the sounding board and sing the vowels loudly to *B*♭. The echo will be very true, much more so than on a piano as usually tuned, where the strings are only approximately sympathetic, and hence do not vibrate so powerfully in sympathy. Such an instrument may evidently be made available for varying many experiments previously described. The caution in the next paragraph of the text must, however, not be overlooked.—*Translator.*]

any but their prime tones, by simply loading the strings slightly in the middle. This would make their higher proper tones inharmonic to their primes.

Now suppose we were able to connect every string of a piano with a nervous fibre in such a manner that this fibre would be excited and experience a sensation every time the string vibrated. Then every musical tone which impinged on the instrument would excite, as we know to be really the case in the ear, a series of sensations exactly corresponding to the pendular vibrations into which the original motion of the air had to be resolved. By this means, then, the existence of each partial tone would be exactly so perceived, as it really is perceived by the ear. The sensations excited by the different higher partials would under the supposed

FIG. 36.

conditions fall to the lot of different nervous fibres, and hence be produced perfectly, separately, and independently.

Now, as a matter of fact, later microscopic discoveries respecting the internal construction of the ear, lead to the hypothesis, that arrangements exist in the ear similar to those which we have imagined. The end of every fibre of the auditory nerve is connected with small elastic parts, which we cannot but assume to be set in sympathetic vibration by the waves of sound.

The construction of the ear may be briefly described as follows:—The fine ends of the fibres of the auditory nerves are expanded on a delicate membrane in a cavity filled with fluid. Owing to its involved form this cavity is known as the *labyrinth*.

To conduct the vibrations of the air with sufficient force into the fluid of the labyrinth is the office of a second portion of the ear, the *tympănum* or *drum* and its parts. Fig. 36 (p. 190) is a diagrammatic section, of the size of life, shewing the cavities belonging to the auditory apparatus. A is the labyrinth, B B the cavity of the *tympănum* or drum, D the funnel-shaped entrance into the *meātus* or external auditory passage, narrowest in the middle and expanding slightly towards its extremity. This *meātus*, in the ear or passage, is a tube formed partly of cartilage or gristle and partly of bone, and it is separated from the tympănum or drum, by a thin circular membrane, the *membrāna tympănī* or *drumskin*[1] c c, which is rather laxly stretched in a bony ring. The *drum* (tympănum) B lies between the outer passage (meātus) and the labyrinth. The drum is separated from the labyrinth by bony walls, pierced with two holes, closed by membranes. These are the so-called *windows* (*fenes'trae*) of the labyrinth. The upper one o, called the *oval* window (*fenes'tra ōvālis*), is connected with one of the ossicles or little bones of the ear called the stirrup. The lower or *round* window r (*fenes'tra rotun'da*) has no connection with these ossicles.

The drum of the ear is consequently completely shut off from the external passage and from the labyrinth. But it has free access to the upper part of the pharynx or throat, through the so-called *Eustachian*[2] *tube* E, which in Germany is termed a *trumpet*, because of the trumpet-like expansion of its pharyngeal extremity and the narrowness of its opening into the drum. The end which opens into the drum is formed of bone, but the expanded pharyngeal end is formed of thin flexible cartilage or gristle, split along its upper side. The edges of the split are closed by a sinewy membrane. By closing the nose and mouth, and either condensing the air in the mouth by pressure, or rarefying it by suction, air can be respectively driven into or drawn out of the drum through this tube. At the entrance of air into the drum, or its departure from it, we feel a sudden jerk in the ear, and hear a dull crack. Air passes from the pharynx to the drum,

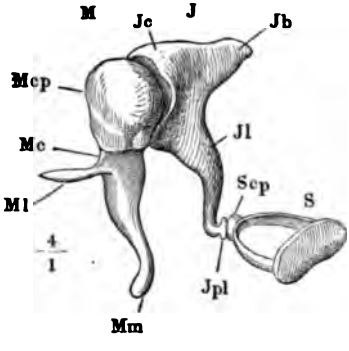
[1] [In common parlance the *drumskin* of the ear, or *tympanic membrane*, is spoken of as the drum itself. Anatomists as well as drummers distinguish the membranous cover (drumskin) which is struck, from the hollow cavity (drum) which contains the resonant air. The quantities of the Latin words are marked, as I have heard musicians give them incorrectly.—*Translator.*]

[2] [Generally pronounced *yoo-stāi'-kī-ăn*, but sometimes *yoo-stāi'-shī-ăn*.—*Translator.*]

or from the drum to the pharynx only at the moment of making the motion of swallowing. When the air has entered the drum it remains there, even after nose and mouth are opened again, until we make another motion of swallowing. Then the air leaves the drum, as we perceive by a second cracking in the ear, and the cessation of the feeling of tension in the drumskin which had remained up till that time. These experiments shew that the tube is not usually open, but is opened only during swallowing, and this is explained by the fact that the muscles which raise the *vēlum palātī* or soft palate, and are set in action on swallowing, arise from part of the cartilaginous extremity of the tube. Hence the drum is generally quite closed, and filled with air, which has a pressure equal to that of the external air, because it has from time to time, that is whenever we swallow, the means of equalising itself with the same by free communication.

In two places, this air in the drum is likewise separated from the fluid of the labyrinth merely by a thin stretched membrane, which closes the two *windows of the labyrinth*, already mentioned, namely, the *oval* window (o, fig. 36, p. 190) and the round window (r). Both of these membranes are in contact on their outer side with the air of the drum, and on their inner side with the water of the labyrinth. The membrane of the round window is free, but that of the oval window is connected by a series of three little bones or ossicles, jointed together, with the drumskin of the ear. Fig. 37 shews the three ossicles in their natural connection, enlarged four diameters. They are the *hammer* (mal'leus) M, the *anvil* (in'cus) J, and the stirrup (sta'pes[1]) S. The first is attached to the drumskin, and the last to the membrane of the oval window.

FIG. 37.

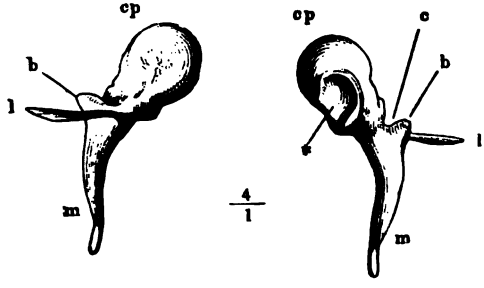


Ossicles of the ear in mutual connection, seen from the front, and from the right side of the head, turned a little to the right round a vertical axis. M hammer or *malleus*. J anvil or *incus*. S stirrup or *stapes*. Mcp head, Mc neck, Ml long process or *processus gracilis*, Mm handle or *manubrium* of the hammer.—Jc body, Jb short process, Jl long process, Jbl orbicular process or *os orbiculāre* or *processus lenticulāris*, of the anvil.—Scp head or *capitulum* of the stirrup.

[1] [*Stapes* is usually called *stāi'pēez*. It is not a classical word, and is usually received as a contraction for *stātīpĕs* or foot-rest, also not classical.—*Translator*.]

The hammer, shewn separately in fig. 38, has a thick, rounded upper extremity, the *head* cp, and a thinner lower extremity, the *handle* m. Between these two is a contraction c, the *neck*. At the back of the head is the surface of the *joint*, by means of which it fits on to the anvil. Below the neck, where the handle begins, project two processes, the *long* l, also called *processus Foliānus* and *pr. grăcilis*, and the short b, also called *pr. bre'vis*. The long process has the proportionate length shewn in the figure, in children only; in adults it appears to be absorbed down to a little stump. It is directed forwards, and is covered by the bands which fasten the hammer in front. The *short process* b, on the other hand, is directed towards the drumskin, and presses its upper part a little forwards. From the point of this

A FIG. 38. B



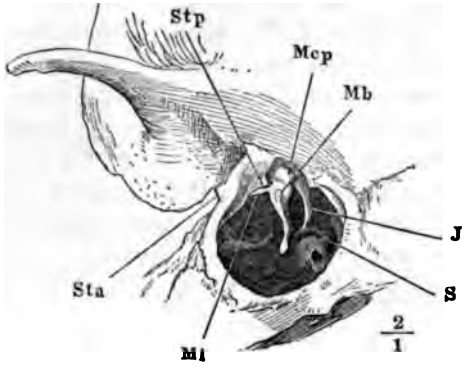
Right hammer, A from the front, B from behind. cp head, c neck, b short, l long process, m handle. * Surface of the joint.

process b to the point of the handle m the hammer is attached to the upper portion of the drumskin, in such a manner that the point of the handle draws the drumskin strongly inwards.

Fig. 39 (page 194) shews the hammer in its natural position as seen from without, after the drumskin has been removed, and fig. 40 shows the hammer, lying against the drumskin as seen from within. The hammer is fastened along the upper margin of the drumskin by a fold of mucous membrane, within which run a series of rather stiff bundles of tendinous fibres. These straps arise in a line which passes from the processus Folianus (fig. 38 l), above the contraction of the neck, towards the lower end of the surface of the joint for the anvil, and in elderly people is developed into a prominent ridge of bone.

The tendinous bands or ligaments are strongest and stiffest at the front and back end of this line of insertion. The front

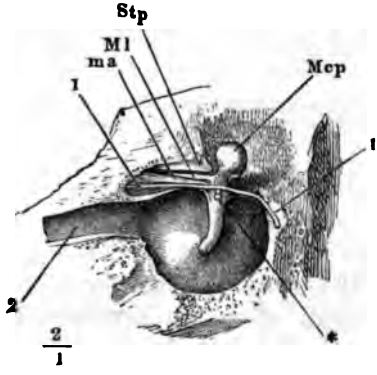
FIG. 39.



Left temporal bone of a newly-born child, with the auditory ossicles *in situ*. Sta, spina tympanica anterior. Stp, spina tympanica posterior. Mcp, head of the hammer. Mb short, Ml long process of hammer. J anvil. S stirrup.

portion of the ligament, lig. mallei anterius (fig. 40, ma), surrounds the processus Folianus, and is attached partly to a bony

FIG. 40.



Right drumskin with the hammer, seen from the inside. The inner layer of the fold of mucous membrane belonging to the hammer (see below) is removed. Stp, spina tympanica post. Mcp, head of the hammer. Ml, long process of hammer. ma, ligamen'tum mallei ant. 1 chorda tympani. 2 Eustachian tube. * Tendon of the m tensor tympani, cut through close to its insertion.

spine (figs. 39 and 40, Stp) of the osseous ring of the drum, which projects close to the neck of the hammer, and partly to its under edge, and partly falls into a bony fissure which leads

towards the articulation of the jaw. The back portion of the same ligament, on the other hand, is attached to a sharp-edged bony ridge projecting inwards from the drumskin, and parallel to it, a little above the opening, through which a traversing nerve the chorda tympănī (fig. 40, 1, 1, p. 194) enters the bone. This second bundle of fibres may be called the lig. malleī posterĭus. In fig. 39 (page 194) the origin of this ligament is seen as a little projection of the ring to which the drumskin is attached. This projection bounds towards the right the upper edge of the opening for the drumskin, which begins to the left of Stp, exactly at the place where the long process of the anvil makes its appearance in the figure. These two ligaments, front and back, taken together form a moderately tense sinewy chord, round which the hammer can turn as an axis. Hence even when the two other ossicles have been carefully removed, without loosing these two ligaments, the hammer will remain in its natural position, although not so tightly as before.

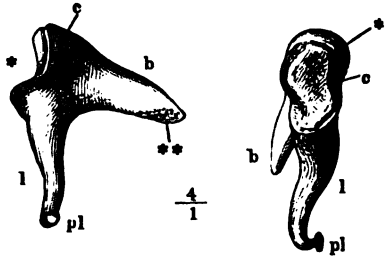
The middle fibres of the broad ligamentous band above-mentioned pass outwards towards the upper bony edge of the drumskin. They are comparatively short, and are known as lig. mallei externum. Arising above the line of the axis of the hammer, they prevent the head from turning too far inwards, and the handle with the drumskin from turning too far outwards, and oppose any down-dragging of the ligament forming the axis. The first effect is increased by a ligament (lig. mallei superius) which passes from the processus Folianus, upwards, into the small slit, shewn in fig. 40 (page 194), between the head of the hammer and the wall of the drum.

It must be observed that in the upper part of the channel of the tube, there is a *muscle for tightening the drumskin* (m. tensor tympănī), the tendon of which passes obliquely across the cavity of the drum and is attached to the upper part of the handle of the hammer (at*, fig. 40, page 194). This muscle must be regarded as a moderately stretched elastic band, and may have its tension temporarily increased by active contraction. The effect of this muscle is also principally to draw the handle of the hammer inwards, together with the drumskin. But since its point of attachment is so close to the ligamentous axis, the chief part of its pull acts on this axis, stretching it as it draws it inwards Here we must observe that in the case of a rectilinear inextensible

cord, which is moderately tight, such as the ligamentous axis of the hammer, a slight force which pulls it sideways, suffices to produce a very considerable increase of tension. This is the present case. It should also be remembered that quiescent muscles not excited by innervation, are always stretched elastically in the living body, and act like elastic bands. This elastic tension can of course be considerably increased by the innervation which brings the muscles into action, but such tension is never entirely absent from the majority of our muscles.

The *anvil*, which is shown separately in fig. 41, resembles a double tooth with two fangs; the surface of its joint with the hammer (at*, fig. 41), replacing the masticating surface. Of the two roots of the tooth which are rather widely separated, the upper, directed backwards, is called the *short process* b; the other, thinner and directed downwards, the *long process of the anvil* l. At the tip of the latter is the knob which articulates with the stirrup. The tip of the short process, on the other hand, by means of a short ligament and an imperfectly developed joint at its under surface is connected with the back wall of the cavity of the drum, at the spot where this passes backwards into the air-cavities of the mastoid process behind the ear. The joint between anvil and hammer is a curved depression of a rather irregular form, like a saddle. In its action it may be compared with the joints of the well-known Breguet watchkeys, which have rows of interlocking teeth, offering scarcely any resistance to revolution in one direction, but allowing no revolution whatever in the other. Interlocking teeth of this kind are developed upon the under side of the joint between hammer and anvil. The tooth on the hammer projects towards the drumskin, that of the anvil lies inwards; and, conversely, towards the upper end of the hollow of the joint, the anvil projects outwards, and the hammer inwards. The consequence of

A FIG. 41. B



Right anvil. A middle surface. B front view. c body. b short, l long process. pl Proc. lenticulāris or os orbiculare. * Articulation with the head of the hammer. ** Surface resting on the wall of the drum.

this arrangement is that when the hammer is drawn inwards by the handle, it bites the anvil firmly and carries it with it. Conversely, when the drumskin, with the hammer, is driven outwards, the anvil is not obliged to follow it. The interlocking teeth of the surfaces of the joint then separate, and the surfaces glide over each other with very little friction. This arrangement has the very great advantage of preventing any possibility of the stirrup's being torn away from the oval window, when the air in the auditory passage is considerably rarefied. There is also no danger from driving in the hammer, as might happen when the air in the auditory passage was condensed, because it is powerfully opposed by the tension of the drumskin, which is drawn in like a funnel.

When air is forced into the cavity of the drum in the act of swallowing, the contact of hammer and anvil is loosened. Weak tones in the middle and upper regions of the scale are then not heard much more weakly than usual, but stronger tones are very sensibly damped. This may perhaps be explained by supposing that the adhesion of the articulating surfaces suffices to transfer weak motions from one bone to the other, but that strong impulses cause the surfaces to slide over one another, and hence the tones due to such impulses must be enfeebled.

Deep tones are damped in this case, whether they are strong or weak, perhaps because these always require larger motions to become audible.[1]

Another important effect on the apprehension of tone, which is due to the above arrangement in the articulation of hammer and anvil, will have to be considered in relation to combinational tones.

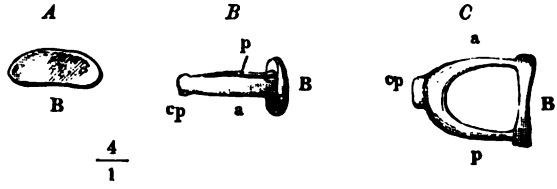
Since the attachment of the tip of the short process of the anvil lies sensibly inwards and above the ligamentous axis of the hammer, the head of the hammer separates from the articulating surface between hammer and anvil, when the head is driven outwards, and therefore the handle and drumskin are driven inwards. The consequence is that the ligaments holding the anvil against the hammer, and on the tip of the short process of the anvil, are sensibly stretched, and hence the tip is raised from its osseous support. Consequently in the normal position of the ossicles for

[1] On this point see Part II. Chap. IX.

hearing, the anvil has no contact with any other bone but the hammer, and both bones are held in position only by stretched ligaments, which are tolerably light, so that only the revolution of the hammer about its ligamentous axis remains comparatively free.

The third ossicle, the *stirrup*, shewn separately in fig. 42, has really a most striking resemblance to the implement after which it has been named. The foot B is fastened into the membrane of the oval window, and fills it all up, with the exception of a narrow edging. The head cp, has an articulating hole for the tip of the long process of the anvil (processus lenticulāris, or os orbiculāre). The joint is surrounded by a lax membrane. When the drumskin is normally drawn inwards, the anvil presses on the stirrup, so that no tighter ligamentous fastening of the joint is necessary. Every

FIG. 42.



Right stirrup: seen, *A* from within, *B* from front, *C* from behind. B Foot. cp, head or capitulum. a Front, p back limb.

increase in the push on the hammer arising from the drumskin also occasions an increase in the push of the stirrup against the oval window; but in this action the upper and somewhat looser edge of its foot is more displaced than the under, so that the head rises slightly; this motion again causes a slight elevation of the tip of the long process in the anvil, in the direction conditioned by its position, inwards and underneath the ligamentous axis of the hammer.

The excursions of the foot of the stirrup are always very small, and according to my measurements[1] never exceed one-tenth of a millimetre (·0039 or about $\frac{1}{250}$ of an inch). But the hammer when freed from anvil and stirrup, with its handle moving out-

[1] Helmholtz, 'Mechanism of the Auditory Ossicles,' in *Pflueger's* 'Archiv für Physiologie,' vol. i, pp. 34–43. In this paper an attempt is made to prove the correctness of the account of this mechanism given in the text.

wards, and sliding over the articulating surface of the anvil, can make excursions at least nine times as great as it can execute when acting in connection with anvil and stirrup.

The first advantage of the apparatus belonging to the drum of the ear, is that the whole resonant motion of the comparatively wide surface of the drumskin (vertical diameter 9 to 10 millimetres, or 0·35 to 0·39 inches, just over one-third of an inch; horizontal diameter, 7·5 to 9 millimetres, or 0·295 to 0·35 inches, that is about five-sixths of the former dimensions) is collected and transferred by the ossicles to the relatively much smaller surface of the oval window or of the foot of the stirrup, which is only 1·5 to 3 millimetres (0·06 to 0·12 inches) in diameter. The surface of the drumskin is hence 15 to 20 times larger than that of the oval window.

In this transference of the vibrations of air into the labyrinth it is to be observed that though the particles of air themselves have a comparatively large amplitude of vibration, yet their density is so small that they have no very great moment of inertia, and consequently when their motion is impeded by the drumskin of the ear, they are not capable of presenting much resistance to such an impediment, or of exerting any sensible pressure against it. The fluid in the labyrinth, on the other hand, is much denser and heavier than the air in the auditory passage, and for moving it rapidly backwards and forwards as in resonant oscillations, a far greater exertion of pressure is required than was necessary for the air in the auditory passage. On the other hand the amplitude of the vibrations performed by the fluid in the labyrinth are relatively very small, and extremely minute vibrations will in this case suffice to give excitory motion to the terminations and appendages of the nerves, which lie on the very limits of microscopic vision.

The mechanical problem which the apparatus within the drum of the ear had to solve, was to transform a motion of great amplitude and little force, such as impinges on the drumskin, into a motion of small amplitude and great force, such as had to be communicated to the fluid in the labyrinth.

A problem of this sort can be solved by various kinds of mechanical apparatus, such as levers, trains of pulleys, cranes, and the like. The mode in which it is solved by the apparatus in the drum of the ear, is quite unusual, and very peculiar.

A leverage is certainly employed, but only to a moderate extent. The tip of the handle of the hammer, on which the pull of the drumskin first acts, is about once and a half as far from the axis of rotation as that point of the handle which presses on the stirrup (see fig. 39, p. 194). The handle of the hammer consequently forms the longer arm of a lever, and the pressure on the stirrup will be once and a half as great as that which drives in the hammer.

The chief means of reinforcement is due to the form of the drumskin. It has been already mentioned that its middle or *navel* (umbĭlĭcus) is drawn inwards by the handle, so as to present a funnel shape. But the meridian lines of this funnel drawn from the navel to the circumference, are not straight lines; they are slightly convex on the outer side. A diminution of pressure in the auditory passage increases this convexity, and an augmentation diminishes it. Now the tension caused in an inextensible thread, having the form of a flat arch, by a force acting perpendicular to its convexity, is very considerable. It is well known that a sensible force must be exerted to stretch a long thin string into even a tolerably straight horizontal line. The force is indeed extremely greater than the weight of the string which pulls the string from the horizontal position.[1] In the case of the drumskin, it is not gravity which prevents its radial fibres from straightening themselves, but partly the pressure of the air, and partly the elastic pull of the circular fibres of the membrane. The latter tend to contract towards the axis of the funnel-shaped membrane, and hence produce the inflection of the radial fibres towards this axis. By means of the variable pressure of air during the resonant vibrations of the atmosphere this pull exerted by the circular fibres is alternately strengthened and weakened, and produces an effect on the point where the radial fibres are attached to the tip of the handle of the hammer, similar to that which would happen if we could alternately increase and diminish the weight of a string stretched horizontally, for this would produce a

[1] [The following quatrain, said to have been unconsciously produced by Vince, as a corollary to one of the propositions in his mechanics, will serve to impress the fact on a non-mathematical reader.

Hence no force, however great,
Can stretch a cord, however fine,
Into a horizontal line,
So as to make it truly straight.—*Translator.*]

proportionate increase and decrease in the pull exerted by the hand which stretched it.

In a horizontally stretched string such as has been just described, it should be further remarked that an extremely small relaxation of the hand is followed by a considerable fall in the middle of the string. The relaxation of the hand, namely, takes place in the direction of the chord of the arc, and easy geometrical considerations shew that chords of arcs of the same length and different, but always very small curvature, differ very slightly indeed from each other and from the length of the arcs itself.[1] This is also the case with the drumskin. An extremely little yielding in the handle of the hammer admits of a very considerable change in the curvature of the drumskin. The consequence is that, in resonant vibrations, the parts of the drumskin which lie between the inner attachment of this membrane to the hammer and its outer attachment to the ring of the drum, are able to follow the oscillations of the air to a considerable extent, while the motion of the air is transmitted to the handle of the hammer with much diminished amplitude but much increased force. After this, as the motion passes from the handle of the hammer to the stirrup, the leverage already mentioned causes a second and more moderate reduction of the amplitude of vibration with corresponding increase of force.

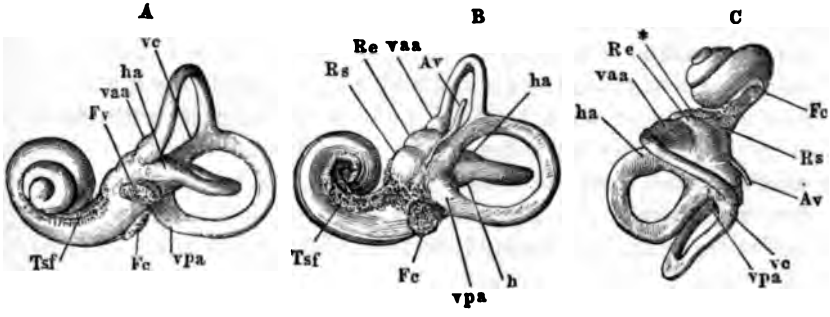
We now proceed to describe the most internal division of the organ of hearing, called the *labyrinth*. Fig. 43 (p. 202) represents a cast of its cavity, as seen from different positions. Its middle portion, containing the *oval window* Fv (fenestra vestĭbŭlī) that receives the foot of the stirrup, is called the *vestibule of the labyrinth*. From the vestibule proceeds forwards and underwards, a spiral canal, the *snailshell* or cochlĕa, at the entrance to which lies the *round window* Fc (fenestra cochlĕae), which is turned towards the cavity of the drum. Upwards and backwards, on the other hand, proceed three semicircular canals from the vestibule, the *horizontal*, *front vertical*, and *back vertical semicircular canals*, each of which debouches with both its mouths in the vestibule, and each of which has at one end a bottle-shaped enlargement, or

[1] The amount of difference varies as the square of the depth of the arc. If the length of the arc be l, and the distance of its middle from the chord be s, the chord is shorter than the arc by the length $\frac{8}{3} \cdot \frac{s^2}{l}$.

ampulla (ha, vaa, vpa). The aquaeductus vestĭbŭlī shewn in the figure carries a vein. The rough places Tsf and * are casts of canals which introduce nerves.

The whole of this cavity of the labyrinth is filled with fluid, and surrounded by the extremely hard close mass of the petrous bone, so that there are only two yielding spots on its walls, the two windows, the oval Fv, and the round Fc. Into the first, as already described, is fastened the foot of the stirrup, by a narrow membranous edging. The second is closed by a membrane. When the stirrup is driven against the oval window, the whole mass of fluid in the labyrinth is necessarily driven against the round window, as the only spot where its walls can give way. If, as

FIG. 43.



A, left labyrinth from without. B, right labyrinth from within. C, left labyrinth from above. Fc, fenestra cochleae or round window. Fv, fenestra vestĭbŭlī, or oval window. Re, recessus ellipticus. Rs, recessus sphaericus. h, horizontal semicircular canal. ha, ampulla of the same. vaa, ampulla of the front vertical semicircular canal. vpa, ampulla of the back vertical semicircular canal. vc, common limb of the two vertical semicircular canals. Av, cast of the aquaeductus vestĭbŭlī. Tsf, tractus spirālis forāmĭnōsus. * Cast of the little canals which debouch on the pyrāmis vestĭbŭlī.

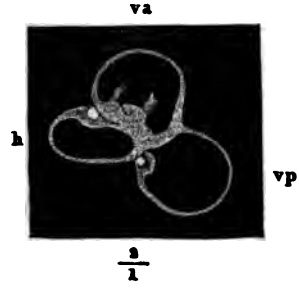
Politzer did, we put a finely drawn glass tube as a manometer into the round window, without in other respects injuring the labyrinth, the water in this tube will be driven upwards as soon as a strong pressure of air acts on the outside of the drumskin and causes the foot of the stirrup to be driven into the oval window.

The terminations of the auditory nerve are spread over fine membranous formations, which lie partly floating and partly expanded in the hollow of the bony labyrinth, and taken together compose the *membranous labyrinth.* This last has on the whole the same shape as the bony labyrinth. But its canals and cavities are smaller, and its interior is divided into two separate sections;

the *utrĭcŭlus* with the *semicircular canals*, and the *saccŭlus* with the *membranous cochlĕa*. Both the utriculus and the sacculus lie in the vestibule of the bony labyrinth; the utriculus opposite to the *recessus elliptĭcus* (Re, fig. 43, p. 202) the sacculus opposite to the *recessus sphaericus* (Rs). These are floating bags filled with water, and only touch the wall of the labyrinth at the point where the nerves enter them.

The form of the utriculus with its membranous semicircular canals is shewn in fig. 44. The ampullae project much more in the membranous than in the bony semicircular canals. According to the recent investigations of Rüdinger, the membranous semicircular canals do not float in the bony ones, but are fastened to the convex side of the latter. In each ampulla there is a pad-like prominence directed inwards, into which fibriles of the auditory nerve enter; and on the utriculus there is a place which is flatter and thickened. The peculiar manner in which the nerves terminate in this place will be described hereafter. In the inside of the utriculus is found the auditory *sand*, consisting of little crystals of lime which are connected by means of a mucous mass with each other and with the thickened places where the nerves are so abundant.

FIG. 44.



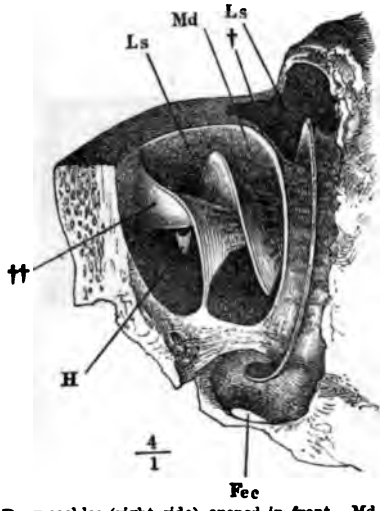
Utriculus and membranous semicircular canals (left side) seen from without. va front, vp back vertical, h horizontal semicircular canal.

In the hollow of the bony vestibule, near the utriculus, and fastened to it, but not communicating with it, lies the sacculus, also provided with a similar thickened spot full of nerves. A narrow canal connects it with the canal of the membranous cochlea. As to the cavity of the cochlea, we see by fig. 43 (p. 202) that it is exactly similar to the shell of a garden snail; but the canal of the cochlea is divided into two almost completely separated galleries, by a transverse partition, partly bony and partly membranous. These galleries communicate only at the vertex of the cochlea through a small opening, the *hĕlĭcotrēma*, bounded by the *hāmŭlus* or hook-shaped termination of its central axis or *mŏdĭŏlus*. Of the two galleries into which the cavity of the bony cochlea is divided, one communicates directly with the vestibule and is hence called the *vestibule-gallery* (scāla vestĭbŭlī). The other gallery is

cut off from the vestibule by the membranous partition, but just at its base, where it begins, is the round window, and the yielding membrane, which closes this, allows the fluid in the gallery to exchange vibrations with the air in the drum. Hence this is called the *drum-gallery* (scāla tympănī).

Finally, it must be observed that the membranous partition of the cochlea is not a single membrane, but a membranous canal (ductus cochleāris). Its inner margin is turned towards the central axis or mŏdĭŏlus, and attached to the rudimentary bony partition (lāmĭna spīrālis). A part of the opposite external surface is attached to the inner surface of the bony gallery. Fig. 45 shews the bony parts of a cochlea which has been laid open, and fig. 46 (p. 205), a transverse section of the canal (which is imperfect on the left hand at bottom). In both figures Ls denotes the bony part of the partition, and in fig. 46 v and b are the two unattached parts of the membranous canal. The transverse section of this canal is, as the figure shews, nearly triangular, so that an angle of the triangle near L l s is attached to the edge of the bony partition. The commencement of the ductus cochlearis at the base of the cochlea, communicates, as already stated, by means of a narrow membranous canal with the sacculus in the vestibule. Of the two unattached strips in its walls, that facing the vestibule gallery is a soft membrane, incapable of offering much resistance—*Reissner's membrane* (membrāna vestibulāris, v, fig. 46, p. 205); but the other, the *membrāna băsĭlāris* (b) is a firm, tightly stretched, elastic membrane, striped radially, corresponding to its radial fibres. It splits easily in the direction of these fibres, shewing that it is loosely connected in a direction transverse to them. The terminations of the nerves of the cochlea and their appendages, are attached

FIG. 45.

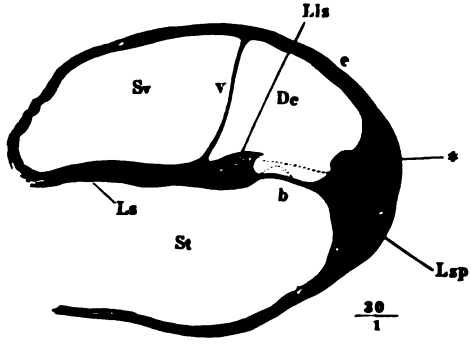


Bony cochlea (right side) opened in front. Md, mŏdĭŏlus. Ls, lāmĭna spīrālis. H, hāmulus. Fec, fenestra cochleae. † Section of the partition of the cochlea. †† Upper extremity of the same.

to the membrāna bāsĭlāris, as is shewn by the dotted lines in fig. 46.

When the drumskin is driven inwards by increased pressure of air in the auditory passage, it also forces the auditory ossicles inwards, as already explained, and as a consequence the foot of the stirrup penetrates deeper into the oval window. The fluid of the labyrinth, being surrounded in all other places by firm bony walls, has only one means of escape,—the round window with its yielding membrane. To reach it, the fluid of the labyrinth must either pass through the hĕlĭcŏtrēma, the narrow opening at the vertex of the cochlea, flowing over from the vestibule gallery into the drum gallery, or, as it would probably not have sufficient time to do this

FIG. 46.



Transverse section of a spire of a cochlea softened in hydrochloric acid. Ls, lāmĭna spĭrālis. Lls, limbus lāmĭnae spĭrālis. Sv, scāla vestĭbŭli. St, scala tympāni. Dc, ductus cochleāris. Lsp, lĭgāmentum spĭrāle. v, membrāna vestibulāris. b, membrāna bāsĭlāris. e, outer wall of the ductus cochleāris. * its fillet. The dotted lines shew sections of the membrāna tectōria and the auditory rods.

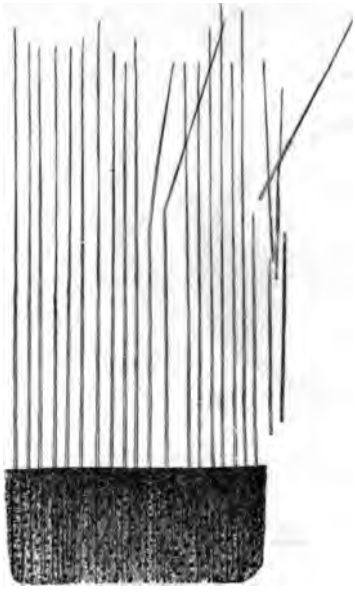
in the case of resonant vibrations, press the membranous partition of the cochlea against the drum gallery. The converse action must take place when the air in the auditory passage is rarefied.

Hence the resonant vibrations of the air in the outer auditory passage are finally transferred to the membranes of the labyrinth, more especially those of the cochlea, and to the expansions of the nerves upon them.

The terminal expansions of these nerves, as I have already mentioned, are connected with very small elastic appendages, which appear adapted to excite the nerves by their vibrations.

The nerves of the vestibule terminate in the thickened places of the bags of the membranous labyrinth, already mentioned (p. 203), where the tissue has an almost cartilaginous consistency. One of these places provided with nerves, projects like a fillet into the inner part of the ampulla of each semicircular canal, and another lies on each of the little bags in the vestibule. The nerve fibres here enter between the soft cylindrical cells of the fine cuticle (ĕpĭthēlĭum) which covers the internal surface of the fillets. Projecting from the internal surface of this epithelium in the ampullae, Max Schultze discovered a number of very peculiar, stiff, elastic hairs, shewn in fig. 47. They are much longer than the vibratory hairs of the ciliated epithe'lium ($\frac{1}{25}$ of a Paris line, or ·60355 English inch, long in the ray fish), brittle, and running to a very fine point. It is clear that fine stiff hairs of this kind are extremely well adapted for moving sympathetically with the motion of the fluid, and hence for producing mechanical irritation in the nerve fibres which lie in the soft epithelium between their roots.

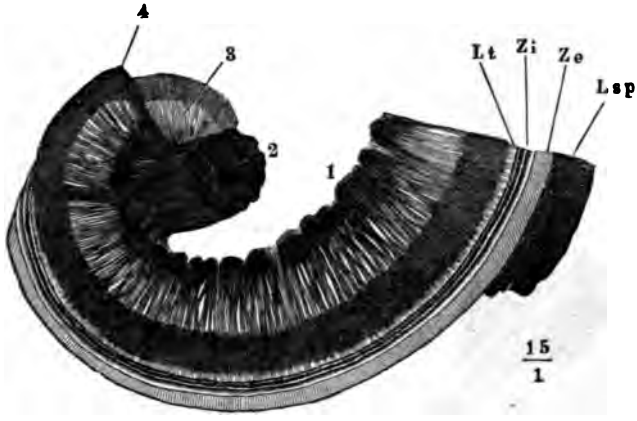
FIG. 47.



According to Max Schultze, the corresponding thickened fillets in the vestibules, where the nerves terminate, have a similar soft epithelium, and have short hairs which are easily destroyed. Close to these surfaces which are covered with nerves, lie the calcareous concretions, called *auditory stones* (ōtŏlĭths), which in fishes form connected convexo-concave solids, shewing on their convex sides an impression of the nerve fillet. In human beings, on the other hand, the otoliths are heaps of little crystalline bodies, of a longish angular form, lying close to the membrane of the little bags, and apparently attached to it. These otoliths seem also extremely well suited for producing a mechanical irritation of the nerves whenever the fluid in the labyrinth is suddenly agitated. The fine light membrane, with its interwoven nerves, probably in-

stantly follows the motion of the fluid, whereas the heavier crystals are set more slowly in motion, and hence also yield up their motion more slowly, and thus partly drag and partly squeeze the adjacent nerves. This would satisfy the same conditions of exciting nerves, as Heidenhain's *tetănomōtor*. By this instrument the nerve which acts on a muscle is exposed to the action of a very rapidly vibrating ivory hammer, which at every blow squeezes without bruising the nerve. A powerful and continuous excitement of the nerve is thus produced, which is shewn by a powerful and continuous contraction of the corresponding muscle. The above parts of the ear seem to be well suited to produce similar mechanical excitement.

FIG. 48.

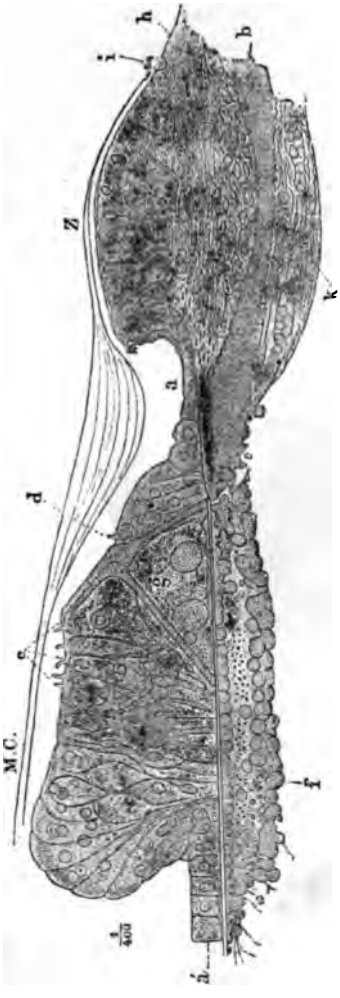


The construction of the cochlea is much more complex. The nerve fibres enter through the axis or modiolus of the cochlea into the bony part of the partition, and then come on the membranous part. Where they reach this, peculiar formations were discovered quite recently by the Marchese *Corti*, and have been named after him. On these the nerves terminate.

The expansion of the cochlean nerve is shewn in fig. 48. It enters through the axis (2) and sends out its fibres in a radial direction from the axis through the bony partition (1, 3, 4), as far as its margins. At this point the nerves pass under the commencement of the membrāna basilāris, penetrate this in a series of openings, and thus reach the ductus cochleāris and those nervous, elastic formations which lie on the inner zone (Zi) of the membrane.

The margin of the bony partition (a to b, fig. 49), and the inner zone of the membrana basilaris (a a′) are shewn in fig. 49 after Hensen. The under side of the figure[1] corresponds with the scala tympăni, the upper with the ductus cochleāris. Here h at the top and k at the bottom, are the two plates of the bony partition, between which the expansion of the nerve b proceeds. The upper side of the bony partition bears a fillet of close ligamentous tissue (Z, fig. 49, also shewn at Lls, fig. 46, p. 205), which, on account of the tooth-like impressions on its upper side, is called the toothed layer (*zo′na denticula′ta*), and which carries a peculiar elastic pierced membrane, Corti's membrane, M. C. fig. 49. This membrane is stretched parallel to the membrana basilāris as far as the bony wall on the outer side of the duct, and is there attached a little above the other. Between these two membranes lie the parts in and on which the nerve fibres terminate.

FIG. 49.



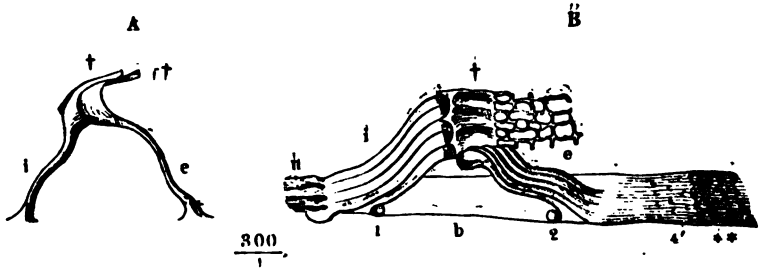
Among these *Corti's arches* (over g in fig. 49) are relatively the most solid formations. The series of these contiguous arches consists of two series of *rods* or *fibres*, an external and an internal. A single pair of these is shewn in fig. 50, A (p. 209), and a short series of them in fig. 50, B, attached to the membrana basilaris, and at † also connected with the pierced tissue, into which fit the terminal cells

[1] [As the engraving would have been too wide for the page if placed in its proper horizontal position, it has been printed vertically; the *left* side consequently corresponds to the *upper*, and its *right* to the *under* side.—*Translator*.]

of the nerves (fig. 49, c, p. 208) to be more fully described presently. These formations are shewn in fig. 51, (p. 210), as seen from the vestibule gallery; a is the *denticulated layer*, c the openings for the nerves on the internal margin of the membra basilaris, its external margin being visible at u u; d is the internal series of Corti's rods, e the external; over these, between e and x is seen the pierced membrane, against which lie the terminal cells of the nerves.

The fibres of the first, or external series, are weak S-shaped formations, having a swelling at the spot where they rise from the membrane to which they are attached, and ending in a kind of articulation which serves to connect them with the second or internal series. In fig. 51 (p. 210), at d will be seen a great number of these ascending fibres, lying beside each other in regular

FIG. 50.



A, external and internal rod in connection seen in profile. B, membrana basilaris (b) with the terminal fascicili of nerves (n), and the external and internal rods (i and e). 1 external, 2 internal cells of the floor, 4' attachments of the cells of the cover. * * epithelium.

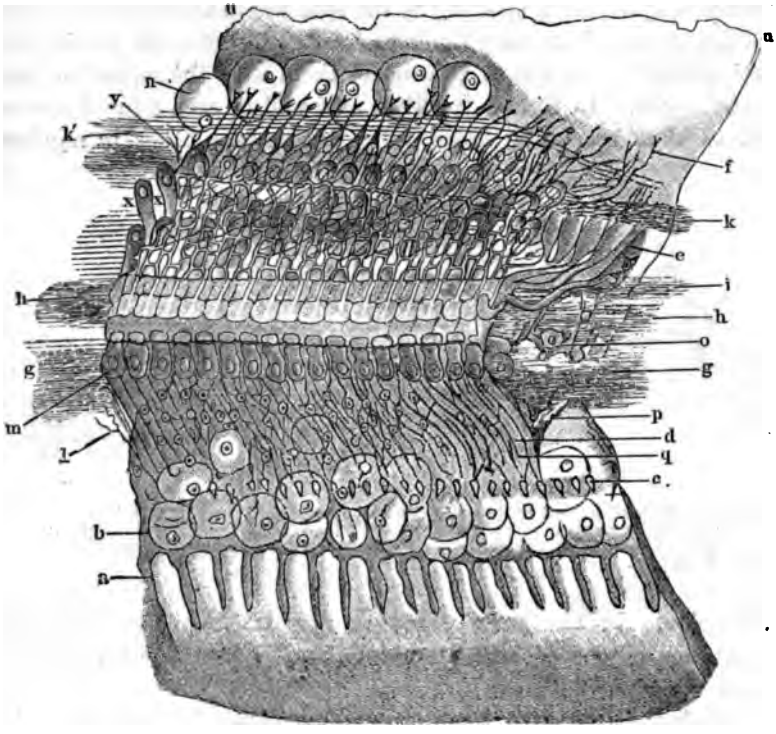
succession. In the same way they may be seen all along the membrane of the cochlea, close together, so that there must be many thousands of them.

Their sides lie close together, and even seem to be connected, leaving however occasional gaps in the line of connection, and these gaps are probably traversed by nerve fibres. Hence the fibres of the first series as a whole form a stiff layer, which endeavours to erect itself when the natural fastenings no longer resist, but allows the membrane on which they stand to crumple up between the attachments d and e of Corti's arches.

The fibres of the second or internal series, which form the descending part of the arch e, fig. 50 (p. 209), are smooth, flexible,

cylindrical threads with thickened ends. The upper extremity forms a kind of joint to connect them with the fibres of the first series, the lower extremity is enlarged in a bell shape and is attached closely to the membrane at the base. In the microscopic preparations they generally appear bent in various ways; but there can be no doubt that in their natural condition they are stretched with some degree of tension, so that they pull down the

FIG. 51.



upper jointed ends of the fibres of the first series. The fibres of the first series arise from the inner margin of the membrane, which can be relatively little agitated, but the fibres of the second series are attached nearly in the middle of the membrane, and this is precisely the place where its vibrations will have the greatest excursions. When the pressure of the fluid in the drum gallery of the labyrinth is increased by driving the foot of the stirrup against

the oval window, the membrane at the base of the arches will sink downwards, the fibres of the second series be more tightly stretched, and perhaps the corresponding places of the fibres of the first series be bent a little downwards. It does not, however, seem probable that the fibres of the first series themselves move to any great extent, for their lateral connections are strong enough to make them hang together in masses like a membrane, when they have been released from their attachment in anatomical preparations. On reviewing the whole arrangement, there can be no doubt that Corti's organ is an apparatus adapted for receiving the vibrations of the membrana basilaris, and for vibrating of itself, but our present knowledge is not sufficient to determine with accuracy the manner in which these vibrations take place. For this purpose we require to estimate the stability of the several parts and the degree of tension and flexibility, with more precision than can be deduced from such observations as have hitherto been made on isolated parts, as they casually group themselves under the microscope.

Corti's fibres are wound round and covered over with a multitude of very delicate frail formations, fibres and cells of various kinds, partly the finest terminational runners of nerve fibres with appended nerve cells, partly fibres of ligamentous tissue, which appear to serve as a support for fixing and suspending the nerve formations.

The connection of these parts is best shewn in fig. 49 (p. 208). They are grouped like a pad of soft cells on each side of and within Corti's arches. The most important of them appear to be the cells c and d which are furnished with hairs, precisely resembling the ciliated cells in the ampullae and utriculus. They appear to be directly connected with fine varicose nerve fibres, and constitute the most constant part of the cochlean formations; for with birds and reptiles, where the structure of the cochlea is much simpler, and even Corti's arches are absent, these little ciliated cells are always to be found, and their hairs are so placed as to strike against Corti's membrane during the vibration of the membrana basilaris. The cells at a and a′, fig. 49 (p. 208), which appear in an enlarged condition at b and n in fig. 51 (p. 210), seem to have the character of an epithelium. In fig. 51 there will also be observed bundles and nets of fibres, which may be partly merely supporting fibres of a ligamentous nature, and partly, by appearing

as strings of beads, possess the character of bundles of the fibriles of nerves. But these parts are all so frail and delicate there is still much doubt as to their connection and office.

The essential result of our description of the ear consequ consists in the constant connection of the termination of auditory nerves with a peculiar auditory apparatus, partly el partly firm, which may be put in sympathetic vibration u the influence of external vibration, and will then probably ag and excite the mass of nerves. Now it was shewn in Chap. that the process of sympathetic vibration was observed to according as the bodies put into sympathetic vibration were as when once put in motion sounded for a long time, or soon their motion, p. 62. Bodies like tuning-forks which, when struck, go on sounding for a long time, are susceptible of sy thetic vibration in a high degree notwithstanding the difficul putting their mass in motion, because they admit of a long mulation of impulses in themselves minute, produced in the each separate vibration of the exciting tone. But precisel this reason there must be the exactest agreement betwee pitches of the proper tone of the fork and of the exciting because otherwise subsequent impulses given by the motion o air could not constantly recur in the same phase of vibration thus be suitable for increasing the subsequent effect of the prece impulses. On the other hand if we take bodies for which the rapidly dies away, such as stretched membranes or thin strings, we find that they are not only susceptible of sympat vibration, when vibrating air is allowed to act on them, but this sympathetic vibration is not so limited to a particular as in the other case, and they can therefore be easily set in m by tones of different kinds. For if an elastic body on being struck and allowed to sound freely, loses nearly the whole motion after ten vibrations, it will not be of much importance any fresh impulses received after the expiration of this time, sh agree exactly with the former, as it would be of great impor in the case of a sonorous body for which the motion generat the first impulse would remain nearly unchanged up to the that the second impulse was applied. In the latter case the se impulse could not increase the amount of motion, unless it upon a phase of the vibration which had precisely the same d tion of motion as itself.

The connection between these two relations can be calculated independently of the nature of the body put into sympathetic vibration,[1] and as the results are important to enable us to form a judgment on the state of things going on in the ear, a short table is annexed. Suppose that a body which vibrates sympathetically has been set into its state of maximum vibration by means of an exact unison, and that the exciting tone is then altered till the sympathetic vibration is reduced to $\frac{1}{10}$ of its former amount. The amount of the required difference of pitch is given in the first column in terms of an equally tempered Tone, [which is $\frac{1}{6}$ of an octave]. Now let the same resonant body be struck, and let its sound be allowed to die away gradually. The number of vibrations which it has made by the time that its intensity is reduced to $\frac{1}{10}$ of its original amount is noted, and given in the second column.

Difference of Pitch, in terms of an equally tempered Tone, necessary to reduce the intensity of sympathetic vibration to $\frac{1}{10}$ of that produced by perfect unisonance	Number of vibrations after which the intensity of tone in a resonant body whose sound is allowed to die out, reduces to $\frac{1}{10}$ of its original amount
1. One eighth of a Tone	38·00
2. One quarter of a Tone	19·00
3. One Semitone	9·50
4. Three quarters of a Tone	6·33
5. A whole Tone	4·75
6. A Tone and a quarter	3·80
7. A tempered minor Third or a Tone and a half	3·17
8. A Tone and three quarters	2·71
9. A tempered major Third or two whole Tones	2·37

Now, although we are not able exactly to discover how long the ear and its individual parts, when made to resound, will continue their resonance, yet well-known experiments allow us to form some sort of judgment as to the position which the parts of the ear must occupy in the scale exhibited in this table. Thus, there cannot possibly be any parts of the ear which continue to sound so long as a tuning-fork, for that would be patent to the commonest observation. But even if there were any parts in the ear answering to the first degree of our table, that is requiring 38 vibrations to be reduced to $\frac{1}{10}$ of their force,—we should recognize this in the deeper tones, because 38 vibrations last $\frac{1}{3}$ of a second for *A*

[1] The mode of calculation is explained in Appendix No. X.

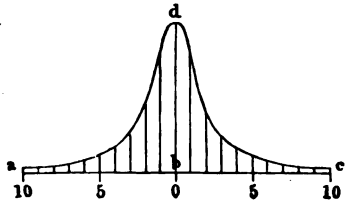
$\frac{1}{6}$ for a, $\frac{1}{12}$ for a' &c., and such a long endurance of sensible sound would render rapid musical passages impossible in the unaccented and once accented Octaves. Such a state of things would disturb musical effect as much as the strong resonance of a vaulted room, or as raising the dampers on a piano. When making a shake, we can readily strike 8 or 10 notes in a second, so that each tone separately is struck from 4 to 5 times. If then the sound of the first tone had not died off in our ear before the end of the second sound, at least to such an extent as not to be sensible when the latter was sounding, the tones of the shake, instead of being individually distinct, would merge into a continuous mixture of both. Now shakes of this kind, with 10 tones to a second, can be clearly and sharply executed throughout almost the whole scale, although it must be owned that from A downwards, in the great and contra octaves they sound bad and rough, and their tones begin to mix. Now it can be easily shewn that this is not due to the mechanism of the instrument. Thus if we shake on the physharmonica (harmonium &c.), the keys of the lower notes are just as accurately constructed and just as easy to move as those of the higher ones. Each separate tone is completely cut off with perfect certainty at the moment the valve falls on the air passage, and each speaks at the moment the valve is raised, because during so brief an interruption the tongues remain in a state of vibration. Similarly for the violoncello. At the instant when the finger which makes the shake falls on the string, the latter must commence a vibration of a different periodic time, due to its length; and the instant that the finger is removed, the vibration belonging to the deeper tone must return. And yet the shake in the bass is as imperfect on the violoncello as on any other instrument. Runs and shakes can be relatively best executed on a pianoforte because, at the moment of striking, the new tone sounds with great but rapidly decreasing intensity. Hence, in addition to the inharmonic noise produced by the simultaneous continuance of the two tones, we also hear a distinct prominence given to each separate tone. Now, since the difficulty of shaking in the bass is the same for all instruments, and for individual instruments is demonstrably independent of the manner in which the tones are produced, we are forced to conclude that the difficulty lies in the ear itself. We have then a plain indication that the vibrating parts of the ear are not damped with sufficient force and rapidity to allow of successfully effecting such a rapid alternation of tones.

Nay more, this fact further proves that *there must be different parts of the ear which are set in vibration by tones of different pitch and which receive the sensation of these tones.* Thus, it might be supposed that as the vibratory mass of the whole ear, the drum-skin, auditory ossicles, and fluid in the labyrinth, were vibrating at the same time, the inertia of this mass was the cause why the resonant vibrations in the ear were not immediately extinguished. But this hypothesis would not suffice to explain the fact observed. For an elastic body set into sympathetic vibration by any tone, vibrates sympathetically in the pitch or with the vibrational number of the exciting tone; but as soon as the exciting tone ceases, it goes on sounding in the pitch or vibrational number of its own proper tone. This fact which is derived from theory, may be perfectly verified on tuning-forks by means of the vibration microscope. If then the ear vibrated as a single system, and were capable of continuing its vibration for a sensible time, it would have to do so at its own pitch or with its own vibrational number, which is totally independent of the vibrational number of the former exciting tone. The consequence is that shakes would be equally difficult upon both high and low tones, and next that the two tones of the shake would not mix with each other, but that each would mix with a third tone, due to the ear itself. We became acquainted with such a tone in the last chapter, the high *f'''*, p. 167. The result, then, under these circumstances would be quite different from what is observed.

Now if a shake of 10 notes in a second, be made on *A*, of which the vibrational number is 110, this tone would be struck every $\frac{1}{5}$ of a second. We may justly assume that the shake would not be clear, if the intensity of the expiring tone were not reduced to $\frac{1}{10}$ of its original amount in this $\frac{1}{5}$ of a second. In this case after at least 22 vibrations, the parts of the ear which vibrate sympathetically with *A* must descend to at least $\frac{1}{10}$ of their intensity of vibration as their tone expires, so that their power of sympathetic vibration cannot be of the first degree in the table on p. 213, but may belong to the second, third, or some other higher degree. That the degree cannot be any much higher one, is shewn in the first place by the fact that shakes and runs begin to be difficult even on tones which do not lie very low. This we shall see by observations on beats subsequently detailed. We may on the whole assume that the parts of the ear which vibrate sympathetically

have an amount of damping power corresponding to the third degree of our table, where the intensity of sympathetic vibration with a Semitone difference of pitch is only $\frac{1}{10}$ of what it is for a complete unison. Of course there can be no question of exact determinations, but it is important for us to be able to form at least an approximate conception of the influence of damping on the sympathetic vibration of the ear, as it has great significance in the relations of consonance. Hence when we hereafter speak of individual parts of the ear vibrating sympathetically with a determinate tone, we mean that they are set into strongest motion by that tone, but are also set into vibration less strongly by tones of nearly the same pitch, and that this sympathetic vibration is still sensible for the interval of a Semitone. Fig. 52 may serve to give a general conception of the law by which the intensity of the sympathetic vibration decreases, as the difference of pitch increases. The horizontal line abc represents a portion of the musical scale, each of the lengths ab and bc standing for a whole (equally tempered) Tone. Suppose that the body which vibrates sympathetically has been tuned to the tone b and that the vertical line bd represents the maximum of intensity of tone which it can attain when excited by a tone in perfect unison with it. On the base line, intervals of $\frac{1}{10}$ of a whole tone are set off, and the vertical lines drawn through them shew the corresponding intensity of the tone in the body which vibrates sympathetically, when the exciting tone differs from a unison by the corresponding interval. The following are the numbers from which fig. 52 was constructed.

FIG. 52.



Difference of Pitch	Intensity of Sympathetic Vibration
0·0	100
0·1	74
0·2	41
0·3	24
0·4	15
Semitone	10
0·6	7·2
0·7	5·4
0·8	4·2
0·9	3·3
Whole Tone	2·7

Now we cannot precisely ascertain what parts of the ear actually vibrate sympathetically with individual tones. The particles of auditory sand, being suspended in a mucous fluid, are scarcely capable of really regular vibrations; but seem rather suited for yielding to single impulses and transmitting them to the nerves. The same holds for the ciliae in the ampullae, because corpuscles of such minute mass, cannot long retain their motion. The form of the ampullae, which are wide cavities with two comparatively narrow exits, appears suitable for producing a distinct central current, which partly resolves sideways into eddies.[1] Currents of moving fluid, separating themselves from the surrounding motionless fluid, are formed wherever a fluid passes through an opening, or out of a canal with sharp edges, into a wider space. Every chimney out of which proceeds a column of air impregnated with smoke, shews this phenomenon. The current of smoke remains for a short distance distinct from the surrounding purer air, and then resolves into eddies. Now if nervous excitement depends upon deflecting the ciliae of a cell which forms the termination of the nerve, towards the cell, an oscillation of the whole mass of the cell floating in water would not suffice to effect this, but it would be necessary for the individual parts to be struck by streams of fluid of different strength and in different directions. The auditory sand and ampullae are admirably adapted for producing discontinuous streams of this kind.

On the other hand the whole construction of the partition of the cochlea, and of Corti's arches upon it, appears suitable for executing independent vibrations. It is not necessary for these parts to be able to continue their vibrations for a long time without assistance. It must certainly be very important for the sense of hearing that the nerves terminate in such different kinds of apparatus. Elastic formations, readily damped, are proportionably more strongly affected by brief transient impulses and streams of the fluid of the labyrinth, than by musical tones. They are therefore suitable for the apprehension of irregular agitations, of short duration, and hence for the sensation of noises. On the other hand elastic bodies which are not so readily damped, are much more strongly excited by a musical tone of the corresponding pitch than by any single impulses. Our ear is capable of doing

[1] See my notice on the discontinuous motions of fluids in the monthly proceedings (*Monatsberichte*) of the Berlin Academy, 23 April 1868.

both, and we may consequently suppose that this capability depends on the existence of the different kinds of terminational organs, and that consequently the expansions of the nerves in the vestibule and ampullae serve for the apprehension of noises, while Corti's rods and the *membrana basilaris* are used for musical tones.

But if these formations are to serve for distinguishing tones of different pitch, and if tones of different pitch are to be equally well perceived in all parts of the scale, the elastic formations in the cochlea, which are connected with different nerve fibres, must be differently tuned, and their proper tones must form a regularly progressive series of degrees through the whole extent of the musical scale.

According to the recent anatomical researches of V. Hensen and C. Hasse, it is probably the breadth of the membrana basilaris in the cochlea, which determines the tuning.[1] Opposite the oval window, it is comparatively narrow, and it continually increases in width as it approaches the apex of the cochlea. The following measurements of the membrane in a newly born child, from the line where the nerves pass through on the inner edge, to the attachment to the ligamentum spirale on the outer edge, are given by V. Hensen.

Place of Section	Breadth of Membrane or Length of Transverse Fibres	
	Millimetres	Inches
0·2625 mm [= 0·010335 in.] from root	0·04125	·00162
0·8626 mm [= 1·034061 in.] from root	0·0825	·00325
Middle of the first spire	0·169	·00665
End of first spire	0·3	·01181
Middle of second spire	0·4125	·01624
End of second spire	0·45	·01772
At the hamulus	0·495	·01949

The breadth therefore increases more than twelvefold from the beginning to the end.

Corti's rods also exhibit an increase of size as they approach

[1] In the first edition of this book (1863), which was written at a time when the more delicate anatomy of the cochlea was just beginning to be developed, I supposed that the different degrees of stiffness and tension in Corti's rods themselves might furnish the reason of their different tuning. By Hensen's measures of the breadth of the membrana basilaris (Zeitschrift für wissensch. Zoologie, vol. xiii. p. 492) and Hasse's proof that Corti's rods are absent in birds and amphibia, far more definite foundations for forming a judgment, have been furnished, than I then possessed.

the vertex of the cochlea, but in a much less degree than the membrana basilaris. The following are Hensen's measurements:

	at the round window		at the hamulus	
	mm.	inch	mm.	inch
Length of inner rod .	0·048	0·00189	0·0855	0·00337
Length of outer rod .	0·048	0·00189	0·098	0·00386
Span of the arch . .	0·019	0·00075	0·085	0·00335

Hence it follows, as Henle has also proved, that the greatest increase of breadth falls on the outer zone of the basilar membrane, beyond the line of the attachment of the outer rods. This increases from 0·023 mm [=·000905 in.] to 0·41 mm [=·016142 inch] or nearly twentyfold.

In accordance with these measures, the two rows of Corti's rods are almost parallel and upright near to the round window, but they are bent much more strongly towards one another near the vertex of the cochlea.

It has been already mentioned that the membrana basilaris of the cochlea breaks easily in the radial direction, but that its radial fibres have considerable tenacity. This seems to me to furnish a very important mechanical relation, namely that this membrane in its natural connection admits of being tightly stretched in the transverse direction from the modiolus to the outer wall of the cochlea, but can have only little tension in the direction of its length, because it could not resist a strong pull in this direction.

Now the mathematical theory of the vibration of a membrane with different tensions in different directions shews that it behaves very differently from a membrane which has the same tension in all directions.[1] On the latter, vibrations produced in one part, spread uniformly in all directions, and hence if the tension were uniform it would be impossible to set one part of the basilar membrane in vibration, without producing nearly as strong vibrations (disregarding individual nodal lines) in all other parts of the membrane.

But if the tension in direction of its length is infinitesimally small in comparison with the tension in direction of the breadth, then the radial fibres of the basilar membrane may be approxima-

[1] See Appendix No. XI.

tively regarded as forming a system of stretched strings, and the membranous connection as only serving to give a fulcrum to the pressure of the fluid against these strings. In that case the laws of their motion would be the same as if every individual string moved independently of all the others, and obeyed, by itself, the influence of the periodically alternating pressure of the fluid of the labyrinth contained in the vestibule gallery. Consequently any exciting tone would set that part of the membrane into sympathetic vibration, for which the proper tone of one of its radial fibres that are stretched and loaded with the various appendages already described, corresponds most nearly with the exciting tone; and thence the vibrations will extend with rapidly diminishing strength on to the adjacent parts of the membrane. Fig. 52, on p. 216, might be taken to represent, on an exaggerated scale of height, a longitudinal section of that part of the basilar membrane in which the proper tone of the radial fibres of the membrane are nearest to the exciting tone.

The strongly vibrating parts of the membrane would, as has been explained in respect to all bodies which vibrate sympathetically, be more or less limited, according to the degree of damping power in the adjacent parts, by friction against the fluid in the labyrinth and in the soft gelatinous parts of the nerve fillet.

Under these circumstances the parts of the membrane in unison with higher tones must be looked for near the round window, and those with the deeper, near the vertex of the cochlea, as Hensen also concluded from his measurements. That such short strings should be capable of corresponding with such deep tones, must be explained by their being loaded in the basilar membrane with all kinds of solid formations; the fluid of both galleries in the cochlea must also be considered as weighting the membrane, because it cannot move without a kind of wave motion in that fluid.

The observations of Hasse shew that Corti's arches do not exist in the cochlea of birds and amphibia, although the other essential parts of the cochlea, as the basilar membrane, the ciliated cells in connection with the terminations of the nerves, and Corti's membrane which stands opposite the ends of these ciliae, are all present. Hence it becomes very probable that Corti's arches play only a secondary part in the office of the cochlea. Perhaps we might look for the effect of Corti's arches in their power, as relatively firm

objects, of transmitting the vibrations of the basilar membrane to small limited regions of the upper part of the relatively thick nervous fillet, better than it could be done by the immediate communication of the vibrations of the basilar membrane through the soft mass of this fillet. Close to the outside of the upper end of the arch, connected with it by the stiffer fibriles of the membrana reticulāris, are the ciliated cells of the nervous fillet (see c in fig. 49, p. 208). In birds, on the other hand, the ciliated cells form a thin stratum upon the basilar membrane, which can readily receive limited vibrations from the membrane, without communicating them too far sideways.

According to this view Corti's arches, in the last resort, will be the means of transmitting the vibrations received from the basilar membrane to the terminal appendages of the conducting nerve. In this sense the reader is requested hereafter to understand references to the vibrations, proper tone, and intonation of Corti's arches; the intonation meant is that which they receive through their connection with the corresponding part of the basilar membrane.

According to Kölliker there are about 3000 of Corti's arches in the human cochlea. If we deduct 200 for the simple tones which lie beyond musical limits, and cannot have their pitch perfectly apprehended, there remain 2800 for the seven octaves of musical instruments, that is, 400 for every octave, $33\frac{1}{3}$ for every interval of a semitone; certainly quite enough to explain the power of distinguishing small parts of a semitone. According to E. H. Weber's investigations practised musicians can perceive even a difference of pitch for which the vibrational numbers are as 1000 to 1001. This would be about $\frac{1}{64}$ of a semitone, which is a smaller interval than that between the pitch of two of Corti's arches. But that does not prejudice our assumption. For if a simple tone is struck having a pitch between those of two of Corti's arches, it would set them both in sympathetic vibration, and that arch would vibrate the more strongly which was nearest in pitch to the proper tone. The amount of power in discriminating small intervals between two pitches which lie between those of two arches will therefore finally depend upon the delicacy with which the different forces of the vibrations excited can be compared. And we have thus also an explanation of the fact that as the pitch of an external tone rises continuously, our sensations also alter continuously and not

by jumps, as must be the case if only one of Corti's arches were set in sympathetic motion at once.

To draw further conclusions from our hypothesis, when a simple tone is presented to the ear, those Corti's arches which are nearly or exactly in unison with it will be strongly excited, and the rest only slightly or not at all. Hence every simple tone of determinate pitch will be felt only by certain nerve fibres, and simple tones of different pitch will excite different fibres. When a compound musical tone or chord is presented to the ear, all those elastic bodies will be excited, which have a proper pitch corresponding to the various individual simple tones contained in the whole mass of tones, and hence by properly directing attention, all the individual sensations of the individual simple tones can be perceived. The chord must be resolved into its individual compound tones, and the compound tone into its individual harmonic partial tones.

This also explains how it is that the ear resolves a motion of the air into pendular vibrations and no other. Any particle of air can of course execute only one motion at one time. That we considered such a motion mathematically as a sum of pendular vibrations, was in the first instance merely an arbitrary assumption to facilitate theory, and had no meaning in nature. The first meaning in nature that we found for this resolution came from considering sympathetic vibration, when we discovered that a motion, which was not pendular could produce sympathetic vibrations in bodies of those different pitches, which corresponded to the harmonic upper partial tones. And now our hypothesis has also reduced the phenomenon of hearing to that of sympathetic vibration, and thus furnishes a reason why an originally simple periodic vibration of the air produces a sum of different sensations, and hence also appears as compound to our perceptions.

The sensation of different pitch would consequently be a sensation in different nerve fibres. The sensation of a quality of tone would depend upon the power of a given compound tone to set in vibration not only those Corti's arches which correspond to its prime tone, but also a series of other arches, and hence to excite sensation in several different groups of nerve fibres.

Physiologically it should be observed that the present assumption reduces sensations which differ qualitatively according to pitch and quality of tone, to a difference in the nerve fibres which are excited. This is a step similar to that taken in a wider field by Johannes Müller in his theory of the specific energies of sense.

He has shewn that the difference in the sensations due to various senses, does not depend upon the actions which excite them, but upon the various nervous arrangements which receive them. We can convince ourselves experimentally that in whatever manner the optic nerve and its expansion, the retina of the eye, may be excited, by light, by twitching, by pressure, or by electricity, the result is never anything but a sensation of light, and that the tactual nerves on the contrary never give us sensations of light or of hearing or of taste. The same solar rays which are felt as light by the eye, are felt by the nerves of the hand as heat; the same agitations which are felt by the hand as flutters, are tone to the ear.

Just as the ear apprehends vibrations of different periodic time as tones of different pitch, so does the eye perceive luminiferous vibrations of different periodic time, as different colours, the quickest giving violet and blue, the mean green and yellow, the slowest red. The laws of the mixture of colours led Thomas Young to the hypothesis that there were three kinds of nerve fibres in the eye, with different powers of sensation, for feeling red, for feeling green, and for feeling violet. In reality this assumption gives a very simple and perfectly consistent explanation of all the optical phenomena depending on colour. And by this means the qualitative differences of the sensations of sight are reduced to differences in the nerves which receive the sensations. For the sensations of each individual fibre of the optic nerve there remains only the quantitative differences of greater or less irritation.

The same result is obtained for hearing by the hypothesis to which our investigation of quality of tone has led us. The qualitative difference of pitch and quality of tone is reduced to a difference in the fibres of the nerve receiving the sensation, and for each individual fibre of the nerve there remains only the quantitative differences in the amount of excitement.

The processes of irritation within the nerves of the muscles, by which their contraction is determined, have hitherto been more accessible to physiological investigation than those which take place in the nerves of sense. In those of the muscle, indeed, we only find quantitative differences of more or less excitement, and no qualitative differences at all. In them we are able to establish, that during excitement the electrically active particles of the nerves undergo determinate changes, and that these changes ensue in exactly the same way whatever be the excitement which

causes them. But precisely the same changes also take place in an excited nerve of sense, although their consequence in this case is a sensation, while in the other it was a motion; and hence we see that the mechanism of the process of irritation in the nerves of sense must be in every respect similar to that in the nerves of motion. The two hypotheses just explained really reduce the processes in the nerves of man's two principal senses, notwithstanding their apparently involved qualitative differences of sensations, to the same simple scheme with which we are familiar in the nerves of motion. Nerves have been often and not unsuitably compared to telegraph wires. Such a wire conducts one kind of electric current and no other; it may be stronger, it may be weaker, it may move in either direction; it has no other qualitative differences. Nevertheless, according to the different kinds of apparatus with which we provide its terminations, we can send telegraphic dispatches, ring bells, explode mines, decompose water, move magnets, magnetise iron, develop light, and so on. So with the nerves. The condition of excitement which can be produced in them and is conducted by them, is, so far as it can be recognised in isolated fibres of a nerve, everywhere the same, but when it is brought to various parts of the brain, or the body, it produces motion, secretions of glands, increase and decrease of the quantity of blood, of redness and of warmth of individual organs, and also sensations of light, of hearing, and so forth. Supposing that every qualitatively different action is produced in an organ of a different kind, to which also separate fibres of nerve must proceed, then the actual process of irritation in individual nerves may always be precisely the same, just as the electrical current in the telegraph wires remains one and the same notwithstanding the various kinds of effects which it produces at its extremities. On the other hand, if we assume that the same fibre of a nerve is capable of conducting different kinds of sensation, we should have to assume that it admits of various kinds of processes of irritation, and this we have been hitherto unable to establish.

In this respect then the view here proposed, like Young's hypothesis for the difference of colours, has a still wider signification for the physiology of the nerves in general.

Since the first publication of this book, the theory of auditory sensation here explained, has received an interesting confirmation from the observations and experiments made by V. Hensen[1] on the auditory apparatus of the Crustaceae. These animals have bags of auditory stones (otoliths), partly closed, partly opening outwards, in which these stones float freely in a watery fluid and are supported by hairs of a peculiar formation, attached to the stones at one end, and, partly, arranged in a series proceeding in order of magnitude, from larger and thicker to shorter and thinner. In many crustaceans also we find precisely similar hairs on the open surface of the body, and these must be considered as auditory hairs. The proof that these external hairs are also intended for hearing, depends first on the similarity of their construction with that of the hairs in the bags of otoliths; and secondly on Hensen's discovery that the sensation of hearing remained in the *Mysis* (opossum shrimp) when the bags of otoliths had been extirpated, and the external auditory hairs of the antennae were left.

Hensen conducted the sound of a keyed horn through an apparatus formed on the model of the drumskin and auditory ossicles of the ear into the water of a little box in which a specimen of Mysis was fastened in such a way as to allow the external auditory hairs of the tail to be observed. It was then seen that certain tones of the horn set some hairs into strong vibration, and other tones other hairs. Each hair answered to several notes of the horn, and from the notes mentioned we can approximatively recognise the series of subtones of one and the same simple tone. The results could not be quite pure, because the resonance of the apparatus must have had some influence.

Thus one of these hairs answered strongly to $d\sharp$ and $d'\sharp$, more weakly to g, and very weakly to G. This leads us to suppose that it was tuned to some pitch between d'' and $d''\sharp$. In that case it answered to the second partial of d' to $d'\sharp$, the third of g to $g\sharp$, the fourth of d to $d\sharp$, and the sixth of G to $G\sharp$. A second hair answered strongly to $a\sharp$ and the adjacent tones, more weakly to $d\sharp$ and $A\sharp$. Its proper tone therefore seems to have been $a\sharp$.

By these observations the existence of such relations as we have supposed in the case of the human cochlea, have been directly

[1] 'Studien über das Gehörorgan der Decapoden,' Leipzig, 1863. Reprinted from Siebold and Kölliker's 'Zeitschrift für wissenschaftliche Zoologie,' vol. xiii.

CHAPTER VII.

COMBINATIONAL TONES.

IN the first part of this book we had to enunciate and constantly apply the proposition that oscillatory motions of the air and other elastic bodies, produced by several sources of sound acting simultaneously, are always the exact sum of the individual motions producible by each source separately. This law is of extreme importance in the theory of sound, because it reduces the consideration of compound cases to those of simple ones. But it must be observed that this law holds strictly only in the case where the vibrations in all parts of the mass of air and of the resonant elastic bodies are of *infinitely small dimensions*; that is to say, only when the alterations of density of the elastic bodies are so small that they may be disregarded in comparison with the whole density of the same body; and in the same way, only when the displacements of the vibrating particles vanish as compared with the dimensions of the whole elastic body. Now certainly in all practical applications of this law to resonant bodies, the vibrations are always *very small*, and near enough to being *infinitesimally small* for this law to hold with great exactness even for the real resonant vibrations of musical tones, and by far the greater part of their phenomena can be deduced from that law in conformity with observation. Still, however, there are certain phenomena which result from the fact that this law does *not* hold with perfect exactness for vibrations of elastic bodies, which, though *very small*, are far from being *infinitely small*.[1] One of these phenomena with which we are here interested, is the occurrence of *Combinational Tones*, which were first discovered in 1740 by Sorge;[2] a

[1] Helmholtz, on 'Combinational Tones,' in Poggendorff's 'Annalen,' vol. xcix., p. 497.—'Monatsberichte' of the Berlin Academy, May 22, 1856. From this an extract is given in Appendix No. XII.

[2] 'Vorgemach musikalischer Composition' (Antechamber of musical composition).

German organist, and were afterwards generally known, although their pitch was often wrongly assigned, through the Italian violinist Tartini, from whom they are often called *Tartini's tones.*[1]

These tones are heard whenever two musical tones of different pitches are sounded together, loudly and continuously. The pitch of a combinational tone is generally different from that of either of the generating tones, or of their harmonic upper partials. In experiments, then, the combinational are readily distinguished from the upper partial tones, by not being heard when only one generating tone is sounded, and by appearing simultaneously with the second tone. Combinational tones are of two kinds. The first class, discovered by Sorge and Tartini, I have termed *differential tones*, because their vibrational number is the *difference* of the vibrational numbers of the generating tones. The second class of *summational tones*, having their vibrational number equal to the *sum* of the vibrational numbers of the generating tones, were discovered by myself.

On investigating the combinational tones of two compound musical tones, we find that both the primary and upper partial tones may give rise to both differential and summational tones. In such cases the number of combinational tones is very great. But it must be observed that generally the differential are stronger than the summational tones, and that the stronger generating simple tones produce stronger combinational tones. The combinational tones, indeed, increase in a much greater ratio than the generating tones, and diminish also more rapidly. Now since in musical compound tones the prime generally predominates over the partials, the differential tones of the two primes are generally heard more loudly than all the rest, and were consequently first discovered. They are most easily heard when the two generating tones are less than an Octave apart, because in that case the differential is deeper than either of the two generating tones. To hear it at first, choose two tones which can be held with great force for some time, and form a justly intoned harmonic interval. First sound the low tone and then the high one. On properly directing

[1] [In England they have hence been often called by Tartini's name, *terzi tuoni*, or third tones, resulting from the combination of two. A commoner English name is *grave harmonics*, which is inapplicable, as they are not necessarily graver than both of the generating tones. Prof. Tyndall calls them *resultant tones*. I prefer retaining the Latin expression used by Prof. Helmholtz and Scheibler.—*Translator.*]

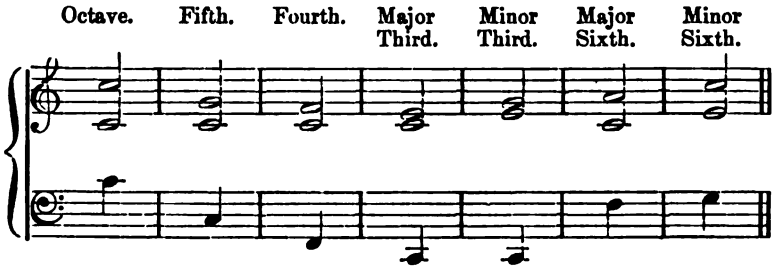
attention, a weaker low tone will be heard at the moment that the higher note is struck; this is the required combinational tone.[1] For particular instruments, as the harmonium, the combinational tones can be made more audible by properly tuned resonators. In this case the tones are generated in the air contained within the instrument. But in other cases, where they are generated solely within the ear, the resonators are of little or no use.

The following table gives the first differential tones of the usual harmonic intervals:—

Intervals	Ratio of the vibrational numbers	Difference of the same	The combinational tone is deeper than the deeper generating tone by
Octave . . .	1 : 2	1	a Unison
Fifth . . .	2 : 3	1	an Octave
Fourth . . .	3 : 4	1	a Twelfth
Major Third . .	4 : 5	1	Two Octaves
Minor Third . .	5 : 6	1	Two Octaves and a major Third
Major Sixth . .	3 : 5	2	a Fifth
Minor Sixth . .	5 : 8	3	a major Sixth

or in ordinary musical notation, the generating tones being written as minims and the differential tones as crotchets—

[1] [I have found that combinational tones can be made quite audible to a hundred people at once, by means of two flageolet fifes or whistles, blown as strongly as possible. I choose very close dissonant intervals because the great depth of the low tone is very striking, being very far below anything that can be touched by the instrument itself. Thus *g''''* being sounded loudly on one pipe by an assistant, I give *f'''*♯, when a deep note is instantly heard which, if the interval were pure, would be *g*, and is sufficiently near to *g* to be recognised as extremely deep. As a second experiment, the *g''''* being held as before, I give first *f''''*♯ and then *e''''* in succession. If the intervals were pure the combinational tones would jump from *g* to *c'*, and in reality, the interval is very nearly the same and quite appreciable. The differential tones are well heard on the English concertina, especially if the ear be placed against the bellows and the interval be small. Semitones tell well. The tones *f'''*♯ and *g'''*, and *e'''* and *g'''* are here also instructive, but are not so well fitted for a large audience as the fifes. It is also convenient to choose these dissonant intervals for first examples in order to dissipate the notion that the 'grave harmonic' is the 'true fundamental bass' of the 'chord.' Again, to dissipate the notion that the combinational tones arise from the number of beats produced (p. 235, line 1), I find it best to use on the English concertina, *g''* and *g''*♯ or *f''* and *f''*♯ as generators. By proper attention it is then quite possible to hear the *booming* of the very low differential tones in the contra octave, and the *rattle* of the beats, *at the same time.* Generally when the ear has come to recognise the combinational tones, the performer himself will find them inconveniently prominent on the English concertina.—*Translator.*]



When the ear has learned to hear the combinational tones of pure intervals and sustained tones, it will be able to hear them from inharmonic intervals and in the rapidly dying notes of a pianoforte. The combinational tones from inharmonic intervals are more difficult to hear, because these intervals beat more or less strongly, as we shall have to explain hereafter. Those arising from tones which rapidly die off, as those of the pianoforte, are not strong enough to be heard except at the first instant, and die off more rapidly than the generating tones. Combinational tones are also in general easier to hear from the simple tones of tuning-forks and stopped organ pipes than from compound tones where a number of other secondary tones are also present. These compound tones, as has been already said, also generate a number of differential tones by their harmonic upper partials, and these easily distract attention from the differential tones of the primes. Combinational tones of this kind, arising from the upper partials, are frequently heard from the violin and harmonium.

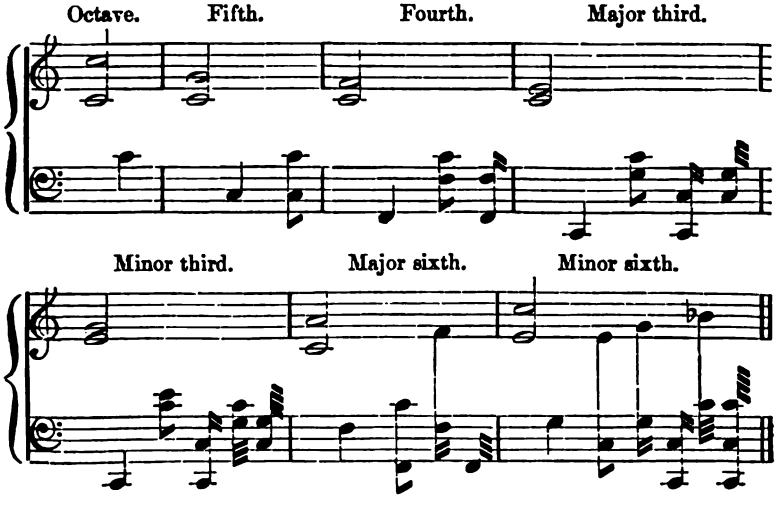
Example.—Take the major Third $c'e'$, ratio of vibrational numbers 4 : 5. First difference 1, that is C. The first harmonic upper partial of c' is c'', vibrational number 8. Ratio of this and e', 5 : 8, difference 3, that is g. The first upper partial of e' is e'', vibrational number 10; ratio for this and c', 4 : 10, difference 6, that is g'. Then again c'' e'' have ratio 8 : 10, difference 2, that is c. Hence, taking only the first upper partials we have the series of combinational tones 1, 3, 6, 2 or C, g, g', c. Of these the tone 3, or g, is often easily perceived.

These multiple combinational tones cannot in general be distinctly heard, except when the generating compound tones contain audible harmonic upper partials. Yet we cannot assert that the combinational tones are absent, where such partials are absent; but in that case they are so weak that the ear does not readily recognise them beside the loud generating tones and first differential. In the first place theory leads us to conclude that

they do exist in a weak state, and in the next place the beats of impure intervals, to be discussed presently, also establish their existence. In this case we may, as Hallstroem suggests,[1] consider the multiple combinational tones to arise thus: the first differential tone, or *combinational tone of the first order*, by combination with the generating tones themselves, produce other differential tones, or *combinational tones of the second order*; these again produce new ones with the generators and differentials of the first order, and so on.

Example.—Take two simple tones c' and e', ratio 4 : 5, difference 1, differential tone of the first order C. This with the generators gives the ratios 1 : 4 and 1 : 5, differences 3 and 4, differential tones of the second order g, and c' once more. The new tone 3, gives with the generators the ratios 3 : 4 and 3 : 5, differences 1 and 2, giving the differential tones of the third order C and c, and the same tone 3 gives with the differential of the first order 1, the ratio 1 : 3, difference 2, and hence as a differential of the fourth order c once more, and so on. The differential tones of different orders which coincide when the interval is perfect, as it is supposed to be in this example, no longer exactly coincide when the generating interval is not pure; and consequently such beats are heard, as would result from the presence of these tones. More on this hereafter.

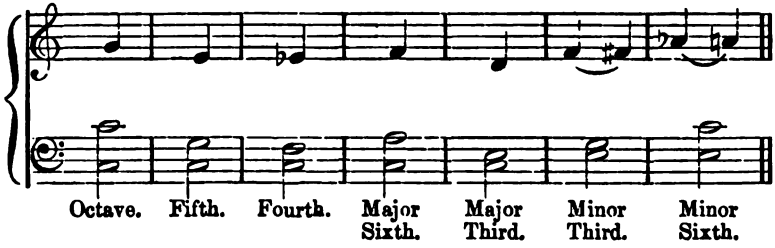
The differential tones of different orders for different intervals are given in the following notes, where the generators are minims, the combinational tones of the first order crotchets, of the second quavers, and so on. The same tones also occur with compound generators as combinational tones of their upper partials.



[1] Poggendorff's 'Annalen,' vol. xxiv. p. 438.

The series are broken off as soon as the last order of differentials furnishes no fresh tones. In general these examples shew that the complete series of harmonic partial tones 1, 2, 3, 4, 5, &c., up to the generators themselves, is produced.

The second kind of combinational tones, which I have distinguished as *summational*, is generally much weaker in sound than the first, and is to be heard with decent ease almost only on the harmonium and polyphonic syren. Scarcely any but the first summational tone can be perceived, having a vibrational number equal to the sum of those of the generators. Of course summational tones may also arise from the harmonic upper partials. Since their vibrational number is always equal to the sum of the other two, they are always higher in pitch than either of the two generators. The following notes will shew their nature for the simple intervals.



For the two last intervals the summational tone in each case lies between the two notes written. In relation to music I will here remark at once that many of these summational tones form extremely inharmonic intervals with the generators. Were they not generally so weak on most instruments, they would give rise to intolerable dissonances. In reality, the major and minor Third, and the minor Sixth, sound very badly indeed on the polyphonic siren, where all combinational tones are remarkably loud, whereas the Octave, Fifth, and major Sixth are very beautiful. Even the Fourth on this siren has only the effect of a tolerably harmonious chord of the minor Seventh.

It was formerly believed that the combinational tones were purely subjective, and were produced in the ear itself. Differential tones alone were known, and these were connected with the beats which usually result from the simultaneous resonance of two tones of nearly the same pitch, a phenomenon to be considered in

the following chapters. It was believed that when these beats occurred with sufficient rapidity, the individual increments of loudness might produce the sensation of a new tone, just as numerous ordinary impulses of the air, and that the vibrational number of such a tone would be equal to the number of beats. But this supposition, in the first place, does not explain the origin of summational tones, being confined to the differentials; secondly, it may be proved that under certain conditions the combinational tones exist objectively, independently of the ear which would have had to gather the beats into a new tone; and thirdly, this supposition cannot be reconciled with the law confirmed by all other experiments, that the only tones which the ear hears, correspond to pendular vibrations of the air.

And in reality a different cause for the origin of combinational tones can be established, which has already been mentioned in general terms. Whenever the vibrations of the air or of other elastic bodies which are set in motion at the same time by two generating simple tones, are so powerful that they can no longer be considered infinitely small, mathematical theory shews that vibrations of the air must arise which have the same vibrational numbers as the combinational tones.

Particular instruments give very powerful combinational tones. The condition for their generation is that the same mass of air should be violently agitated by two simple tones simultaneously. This takes place most powerfully in the polyphonic siren,[1] in which the same rotating disc contains two or more series of holes which are blown upon simultaneously from the same wind chest. The air of the wind chest is condensed whenever the holes are closed; on the holes being opened, a large quantity of air escapes, and the pressure is considerably diminished. The mass of air in the wind chest, and partly even in the bellows, as can be easily felt, comes into violent vibration. If two rows of holes are blown, vibrations arise in the air of the wind chest corresponding to both tones, and each row of openings gives vent not to a stream of air uniformly supplied, but to a stream of air already set in vibration by the other tone. Under these circumstances the combinational tones are extremely powerful, almost as powerful, indeed, as the generators. Their objective existence in the mass of air can be proved by

[1] A detailed description of this instrument will be given in the next chapter.

vibrating membranes tuned to be in unison with the combinational tones. Such membranes are set in sympathetic vibration immediately upon both generating tones being sounded simultaneously, but remain at rest if only one or other of them is sounded. Indeed, in this case the summational tones are so powerful that they make all chords extremely unpleasant which contain Thirds or minor Sixths. Instead of membranes it is more convenient to use the resonators already recommended for investigating harmonic upper partial tones. But resonators are also unable to reinforce a tone when no pendular vibrations actually exist in the air; they have no effect on a tone which exists only in auditory sensation, and hence they can be used to discover whether a combinational tone is objectively present. They are much more sensitive than membranes, and are well adapted for the clear recognition of very weak objective tones.

The conditions in the harmonium are similar to those in the siren. Here too, there is a common wind chest, and when two keys are pressed down, we have two openings which are closed and opened rhythmically by the tongues. In this case also the air in the common receptacle is violently agitated by both tones, and air is blown through each opening which has been already set in vibration by the other tongue. Hence in this instrument also the combinational tones are objectively present, and comparatively very distinct, but they are far from being as powerful as on the siren, probably because the wind chest is very much larger in proportion to the openings, and hence the air which escapes during the short opening of an exit by the oscillating tongue cannot be sufficient to diminish the pressure sensibly. In the harmonium also the combinational tones are very clearly reinforced by resonators tuned to be in unison with them, especially the first and second differential and the first summational tone. Nevertheless I have convinced myself by particular experiments, that even in this instrument the greater part of the force of the combinational tone is generated in the ear itself. I arranged the portvents in the instrument so that one of the two generators was supplied with air by the bellows moved below by the foot, and the second generator was blown by the reserve bellows, which was first pumped full and then cut off by drawing out the so-called expression-stop, and I then found that the combinational tones were not much weaker than for the usual arrangement. But the objective portion

which the resonators reinforce was much weaker. The noted examples given above will easily enable any one to find the digitals which must be pressed down in order to produce a combinational tone in unison with a given resonator.

On the other hand, when the places in which the two tones are struck are entirely separate and have no mechanical connection, as, for example, if they come from two singers, two separate wind instruments, or two violins, the reinforcement of the combinational tones by resonators is small and dubious. Here then there does not exist in the air any clearly sensible pendular vibration corresponding to the combinational tone, and we must conclude that such tones, which are often powerfully audible, are really produced in the ear itself. But analogously to the former cases we are justified in assuming in this case also that the external vibrating parts of the ear, the drumskin and auditory ossicles, are really set in a sufficiently powerful combined vibration to generate combinational tones, so that the vibrations which correspond to combinational tones may really exist objectively in the parts of the ear without existing objectively in the external air. A slight reinforcement of the combinational tone in this case by the proper unisonant resonator may, therefore, arise from the drumskin of the ear communicating to the air in the resonator those particular vibrations which correspond to the combinational tone.[1]

Now it so happens that in the construction of the external parts of the ear for conducting sound, there are certain conditions which are peculiarly favourable for the generation of combinational tones. First we have the unsymmetrical form of the drumskin itself. Its radial fibres, which are externally convex, undergo a much greater alteration of tension when they make an oscillation of moderate amplitude towards the inside, than when the oscillation takes place towards the outside. For this purpose it is only necessary that the amplitude of the oscillation should not be too small a fraction of the minute depth of the arc made by these radial fibres. Under these circumstances deviations from the simple superposition of vibrations arise for very much smaller amplitudes than is the case when the vibrating body is symmetrically constructed on both sides.[2]

[1] [See latter half of Appendix XVI.—*Translator.*]

[2] See my paper on combinational tones already cited, and Appendix No. XII. For unsymmetrical vibrating bodies the disturbances are proportional to the first

But a more important circumstance, as it seems to me, when the resonance is powerful, is the loose formation of the joint between the hammer and anvil (p. 197). If the handle of the hammer is driven inwards by the drumskin, the anvil and stirrup must follow the motion unconditionally. But that is not the case for the subsequent outward motion of the handle of the hammer, during which the teeth of the two ossicles need not catch each other. In this case the ossicles may *click*. Now I seem to hear this clicking in my own ear whenever a very strong and deep tone is brought to bear upon it, even when, for example, it is produced by a tuning-fork held between the fingers, in which there is certainly nothing than can make any click at all.

This peculiar feeling of mechanical tingling in the ear had long ago struck me when two clear and powerful soprano voices executed passages in Thirds, in which case the combinational tone comes out very distinctly. If the phases of the two tones are so related that after every fourth oscillation of the deeper and every fifth of the higher tone, there ensues a considerable outward displacement of the drumskin, sufficient to cause a momentary loosening in the joint between the hammer and anvil, a series of blows will be generated between the two bones, which would be absent if the connection were firm and the oscillation regular, and these blows taken together would exactly generate the first differential tone of the interval of a major Third. Similarly for other intervals.

It must also be remarked that the same peculiarities in the construction of a sonorous body which makes it suitable for allowing combinational tones to be heard when it is excited by two waves of different pitch, must also cause a single simple tone to excite vibrations in it, which correspond to its harmonic upper partials; the effect being the same as if this tone then formed summational tones with itself.

This result ensues because a simple periodical force, corresponding to pendular vibrations, cannot excite similar pendular vibrations in the elastic body on which it acts, unless the elastic forces called into action by the displacements of the excited body from its position of equilibrium, are proportional to these displacements themselves. This is always the case so long as these displacements

power of the amplitude, whereas for symmetrical ones they are proportional to only the second power of this magnitude, which is very small in both cases.

are infinitesimal. But if the amplitude of the oscillations is great enough to cause a sensible deviation from this proportionality, then the vibrations of the exciting tone are increased by others which correspond to its harmonic upper partial tones. That such harmonic upper partials are occasionally heard when tuning-forks are strongly excited, has been already mentioned (p. 88).[1]

The human ear must of course act in the same way, just because it so easily produces combinational tones, and this is the reason why any powerful simple tone is, when heard, necessarily accompanied by (very faint) harmonic upper partial tones.

The importance of combinational tones in the construction of chords will appear hereafter. We have, however, first to investigate a second phenomenon of simultaneous resonance, the so-called *beats*.

[1] [By striking a tuning-fork strongly and holding it in rapid succession over resonance cylinders tuned to the fundamental note and its Octave, I have generally succeeded in hearing both tones quite distinctly.—*Translator*.]

CHAPTER VIII.

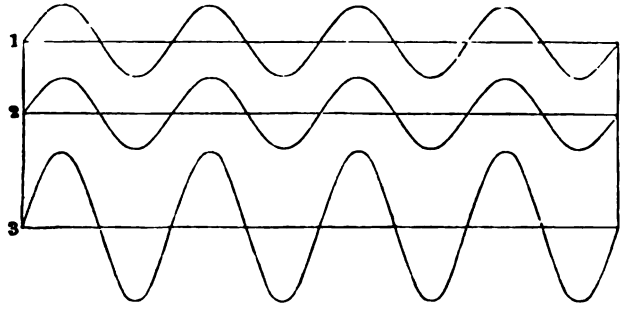
ON THE BEATS OF SIMPLE TONES.

WE now pass to the consideration of other phenomena accompanying the simultaneous resonance of two simple tones, in which, as before, the motions of the air and of the other co-operating elastic bodies without and within the ear may be conceived as an undisturbed coexistence of two systems of vibrations corresponding to the two tones, but where the auditory sensation no longer corresponds to the sum of the two sensations which the tones would excite singly. Combinational tones and beats, which have now to be considered, are essentially distinguished thus :—In combinational tones the composition of vibrations in the vibrating bodies which are either within or without the ear, undergoes certain disturbances, although the ear finally resolves the motion conducted to it, into a series of simple tones, according to the usual law. In beats, on the contrary, the objective motions of the elastic bodies follow the simple law; but the composition of the sensations is disturbed. As long as several simple tones of a sufficiently different pitch enter the ear together, the sensation due to each remains undisturbed in the ear, probably because entirely different bundles of nerve fibres are affected. But tones of the same, or of nearly the same pitch, which therefore affect the same nerve fibres, do not produce a sensation which is the sum of the two they would have separately excited, but new and peculiar phenomena arise which we term *interference*, when caused by two perfectly equal simple tones, and *beats* when due to two nearly equal simple tones.

We will begin with *interference*. Suppose that a point in the air or ear is set in motion by some resonant force, and that its motion is represented by the curve 1, fig. 53 (p. 241). Let the second motion be precisely the same at the same time, and be represented by the curve 2, so that the crests of 2 fall on those of 1, and also the trough of 2 on the trough of 1. If both motions

proceed at once, the whole motion will be their sum, represented by 3, a curve of the same kind but with crests twice as high and

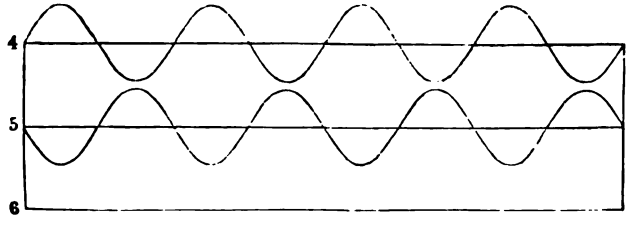
FIG. 53.



troughs twice as deep as those of either of the others. Since the intensity of sound is proportional to the square of the amplitude, we have consequently a tone not of twice but of four times the loudness of either of the others.

But now suppose the vibrations of the second motion to be displaced by half the periodic time. The curves to be added will stand under one another, as 4 and 5 in fig. 54, and when we come to add them, the heights of the second curve will be still the same as those of the first, but, being always in the contrary

FIG. 54.



direction, will mutually destroy each other, giving as their sum the straight line 6, or no vibration at all. In this case the crests of 4 are added to the troughs of 5, and conversely, so that the crests fill up the troughs, and crests and troughs mutually annihilate each other. The intensity of sound also becomes nothing, and when motions are thus cancelled within the ear, sensation also ceases; and although each single motion acting alone would excite

the corresponding auditory sensation, when both act together there is no auditory sensation at all. One sound in this case completely cancels what appears to be an equal sound. This appears extraordinarily paradoxical to ordinary contemplation because our natural consciousness apprehends sound, not as the motion of particles of the air, but as something really existing, analogous to the sensation of sound, and as the sensation of a simple tone of the same pitch shews no oppositions of positive and negative, it naturally appears impossible for one positive sensation to cancel another. But the really cancelling things in such a case are the vibrational impulses which the two sources of sound exert on the ear. When it so happens that the vibrational impulses due to one source constantly coincide with opposite ones due to the other, and exactly counterbalance each other, no motion can possibly ensue in the ear, and hence the auditory nerve can experience no sensation.

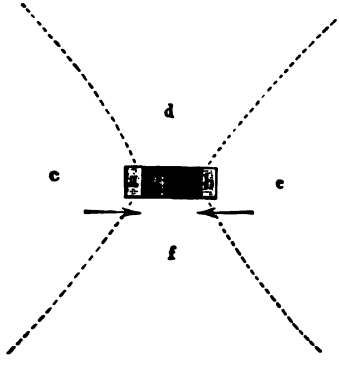
The following are some instances of sound cancelling sound.

Put two perfectly similar stopped organ pipes of the same pitch close beside each other on the same portvent. Each one blown separately gives a powerful tone; but when they are blown together, the motion of the air in the two pipes takes place in such a manner that as the air streams out of one it streams into the other, and hence an observer at a distance hears no tone, but at most the rushing of the air. On bringing the fibre of a feather near to the lips of the pipes, this fibre will vibrate in the same way as if each pipe were blown separately. Also if a tube be stretched from the ear to the mouth of one of the pipes, the tone of that pipe is heard so much more powerfully that it cannot be entirely destroyed by the tone of the other.

Every tuning-fork also exhibits phenomena of interference, because the prongs move in opposite directions. On striking a tuning-fork and slowly revolving it about its longitudinal axis close to the ear, it will be found that there are four positions in which the tone is heard clearly; and four intermediate positions in which it is inaudible. The four positions of strong sound are those in which either one of the prongs, or one of the side surfaces of the fork is turned towards the ear. The positions of no sound lie between the former, almost in planes which make an angle of 45° with the surfaces of the prongs, and pass through the axis of the fork. If in fig. 55, p. 243, a and b are the ends of the fork seen from above, c, d, e, f will be the four places of strong sound,

and the dotted lines the four places of silence. The arrows under a and b shew the mutual motion of the two prongs. Hence while the prong a gives the air about c an impulse in the direction c a, the prong b gives it an opposite one. Both impulses only partially cancel one another at c, because a acts more powerfully than b. But the dotted lines shew the places where the opposite impulses from a and b are equally strong, and consequently completely cancel each other. If the ear be brought into one of these places of silence and a narrow tube be slipped over one of the prongs a or b, taking care not to touch it, the sound will be immediately augmented, because the influence of the covered prong is almost entirely destroyed, and the uncovered prong therefore acts singly and undisturbed.[1]

FIG. 55.



A double siren which I have had constructed is very convenient for the demonstration of these relations.[2] Fig. 56 (p. 245) is a perspective view of this instrument. It is composed of two of Dove's polyphonic sirens, of the kind previously mentioned, p. 20; a_0 and a_1 are the two wind-chests, c_0 and c_1 the discs attached to a common axis, on which a screw is introduced at k, to drive a counting apparatus which can be introduced, as described on p. 20. The upper box a_1 can be turned round its axis, by means of a toothed wheel, in which works a smaller wheel e provided with the driving handle d. The axis of the box a_1 round which it turns, is a prolongation of the upper pipe g_1, which conducts the wind. On each of the two discs of the siren are four rows of holes, which can be either blown separately or together in any combination at pleasure, and at i are the studs for opening and closing the series of holes by a peculiar arrangement.[3] The lower disc has four rows of 8, 10, 12, 18 holes, the upper of 9, 12,

[1] [If instead of bringing the tuning-fork to the ear, it be slowly turned before a proper resonance chamber, the alternation of sound and silence, &c., can be made audible to many persons at once.—*Translator.*]

[2] Constructed by the mechanician Sauerwald in Berlin.

[3] Described in Appendix No. XIII.

15, 16. Hence if we call the tone of 8 holes c, the lower disc gives the tones c, e, g, d' and the upper d, g, b, c'. We are therefore able to produce the following intervals.

1. Unison: gg on the two discs simultaneously.

2. Octaves: cc' and dd' on the two.

3. Fifths: cg and gd' either on the lower disc alone or on both discs together.

4. Fourths: dg and gc' on the upper disc alone or on both together.

5. Major Third: ce on the lower alone, and gb on the upper alone, or gb on both together.

6. Minor Third: eg on the lower, or on both together; bd' on both together.

7. Whole Tone (great Tone): cd and $c'd'$ on both together.

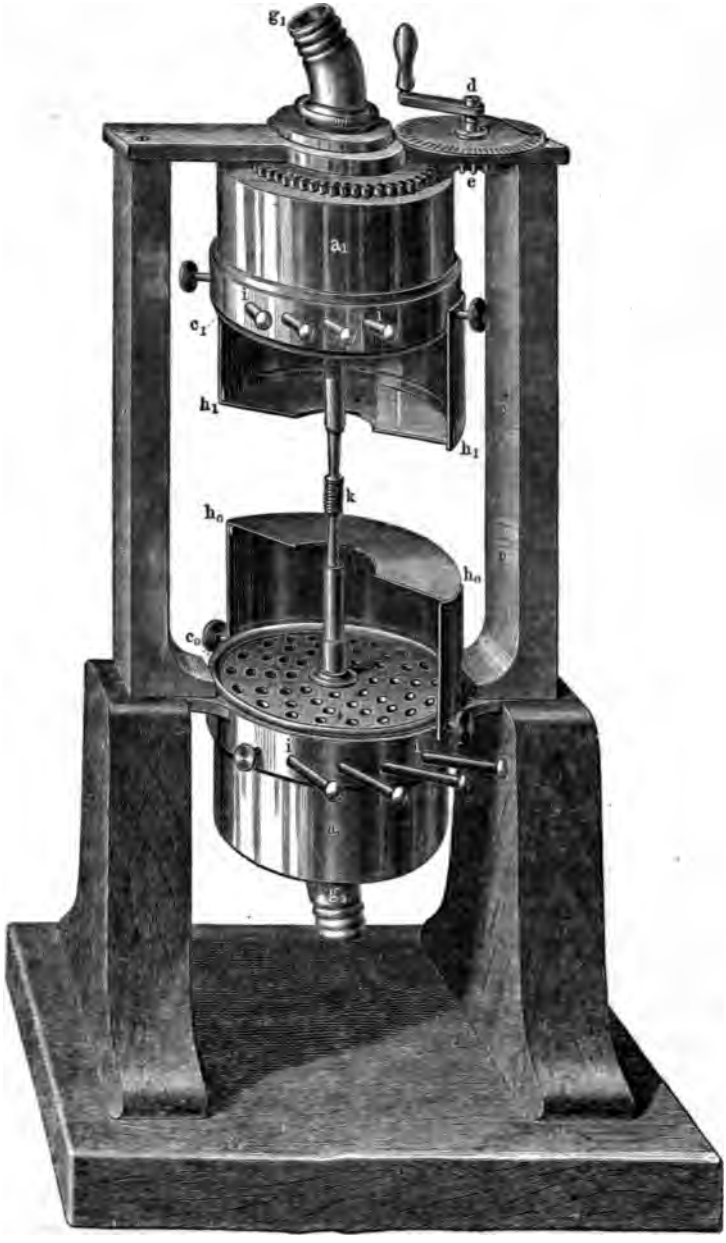
8. Semitone (greater Semitone): bc' on the upper.

When both tones are produced from the same disc the objective combinational tones are very powerful as has been already remarked, p. 236. But if the tones are produced from different discs, the combinational tones are weak. In the latter case, (and this is the chief point of interest to us at present) the two tones can be made to act together with any desired difference of phase. This is effected by altering the position of the upper box.

We have first to investigate the phenomena as they occur in the unison gg. The effect of the interference of the two tones in this case is complicated by the fact that the siren produces compound and not simple tones and that the interference of individual partial tones is independent of that of the prime tone and of one another. In order to damp the upper partial tones in the siren by means of a resonance chamber, I caused cylindrical boxes of brass to be made, of which the back halves are shewn at h_1 h_1 and h_0 h_0 fig. 56, p. 245. These boxes are each made in two sections, so that they can be removed, and be again attached to the windchest by means of screws. When the tone of the siren approaches the prime tone of these boxes, its quality becomes full, strong and soft, like a fine tone on the French horn; otherwise the siren has rather a piercing tone. At the same time we use a small quantity of air, but considerable pressure. The circumstances are of the same nature as when a tongue is applied to a resonance chamber of the same pitch. Used in this way the siren is very well adapted for experiments on interference.

If the boxes are so placed that the puffs of air follow at

FIG. 56.



exactly equal intervals from both discs, similar phases of the prime tone and of all partials coincide, and all are reinforced.

If the handle is turned round half a right angle, the upper box is turned round $\frac{1}{6}$ of a right angle, or $\frac{1}{24}$ of the circumference, that is half the distance between the holes in the series of 12 holes which is in action for *g*. Hence the difference in the phase of the two primes is half the periodic time, the puffs of air in one box occur exactly in the middle between those of the other, and the two prime tones mutually destroy each other. But under these circumstances the difference of phase in the upper Octave is precisely the whole of their periodic time; that is, they reinforce each other, and similarly all the even harmonic upper partials reinforce each other in the same position, and the uneven ones destroy each other. Hence in the new position the tone is weaker, because deprived of several of its partials; but it does not entirely cease; it rather jumps up an Octave. If we turn the handle through half a right angle so that the box is turned through a whole right angle, the puffs of the two discs again agree completely, and the tones reinforce one another. Hence in a complete revolution of the handle there are four positions where the whole tone of the siren appears reinforced, and four intermediate positions where the prime tone and all uneven upper partials vanish, and consequently the Octave occurs in a weaker form accompanied by the even upper partial tones. If we attend to the first upper partial, which is the Octave of the prime, by listening to it through a proper resonator, we find that it vanishes after turning through a quarter of a right angle, and is reinforced after turning through half a right angle, and hence for every complete revolution of the handle it vanishes 8 times, and is reinforced 8 times. The third partial, (or second *upper* partial,) the Twelfth of the prime tone, vanishes in the same time 12 times, the fourth partial 16 times and so on.

Other compound tones behave like those of the siren. When two tones of the same pitch are sounded together having differences of phase corresponding to half the periodic time, the tone does not vanish, but jumps up an Octave. When for example two open organ pipes, or two reed pipes of the same construction and pitch, are placed beside each other on the same portvent, their vibrations generally accommodate themselves in such a manner that the stream of air enters first one and then the other alternately; and while the tone of stopped pipes, which have only uneven partials,

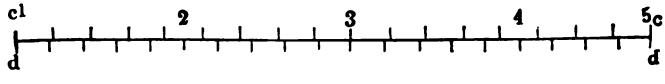
is then almost entirely destroyed, the tone of the open pipes and reed pipes falls into the upper Octave. This is the reason why no reinforcement of tone can be effected on an organ or harmonium by combining tongues or pipes of the same kind.

So far we have combined tones of precisely the same pitch, now let us inquire what happens when the tones have slightly different pitch. The double siren just described is also well fitted for explaining this case, for we can slightly alter the pitch of the upper tone by slowly revolving the upper box by means of the handle, the tone becoming flatter when the direction of revolution is the same as that of the disc, and sharper when it is opposite to the same. The periodic time of a tone of the siren is equal to the time required for a hole in the rotating disc to pass from one hole in the windbox to the next. If the hole of the box advances to meet the hole of the disc through the rotation of the box, the two holes come into coincidence sooner than if the box were at rest; and hence the periodic time is shorter, and the tone sharper. The converse takes place when the revolution is in the opposite direction. These alterations of pitch are easily heard when the box is revolved rather quickly. Now produce the tones of twelve holes on both discs. These will be in absolute unison as long as the upper box is at rest. The two tones constantly reinforce or enfeeble each other according to the position of the upper box. But on setting the upper box in motion, the pitch of the upper tone is altered, while that of the lower tone, which has an immovable windbox, is unchanged. Hence we have now two tones of slightly different pitch sounding together. And we hear the so-called *beats* of the tones, that is, the intensity of the tone will be alternately greater and less in regular succession. The arrangement of our siren makes the reason of this readily intelligible. The revolution of the upper box brings it alternately in positions which as we have seen correspond to stronger and weaker tones. When the handle has been turned through a right angle, the windbox passes from a position of loudness through a position of weakness to a position of strength again. Consequently every complete revolution of the handle gives us four beats, whatever be the rate of revolution of the discs, and hence however low or high the tone may be. If we stop the box at the moment of maximum loudness, we continue to hear the loud tone; if at a moment of minimum force, we continue to hear the weak tone.

The mechanism of the instrument also explains the connection between the number of beats and the difference of the pitch. It is easily seen that the number of the puffs is increased by one for every quarter revolution of the handle. But every such quarter revolution corresponds to one beat. Hence *the number of beats in a given time is equal to the difference of the numbers of vibrations executed by the two tones in the same time.* This is the general law which determines the number of beats, for all kinds of tones. This law results immediately from the construction of the siren; in other instruments it can only be verified by very accurate and laborious measurements of the numbers of vibrations.

The process is shewn graphically in fig. 57. Here c c represents the series of puffs belonging to one tone, and d d those belonging to the other. The distance for c c is divided into 18 parts, the same distance is divided into 20 parts for d d. At

FIG. 57.



1, 3, 5, both puffs concur, and the tone is reinforced. At 2 and 4 they are intermediate and mutually enfeeble each other. The number of beats for the whole distance is 2, because the difference of the numbers of parts, each of which correspond to a vibration, is also 2.

The intensity of tone varies; swelling from a minimum to a maximum, and lessening from the maximum to the minimum. It is the places of maximum intensity which are properly called *beats*, and these are separated by more or less distinct pauses.[1]

Beats are easily produced on all musical instruments, by striking two notes of nearly the same pitch. They are heard best from the simple tones of tuning-forks or stopped organ pipes, because here the tone really vanishes in the pauses. A little fluctuation in the pitch of the beating tone may then be remarked.[2] For the compound tones of other instruments the

[1] [In German, *Schwebungen*, literally librations, as in a balance set in oscillation, represents the whole phenomenon; and *Schläge*, blows, the maxima. In English we are accustomed to speak of *beats* only.—*Translator.*]

[2] See the explanation of this phenomenon which was given me by M. G. Guéroult, in Appendix No. XIV.

upper partial tones are heard in the pauses, and hence the tone jumps up an Octave, as in the case of interference already described. If we have two tuning-forks of exactly the same pitch, it is only necessary to stick a little wax on to the end of one, to strike both, and bring them near the same ear or to the surface of a table, or sounding-board. To make two stopped pipes beat, it is only necessary to bring a finger slowly near to the lip of one, and thus flatten it. The beats of compound tones are heard by striking any note on a pianoforte out of tune when the two strings belonging to the same note are no longer in unison; or if the piano is in tune it is sufficient to attach a piece of wax, about the size of a pea to one of the strings. This puts them sufficiently out of tune. More attention however is required for compound tones because the enfeeblement of the tone is not so striking. The beat in this case resembles a fluctuation in pitch and quality. This is very striking on the siren according as the brass resonance cylinders (h_0 h_0 and h_1 h_1 of fig. 56, p. 245), are attached or not. These make the prime tone relatively strong. Hence when beats are produced by turning a handle, the decrease and increase of loudness in the tone is very striking. On removing the resonance cylinders, the upper partial tones are relatively very strong, and since the ear is very uncertain when comparing the loudness of tones of different pitch, the alteration of force during the beats is much less striking than that of pitch and quality of tone.

On listening to the upper partials of compound tones which beat, it will be found that these beat also, and that for each beat of the prime tone there are two of the second partial, three of the third, and so on. Hence when the upper partials are strong, it is easy to make a mistake in counting the beats, especially when the beats of the primes are very slow, so that they occur at intervals of a second or two. It is then necessary to pay great attention to the pitch of the beats counted, and sometimes to apply a resonator.

It is possible to render beats visible by setting a suitable elastic body into sympathetic vibration with them. Beats can then occur only when the two exciting tones lie near enough to the prime tone of the sympathetic body for the latter to be set into sensible sympathetic vibration by both the tones used. This is most easily done with a thin string which is stretched on a sounding-board on which have been placed two tuning-forks, both of very nearly the

same pitch as the string. On observing the vibrations of the string through a microscope, or attaching a fibril of a goosefeather to the string which will make the same excursions on a magnified scale, the string will be clearly seen to make sympathetic vibrations with alternately large and small excursions, according as the tone of the fork is at its maximum or minimum.

The same effect can be obtained from the sympathetic vibration of a stretched membrane. Fig. 58 is the copy of a drawing

FIG. 58.

made by a vibrating membrane of this sort, used in the phonautograph of Messrs. Scott & König, of Paris. The membrane of this instrument, which resembles the drumskin of the ear, carries a small stiff style, which draws the vibrations of the membrane upon a rotating cylinder. In the present case the membrane was set in motion by two organ pipes, that beat. The undulating line, of which only a part is here given, shews that times of strong vibration have alternated with times of almost entire rest. In this case then the beats are also sympathetically executed by the membrane. Similar drawings again have been made by Dr. Politzer, who attached the writing style to the auditory bone (the columella) of a duck, and then produced a beating tone by means of two organ pipes. This experiment shewed that even the auditory bones follow the beats of two tones.[1]

Generally this must always be the case when the pitches of the two tones struck differ so little from each other and from that of the proper tone of the sympathetic body, that the latter can be put into sensible vibration by both tones at once. Sympathetic bodies which do not damp readily, such as tuning-forks, consequently require two exciting tones which differ extraordinarily little in pitch,

[1] The beats of two tones are also clearly shewn by the vibrating flame described at the end of Appendix No. II. The flame must be connected with a resonator having a pitch sufficiently near to those of the two generating tones. Even without using the rotating mirror for observing the flames, we can easily recognise the alterations in the shape of the flame which take place isochronously with the audible beats.

in order to shew visible beats, and the beats must the slow. For bodies readily damped, as membranes, str difference of the exciting tones may be greater, and the beats may succeed each other more rapidly.

This holds also for the elastic terminal form auditory nerve fibres. Just as we have seen that visible beats of the auditory bones, Corti's arches made to beat by two tones sufficiently near in pi same Corti's arches in sympathetic vibration at th If then, as we have previously supposed, the inte tory sensation in the proper nerve fibres increases with the intensity of the elastic vibrations, the st sensation must also increase and diminish in the as the vibrations of the corresponding elastic appe nerves. In this case also the motion of Corti's arc be considered as compounded of the motions which would have produced if they had acted separately. these motions are directed in the same or in oppos they will reinforce or enfeeble each other by (algebra It is not till these motions excite sensation in the n deviation from this law occurs, whereby each of the each of the two sensations of tones subsist side b disturbance.

We now come to a part of the investigation important for the theory of musical consonance, and fortunately been little regarded by acousticians. Tl what becomes of the beats when they grow faster ar to what extent may their number increase without unable to perceive them? Most acousticians were pro to agree with the hypothesis of Thomas Young, t beats became very quick they gradually passed over national tone (the first differential). Young imag pulses of tone which ensue during beats, might h effect on the ear as elementary pulses of air (in example), and that just as 30 puffs in a second thr would produce the sensation of a deep tone, so wou a second resulting from any two higher tones prod sensation of a deep tone. Certainly this view is v by the fact that the vibrational number of the first combinational tone is actually the number of beats

the two tones in a second. It is however of much importance to remember that there are other combinational tones (my summational tones), which will not agree with this hypothesis in any respect, but on the other hand are readily deduced from the theory of combinational tones which I have proposed (in Appendix XII.). It is moreover an objection to Young's theory, that in many cases the combinational tones exist externally to the ear, and are able to set properly tuned membranes or resonators into sympathetic vibration, because this could not possibly be the case, if the combinational tones were nothing but the series of beats with undisturbed superposition of the two waves. For the mechanical theory of sympathetic vibration shews that a motion of the air compounded of two simple vibrations of different periodic times, are capable of putting such bodies only into sympathetic vibration as have a proper tone corresponding to one of the two given tones, provided no conditions intervene by which the simple superposition of two wave systems might be disturbed; and the nature of such a disturbance was investigated in the last chapter. Hence we may consider combinational tones as an accessory phenomenon, by which, however, the course of the two primary wave systems and of their beats is not essentially interrupted.

Against the old opinion we may also adduce the testimony of our senses, which teaches us that a much greater number of beats than 30 in a second can be distinctly heard. To obtain this result we must pass gradually from the slower to the more rapid beats, taking care that the tones chosen for beating are not too far apart from each other in the scale, because audible beats are not produced unless the tones are so near to each other in the scale that they can both make the same elastic appendages of the nerves vibrate sympathetically. The number of beats, however, can be increased without increasing the interval between the tones, if both tones are taken in the higher octaves. The observations are best begun by producing two simple tones of the same pitch, say from the once accented octave by means of tuning-forks or stopped organ pipes, and slowly altering the pitch of one. This is effected by sticking more and more wax on one of the forks; or more and more covering the mouth of one of the pipes. Stopped organ pipes are also generally produced with a movable plug or lid at

the stopped end, in order to tune them; by pulling this out we deepen, by pushing it in we heighten the tone.[1]

When a slight difference in pitch has been thus produced, the beats are heard at first as long drawn out fluctuations alternately swelling and vanishing. Slow beats of this kind are by no means disagreeable to the ear. In executing music containing long sustained chords, they may even produce a solemn effect, or else give a more lively, tremulous or agitating expression. Hence we find in modern organs and harmoniums, a stop with two pipes or tongues, adjusted to beat. This imitates the trembling of the human voice and of violins which, appropriately introduced in isolated passages, may certainly be very expressive and effective, but applied continuously, as is unfortunately too common, is a detestable malpractice.

The ear easily follows slow beats of not more than 4 to 6 in a second. The hearer has time to apprehend all their separate phases, and become conscious of each separately, he can even count them without difficulty.[2] But when the interval between the two tones increases to about a Semitone, the number of beats becomes 20 or 30 in a second, and the ear is consequently unable to apprehend them singly, and therefore to count them. If, however,

[1] [A cheap apparatus, useful for demonstration of the following facts, was constructed for me by Alex. Liddle, musical instrument maker, of 35 Devonshire Street, Queen Square. He sells 'pitch pipes' consisting of an extensible stopped pipe, which has the compass of the once accented Octave and is blown as a whistle. I had two of these connected with a single mouth-piece. By carefully adjusting the lengths of the pipes, I was first able to produce complete destruction of the tone by interference, the sound returning immediately when the mouth of one whistle was stopped by the finger. Then gradually lengthening one of the pipes the beats began slowly, and increased in rapidity. The tone being nearly simple they are well heard. But good combinational tones are not thus produced. The flageolet fifes for the demonstration of those tones mentioned in a former note, p. 231, were obtained of the same maker.—*Translator.*]

[2] [Scheibler, who carried out the system of tuning by beats most successfully, considers 4 beats in a second the easiest to count correctly. His final plan was to tune a set of 12 tuning-forks having their vibrational numbers less by 4 than the pitches of the notes required for the organ; then to tune the pipe approximatively to the fork by ear, and finally sharpen the pipe till it beat exactly 4 times in a second with the fork. See H. Scheibler's 'Tracts on musical and physical measurement of pitch and its application to the tuning of Pianofortes and Organs' (*Schriften über musikalische und physikalische Tonmessung und deren Anwendung auf Pianoforte- und Orgelstimmung*, Crefeld, 1838), a series of little tracts bound together, and containing the most practical and correct methods for tuning organs yet published, but disfigured by all manner of awkwardness in their presentment.—*Translator.*]

we begin with hearing slow beats, and then increase their rapidity more and more, we cannot fail to recognise that the sensational impression on the ear preserves precisely the same character, although of course when 20 or 30 beats take place in a second, we have no longer time to notice each beat as it passes, fix it in our consciousness and count it.

But while the hearer in this case is quite capable of distinguishing that his ear now hears 30 beats of the same kind as the 4 or 6 in a second which he heard before, the effect of the collective impression of such a rapid beat is quite different. In the first place the mass of tone becomes confused, which I principally refer to the psychological impressions. We actually hear a series of pulses of tone, and are able to recognise it as such, although no longer capable of following each singly, or separating one from the other. But besides this psychological consideration, the sensible impression is also unpleasant. Such rapidly beating tones are jarring and rough. The distinctive property of jarring, is the intermittent character of the sound. We think of the letter R as a characteristic example of a jarring tone. It is well known to be produced by interposing the uvula, or else the thin tip of the tongue in the way of the stream of air passing out of the mouth, in such a manner as only to allow the air to force its way through in separate pulses, the consequence being that the voice at one time sounds freely, and at another is cut off.[1]

Intermittent tones were also produced on the double siren just described by using a little reed pipe instead of the wind-conduit of the upper box, and driving the air through this reed pipe. The tone of this pipe can be heard externally only when the revolution of the disc brings its holes before the holes of the box and open an exit for the air. Hence, if we let the disc revolve while air is driven through the pipe, we obtain an intermittent tone, which sounds exactly like beats arising from two tones sounded at once, although the intermittence is produced by purely mechanical means. Such effects may also be produced in another way on the siren. Remove the lower wind-box and retain only its pierced

[1] [Phonautographic figures of the effect of *r*, resemble those of fig. 58, p. 250. Six varieties of these figures are given on p. 19 of Donders's important little tract, on 'The physiology of Speech sounds, and especially of those in the Dutch Language' (*De Physiologie der Spraakklanken, in het bijzonder van die der nederlandsche taal.* Utrecht 1870, pp. 24.—*Translator.*]

cover, over which the disc revolves. At the under part apply one extremity of an india-rubber tube against one of the holes in the cover, the other end being conducted by a proper ear-piece to the observer's ear. The revolving disc alternately opens and closes the hole to which the india-rubber tube has been applied. Hold a tuning-fork in action or some other suitable musical instrument above and near the rotating disc. Its tone will be heard intermittently and the number of intermissions can be regulated by altering the velocity of the rotation of the disc.

In both ways then we obtain intermittent tones. In the first case the tone of the reed pipe as heard in the outer air is interrupted, because it can only escape from time to time. The intermittent tone in this case can be heard by any number of listeners at once. In the second case the tone in the outer air is continuous, but reaches the ear of the observer, who hears it through the disc of the syren, intermittently. It can certainly be heard by one observer only, but then all kinds of musical tones of the most diverse pitch and quality may be employed for the purpose. The intermission of their tones gives them all exactly the same kind of roughness which is produced by two tones which beat rapidly together. We thus come to recognise clearly that beats and intermissions are identical, and that both when succeeding each other fast enough produce what is termed a jar or rattle.

Beats produce intermittent excitement of certain auditory nerve fibres. The reason why such an intermittent excitement acts so much more unpleasantly than an equally strong or even a stronger continuous excitement, may be gathered from the analogous action of other human nerves. Any powerful excitement of a nerve deadens its excitability, and consequently renders it less sensitive to fresh irritants. But after the excitement ceases, and the nerve is left to itself, irritability is speedily re-established in a living body by the influence of arterial blood. Fatigue and refreshment apparently supervene in different organs of the body with different velocities; but they are found wherever muscles and nerves have to operate. The eye, which has in many respects the greatest analogy to the ear, is one of those organs in which both fatigue and refreshment rapidly ensue. We need to look at the sun but an instant to find that the portion of the retina, or nervous expansion of the eye, which was affected by the solar light has become less sensitive for other light. Immediately afterwards

on turning our eyes to a uniformly illuminated surface, as the sky, we see a dark spot of the apparent size of the sun; or several such spots with lines between them, if we had not kept our eye steady when looking at the sun but had moved it right and left. An instant suffices to produce this effect; nay, an electric spark, that lasts an immeasurably short time, is fully capable of causing this species of fatigue.

When we continue to look at a bright surface, the impression is strongest at first, but this impression at the same time blunts the sensibility of the eye, and consequently the impression becomes weaker, the longer we allow the eye to act. On coming out of darkness into full daylight we feel blinded; but after a few minutes, when the sensibility of the eye has been blunted by the irritation of the light,—or, as we say, when the eye has grown accustomed to the glare,—this degree of brightness is found very pleasant. Conversely, in coming from full daylight into a dark vault, we are insensible to the weak light about us, and can scarcely find our way about, yet after a few minutes when the eye has rested from the effect of the strong light, we are able to see very well in the semi-dark room.

These phenomena and the like can be conveniently studied in the eye, because individual spots in the eye may be excited and others left at rest, and the sensations of each may be afterwards compared. Put a piece of black paper on a tolerably well-lighted white surface, look steadily at a point on or near the black paper, and then withdraw the paper suddenly. The eye sees a secondary image of the black paper on the white surface, consisting of that portion of the white surface where the black paper lay, which now appears brighter than the rest. The place in the eye where the image of the black paper had been formed, has been rested in comparison with all those places which had been affected by the white surface, and on removing the black paper this rested part of the eye sees the white surface in its first fresh brightness, while those parts of the retina which had been already fatigued by looking at it, see a decidedly greyer tinge on the white surface.

Hence by the continuous uniform action of the irritation of light, this irritation itself blunts the sensibility of the nerve, and thus effectually protects this organ against too long and too violent excitement.

It is quite different when we allow intermittent light to act

on the eye, such as flashes of light with intermediate pauses. During these pauses the sensibility is again somewhat re-established, and the new irritation consequently acts much more intensely than if it had lasted with the same uniform strength. Every one knows how unpleasant and annoying is any flickering light, even if it is relatively very weak, coming, for example, from a little flickering taper or rushlight.

The same thing holds for the nerves of touch. Scraping with the nail is far more annoying to the skin than constant pressure on the same place with the same pressure of the nail. The unpleasantness of scratching, rubbing, tickling, depends upon the intermittent excitement which they produce in the nerves of touch.

A jarring intermittent tone is for the nerves of hearing what a flickering light is to the nerves of sight, and scratching to the nerves of touch. A much more intense and unpleasant excitement of the organs is thus produced than would be occasioned by a continuous uniform tone. This is even shewn when we hear very weak intermittent tones. If a tuning-fork is struck and held at such a distance from the ear that its sound cannot be heard, it becomes immediately audible if the handle of the fork be revolved by the fingers. The revolution brings it alternately into positions where it can and cannot transmit sound to the ear, and this alternation of strength is immediately perceptible by the ear. For the same reason one of the most delicate means of hearing a very weak, simple tone consists in sounding another of nearly the same strength, which makes from 2 to 4 beats in a second with the first. In this case the strength of the tone varies from nothing to 4 times the strength of the single simple tone, and this increase of strength combines with the alternation to make it audible.

Just as this alternation of strength will serve to strengthen the impression of the very weakest musical tones upon the ear, we must conclude that it must also serve to make the impression of stronger tones much more penetrating and violent, than they would be if their loudness were continuous.

We have hitherto confined our attention to cases where the number of beats did not exceed 20 or 30 in a second. We saw that the beats in the middle part of the scale are still quite audible and form a series of separate pulses of tone. But this does not furnish a limit to their number in a second.

The interval b' c'' gave us 33 beats in a second and the effect

of sounding the two notes together was very jarring. The interval of a whole tone $b'\flat\ c''$ gives nearly twice as many beats, but these are no longer so cutting as the former. The rule assigns 88 beats in a second to the minor Third $a'\ c''$, but in reality this interval scarcely shews any of the roughness produced by beats from tones at closer intervals. We might then be led to conjecture that the increasing number of beats wiped off their impression and made them inaudible. This conjecture would find an analogy in the impossibility of separating a series of rapidly succeeding impressions of light on the eye, when their number in a second is too large. Think of light swung round in a circle. If it executes 10 or 15 revolutions in a second, the eye believes it sees a continuous circle of fire. Similarly for colour-tops, with which most of my readers are familiar. If the top be spun at the rate of more than 10 revolutions in a second, the colours upon it mix and form a perfectly fixed impression of a mixed colour. It is only for very intense light that the alternations of the various fields of colour have to take place more quickly, 20 to 30 times in a second. Hence the phenomena are quite analogous for ear and eye. When the alternation between irritation and rest is too fast, the alternation ceases to be felt, and sensation becomes continuous and lasting.

However, we may convince ourselves that in the case of the ear, an increase of the number of beats in a second is not the only cause of the disappearance of the corresponding sensation. As we passed from the interval of a Semitone $b'\ c''$ to that of a minor Third $a'\ c''$, we not only increased the number of beats, but the breadth of the interval. Now we can increase the number of beats without increasing the interval by taking it in a higher Octave. Thus taking $b'\ c''$ an Octave higher we have $b''\ c'''$ with 66 beats, and another Octave would give us $b'''\ c''''$ with as many as 132 beats, and these are really audible in the same way as the 33 beats of $b'\ c''$, although they certainly become weaker in the higher positions. Nevertheless the 66 beats of the interval $b''\ c'''$ are much more distinct and penetrating than the same number in the whole Tone $b'\flat\ c''$, and the 88 of the interval $e'''\ f'''$ are still quite evident, while the 88 of the minor Third $a'\ c''$ are practically inaudible. My assertion that as many as 132 beats in a second are audible will perhaps appear very strange and incredible to acousticians. But the experiment is easy to repeat, and if on an instrument which gives continued tones, as an organ or harmonium, we

strike a series of intervals of a semitone each, beginning low down and proceeding higher and higher, we shall hear in the lower parts very slow beats (B, C gives $4\frac{1}{8}$, $B\,c$ gives $8\frac{1}{4}$, $b\,c'$ gives $16\frac{1}{2}$ beats in a second), and as we ascend the rapidity will increase but the character of the sensation remain unaltered. And thus we can pass gradually from 4 to 132 beats in a second, and convince ourselves that though we become incapable of counting them, their character as a series of pulses of tone, producing an intermittent sensation, remains unaltered. It must be observed, however, that the beats, even in the higher parts of the scale, become much cleaner and clearer, when their number is diminished by taking intervals of quarter tones or less.[1] The most penetrating roughness arises even in the upper parts of the scale from beats of 30 to 40 in a second. Hence high tones in a chord are much more sensitive to an error in tuning amounting to the fraction of a Semitone, than deep ones. While two c' which differ from one another by the tenth part of a Semitone, produce about one beat in a second, which cannot be observed without considerable attention, and, at least, gives no feeling of roughness, two c'' with the same error give 4, and two c''' 8 beats in a second, which becomes very disagreeable. The character of the roughness also alters with the number of beats. Slow beats give, so to speak, roughness of a coarse grain, which may be called chattering or jarring; and quicker ones have a finer but more cutting grain.

Hence it is not, or at least not solely, the large number of beats which renders them inaudible. The magnitude of the interval is a factor in the result, and consequently we are able with high tones to produce more rapid audible beats than with low tones.

Observation shews us, then, on the one hand, that equally large intervals by no means give equally distinct beats in all parts of the scale. The increasing number of beats in a second renders the beats in the upper part of the scale less distinct. The beats of a Semitone remain distinct to the upper limits of the four times accented octave, and this is also about the limit for musical tones fit for the combinations of harmony. The beats of a whole tone,

[1] [The justly intoned concertina described in Appendix XIX. is exceedingly convenient for making these observations, as it has d', d'', d''', and $\ddagger d'$, $\ddagger d''$, $\ddagger d'''$, which are flatter than the former by Comma, and the intervals d' $\ddagger d'$ ♯, d'' $\ddagger d''$ ♯, d''' ♯ $\ddagger d'''$ ♯ may be contrasted with the intervals d' $\ddagger d'$, d'' $\ddagger d''$, d''' $\ddagger d'''$.—*Translator.*]

which in deep positions are very distinct and powerful, are scarcely audible at the upper limit of the thrice-accented octave. The major and minor Third, on the other hand, which in the middle of the scale may be regarded as consonances, and when justly intoned scarcely shew any roughness, are decidedly rough in the lower Octaves and produce distinct beats.

On the other hand we have seen that distinctness of beating and the roughness of the combined sounds do not depend solely on the number of beats. For if we could disregard their magnitudes all the following intervals, which by calculation should have 33 beats, would be equally rough:

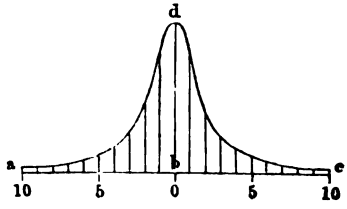
the Semitone . . .	$b' c''$
the whole Tones . . .	$c' d'$ and $d' e'$
the minor Third . . .	$e g$
the major Third . . .	$c e$
the Fourth . . .	$G c$
the Fifth	$C G$

and yet we find that these deeper intervals are more and more free from roughness.

The roughness of a combination of two tones depends, then, in a compound manner on the magnitude of the interval and the number of beats produced in a second. On seeking for the reason of this dependence, we observe that, as before remarked, beats in the ear can exist only when two tones are produced sufficiently near in the scale to set the same elastic appendages of the auditory nerve in sympathetic vibration at the same time. When the two tones produced are too far apart, the vibrations excited by both of them at once in Corti's organs are too weak to admit of their beats being sensibly felt, supposing of course that no upper partial or combinational tones come into play. According to the assumptions made in the last chapter respecting the degree of damping possessed by Corti's organs (p. 216), it would result, for example, that for the interval of a whole Tone $c\,d$, such of Corti's fibres as have the proper tone $c\sharp$, would be excited by each of the tones with $\frac{1}{10}$ of its own intensity; and these fibres will therefore fluctuate between the intensity of vibration 0 and $\frac{4}{10}$. But if we strike the simple tones c and $c\sharp$, it follows from the table there given that Corti's fibres which correspond to the middle between c and $c\sharp$ will alternate between the intensities 0 and $\frac{12}{10}$. Conversely the same intensity of beats would for a minor Third amount to only 0·194,

and for a major Third to only 0·108, and hence would be scarcely perceptible beside the two primary tones of the intensity 1. Fig. 59, which we used on p. 216 to express the intensity of the sympathetic vibration of Corti's fibres for an increasing interval of tone, may here serve to shew the intensity of the beats which two tones excite in the ear when forming different intervals in the scale. But the parts on the base line must now be considered to represent fifths of a *whole* Tone, and not as before of a Semitone. In the present case the distance of the two tones from each other is doubly as great as that between either of them and the intermediate Corti's fibres.

FIG. 59.



If the damping of Corti's organs had been equally great at all parts of the scale, and had the number of beats no influence on the roughness of the sensation, equal intervals in all parts of the scale would have given equal roughness to the combined tones. But as this is not the case, as the same intervals diminish in roughness as we ascend in the scale, and increase in roughness as we descend, we must either assume that the damping power of Corti's organs of higher pitch is less than that of those of deeper pitch, or else that the discrimination of the more rapid beats meets with certain hindrances in the nature of the sensation itself.

At present I see no way of deciding between these two suppositions; but the former is possibly the more improbable, because, at least with our artificial musical instruments, the higher the pitch of a vibrating body, the more difficulty is experienced in isolating it sufficiently to prevent it from communicating its vibrations to its environment. Very short, high-pitched strings, little metal tongues or plates, &c., yield high tones which die off with great rapidity, whereas it is easy to generate deep tones with correspondingly greater bodies which shall retain their tone for a considerable time. On the other hand the second supposition is favoured by the analogy of another nervous apparatus, the eye. As has been already remarked, a series of impressions of light, following each other rapidly and regularly, excite a uniform and continuous sensation of light in the eye. When the separate luminous irritations follow one another very quickly, the impression

produced by each one lasts unweakened in the nerves till the next supervenes, and thus the pauses can no longer be distinguished in sensation. In the eye, the number of separate irritations cannot exceed 24 in a second without being completely fused into a single sensation. In this respect the eye is far surpassed by the ear, which can distinguish as many as 132 intermissions in a second, and probably even that is not the extreme limit. Much higher tones of sufficient strength would probably allow us to hear still more.[1] It lies in the nature of the thing, that different kinds of apparatus of sensation should shew different degrees of mobility in this respect, since the result does not depend simply on the mobility of the molecules of the nerves, but also depends upon the mobility of the auxiliary apparatus through which the excitement is induced or expressed. Muscles are much less mobile than the eye; ten electrical discharges in a second directed through them generally suffice to bring the voluntary muscles into a permanent state of contraction. For the muscles of the involuntary system, of the bowels, the vessels &c., the pauses between the irritations may be even one, or even several seconds, without any intermission in the continuity of contraction.

The ear is greatly superior in this respect to any other nervous apparatus. It is eminently the organ for small intervals of time, and has been long used as such by astronomers. It is well known that when two pendulums are ticking near one another, the ear can distinguish whether the ticks are or are not coincident, within one hundredth of a second. The eye would certainly fail to determine whether two flashes of light coincided within $\frac{1}{24}$; and probably within a much larger fraction of a second.[2]

[1] [In the two high notes $g''''' f''''' \sharp$, of the flageolet fifes (p. 231, note), which if justly intoned should give 198 beats in a second, I could hear none, though the tones were very powerful, and the *scream* was very cutting indeed.—In the case of $b''' c''''$, which on my concertina are tuned to make (and probably make very nearly) 1,920 and 2,048 vibrations in a second, the rattle of the 128 beats, or thereabouts, is quite distinct, and the differential tone very powerful at the same time.—*Translator.*]

[2] [The following is an interesting comparison between eye and ear, and eye and hand. The usual method of observing transits is by counting the pendulum ticks of an astronomical clock, and by observing the distances of the apparent positions of a star before and after passing each bar of the transit instrument at the moments of ticking, to estimate the moment at which it had passed each bar. This is done for five bars and a mean is taken. But within the last few years a chronograph has been introduced at Greenwich Observatory, consisting of a uniformly revolving cylinder in which a point pricks a hole every second. Electrical communication being established with

CHAPTER IX.

DEPTH AND DEEPEST TONES.

BEATS give us an important means of determining the limit of the deepest tones, and of accounting for certain peculiarities of the transition from the sensation of separate pulses of air to a perfectly continuous musical tone, and to this inquiry we now proceed.

The question: what is the smallest number of vibrations in a second which can produce the sensation of a musical tone? has hitherto received very contradictory replies. The estimates of different observers fluctuate between eight (Savart) and about thirty. The contradiction is explained by the existence of certain difficulties in the experiments.

In the first place it is necessary that the strength of the vibrations of the air for very deep tones should be extremely greater than for high tones, if they are to make as strong an impression on the ear. Several acousticians have occasionally started the hypothesis that, *caeteris paribus*, the strength of tones of different heights is directly proportional to the *vis viva* of the motion of the air, or, which comes to the same thing, to the amount of the mechanical work applied for producing it. But a simple experiment with the siren shews that when equal amounts of mechanical work are applied to produce deep and high tones under conditions otherwise alike, the high tones excite a very much more powerful sensation than the deep ones. Thus, if the siren is blown by a bellows, making its disc revolve with increasing rapidity, and if we take care to keep up a perfectly uniform motion of the bellows by raising its handle by the same amount the same number of times in a minute, so as to keep its bag equally filled, and drive the same amount of air under the same pressure through the siren in the same time, we hear at first, while the revolution is slow, a weak deep tone, which continually ascends, but at the same time gains strength at an extraordinary rate, till when the highest tones

producible on my double siren (near to a'', with 880 vibrations in a second) are reached, their strength is almost insupportable. In this case by far the greatest part of the uniform mechanical work is applied to the generation of resonant motion, and only a small part can be lost by the friction of the revolving disc on its axial supports, and the air which it sets into a vortical motion at the same time; and these losses must even be greater for the more rapid rotation than for the slower, so that for the production of the high tones less mechanical work remains applicable than for the deep ones, and yet the higher tones appear to our sensation extraordinarily more powerful than the deep ones. How far upwards this increase may extend, I have as yet been unable to determine, for the rapidity of my siren cannot be further increased with the same pressure of air.

The increase of strength with height of tone is especially important in the deepest part of the scale. It follows that in compound tones of great depth, the upper partial tones may be superior to the prime in strength, even when in musical tones of the same description, but of greater height, the strength of the prime may greatly predominate. This is easily proved on my double siren, because by means of the beats it is easy to determine whether any partial tone which we hear is the prime, or the second or third partial tone of the compound under examination. For when the series of twelve holes are open in both wind-boxes, and the handle, which moves the upper wind-box, is rotated once, we shall have, as already shown, four beats for the primes, eight for the second partials, and twelve for the third partials. Now we make the discs revolve more slowly than usual, by allowing a well-oiled steel spring to rub against the edge of one disc with different degrees of pressure, and thus we can easily produce series of puffs which correspond to very deep tones, and then, turning the handle, we can count the beats. By allowing the rapidity of the revolution of the discs to increase gradually, we find that the first audible tones produced make twelve beats for each revolution of the handle, the number of puffs being from thirty-six to forty in the second. For tones with from forty to eighty puffs, each revolution of the handle gives eight beats. In this case, then, the upper octave of the prime is the strongest tone. It is not till we have eighty puffs in a second that we hear the four beats of the primes.

It is proved by these experiments that motions of the air, which

do not take the form of pendular vibrations, can excite distinct and powerful sensations of tone, of which the vibrational number is two or three times the number of the pulses of the air, and yet that the prime tone is not heard through them. Hence, when we continually descend in the scale, the strength of our sensation decreases so rapidly that the sound of the prime tone, although its *vis viva* is independently greater than that of the upper partials, as is shewn in higher positions of a musical tone of the same composition, is overcome and concealed by its own upper partials. Even when the action of the compound tone on the ear is much reinforced, the effect remains the same. In the experiments with the siren the uppermost plate of the bellows is violently agitated for the deep tones, and when I laid my head on it, my whole head was set into such powerful sympathetic vibration that the holes of the rotating disc, which vanish to an eye at rest, became again separately visible, through an optical action similar to that which takes place in stroboscopic discs. The row of holes in action appeared to stand still, the other rows seemed to move partly backwards and partly forwards, and yet the deepest tones were no more distinct than before. At another time I connected my ear by means of a properly introduced tube, with an opening leading to the interior of the bellows. The agitation of the drumskin of the ear was so great that it produced an intolerable itching, and yet the deepest tones remained as indistinct as ever.

In order, then, to discover the limit of deepest tones, it is necessary not only to produce very violent agitations in the air, but to give these the form of simple pendular vibrations. Until this last condition is fulfilled we cannot possibly say whether the deep tones we hear belong to the prime tone or to an upper partial tone of the motion of the air.[1] Among the instruments hitherto employed the wide-stopped organ pipes are the most suitable for this purpose. Their upper partial tones are at least extremely weak, if not quite absent. Here we find that even the lower tones of the sixteen-foot octave, $C_{,}$ to $E_{,}$, begin to pass over into a droning noise, so that it becomes difficult for even a prac-

[1] Thus Savart's instrument, where a rotating rod strikes through a narrow slit, is totally unsuitable for making the lowest tone audible. The separate puffs of air are here very short in relation to the whole periodic time of the vibration, and consequently the upper partial tones must be very strongly developed, and the deepest tones, which are heard for 8 to 16 passages of the rod through the hole in a second, can be nothing but upper partials.

tised musical ear to assign their pitch with certainty; and, indeed, they cannot be tuned by the ear alone, but only indirectly by means of the beats which they make with the tones of the upper octaves. We observe a similar effect on the same deep tones of a piano or physharmonica; they form drones and seem out of tune, although their musical character is on the whole better established than in the pipes, because of their accompanying upper partial tones. In music, as artistically applied in an orchestra, the deepest tone used is, therefore, the $E_{,}$ of the double bass, with forty-one vibrations in a second, and I think I may predict with certainty that all efforts of modern art applied to produce good musical tones of a lower pitch must fail, not because proper means of agitating the air cannot be discovered, but because the human ear cannot hear them. The 16-foot $C_{,}$ of the organ, with thirty-three vibrations in a second, certainly gives a tolerably continuous sensation of drone, but does not allow us to give it a definite position in the musical scale. We almost begin to observe the separate pulses of air, notwithstanding the regular form of the motion. In the upper half of the 32-foot octave, the perception of the separate pulses becomes still clearer, and the continuous part of the sensation, which may be compared with a sensation of tone, continually weaker, and in the lower half of the 32-foot octave we can scarcely be said to hear anything but the individual pulses, or if anything else is really heard, it can only be weak upper partial tones, from which the musical tones of stopped pipes are not quite free.

I have tried to produce deep simple tones in another way. Strings which are weighted in their middle with a heavy piece of metal, on being struck give a compound tone consisting of many simple tones which are mutually inharmonic. The prime tone is separated from the nearest upper partials by an interval of several octaves, and hence there is no danger of confusing it with any of them; besides, the upper tones die away rapidly, but the deeper ones continue for a very long time. A string of this kind[1] was stretched on a sounding box having a single opening which could

[1] It was a thin brass pianoforte string. The weight was a copper kreutzer piece, [pronounce *kroitser*; three kreutzers make a penny at Heidelberg, where the experiment was probably tried,] pierced in the middle by a hole through which the wire passed and then made to grip the wire immovably by driving a steel tack through the kreutzer near the hole.

be connected with the auditory passage, so that the air of the sounding box could escape nowhere else but into the ear. The tones of a string of customary pitch are under these circumstances insupportably loud. But for $D_{,}$ with 37 vibrations in a second, there was only a very weak sensation of tone, and even this was rather jarring, leading to the conclusion that the ear began even here to feel the separate pulses separately, notwithstanding their regularity. At $B_{,,}$ with 34 vibrations in a second, there was scarcely anything audible left. It appears, then, that those nerve fibres which perceive such tones begin as early as at this note to be no longer excited with a uniform degree of strength during the whole time of a vibration, whether it be the phases of greatest velocity or the phases of greatest deviation from its mean position in the vibrating body which effect the excitement.[1]

We are, then, justified in asserting that sensation of musical tone begins at about 30 vibrations in a second, but that a determinate musical pitch is not perceived till about 40 vibrations are performed in a second. These facts will agree with the hypothesis concerning the elastic appendages to the auditory nerves, on remembering that the deeply intoned fibres of Corti may be set in sympathetic vibration by still deeper tones, although with rapidly decreasing strength, so that sensation of tone, but no discrimination of pitch, is possible. If the most deeply intoned of Corti's fibres lie at greater intervals from each other in the scale, but at the same time their power of damping is so great that every tone which corresponds to the pitch of a fibre, also pretty strongly affects the neighbouring fibres, there will be no safe distinction of pitch in their vicinity, but it will proceed continuously without jumps, while the intensity of the sensation must at the same time become small.

Whilst simple tones in the upper half of the 16-foot octave are perfectly continuous and musical, yet for aerial vibrations of a different form, for example when compound tones are used, dis-

[1] Subsequently I obtained two large tuning-forks from Herr Koenig in Paris, with sliding weights on their prongs. By altering the position of the weights, the pitch was changed, and the corresponding number of vibrations was given on a scale which runs along the prongs. One fork gave 24 to 35, the other 35 to 61 vibrations. The sliding weight is a plate, 5 centimètres [nearly two inches] in diameter, and forms a mirror. On bringing the ear close to these plates the deep tones are well heard. For 30 vibrations I could still hear a weak drone, for 28 scarcely a trace, although this arrangement made it easily possible to form oscillations of 9 millimètres [about ⅓ inch] in amplitude, quite close to the ear.

continuous pulses of sound are still heard even within this octave. For example, blow the disc of the siren with gradually increasing speed. At first only pulses of air are heard; but after reaching 36 vibrations in a second, weak tones sound with them, which, however, are at first upper partials. As the velocity increases the sensation of the tones becomes continually stronger, but it is a long time before we cease to perceive the discontinuous pulses of air, although these tend more and more to coalesce. It is not till we reach 110 or 120 vibrations in a second (A or $B\flat$ of the great octave) that the tone is tolerably continuous. It is just the same on the harmonium, where, in the French horn stop, c with 132 vibrations in a second still jars a little, and in the bassoon stop we observe the same jarring even in c' with 264 vibrations in a second. Generally the same observation can be made on all cutting, snarling, or braying tones, which, as has been already mentioned, are always provided with a very great number of distinct upper partial tones.

The cause of this phenomenon must be looked for in the beats produced by the high upper partials of such compound tones, which are too nearly of the same pitch. If the 15th and 16th partial of a compound tone are still audible, they form the interval of a semitone, and naturally produce the sharp beats of this dissonance. That it is really the beats of these tones which cause the roughness of the whole compound tone, can be easily felt by using a proper resonator. If $G_{,}$ is struck, having $49\frac{1}{2}$ vibrations in a second, the 15th partial is $f''\sharp$, the 16th g'', and the 17th $g''\sharp$ (nearly), &c. Now when I apply the resonator g'', which reinforces g'' most, and $f''\sharp$, $g''\sharp$ somewhat less, the roughness of the tone becomes extremely more prominent, and exactly resembles the sharp jar produced when $f''\sharp$ and g'' are themselves sounded. This experiment succeeds on the pianoforte, as well as on both stops of the harmonium. It also distinctly succeeds for higher pitches, as far as the resonators reach. I possess a resonator for g''', and although it only slightly reinforces the tone, the roughness of G, with 99 vibrations in a second, was distinctly increased when the resonator was applied.

Even the 8th and 9th partials of a compound tone, which are a whole tone apart, cannot but produce beats, although they are not so cutting as those from the higher upper partials. But the reinforcement by resonators does not now succeed so well, because the deeper resonators at least are not capable of simultaneously rein-

forcing the tones which differ from each other by a whole tone. For the higher resonators, where the reinforcement is slighter, the interval between the tones capable of being reinforced is greater, and thus I succeeded by means of the resonators g'' and g''', in increasing the roughness of the Tones G to g (having 99 and 198 vibrations in a second respectively), which is due to the 7th, 8th, and 9th partial tones (f'', g'', a'', and f''', g''', a''' respectively). On comparing the tone of G as heard in the resonators with the tone of the dissonances f'' g'' and g'' a'' as struck directly, the ear felt their close resemblance, the rapidity of intermittence being nearly the same.

Hence there can no longer be any doubt that motions of the air corresponding to deep musical tones compounded of numerous partials, are capable of exciting at one and the same time a continuous sensation of deep tones and a discontinuous sensation of high tones, and become rough or jarring through the latter. Herein lies the explanation of the fact already observed in examining qnalities of tone, that compound tones with many high upper partials are cutting, jarring, or braying; and also of the fact that they are more penetrating and cannot readily pass unobserved, for an intermittent impression excites our nervous apparatus much more powerfully than a continuous one, and continually forces itself afresh on our perception.[1] On the other hand simple tones, or compound tones which have only a few of the lower upper partials, lying at wide intervals apart, must produce perfectly continuous sensations in the ear, and make a soft and gentle impression, without much energy, even when they are in reality relatively strong.

We have not been able to determine the upper limit of the number of intermittences perceptible in a second for high notes, and could only draw attention to their becoming more difficult to perceive, and making a slighter impression, as they became more numerous. Hence even when the form of vibration, that is the quality of tone, remains the same, while the pitch is increased, the quality of tone will generally appear to diminish in roughness.

[1] [In Professor Tyndall's paper *On the Atmosphere as a Vehicle of Sound*, read before the Royal Society, Feb. 12, 1874, in trying the distance at which intense sounds could be heard at sea, he says (Philosophical Transactions for 1874, vol. 164, p. 189), 'The influence of "beats" was tried on June 3 [1873] by throwing the horns slightly out of unison; but though the beats rendered the sound characteristic, they did not seem to augment the range.'—*Translator.*]

CHAPTER X.

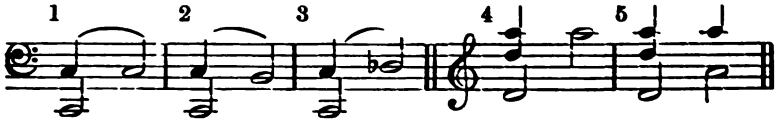
BEATS OF THE UPPER PARTIAL TONES.

THE beats hitherto considered, were produced by two simple tones, without any intervention of upper partial or combinational tones. Such beats could only arise when the two given tones made a relatively small interval with each other. As soon as the interval increased to a minor Third the beats became indistinct in the middle parts of the scale. Now it is well known that beats can also arise from two tones which make a much greater interval with each other, and we shall see hereafter that these beats play a principal part in settling the consonant intervals of our musical scales, and they must consequently be closely examined. The beats heard when the two generating tones are more than a minor Third apart in the scale, arise from upper partial and combinational tones. When the compound tones have distinctly audible upper partials, the beats resulting from them are generally clearer and stronger than those due to the combinational tones, and it is much more easy to determine their source. Hence we begin the investigation of the beats occurring in wide intervals with those which arise from the presence of upper partial tones. It must not be forgotten, however, that beats of combinational tones are much more general than these, as they occur with all kinds of musical tones, both simple and compound, whereas of course those due to upper partial tones are only found when these tones themselves are distinct. But since all tones which are useful for musical purposes are, with rare exceptions, richly endowed with powerful upper partial tones, the beats due to these upper partials are relatively of much greater practical importance than those due to the weak combinational tones.

When two compound tones are sounded at the same time, it is readily seen, from what precedes, that beats may arise whenever any two upper partial tones lie sufficiently near, or when the prime of

one tone approaches an upper partial of the other. The number of beats is of course, as before, the difference of the vibrational numbers of the two partial tones to which the beats are due. When this difference is small, and the beats are therefore slow, they are relatively most distinct to hear and to count and to investigate, precisely as for beats of prime tones. They are also more distinct when the particular partial tones which generate them are loudest. Now, for the tones most used in music, partials with a low ordinal number are loudest, because the intensity of partial tones usually diminishes as their ordinal number increases.

Let us begin, then, with examples like the following, on an organ in its principal or violin stops,[1] or upon an harmonium:



The minims in these examples denote the prime tones of the notes struck, and the crotchets the corresponding upper partial tones. If the octave *C c* in the first example is tuned accurately, no beats will be heard. But if the upper note is changed into *B* as in the second example, or *d*♭ as in the third, we obtain the same beats as we should from the two tones *B c*, or *c d*♭, where the interval is a semitone. The number of beats ($16\frac{1}{2}$ in a second) is the same in each case, but their intensity is naturally less in the former case, because they are somewhat concealed by the strong deep tone *C*, and also because *c*, the second partial of *C*, has generally less force than its prime.

In examples 4 and 5 beats will be heard on keyed instruments tuned according to the usual system of temperament. If the tempered intonation is exact there will be one beat in a second, because the note a'' on the instrument does not exactly agree with the note a'' which is the third partial tone of the note d'. On the other hand the note a'' on the instrument exactly coincides with a'', the second partial tone of the note a' in the fifth example, so that on instruments exactly tuned in any temperament the two examples 4 and 5 should give the same number of beats.

[1] [See preceding notes 1 and 2, p. 141. In English organs the open diapason and cornet (p. 91, note 3) might be used.—*Translator.*]

Since the first upper partial tone makes exactly twice as many vibrations in a second as its prime, the c on the instrument in Ex. 1, is identical with the first upper partial of the prime tone C, provided c makes twice as many vibrations in a second as C. The two notes C, c, cannot be struck together without producing beats, unless this exact relation is maintained. The least deviation from this exact relation is betrayed by beats. In the fourth example the beats will not cease till we tune a'' on the instrument so as to coincide with the third partial tone of the note d, and this can only happen when the vibrational number of a'' is precisely three times that of d'. In the fifth example we have to make the vibrational number of a' half as great as that of a'', which is three times that of d'; that is the vibrational numbers of d' and a' must be exactly as $2:3$, or beats will ensue. Any deviation from this ratio will be detected at once by beats.

Now we have already shewn that the vibrational numbers of two tones which form an octave are as 1 to 2, and those of two which form a fifth as 2 to 3. These ratios were discovered long ago by merely following the judgment of the ear respecting the pleasantness of the concords of two tones. The circumstances just stated furnish the reason why these intervals when tuned according to these simple ratios of numbers, and in no other case, will produce an undisturbed concord, whereas very small deviations from this mathematical intonation will betray themselves by that restless fluctuation of tone known as beats. The d' and a' of the last example, if tuned as a perfect fifth, make $293\frac{1}{3}$ and 440 vibrations in a second respectively; and their common upper partial a'' makes $3 \times 293\frac{1}{3} = 2 \times 440 = 880$ vibrations in a second. In the tempered intonation d' makes as much as $293\frac{2}{3}$ vibrations in a second, and hence its second upper partial (or third partial) tone makes 881 in the same time, and this extremely small difference is betrayed to the ear by one beat in a second. That imperfect Octaves and Fifths will produce beats, was a fact long known to organ builders, who made use of it practically to obtain the required just or tempered intonation with greater ease and certainty. Indeed, there is no more sensitive means of proving the correctness of intervals.

Two musical tones, therefore, which stand in the relation of a perfect Octave, a perfect Twelfth, or a perfect Fifth, go on sounding uniformly without disturbance, and are thus distinguished from the next adjacent intervals, imperfect Octaves and Fifths, for

which a part of the tone breaks up into distinct pulses, and consequently the two tones do not continue to sound without interruption. For this reason the perfect Octave, Twelfth, and Fifth will be called *consonant intervals* in contradistinction to the next adjacent intervals, which are termed *dissonant*. Although these names were given long ago, long before anything was known about upper partial tones and their beats, they give a very correct notion of the essential character of the phenomenon which consists in the undisturbed or disturbed coexistence of sounds.

Since the phenomena just described form the essential basis for the construction of normal musical intervals, it is advisable to establish them experimentally in every possible form.

We have stated that the beats heard are the beats of those partial tones of both compounds which nearly coincide. Now it is not always very easy on hearing a Fifth or an Octave which is slightly out of tune, to recognise clearly with the unassisted ear which part of the whole sound generates the beats. On listening we are apt to feel that the whole sound is alternately reinforced and weakened. Yet an ear accustomed to distinguish upper partial tones, after directing its attention on the common upper partials concerned, will easily hear the strong beats of these particular tones, and recognise the continued and undisturbed sound of the primes. Strike the note d', attend to its upper partial a'', and then strike a tempered fifth a'; the beats of a'' will be clearly heard. To an unpractised ear the resonators already described will be of great assistance. Apply the resonator for a'', and the above beats will be heard with great distinctness. If, on the other hand, a resonator, tuned to one of the prime tones d' or a', be employed, the beats are heard much less distinctly, because the continuous part of the tone is then reinforced.

This last remark must not be taken to mean that no other simple tones beat in this combination except a''. On the contrary, there are other higher and weaker upper partials, and also combinational tones which beat as we shall learn in the next chapter, and these beats coexist with those already described. The beats of the lowest common upper partials are the most prominent, simply because these partials are the loudest and slowest of all.

Secondly, a direct experimental proof is desirable that the numerical ratios here deduced from the vibrational numbers are really those which give no beats. This proof is most easily given

by means of the double siren (fig. 56, p. 245). Set the discs in revolution and open the series of 8 holes on the lower and 16 on the upper, thus obtaining two compound tones which form an Octave. They continue to sound without beats as long as the upper box is stationary. But directly we begin to revolve the upper box, thus slightly sharpening or flattening the tone of the upper disc, beats are heard. As long as the box was stationary the ratio of the vibrational numbers was exactly 1 : 2, because exactly 8 pulses of air escaped on one rotation of the lower, and 16 on one rotation of the upper disc. By diminishing the speed of rotation of the handle this ratio may be altered as slightly as we please, but however slowly we turn it, if it move at all, beats are heard proclaiming the falsification of the interval.

Similarly with the Fifth. Open the series of 12 holes above, and 18 below, and a perfectly unbroken Fifth will be heard as long as the upper windbox is at rest. The ratio of the vibrational numbers, fixed by the holes of the two series, is exactly 2 to 3. On rotating the windchest, beats are heard. We have seen that each revolution of the handle increases or diminishes the number of vibrations of the tone due to the 12 holes by 4. When we have the tone of 12 holes on the lower discs also, we thus obtain 4 beats. But with the Fifth from 12 and 18 holes each revolution of the handle gives 12 beats, because the vibrational number of the third partial tone increases on each revolution of the handle by $3 \times 4 = 12$, when that of the prime tone increases by 4, and we are now concerned with the beats of this partial tone.

In these investigations the siren has the great advantage over all other musical instruments, of having its intervals tuned according to their simple numerical relations with mechanical certainty by the method of constructing the instrument, and we are consequently relieved from the extremely laborious and difficult measurements of the vibrational numbers which would have to precede the proof of our law on any other musical instrument. Yet the law had been already established by such measurements, and the ratios were shewn to approximate more and more closely to those of the simple numbers, as the degree of perfection increased, to which the methods of measuring numbers of vibrations and tuning perfectly had been brought.

Just as the coincidences of the two first upper partial tones led us to the natural consonances of the Octave and Fifth, the coincidences of higher upper partials would lead us to a further

series of natural consonances. Now it must be remarked that in the same proportion that these higher upper partials become weaker, the beats become less perceptible by which the imperfect are distinguished from the perfect intervals, and the error of tuning is shewn. Hence the delimitation of those intervals which depend upon coincidences of the higher upper partials becomes continually more indistinct and indeterminate as the upper partials involved are higher in order. In the following table the first horizontal line and first vertical column contain the ordinal numbers of the coincident upper partial tones, and at their intersection will be found the name of the corresponding interval between the prime tones, and the ratio of the vibrational numbers of the tones composing it. This numerical ratio always results from the ordinal numbers of the two coincident upper partial tones.

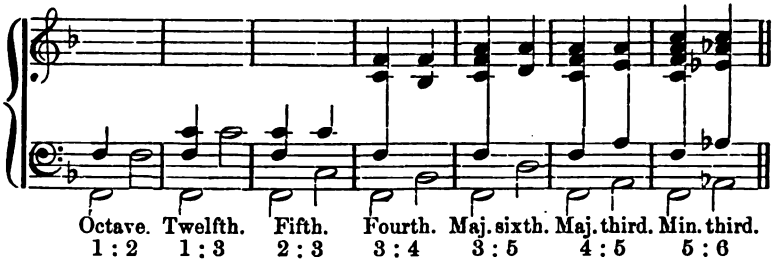
Coincident Partial Tones	1	2	3	4	5
6	2 Octaves and Fifth 1 : 6	Twelfth 1 : 3	Octave 1 : 2	Fifth 2 : 3	Minor Third 5 : 6
5	2 Octaves & Major Third 1 : 5	Major Tenth 2 : 5	Major Sixth 3 : 5	Major Third 4 : 5	
4	Double Octave 1 : 4	Octave 1 : 2	Fourth 3 : 4		
3	Twelfth 1 : 3	Fifth 2 : 3			
2	Octave 1 : 2				

The two lowest lines of this table contain the intervals already considered, the Octave, Twelfth, and Fifth. In the third line from the bottom the 4th partial gives the intervals of the Fourth and double Octave. The 5th partial determines the major Third, either simple or increased by one or two Octaves, and the major Sixth. The 6th partial introduces the minor Third in addition. Here I have stopped, because the 7th partial tone is entirely eliminated, or at least much weakened, on instruments such as the piano, where the quality of tone can be regulated within certain limits. Even the 6th partial is generally very weak, but an endeavour is made to favour all the partials up to the 5th. We shall return hereafter to the intervals characterised by the 7th partial, and to the minor Sixth, which is determined by the 8th.

The following is the order of the consonant intervals beginning with those distinctly characterised, and then proceeding to those which have their limits somewhat blurred, so to speak, by the weaker beats of the higher upper partial tones.

1.	Octave	1 : 2
2.	Twelfth	1 : 3
3.	Fifth	2 : 3
4.	Fourth	3 : 4
5.	Major Sixth	3 : 5
6.	Major Third	4 : 5
7.	Minor Third	5 : 6

The following examples in musical notation shew the coincidences of the upper partials. The primes are as before represented by minims, and the upper partials by crotchets. The series of upper partials is continued up to the common tone only.



We have hitherto confined our attention to beats arising from intervals which differ but slightly from those of perfect consonances. When the difference is small the beats are slow, and hence easy both to observe and count. Of course beats continue to occur when the deviation of the two coincident upper partials increases. But as the beats then become more numerous, their real character is more easily concealed than even the more rapid beats of dissonant primes, under the overwhelming mass of sound of the louder primes. These more rapid beats give a rough effect to the whole mass of sound, but the ear does not readily recognise its cause, unless the experiments have been conducted by gradually increasing the imperfection of an harmonic interval, so as to make the beats gradually more and more rapid, thus leading the observer to mark the intermediate steps between the numerable rapid beats on the one hand, and the roughness of a dissonance on the other, and hence to convince himself that the two phenomena differ only in degree.

In examining the beats arising from two simple tones we saw that their distinctness and roughness depended partly on the magnitude of the interval between the beating tones, and partly upon the rapidity of the beats themselves, so that for high tones this increasing rapidity injured the distinctness of even the beats arising from small intervals, and obliterated them in sensation. At present, as we have to deal with beats of upper partials, which, when their primes lie in the middle region, principally belong to the higher parts of the scale, the rapidity of the beats has a preponderating influence on the distinctness of their definition.

The law determining the number of beats in a second for a given imperfection in a consonant interval, results immediately from the law above assigned for the beats of simple tones. When two simple tones, making a small interval, generate beats, the number of beats in a second is the difference of their vibrational numbers. Let us suppose by way of example that a certain prime tone has the vibrational number 300. The vibrational numbers of the primes which make consonant intervals with it, will be as follows:—

Prime tone = 300			
Upper Octave	= 600	Lower Octave	= 150
„ Fifth	= 450	„ Fifth	= 200
„ Fourth	= 400	„ Fourth	= 225
„ Major Sixth	= 500	„ Major Sixth	= 180
„ Major Third	= 375	„ Major Third	= 240
„ Minor Third	= 360	„ Minor Third	= 250

Now assume that the prime tone has been put out of tune by one vibration in a second, so that its vibrational number becomes 301, then calculating the vibrational number of the coincident upper partial tones, and taking their difference, we find the number of beats thus:—

Interval upwards	Beating Partial Tones		Number of Beats
Prime . . .	1 × 300 = 300	1 × 301 = 301	1
Octave . . .	1 × 600 = 600	2 × 301 = 602	2
Fifth . . .	2 × 450 = 900	3 × 301 = 903	3
Fourth . . .	3 × 400 = 1200	4 × 301 = 1204	4
Major Sixth . .	3 × 500 = 1500	5 × 301 = 1505	5
Major Third . .	4 × 375 = 1500	5 × 301 = 1505	5
Minor Third . .	5 × 360 = 1800	6 × 301 = 1806	6

Interval downwards	Beating Partial Tones		Number of Beats
Prime . . .	1 × 300 = 300	1 × 301 = 301	1
Octave . . .	2 × 150 = 300	1 × 301 = 301	1
Fifth . . .	3 × 200 = 600	2 × 301 = 602	2
Fourth . . .	4 × 225 = 900	3 × 301 = 903	3
Major Sixth . .	5 × 180 = 900	3 × 301 = 903	3
Major Third . .	5 × 240 = 1200	4 × 301 = 1204	4
Minor Third . .	6 × 250 = 1500	5 × 301 = 1505	5

Hence the number of beats which arise from putting one of the generating tones out of tune to the amount of one vibration in a second, is always given by the two numbers which define the interval. The smaller number gives the number of beats which arise from increasing the vibrational number of the upper tone by 1. The larger number gives the number of beats which arise from increasing the vibrational number of the lower tone by 1. Hence if we take the major Sixth *ca*, having the ratio 3 : 5, and sharpen *a* so as to make one additional vibration in a second, we shall have 3 beats in a second; but if we sharpen *c* so as to make one more vibration in a second, we obtain 5 beats in a second, and so on.

Our calculation and the rule based on it shew that if the amount by which one of the tones is put out of tune remains constant, the number of the beats increases according as the interval is expressed in larger numbers. Hence for Sixths and Thirds we must have vibrational numbers of the tones much more nearly in the normal ratio, if we wish to avoid slow beats, than for Octaves and Unisons. On the other hand a slight imperfection in the tuning of Thirds brings us much sooner to the limit where the beats become too rapid to be distinctly separable. By changing the Unison *c″c″*, by flattening one tone, into the Semitone *b′c″*, on sounding the notes together there results a clear dissonance with 33 beats, the number which, as before observed, seems to give the maximum of harshness. But to obtain 33 beats from fifth *f′c″*, it is only necessary to alter *c″* by a quarter of a Tone. If it is changed by a Semitone, so that *f′c″* becomes *f′ b′*, there result 66 beats, and their clearness is already much injured. To obtain 33 beats the *c″* must not be changed in the Fifth *c″g″* by more than one-sixth of a Tone, in the Fourth *c″f″* by more than one-eighth, in the major Third *c″e″* and major Sixth *c″a″* by more than one-tenth, and in the minor Third *c″e″*♭ by more than one-twelfth.

Conversely, if in each of these intervals the vibrational number of c'' be altered by 33, so that c'' becomes b' or $d''\flat$, we obtain the following numbers of beats.

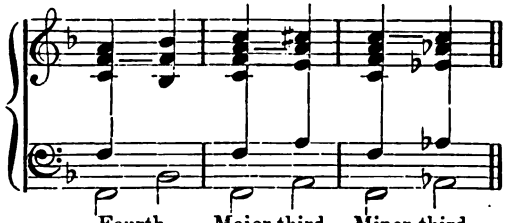
The interval of the		becomes	or	and gives beats
Octave . .	c'' c'''	b' c'''	$d''\flat$ c'''	66
Fifth . . .	c'' g''	b' g''	$d''\flat$ g''	99
Fourth . .	c'' f''	b' f''	$d''\flat$ f''	132
Major Third .	c'' e''	b' e''	$d''\flat$ e''	165
Minor Third .	c'' $e''\flat$	b' $e''\flat$	$d''\flat$ $e''\flat$	198

Now since 99 beats in a second produce very weak effects even under favourable circumstances for simple tones, and 132 beats in a second seem to lie at the limit of audibility, we must not be surprised if such numbers of beats, produced by the weaker upper partials, and smothered by the more powerful prime tones, no longer produce any sensible effect, and in fact vanish so far as the ear is concerned. Now this relation is of great importance in the practice of music, for in the table it will be seen that the mistuned Fifth gives the interval b' g'' which is much used as an imperfect consonance under the name of *minor Sixth*. In the same way we find the major Third $d''\flat$ f'' as a mistuned Fourth, and the Fourth $b'e''$ as a mistuned major Third, and so on. That, at least in this part of the scale, the major Third does not produce the beats of a mistuned Fourth, or the Fourth those of a mistuned major Third, is explained by the great number of beats. In point of fact these intervals in this part of the scale give a perfectly uninterrupted sound, without a trace of beats or harshness, when they are tuned perfectly.

This brings us to the investigation of those circumstances which affect the perfection of the consonance for the different intervals. A consonance has been characterised by the coincidence of two of the upper partial tones of the compounds forming the chord. When this is the case the two compound tones cannot generate any slow beats. But it is possible that some other two upper partial tones of these two compounds may be so nearly of the same pitch that they can generate rapid beats. Cases of this kind occur in the last examples in musical notation (p. 278.) Among the upper partials of the major Third FA occur f' and e', side by side; and among those of the minor Third $FA\flat$ will be found a' and $a'\flat$. In each

case there is the dissonance of a Semitone, and these must produce the same beats as if they had been given directly as simple prime tones. Now although such beats can produce no very prominent impression, partly on account of their rapidity, partly on account of the weakness of the tones which generate them, and partly because the primes are sounding on at the same time unintermittently, they cannot but exert some effect on the harmoniousness of the interval. In the last chapter we found that in certain qualities of tone, where the higher upper partials are strongly developed, sensible dissonances may arise within a single compound tone (p. 269). When two such musical tones are sounded together, there will be not only the dissonances resulting from the higher upper partial tones in each individual compound, but also those which arise from a partial tone of the one forming a dissonance with a partial tone of the other, and in this way there must be a certain increase in roughness.

An easy method of finding those upper partials in each consonant interval which form dissonances with each other, may be deduced from what has been already stated (p. 281) concerning larger imperfections in tuning consonant intervals. We thus found that the major Third might be considered as a mistuned Fourth, and the Fourth again as a mistuned Third. On raising the pitch of a compound tone by a Semitone, we raise the pitch of all its upper partial tones by the same amount. Those upper partials which coincide for the interval of a Fourth, separate by a Semitone when by altering the pitch of one generating tone we convert the Fourth into a major Third, and similarly those which coincide for the major Third differ by a Semitone for the Fourth, as will appear in the following example.



Fourth. Major third. Minor third.

The 4th and 3rd partial in the Fourth of the first example coincide as f'. But if the Fourth $B\flat$ sinks, as in the second ex-

ample, to the major Third A, its 3rd partial f' sinks also to e', and forms a dissonance with the 4th partial f' of F, which was unaltered. On the other hand the 5th and 4th tone of the two compounds, which in the first example formed the dissonance $a'b'\flat$, now coincide as a'. In the same way the consonant unison $a'a'$ of the second example appears as the dissonance $a'a'\flat$ in the third, and the dissonance $c''c''\sharp$ in the second becomes the consonant unison $c''c''$ in the third.

Hence *in each consonant interval those upper partials form a dissonance, which coincide in one of the adjacent consonant intervals,*[1] and in this sense we can say, that every consonance is disturbed by the proximity of the consonances next adjoining it in the scale, and that the resulting disturbance is the greater, the lower and louder the upper partials which by their coincidence characterise the disturbing interval, or, in other words, the smaller the number which expresses the ratio of the vibrational numbers.

The following table gives a general view of this influence of the different consonances on each other. The partials are given up to the 9th inclusive, and corresponding names[2] assigned to the intervals arising from the coincidence of the higher upper partial tones. The third column contains the ratios of their vibrational numbers, which at the same time furnish the number of the order of the coincident partial tones. The fourth column gives the distance of the separate intervals from each other, and the last a measure of the relative strength of the beats resulting from the mistuning of the corresponding interval, reckoned for the quality of tone of the violin.[3] The degree to which any interval disturbs the adjacent intervals, increases with this last number.

[1] [That is, in intervals which differ from the first by raising or depressing one of its tones by a Semitone (either $\frac{16}{15}$ or $\frac{25}{24}$) as in the table on p. 281, or even a Tone ($\frac{9}{8}$). Thus for Fifth, $\frac{3}{2} \times \frac{16}{15} = \frac{8}{5}$ a minor Sixth; and $\frac{3}{2} \times \frac{8}{9} = \frac{4}{3}$ a Fourth. For Fourth, $\frac{4}{3} \times \frac{15}{16} = \frac{5}{4}$ a major Third; and $\frac{4}{3} \times \frac{9}{8} = \frac{3}{2}$ a Fifth. For major Third $\frac{5}{4} \times \frac{16}{15} = \frac{4}{3}$ a Fourth; and $\frac{5}{4} \times \frac{24}{25} = \frac{6}{5}$ a minor Third. For minor Third $\frac{6}{5} \times \frac{25}{24} = \frac{5}{4}$ a major Third, and $\frac{6}{5} \times \frac{15}{16} = \frac{9}{8}$ a major Tone. The adjacency of the consonant intervals is best shewn in fig. 60, A (p. 292), where it appears that the order may be taken at; 1) Unison, 2) minor Third, 3) major Third, 4) Fourth, 5) Fifth, 6) minor Sixth, 7) major Sixth, 8) Octave. In the Table on p. 284, other intervals, not perfectly consonant, are intercalated among these.—*Translator.*]

[2] [For the systematic English names here employed see Appendix XIX., Sect. B, and Table II., ii. (Septimal Intervals.)—*Translator.*]

[3] See Appendix No. XV.

Intervals	Notation	Ratio of the Vibrational Numbers	Relative Distance	Intensity of Influence
Unison	*C*	1 : 1	8 : 9	100·0
Second	*D*	8 : 9	63 : 64	1·4
Super-Second	*D*+	7 : 8	48 : 49	1·8
Sub-minor Third	*E*♭ −	6 : 7	35 : 36	2·4
Minor Third	*E*♭	5 : 6	24 : 25	3·3
Major Third	*E*	4 : 5	35 : 36	5·0
Super-major Third	*E*+	7 : 9	27 : 28	1·6
Fourth	*F*	3 : 4	20 : 21	8·3
Sub-Fifth	*G*♭ −	5 : 7	14 : 15	2·8
Fifth	*G*	2 : 3	15 : 16	16·7
Minor Sixth	*A*♭	5 : 8	24 : 25	2·5
Major Sixth	*A*	3 : 5	20 : 21	6·7
Sub-minor Seventh	*B*♭ −	4 : 7	35 : 36	3·6
Minor Seventh	*B*♭	5 : 9	9 : 10	2·2
Octave	*c*	1 : 2		50·0

The most perfect chord is the *Unison*, for which both compound tones have the same pitch. All its partial tones coincide, and hence no dissonance can occur except such as is contained in each compound separately (p. 269).

It is much the same with the *Octave*. All the partial tones of the higher note of this interval coincide with the even-partial tones of the deeper, and reinforce them, so that in this case also there can be no dissonance between two upper partial tones, except such as already exists, in a weaker form, among those of the deeper note. A note accompanied by its Octave consequently becomes brighter in quality, because the higher upper partial tones on which brightness of quality depends, are partly reinforced by the additional Octave. But a similar effect would also be produced by simply increasing the intensity of the lower note without adding the Octave; the only difference would be, that in the latter case the reinforcement of the different partial tones would be somewhat differently distributed.

The same holds for the *Twelfth* and *double Octave*, and generally for all those cases in which the prime tone of the higher note coincides with one of the partial tones of the lower note, although as the interval between the two notes increases the difference between consonance and disonance tends towards obliteration.

The cases hitherto considered, where the prime of one compound tone coincides with one of the partials of the other, may be termed *absolute consonances*. The second compound tone introduces no new element, but merely reinforces a part of the other.

Unison and Octave disturb the next adjacent intervals considerably, in the sense assigned to this expression on p. 283, so that the minor Second *C D*♭, and the major Seventh *C B*, which differ from the Unison and Octave by a Semitone respectively, are the harshest dissonances in our scale. Even the major Second *C D*, and the minor Seventh *C B*♭, which are a whole Tone apart from the disturbing intervals, must be reckoned as dissonances, although, owing to the greater interval of the dissonant partial tones, they are much milder than the others. In the higher regions of the scale their roughness is materially lessened by the increased rapidity of the beats. Since the dissonance of the minor Seventh is due to the second partial tone, which in most musical qualities of tone is much weaker than the prime, it is still milder than that of the major Second, and hence lies on the very boundary between dissonance and consonance.

To find additional good consonances we must consequently go to the middle of the Octave, and the first we meet is the *Fifth*. Immediately next to it within the interval of a Semitone there are only the intervals 5 : 7 and 5 : 8 in our table, and these cannot much disturb it, because in all the better kinds of musical tones the 7th and 8th partials are either very weak or entirely absent. The next intervals with stronger upper partials are the Fourth 3 : 4 and the major Sixth 3 : 5. But here the interval is a whole Tone, and if the tones 1 and 2 of the interval of the Octave could produce very little disturbing effect in the minor Seventh, the disturbance by the tones 2 and 3, or by the adjacency of the Fifth for the Fourth and major Sixth must be insignificant, and the reaction of these two intervals with the tones 3 and 4 or 3 and 5 on the Fifth must be entirely neglected. Hence the Fifth remains a perfect consonance, in which there is no sensible disturbance of closely adjacent upper partial tones. It is only in harsh qualities of tone (harmonium, double-bass, violoncello, reed organ pipes) with high upper partial tones, and deep primes, when the number of beats is small, that we remark that the Fifth is somewhat rougher than the Octave.[1] Hence the Fifth has been acknowledged as a consonance from the earliest times and by all musicians. On the other hand the intervals next adjacent to the Fifth are those which produce the harshest dissonances after those

[1] [Take the vibrational numbers of the two prime tones which form the first to

next adjacent to the Octave. Of the dissonant intervals next the Fifth, those in which the Fifth is flattened, that is which lie between the Fifth and Fourth, and are disturbed firstly by the tones 2 and 3, and secondly by the tones 3 and 4, are more decidedly dissonant than those in which the Fifth is sharpened and which lie between the Fifth and major Sixth, because for the latter the second disturbance arises from the tone 4 and the weaker tone 5.[1] The intervals between the Fifth and Fourth are conse-

be 2 and 3, and find those of their upper partials thus, assuming *C G* to be the two notes.

Nos.	1	2	3	4	5	6	7	8
Partials of	2	4	6	8	10	12	14	16
Lower note	*C*	*c*	*g*	*c′*	*e′*	*g′*	*b′*♭	*c″*
$\frac{3}{2}$ *Fifth*		*G*	*g*		*d′*	*g′*	*b′*	
Partials		3	6	9		12	15	
Nos.		1	2	3		4	5	

We see that the beating tones are 14 and 15, or *b′*♭ *b′*, the 7th partial of the lower and 5th of the upper; and 15 and 16, or *b′ c″*, the 5th of the upper and 8th of the lower note, and that these beats are unimportant because the 7th and 8th partials are generally weak; but if they are strong these beats being those of a Semitone, and of nearly a Semitone, are very harsh. On a bass concertina with just intonation, where the Twelfth was faultless, the Fifth was extremely harsh. The next beating partial tones are 8 and 9, or *c′ d′*, the 4th partial of the lower and 3rd of the upper note, and these being a whole Tone apart, the beats are not of importance even when strong, and with weak upper partials are insignificant. Similarly for the beats of 9 and 10, or *d′ e′*, the 3rd partial of the upper and 5th of the lower note. Perhaps some readers may find this method of examining the cause of roughness easier than that in the text. The meaning in both cases is the same, and on comparing the text it will be seen that the same intervals are compared in the same order.—*Translator.*]

[1] [Taking the scheme in the last note, and supposing *G* to be altered first to *G*♭ and then to *A*♭, we may write the several schemes thus:

No. of Partials	1	2	3	4	5	6	7	8
Lower Note	*C*	*c*	*g*	*c′*	*e′*	*g′*	*b′*♭	*c″*
$\frac{3}{2}$ *Fifth*		*G*	*g*		*d′*	*g′*	*b′*	
Flattened		*G*♭	*g*♭		*d′*♭	*g′*♭	*b′*♭	
Sharpened		*A*♭	*a*♭		*e′*♭	*a′*♭	*c″*	
		1	2		3	4	5	

If the *G*♭ were made sufficiently flat, we should have its 5th partial *b′*♭ coinciding with the 7th partial of C, which, however, is never felt as a consonance, and the interval then becomes 5 : 7. This, however, never occurs in musical practice, where the *b′*♭ from *G*♭ is always sharper than that from *C*, but this dissonance is not felt, the *g*♭ *g* or tones 2 and 3, and *c′ d′*♭ or tones 3 and 4 producing the chief disturbance.

quently always considered dissonant in musical practice. But between the Fifth and major Sixth lies the interval of the *minor Sixth*, which is treated as an imperfect consonance, and owes this preference mainly to its being the inversion of the major Third. On keyed instruments, as the piano, the same keys will strike notes which at one time represent the consonance *C A*♭, and at another the dissonance *C G*♯.[1]

Next to the Fifth follow the consonances of the *Fourth* 3 : 4 and the *major Sixth*, the chief disturbance of which arises usually from the Fifth. The Fourth is somewhat further from the Fifth (the interval is 8 : 9) than it is from the major Sixth (the interval is 9 : 10), and hence the latter is a less perfect consonance than the former. But close by the Fourth lies the major Third with the 4th and 5th partials coincident, and hence when these partials are strongly developed, the Fourth may lose its advantage over the major Sixth. It is also well known that the old theoretical musicians long disputed as to whether the Fourth should be considered consonant or dissonant. The precedence given to the Fourth over the major Sixth and major Third, is rather due to its being the inversion of the Fifth than to its own inherent harmoniousness. The Fourth, the major Sixth and minor Sixth, are rendered less pleasant by being widened by an Octave (thus becoming the Eleventh, and major and minor Thirteenth), because they then lie near the Twelfth, and consequently the disturbance by the characteristic tones of the Twelfth 1 and 3, is greater, and also the adjacent intervals 2 : 5 for the Eleventh, and 2 : 7 for the Thirteenths, are more disturbing than the 4 : 5 for the Fourth and 4 : 7 for the Sixths in the lower Octave.[2]

If *A*♭ is taken sufficiently sharp for its 5th partial *c″* to coincide with the eighth of *C* we have the interval 5 : 8 or minor Sixth. Here again we have the disturbance from *a*♭ *g* the tones 2 and 3, but the *second* disturbance is now from *e′*♭ *e′* or tones 3 and 5, instead of from *d′*♭ *c′* or tones 3 and 4, and as the tone 5 is weaker the disturbance on the whole is weaker. This is the case in musical practice.—*Translator.*]

[1] [This is the result of temperament. When justly intoned *e′ c″* is a pleasant interval, even on the concertina. But in the bass *E c* is miserable for the same instrument. On the other hand *c′ g′*♯ (where *g′*♯ is a major Third above *e′*) is harshly dissonant, more so than *c′ f′*♯ (where *f′*♯ is the perfect Fifth to *b*).—*Translator.*]

[2] [Treating these intervals as in the preceding notes we have:

No. of Partials	1	2	3	4	5	6	7	8	
Lower note	*C*	*c*	*g*	*c′*	*e′*	*g′*	*b′*♭	*c″*	
Fourth	*F*	*f*		*c′*		*f′*	*a′*	*c″*	
No. of Partials	1	2		3		4	5	6	

Next come the *major* and *minor Third.* The latter is very imperfectly delimited on instruments which, like the pianoforte, do not strongly develop the 6th partial of the compound tone, because it can then be imperfectly tuned without producing sensible beats.[1] The minor Third is sensibly exposed to disturbance from the Unison, and the major Third from the Fourth; and both mutually disturb each other, the minor Third coming off worse than the major.[2] For the harmoniousness of either interval it is necessary

	No. of Partials	1	2	3	4	5	6	7	8		
	Lower note	*C*	*c*	*g*	*c′*	*e′*	*g′*	*b′*♭	*c″*		
	Eleventh		*f*			*f′*			*c″*		
	No. of Partials		1			2			3		
	No. of Partials	1	2	3	4	5	6	7	8		
	Lower note	*C*	*c*	*g*	*c′*	*e′*	*g′*	*b′*♭	*c″*		
$\frac{5}{3}$	*Major Sixth*	*A*		*a*		*e′*	*a′*		*c″*♯		
	No. of Partials	1		2		3	4		5		
	No. of Partials	1	2	3	4	5	6	7	8	9	10
	Lower note	*C*	*c*	*g*	*c′*	*c′*	*g′*	*b′*♭	*c″*	*d″*	*e″*
$\frac{10}{3}$	*Major Thirteenth*			*a*			*a′*				*e″*
	No. of Partials			1			2				3
	No. of Partials	1	2	3	4	5	6	7	8		
	Lower note	*C*	*c*	*g*	*c′*	*e′*	*g′*	*b′*♭	*c″*		
$\frac{8}{5}$	*Minor Sixth*	*A*♭		*a*♭	*e′*♭		*a′*♭		*c″*		
	No. of Partials	1		2	3		4		5		
	No. of Partials	1	2	3	4	5	6	7	8	9	10
	Lower note	*C*	*c*	*g*	*c′*	*e′*	*g′*	*b′*♭	*c″*	*d″*	*e″*
$\frac{16}{5}$	*Minor Thirteenth*			*a*♭			*a′*♭			*e″*♭	
	No. of Partials			1			2			3	

These diagrams will make the text immediately intelligible, but as the notes refer to the ordinary notation the fact that *f* to *g* in the Fourth is a wider interval than *g* to *a* in the major Sixth is not expressed. It is, however, readily seen how much worse is the minor Sixth with *g* to *a*♭, and that in all these cases the disturbance arises from the 2nd and 3rd partials which coincide for the fifth. It is also seen how the disturbance is increased in the Elevenths and Thirteenths on account of one of the disturbing tones becoming a prime, and hence louder.—*Translator.*]

[1] [As the usual tempered tuning of the piano makes the minor Third greatly too flat, this becomes a great advantage on that instrument. On the harmonium even *e′ g′*, *e″ g″* are very harsh, as compared with the same intervals on my justly intoned concertina.—*Translator.*]

[2] [This will be made clearer by the following diagrams:

	No. of Partials	1	2	3	4	5	6	7	8	
	Lower Note	*C*	*c*	*g*	*c′*	*e′*	*g′*	*b′*♭	*c″*	
$\frac{5}{4}$	*Major Third*	*E*	*e*	*b*		*e′*	*g′*♯	*b′*		*d″*
	No. of Partials	1	2	3		4	5	6		

that the disturbing beats should be very rapid. Hence in the upper part of the scale these intervals are pure and good, but in the lower part they are very rough. Hence all antiquity refused to accept Thirds as consonances. It was not till the time of Franco of Cologne (at the end of the twelfth century) that they were admitted as *imperfect consonances*. The reason of this may probably be that musical theory was developed among classical nations and in medieval times principally in respect to men's voices, and in the lower part of this scale Thirds are far from good. With this we must connect the fact that the proper intonation of Thirds was not discovered in early times, and that the *Pythagorean Third*, with its ratio of 64 : 81, was looked upon as the normal form till towards the close of the middle ages.[1]

No. of Partials . . .	1	2	3	4	5	6	7	8		
Lower Note . . .	*C*	*c*	*g*	*c′*	*e′*	*g′*	*b′*♭	*c″*		
$\frac{6}{5}$ *Minor Third* . . .	*E*♭	*e*♭	*b*♭	*e′*♭		*g′*	*b′*♭	*d″*♭		
No. of Partials . . .	1	2	3	4		5	6	7		

The 6th partial of this *E*♭ is not the same as the 7th partial of *C*, although the notation makes it appear so, but it is sharper in the ratio of 36 : 35, and hence if the partials were not so high would be very disturbing. It is seen that *g′ g′*♯ are the 6th and 5th partials for the major Third, and *e′*♭ *e′* the 4th and 5th for the minor Third; the interval being the same (24 : 25), the disturbance is worse in the latter case, because the partials are lower and hence louder.—*Translator.*]

[1] [The ordinary major Third on the harmonium is very little flatter than this, but still it is much less harsh. On my justly intoned concertina it is possible to contrast the just with the Pythagorean Third. The following arrangement of the consonant intervals will shew the beating partials in each case, and the exact ratios of their intervals. The number of the partial is subscribed in each case. The beating interval is inoffensive for 5 : 6, but its action becomes sensible for 7 : 8, 8 : 9, and 9 : 10, and for 14 : 15, 15 : 16, 24 : 25 the effect is decidedly bad if the tones are strong enough and the beats slow enough; the strength depends on the lowness of the ordinal numbers of the beating partials, and the rapidity on their position in the scale. This must be taken into consideration, as in fig. 60, p. 292. The order of the intervals is that of their relative harmoniousness as assigned in my paper 'On the Physical Constitution and Relations of Musical Chords,' in the Proceedings of the Royal Society, June 16, 1864, vol. xiii. p. 392, Table VIII., here re-arranged.

C c	1_1	2_2	3_3	4_4	5_5	6_6	7_7	8_8	9_9	10_{10}		
$\frac{2}{1}$ Octave		2_1		4_2		6_3		8_4		10_5		
C G	2_1		4_2		6_3		8_4		10_5		12_6	14_7
$\frac{3}{2}$ Fifth		3_1			6_2			9_3			12_4	
C e	2_1		4_2		6_3		8_4		10_5		12_6	14_7
$\frac{5}{2}$ Major Tenth				5_1					10_2			

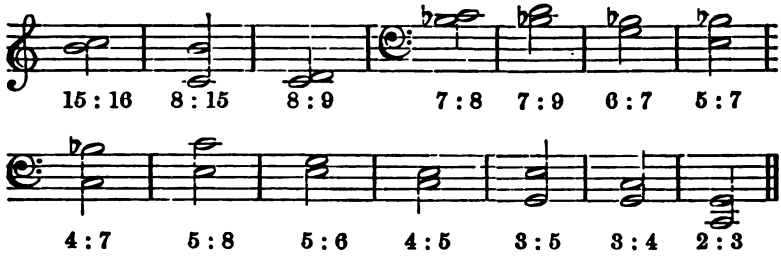
C c	1_1	2_2	3_3	4_4	5_5	6_6	7_7	8_8	9_9	10_{10}		
$\frac{2}{1}$ Octave		2_1		4_2		6_3		8_4		10_5		
C G	2_1		4_2		6_3		8_4		10_5		12_6	14_7
$\frac{3}{2}$ Fifth		3_1			6_2			9_3			12_4	
C e	2_1		4_2		6_3		8_4		10_5		12_6	14_7
$\frac{5}{2}$ Major Tenth				5_1					10_2			

The important influence exercised on the harmoniousness of the consonances, especially the less perfect ones, by the rapidity of the weak beats of the dissonant upper partials, has already been indicated. If we place all the intervals above the same bass note,

C g	1	2	3	4	5	6	7	8	9	10
$\frac{3}{1}$ Twelfth			3_1			6_2			9_3	
C F	3_1	6_2	9_3	12_4	15_5	18_6	21_7	24_8	27_9	30_{10}
$\frac{4}{3}$ Fourth	4_1	8_2		12_3		16_4	20_5	24_6		28_7
C A	3_1	6_2	9_3	12_4	15_5	18_6	21_7	24_8	27_9	30_{10}
$\frac{5}{3}$ Major Sixth	5_1		10_2		15_3		20_4		25_5	30_6
C E	4_1	8_2	12_3	16_4	20_5	24_6	28_7	32_8	36_9	40_{10}
$\frac{5}{4}$ Major Third	5_1	10_2	15_3		20_4		25_5	30_6	35_7	40_8
C E♭	5_1	10_2	15_3	20_4	25_5	30_6	35_7	40_8	45_9	50_{10}
$\frac{6}{5}$ Minor Third	6_1	12_2	18_3	24_4		30_5		36_6	42_7	48_8
C A♭	5_1	10_2	15_3	20_4	25_5	30_6	35_7	40_8	45_9	50_{10}
$\frac{8}{5}$ Minor Sixth	8_1		16_2	24_3		32_4		40_5		48_6
C e♭	5_1	10_2	15_3	20_4	25_5	30_6	35_7	40_8	45_9	50_{10}
$\frac{12}{5}$ Minor Tenth		12_1		24_2				36_3		48_4
C f	3_1	6_2	9_3	12_4	15_5	18_6	21_7	24_8	27_9	30_{10}
$\frac{8}{3}$ Eleventh		8_1				16_2		24_3		
C a	3_1	6_2	9_3	12_4	15_5	18_6	21_7	24_8	27_9	30_{10}
$\frac{10}{3}$ Ma. Thirteenth			10_1				20_2			30_3
C a♭	5_1	10_2	15_3	20_4	25_5	30_6	35_7	40_8	45_9	50_{10}
$\frac{16}{5}$ Mi. Thirteenth			16_1				32_2			48_3

The last four are so rough that they are seldom reckoned as consonances. The above order was determined merely by frequently sounding the intervals in just intonation on my bass and treble concertinas, and relates solely to the effect on my own ear. The greater richness of the major Tenth over the Twelfth made me prefer the former. The effect is very much like that of a compound tone, in which the prime is inaudible; even the tone 3 is supplied partly by a combinational tone. Hence when a man's voice accompanies a woman's at a Third below (that is really a Tenth) the effect is more agreeable than when another woman sings the real Third below, as long as the Thirds are major; the contrary is the case when the Thirds are minor. In ordinary rules for harmony no distinction is made between Tenths and Thirds, Fourths and Elevenths, &c. The above table shews that the differences are of extreme importance. As will be seen hereafter, the minor Tenth, the Eleventh, and both Thirteenths ought to be avoided or else treated as dissonances.—*Translator.*]

the number of their beats in a second varies much, and is much greater for the imperfect than for the perfect consonances. But we can give all the intervals hitherto considered such a position in the scale that the number of their beats in a second should be the same. Since we have found that 33 beats in a second produce about the maximum amount of roughness, I have so chosen the position of the intervals in the following examples in musical notation, as to give that number in every case. The intonation is supposed to be that of the scale of *C major* with just intervals, but *b♭* represents the sub-minor Seventh of *c* (4 : 7). See p. 284.



The prime tones of the notes in this example are all partials of $C_{,}$, which makes 33 vibrations in a second, and hence their own vibrational numbers and those of their upper partials are multiples of 33; consequently the difference of these vibrational numbers, which gives the number of beats, must always be 33, 66, or some higher multiple of 33.[1]

In the low positions here assigned the beats arising from the dissonant upper partials are as effective as their intensity will allow, and in this case the Sixths, Thirds, and even the Fourth are considerably rough. But the major Sixth and major Third shew their superiority over the minor Third and minor Sixth, by descending lower down in the scale, and yet sounding somewhat milder than the others.[2] It is also a well-known practical rule among musicians to avoid these close intervals in low positions, when soft chords are required, though there was no justification for this rule in any previous theory of chords.

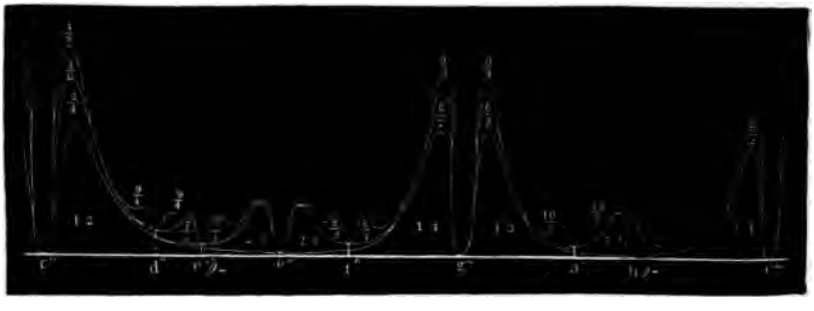
[1] [If we took the notes as simple tones, only those would beat sensibly which are less than a minor Third apart, that is 15 : 16, 9 : 8, 7 : 8, 6 : 7. The rest beat 33 times by upper partials which are at a less distance apart.—*Translator.*]

[2] [Although the number of the beats is the same, they are due to higher upper partials in the two major intervals, and hence are not so loud.—*Translator.*]

My theory of hearing by means of the sympathetic vibration of elastic appendages to the nerves, would allow of calculating the intensity of the beats of the different intervals, when the intensity of the upper partials in the corresponding quality of tone belonging to the instrument used, is known, and the intervals are so chosen that the number of beats in a second is the same. But such a calculation would be very different for different qualities of tone, and holds only for such a particular case as may be assumed.

FIG. 60 A.

FIG. 60 B.



For intervals constructed on the same lower note a new factor comes into play, namely, the number of beats which occur in a second; and the influence of this factor on the roughness of the sensation cannot be calculated directly by any fixed law. But to obtain a general graphical representation of the complicated relations which co-operate to produce the effect, I have made such a calculation, knowing that diagrams teach more at a glance than the most complicated descriptions, and have hence constructed

figs. 60, A and B (p. 292). In order to construct them I have been forced to assume a somewhat arbitrary law for the dependence of roughness upon the number of beats. I chose for this purpose the simplest mathematical formula which would shew that the roughness vanishes when there are no beats, increases to a maximum for 33 beats, and then diminishes as the number of beats increases. Next I have selected the quality of tone on the violin in order to calculate the intensity and roughness of the beats due to the upper partials taken two and two together, and from the final results I have constructed figs. 60, A and B (p. 292). The base lines $c'c''$, $c''c'''$ denote those parts of the musical scale which lie between the notes thus named, but the pitch is taken to increase continuously [as when the finger slides down the violin string], and not by separate steps [as when the finger stops off definite lengths of the violin string]. It is further assumed that the notes or compound tones belonging to any individual part of the scale, are sounded together with the note c', which forms the constant lower note of all the intervals. Fig. 60 A, therefore, shews the roughness of all intervals which are less than an Octave, and fig. 60 B of those which are greater than one and less than two Octaves. Above the base line there are prominences marked with the ordinal numbers of the partials. The height of these prominences at every point of their width is made proportional to the roughness produced by the partial tones denoted by the numbers, when a note of corresponding pitch is sounded at the same time with the note c'. The roughnesses produced by the different pairs of upper partials are erected one over the other.[1] It will be seen that the various roughnesses arising from the different intervals encroach on each other's regions, and that only a few narrow valleys remain, corresponding to the position of the best consonances, in which the roughness of the chord is comparatively small. The deepest valleys in the first Octave c' c'' belong to the Octave c' and the Fifth g'; then comes the Fourth f', the major Sixth a', and the major Third e', in the order already found for these intervals. The minor Third $e'\flat$, and the minor Sixth $a'\flat$, have 'cols' rather than valleys, the bottoms of their depressions lie so high, corresponding to the greater roughness of these intervals. They

[1] [The method in which these diagrams were calculated is shewn at the end of Appendix XV.—*Translator.*]

are almost the same as for the intervals involving 7, as 4 : 7, 5 : 7, 6 : 7.[1]

In the second Octave as a general rule all those intervals of the first Octave are improved, in which the smaller of the two numbers expressing the ratio was even; thus the Twelfth 1 : 3, major Tenth 2 : 5, subminor Fourteenth 2 : 7, and subminor Tenth 3 : 7,[2] are smoother than the Fifth 2 : 3, major Third 4 : 5, subminor Seventh 4 : 7, and subminor Third 6 : 7. The other intervals are relatively deteriorated. The Eleventh or increased Fourth is distinctly worse than the major Tenth; the major Thirteenth, or increased major Sixth, is similarly worse than the subminor Fourteenth. The minor Third, when increased to a minor Tenth,[2] and the minor Sixth, when increased to a minor Thirteenth, fare still worse, on account of the increased disturbance of the adjacent intervals. The conclusions here drawn from calculation are easily confirmed by experiments on justly intoned instruments. That they are also attended to in the practice of musical composition, notwithstanding the theoretical assumption that the nature of a chord is not changed by altering the pitch of any one of its constituents by whole Octaves, we shall see further on, when considering chords and their inversions.

It has already been mentioned that peculiarities of individual qualities of tone may have considerable effect in altering the order of the relative harmoniousness of the intervals. The quality of tone in the musical instruments now in use has been of course selected and altered with a view to their employment in harmonic combinations. The preceding investigation of the qualities of tone in our principal musical instruments has shewn that in what are considered good qualities of tone the Octave and Twelfth of the prime, that is the 2nd and 3rd partial, are powerful, the 4th and 5th partial have only moderate strength, and the higher partials rapidly diminish in force. Assuming such a quality of tone, the results of this chapter may be summed up as follows.

When two musical tones are sounded at the same time, their

[1] [The interval 4 : 7 is over *b'*b — ; the interval 5 : 7 is the 'col' between *f* and *g*, and the interval 6 : 7 is the next 'col' to the left of *e'*b.—*Translator.*]

[2] [In altering the German names of notes, in the above diagrams, some slight mistakes in the original lettering have been corrected. By carrying a line down from *e'*b in fig. 60 A, it will be seen that *e''*b belongs to the little depression to the right of the fraction $\frac{7}{3}$ between *e''*b– and *e''*. The slight depression for *a''*b is just under the fraction $\frac{10}{3}$ to the left of *a''*.—*Translator.*]

united sound is generally disturbed by the beats of the upper partials, so that a greater or less part of the whole mass of sound is broken up into pulses of tone, and the joint effect is rough. This relation is called *Dissonance*.

But there are certain determinate ratios between vibrational numbers, for which this rule suffers an exception, and either no beats at all are formed, or at least only such as have so little intensity that they produce no unpleasant disturbance of the united sound. These exceptional cases are called *Consonances*.

1. The most perfect *consonances* are those that have been here called *absolute*, in which the prime tone of one of the combined notes coincides with some partial tone of the other. To this group belong the *Octave*, *Twelfth*, and *double Octave*.

2. Next follow the *Fifth* and the *Fourth*, which may be called *perfect consonances*, because they may be used in all parts of the scale without any important disturbance of harmoniousness. The Fourth is the less perfect consonance and approaches those of the next group. It owes its superiority in musical practice simply to its being the defect of a Fifth from an Octave, a circumstance to which we shall return in a later chapter.

3. The next group consists of the *major Sixth* and the *major Third*, which may be called *medial consonances*. The old writers on harmony considered them as imperfect consonances. In lower parts of the scale the disturbance of the harmoniousness is very sensible, but in the higher positions it disappears, because the beats are too rapid to be sensible. But each, in good musical qualities of tone has an independent character, because any little defect in its intonation produces sensible beats of the upper partials, and consequently each interval is clearly separated from all adjacent intervals.

4. The *imperfect consonances*, consisting of the *minor Third* and *minor Sixth*, have not in general an independent character, because in good musical qualities of tone the partials on which their definition depends are often not found for the minor Third, and are generally absent for the minor Sixth, so that small imperfections in the intonation of these intervals do not necessarily produce beats.[1] They are all less suited for use in lower parts of the scale

[1] [It must be recollected that in the minor Sixth the 2nd and 3rd partials are within the Semitone 15 : 16, and the 3rd and 5th within the Semitone 24 : 25 (see note p. 290), and that the resulting beats, which in good qualities of tone are never absent,

than the others, and they owe their precedence as consonances over many other intervals which lie on the boundaries of consonance and dissonance, essentially to their being indispensable in the formation of chords because they are defects of the major Sixth and major Third from the Octave or Fifth. The subminor Seventh 4 : 7 is very often more harmonious than the minor Sixth 5 : 8, in fact, it is always so when the third partial tone of the note is strong compared with the second, because then the Fifth has a more powerfully disturbing effect on the intervals distant from it by a Semitone, than the Octave on the subminor Seventh, which is rather more than a whole Tone removed from it.[1] But this subminor Seventh when combined with other consonances in chords produces intervals which are all worse than itself, as 6 : 7, 5 : 7, 7 : 8, &c., and it is consequently not used as a consonance in modern music.[2]

will always be more powerful than those which arise from small errors of intonation, even in qualities of tone in which an 8th partial is well developed.—*Translator.*]

[1] [Reverting to the diagrams before given (p. 290, note), we may compare the effect of these intervals thus:

C A♭	5_1	10_2	15_3	20_4	25_5	30_6	35_7	40_8	45_9	50_{10}
$\frac{8}{5}$ Minor Sixth	8_1		16_2	24_3		32_4		40_5	48_6	
C B♭ —	4_1	8_2	12_3	16_4	20_5	24_6	28_7	32_8	36_9	40_{10}
$\frac{7}{4}$ Subminor Seventh	7_1		14_2		21_3		28_4	35_5		

Hence for the minor Sixth the chief beats arise from the interval 15 : 16, or the 3rd partial of the lower and 2nd of the upper note, that is, from those tones which would coincide for the Fifth. But in the subminor Seventh the chief disturbance is from 7 : 8, or the prime of the upper and 2nd partial of the lower note, which would coincide for the Octave. The beats from the interval 12 : 14 or 6 : 7 are hardly perceptible, but this is the interval which replaces the 15 : 16 in the minor Sixth, being due to those upper partials which would have coincided for the Fifth.—*Translator.*]

[2] [In fig. 60 A (p. 292), the bottom of the valley of 4 : 7 above *b*′—, is just a little lower than that of 5 : 7, between *f*′ and *g*′, and that of 6 : 7, which with that of 7 : 8, lie between *c*′ and *e*′♭. If we take the diagrams for these intervals we have:

C E♭ —	6_1	12_2	18_3	24_4	30_5	36_6	42_7	48_8	54_9	60_{10}
$\frac{7}{6}$ Subminor Third	7_1	14_2	21_3	28_4	35_5		42_6	49_7	56_8	
C G♭ —	5_1	10_2	15_3	20_4	25_5	30_6	35_7	40_8	45_9	50_{10}
$\frac{7}{5}$ Subminor Fifth	7_1	14_2	21_3		28_4		35_5	42_6	49_7	
C D+	7_1	14_2	21_3	28_4	35_5	42_6	49_7	56_8	63_9	70_{10}
$\frac{8}{7}$ Super-Second	8_1	16_2	24_3	32_4	40_5	48_6		56_7	64_8	

5. By increasing the interval by an octave, the Fifth and major Third are improved on becoming the Twelfth and major Tenth. But the Fourth and major Sixth become worse as the Eleventh and major Thirteenth. The minor Third and minor Sixth, however, become still worse as the minor Tenth and minor Thirteenth, so that the latter intervals are far less harmonious than the subminor Fourteenth 2 : 7, and subminor Tenth 3 : 7.

The order of the consonances here proposed is based upon a consideration of the harmoniousness of each individual interval independently of any connection with other intervals, and consequently without any regard to key, scale, and modulation. Almost all writers on musical theory have proposed similar orders for the consonances, agreeing in their general features with each other and with that here deduced from the theory of beats. Thus all put the Unison and Octave first, as the most perfect of all consonances; and next in order comes the Fifth, after which the Fourth is placed by those, who do not include the modulational properties of the Fourth, but restrict their observation to the independent harmoniousness of the interval. There is great diversity in the arrangement of the Sixths and Thirds. The Greeks and Romans did not acknowledge these intervals to be consonances at all, perhaps because in the unaccented Octave, within which their music, arranged for men's voices, usually lay, these intervals really sound badly, and perhaps because their ear was too sensitive to endure the trifling increase of roughness generated by compound tones when sounded together in Thirds and Sixths. In the present century, the Archbishop Chrysanthus of Dyrrhachium declares that modern Greeks have no pleasure in polyphonic music, and consequently he disdains to enter upon it in his book on music, and refers those who are curious to know its rules, to the writings of

We see then that 6 : 7 is disturbed by a continual repetition of this same interval among its lower partials, and also by the interval 28 : 30 = 14 : 15 from the 4th and 5th partials, and by 35 : 36 from the 5th and 6th partials, and on looking at the diagram, fig. 60 A (p. 292), it will be seen that of these three sources the first is chief, but the other two are active. For the subminor Fifth 5 : 7 the great disturbance is from 14 : 15, or the 2nd and 3rd partial, but there is also an active one from 20 : 21 or the 4th and 3rd partial, and these are almost the only ones noted in fig. 60 A. In the Supersecond the continual repetition of the interval 7 : 8 produces the chief effect, but 40 : 42 = 20 : 21, from the 5th and 6th partial, also produces much effect, as shewn in the fig. 60 A.—*Translator.*]

the West.[1] And the Arabs are of the same opinion according to the accounts of all travellers.

This rule remained in force even during the first half of the middle ages, when the first attempts were made at harmonies for two voices. It was not till towards the end of the twelfth century that Franco of Cologne included the Thirds among the consonances. He distinguishes:

1. *Perfect Consonances:* Unison and Octave.
2. *Medial Consonances:* Fifth and Fourth.
3. *Imperfect Consonances:* Major and minor Thirds.
4. *Imperfect Dissonances:* Major and minor Sixth.
5. *Perfect Dissonances:* Minor Second, augmented Fourth, major and minor Seventh.[2]

It was not till the thirteenth and fourteenth centuries that musicians began to include the Sixths among the consonances. Philipp de Vitry and Jean de Muris[3] mention as perfect consonances the Unison, Octave, and Fifth; as imperfect, the Thirds and Sixths. The Fourth has been cut out. The first author opposes the major Third and major Sixth, as more perfect, to the minor Third and minor Sixth. The same order is found in the *Dodecachordon* of Glareanus, 1557, who merely added the intervals increased by an Octave. The reason why the Fourth was not admitted as either a perfect or an imperfect consonant, must be looked for in the rules for the progression of parts. Perfect consonances were not allowed to follow each other between the same parts, still less dissonances; but imperfect consonances, as the Thirds and Sixths, were permitted to do so. But on the other hand the perfect consonances, Octaves, and Fifths were admitted in chords on which the music paused, as in the closing chord. Here, however, the Fourth of the bass could not occur. Again a succession of Fourths for two voices were not admitted, as the Fourth and Fifth lay too close. Hence so far as the progression of parts was concerned, the Fourth shared the properties of dissonances, and it was at once placed among them; it would have been better to have placed it in an intermediate class between perfect and imperfect consonances.

[1] Θεωρητικὸν μέγα τῆς Μουσικῆς παρὰ Χρυσάνθου. Τεργέστη, 1832, cited by *Coussemaker*, Histoire de l'harmonie, p. 5.

[2] Gerbert, 'Scriptores ecclesiastici de Musica Sacra.' Saint-Blaise, 1784, vol. iii., p. 11.—Coussemaker, 'Histoire de l'harmonie,' Paris, 1852, p. 49.

[3] Coussemaker, ibid. p. 66 and p. 68.

As far as harmoniousness is concerned, there can be no doubt that for most qualities of tone, the Fourth is much superior to the major Third and major Sixth, and beyond all doubt better than the minor Third and minor Sixth. But the Eleventh, or Fourth increased by an Octave, sounds far from well when the third partial tone is in any degree strong.[1]

The dispute as to the consonance or dissonance of the Fourth has been continued to the present day. As late as 1840, in Dehn's treatise on harmony, we find it asserted that the Fourth must be treated and resolved as a dissonance; but Dehn certainly puts a totally different interpretation on the question in dispute by laying it down that the Fourth of any bass within its key and independently of the intervals with which it is combined, has to be treated as a dissonance.[2] Otherwise it has been the constant custom in modern music to allow the fundamental bass to occur as the Fourth of the dominant in conjunction with the dominant in final chords, and it was long so used in these chords, even before Thirds were allowed in them, and in this way it came to be recognised as one of the superior consonances.

[1] [For the eleventh, or 3 : 8, gives

3_1 6_2 8_1 9_3 12_4 15_5 16_2 18_6 21_7 24_8 and 24_8

and hence has the interval 8 : 9 between the prime of one tone and the 3rd partial of the other. If the 5th partial is tolerably strong, it has also 15 : 16 very sensibly.—*Translator.*]

[2] [In Mr. W. Chappell's 'History of Music,' already cited (p. 151 note), he distinguishes between the Fourth in general, and the Fourth of the tonic in the scale. 'We sacrifice too much for the sake of making one extra interval of a perfect Fourth from the key-note to its Fourth above, which Nature does not allow. Her perfect Fourths are from the Second and Fifth of the key upwards, as from D and G in the key of C.'—Vol. i. p. 238. That is *D* to *G*, and *G* to *c* are perfect Fourths, but *C* to *F* is not. The ratio of the vibrational numbers is the same in each case. Suppose, as in the key of *B*♭, *C* were the Second of the key, or suppose, as in the key of *F*, *C* were the Fifth of the key, then the identical notes, the identical interval *C* to *F*, which was dissonant in the key of *C*, becomes consonant. It is evident that Mr. Chappell is not arguing upon a physical basis, although he gives as his reason that *F* cannot be obtained as an harmonic on a string tuned to *C* (p. xiv.). But likewise *G* cannot be obtained on a string tuned to *D*. The same objection should apply to the major Sixth *A* in the key of *C*, but I do not find that he has raised it. On p. xiv. he also says: 'Strike C, C, F on the pianoforte. Now take away the lower C, and substitute F for the base. That is what other people [as distinct from Dr. Burney] call concord, and the first they term discord.' It is evident that the *Fourth* is thus left unchanged, but the *Eleventh* is altered into the *Octave*, which accounts for everything. The very sensible harshness of the Eleventh, which is here, as usually in musical language, not distinguished from the Fourth, may have acted strongly in the assignment of a dissonant character to the Fourth.—*Translator.*]

CHAPTER XI.

BEATS DUE TO COMBINATIONAL TONES.

WHEN two or more compound tones are sounded at the same time beats may arise from the combinational as well as from the harmonic upper partial tones. In Chapter VII. it was shewn that the loudest combinational tone resulting from two generating tones is that corresponding to the difference of their vibrational numbers, or the differential tone of the first order. It is this combinational tone, therefore, which is chiefly effective in producing beats. Even this loudest combinational tone is somewhat weak, unless the generators are very loud; the differential tones of higher orders, and the summational tones are still weaker. Beats due to such weak tones as those last-mentioned cannot be observed unless all other beats which would disturb the observer are absent, as in sounding two simple tones, which are entirely free from upper partials. On the other hand the beats of the first differential tones can be heard very well at the same time as those due to the harmonic upper partials of compound tones, by an ear accustomed to hear combinational tones.

The *differential tones of the first order* alone, and independently of the combinational tones of higher orders, are capable of causing beats, (1) when two compound tones sound together, (2) when three or more simple or compound tones sound together. On the other hand beats generated by *combinational tones of higher orders* have to be considered when two simple tones sound together.

We commence with the differential tones of compound tones. In the same way that the prime tones in such cases develop combinational tones, any pair of upper partials of the two compounds will also develop combinational tones, but such tones will diminish very rapidly in intensity as the upper partials become weaker. When one or more of these combinational tones nearly coincide

with other combinational tones or the primes or upper partials the generators, beats ensue. Let us take as an example a slight incorrectly tuned Fifth, having the vibrational numbers 200 a 301, in place of 200 and 300, as in a justly intoned Fifth. V calculate the vibrational numbers of the upper partials by mul plying those of the primes by 1, 2, 3, and so on. We find t vibrational numbers of the differential tones of the first order, subtracting these numbers from each other, two and two. T following table contains in the first horizontal line and vertic column the vibrational numbers of the several partials of the t compound tones, and in their intersections the differences of the numbers, which are the vibrational numbers of the differenti tones due to them.

		Partials of the Fifth			
		301	602	903	
Partials of the Lower Note	200	101	402	703	Combi-national Tones
	400	99	202	503	
	600	299	2	303	
	800	499	198	103	
	1,000	699	398	97	

If we arrange these tones by pitch we find the followi groups:—

2	97	198	299	398	600	699
	99	200	301	400	602	703
	101	202	303	402		
	103					

The number 2 is too small to correspond to a combination tone. It only shews the number of beats due to the two upp partials 600 and 602. In all the other groups, however, tones a found whose vibrational numbers differ by 2, 4 or 6, and hen produce respectively 2, 4, and 6 beats in the same time that t two first-named partials produce 2 beats. The two strongest co binational tones are 101 and 99, and these also are well disti guished from the rest by their low pitch.

We observe in this example that the slowest beats due to t combinational tones are the same in number as those due to t upper partials. This is a general rule and applies to all interva

Further it is easy to see that if in our example we replac

200 and 301, by the numbers 200 and 300 belonging to the perfect Fifth, all the numbers in our table would become multiples of 100, and hence all the different combinational and upper partial tones which now beat would become coincident and not generate any beats. What is here shewn to be the case in this example for the Fifth is also true for all other harmonic intervals.[1]

The first differential tones of compounds cannot generate beats, except when the upper partials of the same compounds generate them, and the rapidity of the beats is the same in both cases, supposing that the series of upper partials is complete. Hence the addition of combinational tones makes no essential difference in the results obtained in the last chapter on investigating the beats due to the upper partials only. There can be only a slight increase in the strength of the beats.

But the case is essentially different when two simple tones are sounded together, so that there are no upper partials to consider. If combinational tones were not taken into account, two simple tones, as those of tuning-forks or stopped organ pipes, could not produce beats unless they were very nearly of the same pitch. These beats are strong when their interval is a minor or major Second, but they are weak for a Third and only recognisable in the lower parts of the scale (p. 260), and they gradually diminish in distinctness as the interval increases, without shewing any special differences for the harmonic intervals themselves. For any larger interval between two simple tones there would be absolutely no beats at all, if there were no upper partial or combinational tones, and hence the consonant intervals discovered in the former chapter would be in no respect distinguished from adjacent intervals; there would in fact be no distinction at all between wide consonant intervals and absolutely dissonant intervals.

Now such wider intervals between simple tones are known to produce beats, although very much weaker than those hitherto considered, so that even for such tones there is a difference between consonances and dissonances, although it is very much more imperfect than for compound tones. And these facts depend, as Scheibler shewed,[2] on the combinational tones of higher orders.

[1] This is proved mathematically in Appendix No. XVI.

[2] ['The physical and musical Tonometer, which makes evident to the eye, by means of the pendulum, the absolute vibrations of the tones, the principal kinds of combinational tones, and also proves the most precise exactness of equally tempered

It is only for the *Octave* that the first differential tone suffices. If the lower note makes 100 vibrations in a second, while the imperfect Octave makes 201, the first differential tone makes 201 − 100 = 101, and hence nearly coincides with the lower note of 100 vibrations, producing one beat for each 100 vibrations. There is no difficulty in hearing these beats, and hence it is easily possible to distinguish imperfect Octaves from perfect ones, even for simple tones, by the beats produced by the former.

For the *Fifth*, the first order of differential tones no longer suffices. Take an imperfect Fifth with the ratio 200 : 301; then the differential tone of the first order is 101, which is too far from either primary to generate beats. But it forms an imperfect Octave with the tone 200, and, as just seen, in such a case beats ensue. Here they are produced by the differential tone 99 arising from the tone 101 and the tone 200, and this tone 99 makes two beats in a a second with the tone 101. These beats then serve to distinguish the imperfect from the justly intoned Fifth even in the case of two simple tones. The number of these beats is also exactly as many as if they were beats due to the upper partial tones. But to observe these beats the two primary tones must be loud, and the ear must not be distracted by any extraneous noise. Under favourable circumstances, however, they are not difficult to hear.[1]

For an imperfect *Fourth*, having, say, the vibrational numbers 300 and 401, the first differential tone is 101; this with the tone 300 produces the differential tone 199 of the second order, and this again with the tone 401 the differential tone 202 of the third order, and this makes 3 beats with the differential tone 199 of the second order, that is, precisely as many beats as would have been generated by the upper partial tones 1200 and 1203, if they had existed. These beats of the Fourth are very weak even when the primary tones are powerful. Perfect quiet and great attention are necessary for observing them.

The beats of an imperfect *major Third* are scarcely recognis-

and mathematical chords, invented and executed by Heinrich Scheibler, silk manufacturer in Crefeld.' ('Der physikalische und musikalische Tonmesser,' &c.) Essen, G. D. Bädeker, 1834, pp. viii. 80, 5 lithographed Tables (called 3 on title page), and an engraving of tuning-forks and waves. A most remarkable pamphlet. This Tonometer (consisting of 65 large tuning-forks, each exactly 4 vibrations sharper than the preceding, from $c' = 264$, to $c'' = 512$, with two additional forks for f and a', being 67 in all) is now made by König in Paris, price 2,000 francs (80*l*.). *Translator.*]

[1] [Scheibler, ibid. p. 21.—*Translator.*]

able, even under the most favourable conditions. If we take as the vibrational numbers of the primary tones 400 and 501, we have:

501 − 400 = 101,	the differential	tone	of the	first order
400 − 101 = 299,	„	„	„	second „
501 − 299 = 202,	„	„	„	third „
400 − 202 = 198,	„	„	„	fourth „

The tones 202 and 198 produce 4 beats. Scheibler succeeded in counting these beats of the imperfect major Third. I have myself believed that I heard them under favourable circumstances. But in any case they are so difficult to perceive that they are not of any importance in distinguishing consonance from dissonance.[1]

Hence it follows that two simple tones making various intervals adjacent to the major Third and sounded together will produce a uniform uninterrupted mass of sound, without any break in their harmoniousness, provided that they do not approach a Second too closely on the one hand or a Fourth on the other. My own experiments with stopped organ pipes justify me in asserting that however much this conclusion is opposed to musical dogmas, it is borne out by the fact, provided that really simple tones are used for the purpose. It is the same with intervals near to the major Sixth; these also shew no difference as long as they remain sufficiently far from the Fifth and Octave. Hence although it is not difficult to tune perfect major and minor Thirds on the harmonium or reed pipes or on the violin, by sounding the two tones together and trying to get rid of the beats, it is perfectly impossible to do so on stopped organ pipes or tuning-forks without the aid of other intervals. It will appear hereafter that the use of more than two tones will allow of these intervals being perfectly tuned even for simple tones.

Intermediate between the compound tones possessing many powerful upper partials, such as those of reed pipes and violins, and the entirely simple tones of tuning-forks and stopped organ pipes, lie those compound tones in which only the lowest of the upper partials are audible, such as those of wide open organ pipes or the human voice when singing some of the obscurer vowels, as *oo* in *too*. For these the partials would not suffice to distinguish all the consonant intervals, but the addition of the first differential tones renders it possible.

[1] [Scheibler, ibid. p. 25.—*Translator.*]

A. *Compound Tones consisting of the prime and its Octave.* These cannot limit Fifths and Fourths by beats of the partials, but are able to do so by those of the first differential tones.

a. *Fifth.* Let the vibrational numbers of the prime tones be 200 and 301, which are accompanied by their Octaves 400 and 602; all four tones are then too far apart to beat. But the differential tones

$$\begin{array}{rr} 301-200= & 101 \\ 400-301= & \underline{\ \ 99} \\ \text{Difference} & 2 \end{array}$$

give two beats. The number of these beats again is precisely the same as if they had been produced by the two next upper partials. Namely

$$2 \times 301 - 3 \times 200 = 2.$$

b. *Fourth.* Let the vibrational numbers of the primes be 300 and 401, and of the first upper partials 600 and 802; these cannot produce any beats. But the first differential tones give 3 beats, thus:—

$$\begin{array}{rr} 600-401= & 199 \\ 802-600= & \underline{202} \\ \text{Difference} & 3 \end{array}$$

For Thirds it would be necessary to take differential tones of the second order into account.

B. *Compound Tones consisting of the prime and Twelfth.* Such tones are produced by the narrow stopped pipes on the organ (*Quintaten*, p. 50, note). These are related in the same way as those which have only the Octave.

a. *Fifth.* Primes 200 and 301, upper partials 600 and 903. First differential tone

$$\begin{array}{rr} 903-600= & 303 \\ \text{Fifth}= & \underline{301} \\ \text{Number of beats} & 2 \end{array}$$

b. *Fourth.* Primes 300 and 401, upper partials 900 and 1203. First differential tone

$$\begin{array}{rr} 1203-900= & 303 \\ \text{Lower prime}= & \underline{300} \\ \text{Number of beats} & 3 \end{array}$$

Beats of the Third cannot be heard except by the help of the weak second differential tones.

C. *Compound Tones having both Octave and Twelfth as audible partials.* Such tones are produced by the wide (wooden) open pipes of the organ (*Principal*, p. 141, note). The beats of the upper partials here suffice to limit the Fifths, but not the Fourths. The Thirds can now be distinguished by means of the first differential tones.

a. *Major Third.* Primes 400 and 501, with the Octaves 800 and 1002, and Twelfths 1200 and 1503. First differential tones

$$\begin{aligned} 1002-800&=202 \\ 1200-1002&=188 \\ \hline \text{Number of beats}\quad & 4 \end{aligned}$$

b. *Minor Third.* Primes 500 and 601, Octaves 1000 and 1202, Twelfths 1500 and 1803. Differential tones

$$\begin{aligned} 1500-1202&=298 \\ 1803-1500&=303 \\ \hline \text{Number of beats}\quad & 5 \end{aligned}$$

c. *Major Sixth.* Primes 300 and 501, Octaves 600 and 1002, Twelfths 900 and 1503. Differential tones

$$\begin{aligned} 600-501&=\ \ 99 \\ 1002-900&=102 \\ \hline \text{Number of beats}\quad & 3 \end{aligned}$$

In fact not only the beats of imperfect Fifths and Fourths, but also those of imperfect major and minor Thirds are easily heard on open organ pipes, and can be immediately used for the purposes of tuning.

Thus, where upper partials, owing to the quality of tone, do not suffice, the combinational tones step in to make every imperfection in the consonant intervals of the Octave, Fifth, Fourth, major Sixth, major and minor Third immediately sensible by means of beats and roughness in the combined sound, and thus to distinguish these intervals from all those adjacent to them. It is only perfectly simple tones that so far make default in determining the

Thirds; and for them also the beats which disturb the harmoniousness of imperfect Fifths and Fourths, are relatively too weak to affect the ear sensibly, because they depend on differential tones of higher orders. In reality, as I have already mentioned, two stopped pipes, giving tones which lie between a major and a minor Third apart, will give just as good a consonance as if the interval were exactly either a major or a minor Third. This does not mean that a practised musical ear would not find such an interval strange and unusual, and hence would perhaps call it false, but the immediate impression on the ear, the simple perception of harmoniousness, considered independently of any musical habits, is in no respect worse than for one of the perfect intervals.

Matters are very different when more than two simple tones are sounded together. We have seen that Octaves are precisely limited even for simple tones by the beats of the first differential tone with the lower primary. Now suppose that an Octave has been tuned perfectly, and that then a third tone is interposed to act as a Fifth; then if the Fifth is not perfect, beats will ensue from the first differential tone.

Let the tones forming the perfect Octave have the vibrational numbers 200 and 400, and let that of the imperfect Fifth be 301. The differential tones are

$$\begin{aligned} 400-301 &= 99 \\ 301-200 &= 101 \\ \hline \text{Number of beats} & \quad 2 \end{aligned}$$

These beats of the Fifth which lies between two Octaves are much more audible than those of the Fifth alone without its Octave. The latter depend on the weak differential tones of the second order, the former on those of the first order. Hence Scheibler some time ago laid down the rule for tuning tuning-forks, first to tune two of them as a perfect Octave, and then to sound them both at once with the Fifth, in order to tune the latter. If Fifth and Octave are both perfect, they also give together the perfect Fourth.

Similarly, when two simple tones have been tuned to be a perfect Fifth, and we interpose a new tone between them to act as a major Third. Let the perfect Fifth have the vibrational numbers

400 and 600. On intercalating the impure major Third with the vibrational number 501 in lieu of 500, the differential tones are

$$\begin{array}{rr} 600-501= & 99 \\ 501-400= & \underline{101} \\ \text{Number of beats} & 2 \end{array}$$

The major Sixth is determined by combining it with the Fourth. Let 300 and 400 be the vibrational numbers of a perfect Fourth, and 501 that of an imperfect major Sixth. The differential tones are

$$\begin{array}{rr} 501-400= & 101 \\ 400-300= & \underline{100} \\ \text{Number of beats} & 1 \end{array}$$

If we tried to intercalate an interval between the tones forming a perfect Third, and having the vibrational numbers 300 and 400, it could only be the subminor Third with the vibrational number 350. Taking it imperfect and $=351$, we have the differential tones

$$\begin{array}{rr} 400-351= & 49 \\ 351-300= & \underline{51} \\ \text{Number of beats} & 2 \end{array}$$

These intervals 8 : 7 and 7 : 6 are, however, too close to be consonances, and hence they can only be used in weak discords (chord of the dominant Seventh.)[1]

Collecting the results of our investigations upon beats, we find that when two or more simple tones are sounded at the same time, they cannot go on sounding without mutual disturbance, unless they form with each other certain perfectly definite intervals. When these intervals exist, and there is no disturbance at all, the result is called a *consonance*. When these intervals do not exist, beats arise, that is, the whole compound tones, or individual partial and combinational tones contained in them or resulting from them, alternately reinforce and enfeeble each other. The tones then do

[1] [In actual practice, for the chord of the dominant Seventh the interval is 4 : 7½ the interval of the just sub-minor Seventh 4 : 7 not being used, even in just intonation.—*Translator.*]

not coexist undisturbed in the ear. They mutually check each other's uniform flow. This process is called *dissonance.*

Combinational tones are the most general cause of beats. They are the sole cause of beats for simple tones which lie as much as, or more than, a minor Third apart. For two simple tones they suffice to determine the limit of the Fifth, perhaps of the Fourth, but certainly not of the Thirds and Sixths. These, however, will have their limits strictly determined when the major Third is added to the Fifth to form the common major chord, and when the Sixth is united with the Fourth to form the chord of the Fourth and Sixth.

Thirds, however, are strictly determined in a chord of two compound tones, each consisting of a prime and the two next partial tones, by means of the beats of imperfect intervals. The beats of such intervals increase in strength and distinctness, with the increase in number and strength of the upper partial tones in the compounds. By this means the difference between dissonance and consonance, and of perfectly from imperfectly tuned intervals, becomes continually more marked and distinct, increasing the certainty with which the hearer distinguishes the correct intervals, and adding much to the powerful and artistic effect of successions of chords. Finally when the high upper partials are relatively too strong (in piercing and braying qualities of tone) each separate tone will by itself generate intermittent sensations of tone, and any combination of two or more compounds of this description produces a sensible harshness, while at the same time the large number of partial and combinational tones renders it difficult for the hearer to follow a complicated arrangement of parts in a musical composition.

These relations are of the utmost importance for the use of different instruments in the different kinds of musical composition. The considerations which determine the selection of the proper instrument for an entire composition or for individual phrases in movements written for an orchestra, are very multifarious. First in rank stands mobility and power of tone; on which there is no need to dwell. The bowed instrument and piano are most mobile, and then flutes and oboes. To these are opposed the trumpets and trombones, which commence sluggishly, but surpass all instruments in power. Another essential consideration is expressiveness, which in general depends on the power of producing with certainty any

degree of rapid alterations in loudness at the pleasure of the player. In this respect also bowed instruments, and the human voice, are pre-eminent. Artificial reed instruments both wooden and brazen, cannot diminish their power much without stopping the action of the reeds. Flutes and organ pipes cannot alter their tone much without at the same time altering their pitch. On the pianoforte the strength with which a tone commences is determined by the player, but not its duration; so that the rhythm can be marked delicately, but real melodic expression is wanting. All these points in the use of the above instruments are easy to observe and have long been known and allowed for. The influence of quality proper was more difficult to define. Our investigations, however, on the composition of musical tones have given us a means of taking into account the principal differences in the effect of the simultaneous action of different instruments and of shewing how the problem is to be solved, although there is still a large field left for a searching investigation in detail.

Let us begin with the simple tones of *wide stopped organ pipes*. In themselves they are very soft and mild, dull in the low notes, and very harmonious in the upper. They are quite unsuited, however, for combinations of harmony as now felt. We have already explained that simple tones of this kind discriminate only the very small interval of a Second by strong beats. Imperfect Octaves, and the dissonant intervals in the neighbourhood of the Octave, (the Sevenths, and Ninths,) beat with the combinational tones, but these beats are weak in comparison with those due to upper partials. The beats of imperfect Fifths and Fourths are entirely inaudible except under the most favourable conditions. Hence in general the impression made on the ear by any dissonant interval except the Second differs very little from that made by consonances, and as a consequence the harmony loses its character and the hearer has no certainty in his perception of the difference of intervals. If polyphonic compositions containing the harshest and most venturesome dissonances are played upon wide stopped organ pipes, the whole is uniformly soft and harmonious, and for that very reason also indefinite, wearisome and weak, without character or energy. Every reader that has an opportunity is requested to try this experiment. There is no better proof of the important part which upper partial tones play in music, than the impression produced by music composed of simple tones, which we

have just described. Hence the wide stopped pipes of the organ are used only to give prominence to the extreme softness and harmoniousness of certain phrases in contradistinction to the harsher effect of other stops, or else in connection with other stops to strengthen their prime tones. Next to the wide organ pipes as regards quality of tone stand *flutes* and the *flute pipes* on organs (open pipes, blown gently). These have the Octave plainly in addition to the prime, and when blown more strongly even produce the Twelfth. In this case the Octaves and Fifths are more distinctly limited by upper partial tones; but the definition of Thirds and Sixths has to depend upon combinational tones, and hence is much less distinct. The musical character of these pipes is therefore not much unlike that of the wide stopped pipes already described. This is well expressed by the old joke that nothing is more dreadful to a musical ear than a flute-concerto, except a concerto for two flutes.[1] But in connection with other instruments which give effect to the connection of the harmony, the flute, from the perfect softness of its tone and its great mobility, is extraordinarily pleasant and attractive, and cannot be replaced by any other instrument. In ancient music the flute played a much more important part than at present, and this seems to accord with the whole ideal of classical art, which aimed at keeping every thing unpleasant from its productions, confining itself to pure beauty, whereas modern art requires more abundant means of expression, and consequently to a certain extent admits into its circle what in itself would be contrary to the gratification of the senses. However this be, the earnest friends of music, even in classical times, contended for the harsher tones of stringed instruments in opposition to the effeminate flute.

The open organ pipes afford a favourable means of meeting the harmonic requirements of polyphonic music, and consequently form the *principal* stops.[2] They make the lower partials distinctly audible, the wide pipes up to the third, the narrow ones (*giegen principal*[3]) up to the sixth partial tone. The wider pipes

[1] [In the original, 'dass einem musikalischen Ohre nichts schrecklicher sei als ein Flötenconcert, ausgenommen ein Concert von zwei Flöten.' The pun on 'Concert,' first as a *concerto* or peculiar piece of music for one instrument, and secondly as a *concert*, or piece of music for several instruments, cannot be properly rendered in the translation.—*Translator.*]

[2] [See p. 141, note.—*Translator.*]

[3] [See p. 91, note.—*Translator.*]

have more power of tone than the narrower; to give them more brightness the 8-foot stops, which contains the 'principal work,' is connected with the 4-foot stops, which add the Octave to each note, or the *principal* is connected with the *giegen principal*, so that the first gives power and the second brightness. By this means qualities of tone are produced which contain the first six partial tones in moderate force, decreasing as the pitch ascends. These give a very distinct feeling for the purity of the consonant intervals, enabling us to distinguish clearly between consonance and dissonance, and preventing the unavoidable but weak dissonances resulting from the higher upper partials in the imperfect consonances from becoming too marked, but at the same time not allowing the hearer's appreciation of the progression of the parts to be disturbed by a multitude of loud accessory tones. In this respect the organ has an advantage over all other instruments, as the player is able to mix and alter the qualities of tone at pleasure, and make them suitable to the character of the piece he has to perform.

The narrow *stopped pipes* (*Quintaten*),[1] for which the prime tone is accompanied by the Twelfth, the *reed-flute* (*Rohrflöte*)[2] where the third and fifth partial are both present, the conical open pipes, as the *goat-horn* (*Gemshorn*),[3] which reinforce certain higher partials more than the lower, and so forth, serve only to give distinctive qualities of tone for particular parts, and thus to separate them from the rest. They are not well adapted for forming the chief mass of the harmony.

Very piercing qualities of tone are produced by the *reed pipes* and *compound stops*[4] on the organ. The latter, as already explained, are artificial imitations of the natural composition of all musical tones, each key bringing a series of pipes into action, which correspond to the first three or first six partial tones of the corresponding note. They can be used only to accompany congregational singing. When employed alone they produce insupportable noise and horrible confusion. But when the singing of the congregation gives overpowering force to the prime tones in the notes of the melody, the proper relation of quality of tone is restored, and the result is a powerful, well-proportioned mass of sound. Without the assistance of these compound stops it would

[1] [See p. 50, note.—*Translator.*]
[2] [See p. 143, note 3.—*Translator.*]
[3] [See p. 142, note 2.—*Translator.*]
[4] [See p. 91, note.—*Translator.*]

be impossible to control a vast body of sound produced by unpractised voices, such as we hear in churches.

The *human voice* is on the whole not unlike the organ in quality, so far as harmony is concerned. The brighter vowels, of course, generate individual high partial tones, but these are so unconnected from the rest that they can have no universal and essential effect on the sound of the chords. For this we must look to the lower partials which are tolerably uniform for all vowels. But of course in particular consonances the characteristic tone of the vowels may play an important part. If, for example, two human voices sing the major Third *b*♭ *d′* on the vowel *a* in *father*, the fourth partial of *b*♭ (or *b″*♭), and the third partial of *d′* (or *a″*), fall among the tones characteristically reinforced by A, and consequently the imperfection of the consonance of a major Third will come out harshly by the dissonance *a″ b″*♭, between these upper partials; whereas if the vowel be changed to *o* in *no*, the dissonance disappears. On the other hand the Fourth *b*♭ *e′*♭ sounds perfectly well on the vowel *a* in *father*, because the higher note *e′*♭ has the same upper partial *b″*♭ as the deeper *b*♭. But if *a* in *father* be inclined towards *a* in *fall*, or *a* in *fat*, the upper partials *f″* and *e″*♭ or else *d‴* and *e‴*♭, might interrupt the consonance. This serves to shew among other things, that the translation of the words of a song from one language into another is not by any means a matter of indifference for its musical effect.[1]

Disregarding then these reinforcements of partials due to the characteristic resonance of each vowel, the musical tones of the human voice are on the whole accompanied by the lower partials in moderate strength, and hence are well adapted for combinations of chords, precisely as for the *principal* stops of the organ. Besides this the human voice has a peculiar advantage over the organ and all other musical instruments in the execution of poly-

[1] [Also, it shews how the musical effect of different stanzas in a ballad, though sung to the same written notes, will vary in effect, quite independently of difference of expression. This is often remarkable on the closing cadence of the stanza. As the vowel changes from *a* in *father*, to *a* in *mat*; *e* in *met*, or *i* in *sit*, or again to *o* in *not*, *u* in *but*, and *u* in *put*, the musical result is totally different, though the pitch remains unaltered. To shew the effect of the different vowels throughout a piece of music, I asked a set of about 8 voices to sing, before about 200 others, the first half of *See the conquering hero comes*, first to *lah*, then to *lee*, and then to *loo*. The difference of effect was almost ludicrous. Much has to be studied in the relation of the qualities of vowels to the effect of the music. In this respect, too, the *pitch* chosen for the tonic will be found of great importance.—*Translator*.]

phonic music. The words which are sung connect the notes belonging to each part, and form a clue which readily guides the hearer to discover and pursue the related parts of the whole body of sound. Hence polyphonic music and the whole modern system of harmony were first developed for the human voice. Indeed, nothing can exceed the musical effect of well harmonised part music perfectly executed in just intonation by practised voices. For the complete harmoniousness of such music it is indispensably necessary that the several musical intervals should be justly intoned, and our present singers[1] unfortunately seldom learn to take just intervals, because they are accustomed from the first to sing to the accompaniment of instruments which are tuned in equal temperament, and hence with imperfect consonances. It is only such singers as have a delicate musical feeling of their own who find out the correct result, which is no longer taught them.

Richer in upper partials, and consequently brighter in tone than the human voice and the *principal* stops on the organ, are the *bowed instruments*, which consequently fill such an important place in music. Their extraordinary mobility and expressiveness give them the first place in instrumental music, and the moderate acuteness of their quality of tone assigns them an intermediate position between the softer flutes and the braying brass instruments. There is a slight difference between the different instruments of this class; the tenor and double-bass have a somewhat acuter and thinner quality than the violin and violoncello, that is, they have relatively stronger upper partials. The audible partials reach to the sixth or eighth, according as the bow is brought nearer the finger-board for piano, or nearer the bridge for forte, and they decrease regularly in force as they ascend in pitch.

[1] [This refers to Germany, not to the English Tonic Sol-faists, nor to the English madrigal singers. On Dec. 27, 1869, at a meeting of the Tonic Solfa College I had an unusual opportunity of contrasting the effect of just and tempered intonation in the singing of the same choir. It was a choir of about 60 mixed voices, which had gained the prize at the International Exhibition at Paris in 1867, and had been kept well together ever since. After singing some pieces without accompaniment, and hence in the just intonation to which the singers had been trained, and with the most delightful effect of harmony, they sang a piece with a pianoforte accompaniment. Of course the pianoforte itself was inaudible among the mass of sound produced by sixty voices. But it had the effect of perverting their intonation, and the whole charm of the singing was at once destroyed. There was nothing left but the everyday singing of an ordinary choir. The disillusion was complete and the effect most unsatisfactory as a conclusion.—*Translator.*]

Hence on bowed instruments the difference between consonance and dissonance is clearly and distinctly marked, and the feeling for the justness of the intervals very assured; indeed it is notorious that practised violin and violoncello players have a very delicate ear for distinguishing differences of pitch. On the other hand the piercing character of the tones is so marked, that soft song-like melodies are not well suited for bowed instruments, and are better given to flutes and clarinets in the orchestra. Full chords are also relatively too rough, since those upper partials which form dissonant intervals in every consonance, are sufficiently strong to make the dissonance obtrusive, especially for Thirds and Sixths. Moreover, where the player forces himself not to play the pure natural intervals which the ear requires, the imperfect Thirds and Sixths of the tempered musical scale are very perceptibly different in effect from the justly intoned Thirds and Sixths on bowed instruments. Hence in compositions for bowed instruments, slow and flowing progressions of chords are introduced by way of exception only, because they are not sufficiently harmonious; quick movements and figures, and arpeggio chords are preferred, for which these instruments are extremely well adapted, and in which the acute and piercing character of their combined sounds cannot be so distinctly perceived.

The beats have a peculiar character in the case of bowed instruments. Regular, slow, numerable beats seldom occur. This is owing to the minute irregularities in the action of the bow on the string, already described, to which is due the well-known scraping effect so often heard. Observations on the vibrational figure shewed that every little scrape of the bow causes the vibrational curve to jump suddenly backwards or forwards, or in physical terms, causes a sudden alteration in the phase of vibration. Now since it depends solely on the difference of phase whether two tones which are sounded at the same time mutually reinforce or enfeeble each other, every minutest catch or scrape of the bow will also affect the flow of the beats, and when two tones of the same pitch are played, every jump in the phase will suffice to produce a change in the loudness, just as if irregular beats were occurring at unexpected moments. Hence the best instruments and the best players are necessary to produce slow beats or a uniform flow of sustained consonant chords. Probably this is one of the reasons why quartets for bowed instruments, when extended

by players who can play solo pieces pleasantly enough, sometimes sound so intolerably rough and harsh that the effect bears no proper ratio to the slight roughness which each individual player produces on his own instrument.[1] When I was making observations on vibrational figures, I found it difficult to avoid the occurrence of one or two jumps in the figure every second. Now in solo-playing the tone of the string is thus interrupted for almost inappreciably minute instants, which the hearer scarcely perceives, but in a quartett when a chord is played for which all the notes have a common upper partial tone, there would be from four to eight sudden and irregular alterations of loudness in this common tone every second, and this could not pass unobserved. Hence for good combined performance, a much greater evenness of tone is required than for solo-playing.

The *pianoforte* takes the first place among stringed instruments for which the strings are struck. The previous analysis of its quality shews that its deeper octaves are rich, but its higher octaves relatively poor, in upper partial tones. In the lower octaves, the second or third partial tone is often as loud as the prime, nay, the second partial is often louder than the prime. The consequence is that the dissonances near the Octave (the Sevenths and Ninths) are almost as harsh as the Seconds, and that diminished and augmented Twelfths and Fifths are rather rough. The 4th, 5th, and 6th partial tones, on the other hand, on which the Thirds depend, decrease rapidly in loudness, so that the Thirds are relatively much less distinctly limited than the Octaves, Fifths, and Fourths. This last circumstance is important, because it makes the sharp Thirds of the equal temperament much more endurable upon the piano than upon other instruments with a more piercing quality of tone, whereas the Octaves, Fifths, and Fourths are quite distinctly and surely bounded. Notwithstanding the relatively large number of upper partial tones on the pianoforte, the impression produced by dissonances is far from being so penetrating as on instruments of long-sustained tones.

[1] [To myself, one of the principal reasons for the painful effect here alluded to, which is unfortunately so extremely well known, is the fact that the players not having been taught the nature of just intonation, do not accommodate the pitches of the notes properly. When quartett players are used to one another they overcome this difficulty. If their music could be previously marked in a way which will be suggested in Appendix XIX. much of this difficulty might be avoided from the first. —*Translator*.]

On the piano the note is powerful only at the moment when it is struck, and rapidly decreases in strength, so that the beats which characterise the dissonances have not time to become sensible; they do not even begin until the tone is greatly diminished in intensity. Hence in the modern music written for the pianoforte, since the time that Beethoven shewed how the characteristic peculiarities of the instrument were to be utilised in compositions, we find an accumulation and reduplication of dissonant intervals which would be perfectly insupportable on other instruments. The great difference becomes very evident when an attempt is made to play recent compositions for the piano on the harmonium or organ.

That instrument-makers, led solely by practised ears, and not by any theory, should have found it most advantageous to arrange the striking place of the hammer so that the seventh partial tone entirely disappears, and the sixth is weak although actually present, is manifestly connected with the structure of our system of musical tones. The fifth and sixth partial tones serve to define the minor Third, and in this way almost all the intervals treated as consonances in modern music are determined on the piano by coincident upper partials; the Octave, Fifth, and Fourth, by relatively loud tones; the major Sixth and major Third by weak ones; and the minor Third by the weakest of all. If the seventh partial tone were also present, the subminor[1] Seventh 4 : 7 would injure the harmoniousness of the minor Sixth, the Subfifth 5 : 7 that of the Fifth and Fourth, and the subminor Third 6 : 7 that of the minor Third, without any gain in the more accurate determination of new intervals suitable for musical purposes.

Mention has already been made of a further peculiarity in the selection of quality of tone on the pianoforte, namely that its upper notes have fewer and weaker upper partial tones than the lower. This difference is much more marked on the piano than on any other instrument, and the musical reason is easily assigned. The high notes are usually played in combination with much lower notes, and the relation between the two groups of notes is given by the high upper partials of the bass tones. When the interval between the bass and treble amounts to two or three Octaves, the second Octave, higher Third and Fifth of the bass note, are in the close neighbourhood of the treble, and form direct

[1] [For these terms see the Table at p. 184 and footnote 2 to p. 183.—Translator.]

consonances and dissonances with it, without any necessity for using the upper partials of the treble note. Hence the only effect of upper partials on the highest notes of the pianoforte would be to give them shrillness, without any gain in respect to musical definition. In actual practice the construction of the hammers on good instruments causes the notes of the highest Octaves to be only gently accompanied by their second partials. This makes them mild and pleasant, with a flutelike tone. Some instrument-makers, however, prefer to make these notes shrill and piercing, like the piccolo flute, by transferring the striking place to the very end of the highest strings. This contrivance succeeds in increasing the force of the upper partial tones, but gives a quality of tone to these strings which does not suit the character of the others, and hence certainly detracts from their charm.

In many other instruments, where their construction does not admit of such absolute control over the quality of tone as on the pianoforte, attempts have been made to produce similar varieties of quality in the high notes, by other means. In the bowed instruments this purpose is served by the resonance box, the proper tones of which lie within the deepest Octaves of the scale of the instrument. Since the partial tones of the resonant strings are reinforced in proportion to their proximity to the partial tones of the resonance box, this resonance will assist the prime tones of the higher notes as contrasted with their upper partials, much more than it will do so for the deep notes. On the contrary the deepest notes of the violin will have not only their prime tones, but also their Octaves and Fifths favoured by the resonance; for the deeper proper tone of the resonance box lies between the prime and second partial, and its higher proper tone between the second and third partials. A similar effect is attained in the *compound stops* of the organ, by making the series of upper partial tones, which are represented by distinct pipes, less extensive for the higher than for the lower notes in the stop. Thus each digital opens six pipes for the lower octaves, answering to the first six partial tones of its note; but in the two upper octaves, the digital opens only three or even two pipes, which give the Octave and Twelfth, or merely the Octave, in addition to the prime.

There is also a somewhat similar relation in the human voice, although it varies much for the different vowels. On comparing the higher and lower notes which are sung to the same vowel, it

will be found that the resonance of the cavity of the mouth generally reinforces relatively high upper partials of the deep notes of the bass, whereas for the soprano, where the note sung comes near to the characteristic pitch of the vowel, or even exceeds it, all the upper partials become much weaker. Hence in general, at least for the open vowels, the audible upper partials of the bass are much more numerous than those of the soprano.

We have still to consider the artificial reed instruments, that is the *wind instruments of wood and brass*. Among the former the clarinet, among the latter the French horn are distinguished for the softness of their tones, whereas the bassoon and hautbois in the first class, and the trombone and trumpet in the second represent the most penetrating qualities of tone used in music.

Notwithstanding that the keyed horns used for so-called concerted music have a far less braying quality of tone than trumpets proper, which have no side holes, yet the number and force of their upper partial tones is far too great for the harmonious effect of the less perfect consonances, and the chords on these instruments are very noisy and harsh, so that they are only endurable in the open air. In artistic orchestral music, therefore, trumpets and trombones, which on account of their penetrative power cannot be dispensed with, are seldom employed for harmonies, except for a few and if possible perfect consonances.

The *clarinet* is distinguished from all other orchestral wind instruments by having no evenly numbered partial tones. To this circumstance must be due many remarkable deviations in the effect of its chords from those of other instruments. When two clarinets are playing together all of its consonant intervals will be defined by combinational tones alone, except the major Sixth 3 : 5, and the Twelfth 1 : 3. But the differential tones of the first order, which are the strongest among all combinational tones, will always suffice to produce the beats of imperfect consonances. Hence it follows that in general the consonances of two clarinets have but little definition, and must be proportionately agreeable. This is really the case, except for the minor Sixth, and subminor Seventh, which are too near the major Sixth, and for the Eleventh and minor Thirteenth, which are too near the Twelfth. On the other hand, when a clarinet is played in combination with a violin or oboe, the majority of consonances will have a perceptibly different effect according as the clarinet takes the upper or the

lower note of the chord. Thus the major Third *d′ f′*♯ will sound better when the clarinet takes *d′* and the oboe *f′*♯, so that the fifth partial of the clarinet coincides with the fourth of the oboe. The 3rd and 4th and the 5th and 6th partials, which are so disturbing in the major Third,[1] cannot here be heard, because the 4th and 6th partials do not exist on the clarinet. But if the oboe takes *d′* and the clarinet *f′*♯, the coincident fourth partial will be absent, and the disturbing third and fifth present. For the same reason it follows that the Fourth and minor Third will sound better when the clarinet takes the upper tone. I have made experiments of this kind with the clarinet and a bright stop of the harmonium, which possessed the evenly numbered partial tones, and was tuned in just intonation and not in equal temperament. When *b*♭ was played on the clarinet, and *e′*♭, *d′*, *d′*♭, in succession on the harmonium, the major Third *b*♭ *d′* sounded better than the Fourth *b*♭ *e′*♭, and much better than the minor Third *b*♭ *d′*♭. If, retaining *b*♭ on the clarinet, I played *f*, *g*♭, *g* in succession on the harmonium, the major Third *g*♭ *b*♭ was rougher, not merely than the Fourth *f b*♭, but even than the minor Third *g b*♭.

This example, to which I was led by purely theoretical considerations that were immediately confirmed by experiment, will serve to shew how the use of exceptional qualities of tone will affect the order of agreeableness of the consonances which was settled for those usually heard.

Enough has been said to shew the readiness with which we can now account for numerous peculiarities in the effects of playing different musical instruments in combination. Further details are rendered impossible by the want of sufficient preliminary investigations, especially into the exact differences of various qualities of tone. But in any case it would lead us too far from our main purpose to pursue a subject which has rather a technical than a general interest.

[1] [See table on p. 290, note.—*Translator.*]

CHAPTER XII.

CHORDS.

WE have hitherto examined the effect of sounding together only two tones which form a determinate interval. It is now easy to discover what will happen when more than two tones are combined. The simultaneous production of more than two separate compound tones is called a *chord*. We will first examine the harmoniousness of chords in the same sense as we examined the harmoniousness of any two tones sounded together. That is, we shall in this section deal exclusively with the isolated effect of the chord in question, quite independently of any musical connection, mode, key, modulation, and so on. The first problem is to determine *under what conditions chords are consonant*, in which case they are termed *concords*. Concords of three tones are readily found by taking two consonant intervals to any one fundamental tone as *c*, and then seeing whether the new third interval between the two new tones, which is thus produced, is also consonant. If this is the case each one of the three tones forms a consonant interval with each one of the other two, and the chord is consonant, or is a concord.

Let us confine ourselves in the first place to intervals which are less than an Octave. The consonant intervals within these limits, we have found to be: 1) the Fifth *c g*, $\frac{3}{2}$; 2) the Fourth *c f*, $\frac{4}{3}$; 3) the major Sixth *c a*, $\frac{5}{3}$; 4) the major Third *c e*, $\frac{5}{4}$; 5) the minor Third *c e♭*, $\frac{6}{5}$; 6) the minor Sixth *c a♭*, $\frac{8}{5}$; to which we may add 7) the subminor or natural Seventh *c b♭*, $\frac{7}{4}$,[1] which approaches to the minor Sixth in harmoniousness. The following table (p. 322) gives a general view of the chords contained within an Octave. The chord is supposed to consist of the fundamental

[1] [The minus sign is supposed to shew that interval is less than that on the proper scale. The vibrational number of *b*b, the Fifth below *f* in the scale, is to that of this *b*b —, as 64 : 63.—*Translator*.]

tone *C*, some one tone of the first horizontal line, and some one tone of the first vertical column. Where the line and column corresponding to these two selected tones intersect, is the name of the interval which these two latter tones form with each other. This name is printed in *italics* when the interval is *consonant*, and in Roman letters when dissonant, so that the eye sees at a glance what *concords* are thus produced.

C	*G* $\frac{3}{2}$	*F* $\frac{4}{3}$	*A* $\frac{5}{3}$	*E* $\frac{5}{4}$	*E*♭ $\frac{6}{5}$	*A*♭ $\frac{8}{5}$
G $\frac{3}{2}$						
F $\frac{4}{3}$	major Second $\frac{9}{8}$					
A $\frac{5}{3}$	major Second $\frac{10}{9}$	*major Third* $\frac{5}{4}$				
E $\frac{5}{4}$	*minor Third* $\frac{6}{5}$	minor Second $\frac{16}{15}$	*Fourth* $\frac{4}{3}$			
E♭ $\frac{6}{5}$	*major Third* $\frac{5}{4}$	major Second $\frac{10}{9}$	superfluous Fourth $\frac{25}{16}$	minor Second $\frac{25}{24}$		
A♭ $\frac{8}{5}$	minor Second $\frac{16}{15}$	*minor Third* $\frac{6}{5}$	minor Second $\frac{25}{24}$	diminished Fourth $\frac{32}{25}$	*Fourth* $\frac{4}{3}$	
B♭ — $\frac{7}{4}$	subminor Third $\frac{7}{6}$	super Fourth $\frac{21}{16}$	subminor Second $\frac{21}{20}$	Subfifth $\frac{7}{5}$	super Fifth $\frac{35}{24}$	super major Second $\frac{35}{32}$

From this it follows that the only consonant *triads* or chords of three notes, that can possibly exist within the compass of an Octave are the following:—

1) *C E G* 2) *C E*♭ *G*
3) *C F A* 4) *C F A*♭
5) *C E*♭ *A*♭ 6) *C E A*.[1]

[1] [The reader ought to hear the whole set of triads that could be formed from the table, at least all exclusive of those formed by the last line. The ordinary tuning of the harmonium, organ, and piano does not permit this. In Chapter XVI. and

The two first of these triads are considered in musical theory as the fundamental triads from which all others are deduced. They may each be regarded as composed of two Thirds, one major and the other minor, superimposed in different orders. The chord *C E G*, in which the major Third is below, and the minor above, is a *major triad*. It is distinguished from all other major triads by having its tones in the closest position, that is, forming the smallest intervals with each other. It is hence considered as the *fundamental chord* or basis of all other major chords. The triad *C E♭ G*, which has the minor Third below, and the major above, is the *fundamental chord* of all *minor triads*.

The next two chords, *C F A* and *C F A♭*, are termed, from their composition, *chords of the Fourth and Sixth*, [*C* to *F* being a Fourth, and *C* to *A* a major, but *C* to *A♭* a minor Sixth]. If we take *G*, instead of *C* for the fundamental or bass tone, these chords of the Fourth and Sixth become *G*, *C E* and *G*, *C E♭*. Hence we may conceive them as having been formed from the fundamental major and minor triads *C E G* and *C E♭ G*, by transposing the Fifth *G* an Octave lower, when it becomes *G*,.

The two last chords, *C E♭ A♭* and *C E A*, are termed *chords of the Third and Sixth*, or simply *chords of the Sixth*, [*C* to *E* being a major, but *C* to *E♭* a minor third; and *C* to *A* a major, but *C* to *A♭* a minor Sixth]. If we take *E* as the bass note of the first, and *E♭* as that of the second, they become *E G c*, *E♭ G c*, respectively. Hence they may be considered as the transpositions or *inversions* of a fundamental major and a fundamental minor chord, *C E G*, *C E♭ G*, in which the bass note *C* is transposed an Octave higher and becomes *c*.

Collecting these inversions, the six consonant triads will assume

Appendix XVII. there will be found a description of an harmonium with just intonation. But for merely experimental purposes, a means of tuning an ordinary five-octave harmonium, and an English concertina (either of which can be obtained for a few pounds), so as to give all the theoretical forms of the major and minor chords in just intonation, will be added in App. XIX. Sect. G. 1, 2. All who wish to have an aural acquaintance with such chords (without which much of the text will be scarcely more than words) are strongly recommended to avail themselves of one or the other of these simple instruments. No instruments are better suited for distinguishing *harshly* between just and tempered intonation. Pianofortes and monochords are practically useless for such purposes. On the experimental harmonium these chords can be played as in the text. On the justly intoned concertina the contrast of major and minor cannot be attained, unless the chords are replaced by 1) *e′ g′♮ b′*, 2) *e′ g′ b′*; 3) *e′ a′ c′♮*, 4) *e′ a′ c″*; 5) *e′ g′ c″*, 6) *e′ g′♮ c″♮*.—*Translator*.]

the following form [the numbers shewing their correspondence with the forms on p. 322]:—

1)	*C*	*E*	*G*		2)	*C*	*E*♭ *G*
5)		*E*	*G*	*c*	6)		*E*♭ *G* *c*
3)			*G*	*c* *e*	4)		*G* *c* *e*♭

We must observe that although the natural or sub-minor seventh B♭ — forms a good consonance with the bass note *C*, a consonance which is indeed rather superior than inferior to the minor Sixth *C A*♭, yet it never forms part of any triad, because it would make worse consonances with all the other intervals consonant to *C* than it does with *C* itself. The best triads which it can produce are *C E B*♭ — and *C G B*♭ —. In the first of these the interval *E B*♭ — occurs between the Fourth and Fifth, in the latter the subminor Third *G B*♭ —.[1] On the other hand the minor Sixth makes a perfect Fourth with the minor Third, so that this minor Sixth is the worst interval in the chords of the Third and Sixth, and of the Fourth and Sixth, for which reason these triads can still be considered as consonant. This is the reason why the natural or subminor Seventh is never used as a consonance in harmony, whereas the minor Sixth can be employed, although, considered independently, it is not more harmonious than the subminor Seventh.

The triad *C E A*♭[2], to which we shall return, is very instructive for the theory of music. It must be considered as a dissonance, because it contains the diminished Fourth *E A*♭, having the vibrational ratio $\frac{32}{25}$. Now this diminished Fourth *E A*♭ is so nearly the same as a major Third *E G*♯♯, that on our keyed instruments, the organ and pianoforte, the two intervals are not distinguished. We have in fact

$$E\ A\flat = \tfrac{32}{25} = \tfrac{5}{4} \cdot \tfrac{128}{125}$$

or, approximatively

$$(E\ A\flat) = (E\ G\sharp\sharp) \cdot \tfrac{43}{42}$$

[1] [Although the experimental harmonium of Appendix XIX. does not contain any sub-minor Seventh, yet if in place of *C E B*♭ — and *C G B*♭ —, we play *D*♭ *F B*, and *D*♭ *A B*, as there intoned, we shall, as will appear hereafter, approximate very closely to the desired effect.—*Translator*.]

[2] [On the justly intoned concertina of Appendix XIX., this triad must be studied as *e′ g′*♯ *c″*, where *e′ g′*♯ is the major Third, and *g′*♯ *c″* the diminished Fourth.—*Translator*.]

On the pianoforte it would seem as if this triad, which for pr tical purposes may be written either *C E* A♭ or *C E* G♯, must consonant, since each one of its tones forms with each of the oth an interval which is considered as consonant on the piano, and this chord is one of the harshest dissonances, as all musicians agreed, and as any one can convince himself immediately. O justly intoned instrument the interval *E* A♭ is immediately cognised as dissonant.[1] This chord is well adapted for shew that the original meaning of the intervals asserts itself even w the imperfect tuning of the piano, and determines the judgm of the ear.

The harmonious effect of the various inversions of triads alre found depends in the first place upon the greater or less perfect of the consonance of the several intervals they contain. We h found that the Fourth is less agreeable than the Fifth, and t minor are less agreeable than major Thirds and Sixths. Now triad

C E G	has	a Fifth,	a major Third, and a minor Third
E G C	„	a Fourth,	a minor Third, and a minor Sixth
G C E	„	a Fourth,	a major Third, and a major Sixth
C E♭ *G*	„	a Fifth,	a minor Third, and a major Third
E♭ *G C*	„	a Fourth,	a major Third, and a major Sixth
G C E♭	„	a Fourth,	a minor Third, and a minor Sixth

For just intervals the Thirds and Sixths decidedly disturb general harmoniousness more than the Fourths, and hence major chords of the Fourth and Sixth are more harmonious t those in the fundamental position, and these again than the cho of the Third and Sixth. On the other hand the *minor* chords of Third and Sixth are more agreeable than those in the fundamer position, and these again are better than the minor chords of Fourth and Sixth. This conclusion will be found perfectly con for the middle parts of the scale, provided the intervals are justly intoned.[2] The chords must be struck separately, and

[1] [When the interval *g*′♯ *c*″ is played alone on the justly intoned concertin Appendix XIX., it will be found very harsh, but the harshness is much increase taking it an Octave lower as *g*♯ *c*′. On the other hand *e*′ *g*′♯ is perfectly agree but if *c*′ is added, as *c*′ *e*′ *g*′♯, forming a second major Third, the effect is very d nant.—*Translator*.]

[2] [On the experimental harmonium of Appendix XIX. all the inversions in

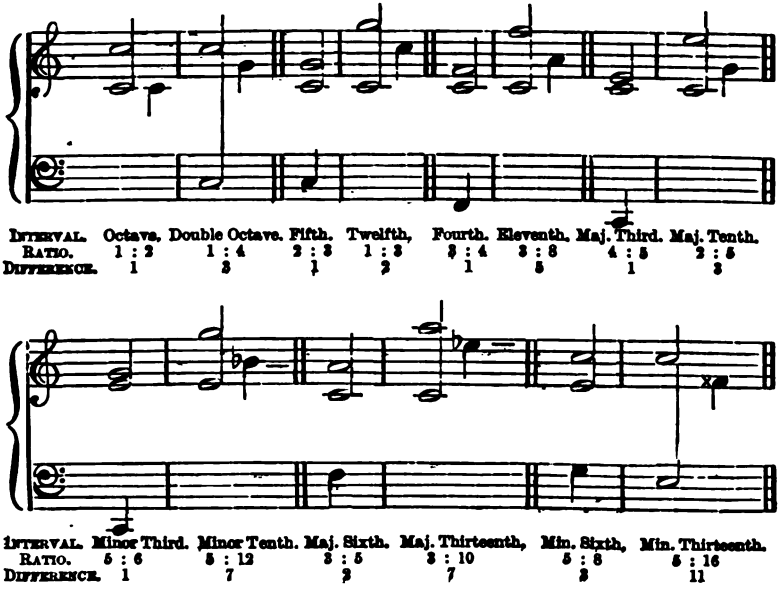
If the three intervals of the triad were consonant before, they will remain so after transposition. We have already seen how the chords of the Third and Sixth, and of the Fourth and Sixth were thus obtained from the fundamental form. It follows further that when larger intervals are admitted, no consonant triads can exist, which are not generated by the transposition of the major and minor triads. Of course if such other chords could exist, we should be able by transposition of their tones to bring them within the compass of an Octave, and we should thus obtain a new consonant triad within this compass, whereas our method of discovering consonant triads enabled us to determine every one that could lie within that compass. It is certainly true that slightly dissonant chords which lie within the compass of an Octave are sometimes rendered smoother by transposing one of their tones. Thus the chord $1 : \frac{7}{6} : \frac{7}{4}$, or C, $E\flat$ —, $B\flat$ —, is slightly dissonant in consequence of the interval $1 : \frac{7}{6}$; the interval $1 : \frac{7}{4}$, or sub-minor Seventh, does not sound worse than the minor Sixth; the interval $\frac{7}{6} : \frac{7}{4}$ is a perfect Fifth. Now transposing the tone $E\flat$ —, an Octave higher to $e\flat$ —, and thus transforming the chord into $1 : \frac{7}{4} : \frac{7}{3}$, we obtain $1 : \frac{7}{3}$ in place of $1 : \frac{7}{6}$, and this is much smoother, indeed it is better than the minor Tenth of our minor scale $1 : \frac{12}{5}$, and a chord thus composed, which I have had carefully tuned on the harmonium, although its unusual intervals produced a strange effect, is not rougher in sound than the worst minor chord, that of the Fourth and Sixth. This chord, C, $B\flat$ —, $e\flat$ —, is also much injured by the unsuitable combinational tones G, and F. Of course it would not be worth while to introduce such strange tones as $B\flat$ —, $e\flat$ —, into the scale for the sake of a chord which in itself is not superior to the worst of our present consonant chords, and for which the tones could not be transposed without greatly deteriorating its effect.

The transposition of tones in a consonant triad, for the purpose of widening their intervals, affects their harmoniousness in the first place by changing the intervals. Major Tenths, as we found in the last chapter, sound better than major Thirds, but minor Tenths worse than minor Thirds, the major and minor Thirteenth worse than the minor Sixth. The following rule embraces all the cases: *Those intervals in which the smaller of the two numbers expressing the ratios of the vibrational numbers is* EVEN, *are* IMPROVED *by having one of their tones transposed by an Octave,* because the numbers expressing the ratio are thus diminished.

The *Fifth* . .	2 : 3	becomes the	*Twelfth* . .	2 : 6	= 1 : 3
The *major Third* .	4 : 5	„	*major Tenth* .	4 : 10	= 2 : 5
The *sub-minor Third*	6 : 7	„	*sub-minor Tenth*	6 : 14	= 3 : 7.

Those intervals in which the smaller of the two numbers expressing the ratio of the vibrational numbers is ODD, *are* MADE WORSE *by having one of their tones transposed by an Octave*, as the Fourth 3 : 4 [which becomes the Eleventh 3 : 8], the minor Third 5 : 6 [which becomes the minor Tenth 5 : 12], and the Sixths 3 : 5 and 5 : 8 [which become the Thirteenths, major 3 : 10 and minor 5 : 16].

Besides this the principal combinational Tones are of essential importance. An example of the first combinational tones of the consonant intervals within the compass of an Octave is given below, the primary tones being represented by minims and the combinational tones by crotchets, as before.



The sign × is here used to denote a degree of sharpening of rather less than half a Tone; and $b'\flat$, $e''\flat$ followed by a line — are the sub-minor Sevenths of *c* and *f*.[1] Below the notes are

[1] [The interval *c* to *f* is 3 : 4 or 24 : 32; the interval *c* to *f* ×, here meant, is 24 : 33, so that the interval *f* to *f* × is 32 : 33, which is flatter than half a Tone or 15 : 16, by

added the names of the intervals, the numbers of the ratios, and the difference of those numbers, giving the vibrational number of the combinational tones.

We find in the first place that the combinational tones of the *Octave*, *Fifth*, *Twelfth*, *Fourth*, and *major Third* are merely transpositions of one of the primary tones by one or more Octaves, and hence introduce no foreign tone. Hence these five intervals can be used in all kinds of consonant triads, without disturbing the effect by the combinational tones which they introduce. In this respect the major Third is really superior to the major Sixth and the Twelfth in the construction of chords, although its independent harmoniousness is inferior to that of either.

The *double Octave* introduces the Fifth as a combinational tone. Hence if the fundamental tone of a chord is doubled by means of the double Octave, the chord is not injured. But injury would ensue if the Third or Fifth of the chord were doubled in the double Octave.

Then we have a series of intervals which are made into complete major triads by means of their combinational tones, and hence produce no disturbance in major chords, but are injurious to minor chords. These are the *Eleventh*, *minor Third*, *major Tenth*, *major Sixth*, and *minor Sixth*.

But the *minor Tenth*, and the *major* and *minor Thirteenth* cannot form part of a chord without injuring its consonance by their combinational tones.

We proceed to apply these considerations to the construction of triads.

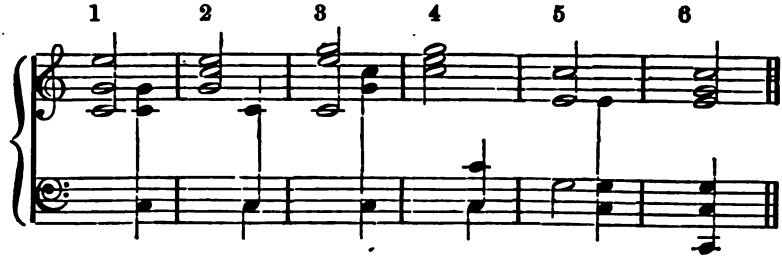
1. Major Triads.

Major triads can be so arranged that the combinational tones remain parts of the chord. This gives the most perfectly harmonious positions of these chords. To find them, remember that no minor Tenths or minor Thirteenths are admissible, so that all minor Thirds and Sixths must be in their closest position. By taking as the uppermost tone first the Third, then the Fifth, and lastly the fundamental tone, we find the following positions of these chords, within a compass of two octaves, in which the combi-

the small interval 495 : 512, because $\frac{33}{32}=\frac{16}{15}\times\frac{495}{512}$. If we take b'♭ to be the Fifth below f'', and e''♭ to be the Fifth below b'♭, these would each have to be flattened by the interval 63 : 64 to reduce them to those in the example.—*Translator.*]

national tones (here written as crotchets as usual) do not disturb the harmony.

The most Perfect Positions of Major Triads.



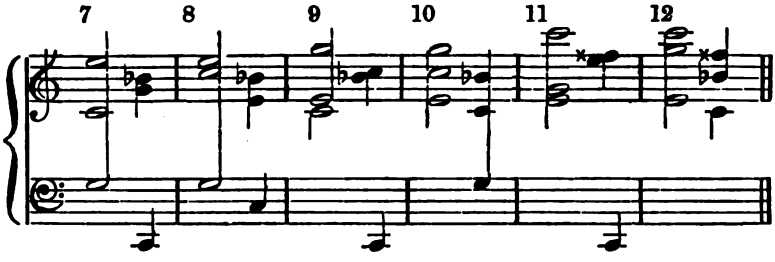
When the Third lies uppermost, the Fifth must not be more than a major Sixth below, as otherwise a Thirteenth would be generated. But the fundamental tone can be transposed. Hence when the Third is uppermost there are only two positions, 1 and 2, which are undisturbed. When the Fifth lies uppermost, the Third must be immediately under it, or otherwise a minor Thirteenth would be produced; but the fundamental tone may be transposed. Finally, when the fundamental tone is uppermost, the major Third can lie only in the position of a minor Sixth below it, but the Fifth may be placed at pleasure. Hence it follows that the only possible positions of the major chord which will be entirely free from disturbance by combinational tones, are the six here presented, among which we find the three *close* positions 2, 4, 6 already mentioned, and three new ones 1, 3, 5. Of these new positions two (1, 3) have the fundamental tone in the bass, just as in the primary form, and are considered as *open* positions of that form, while the third (5) has the Fifth in the bass, just as in the chord of the Fourth and Sixth [of which it is also considered as an *open* position]. The chord of the Third and Sixth (6), on the other hand, admits of no opener position.

The order of these chords in respect to harmoniousness of the intervals is, perhaps, the same as that presented above. The three intervals of 1 (the Fifth, major Tenth, and major Sixth) are the best, and those of 6 (the Fourth, minor Third, and minor Sixth) are relatively the most unfavourable of the intervals that occur in these chords.

The remaining positions of the major triads present individual

unsuitable combinational tones, and on justly intoned instruments are unmistakably rougher than those previously considered, but this does not make them dissonant, it merely puts them in the same category as minor chords. We obtain all of them which lie within the compass of two Octaves, by making the transpositions forbidden in the last cases. They are as follows, in the same order as before, 7 being made from 1, and so on:—

The less Perfect Positions of Major Triads.



Musicians will immediately perceive that these positions of the major triad are much less in use. The combinational tone *b'*♭ —, gives the positions 7 to 10 something of the character of the chord of the dominant Seventh in the key of F major, *c e g b*♭. The two last, 11 and 12, are much the least pleasing; indeed they are decidedly rougher than the better positions of the minor chord.[1]

2. Minor Triads.

No minor chord can be obtained perfectly free from false combinational tones, because its Third can never be so placed relatively to the fundamental tone, as not to produce a combinational tone unsuitable to the minor chord. If only one such tone is admitted, the Third and Fifth of the minor chord must lie close together

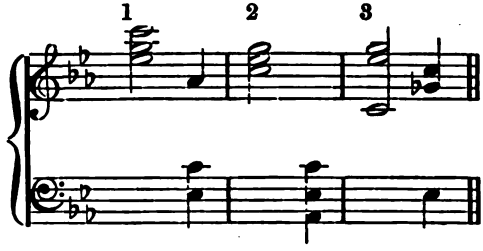
[1] All the above 12 chords can be played either on the experimental harmonium or concertina of Appendix XIX. But for the latter, in order especially to compare them with the positions of the minor triad subsequently given, it is advisable to play them also from the triad *e' g'*♯ *b'*, when they will assume the following forms, on taking the lowest available positions:—The following bracketed numbers refer to the corresponding forms of the minor chords given on p. 336, note.

1)	*e' b' g''*♯	[8]	4)	*e' g'*♯ *b'*	[2]	7)	*b e' g''*♯	[10]	10)	*g*♯ *e' b'*	[7]
2)	*b e' g'*♯	[6]	5)	*b g'*♯ *e''*	[12]	8)	*b e'' g''*♯	[11]	11)	*g*♯ *b e''*	[4]
3)	*e' g''*♯ *b''*	[3]	6)	*g*♯ *b e'*	[1]	9)	*e' g'*♯ *b''*	[9]	12)	*g*♯ *b' e''*	[5]

—*Translator.*]

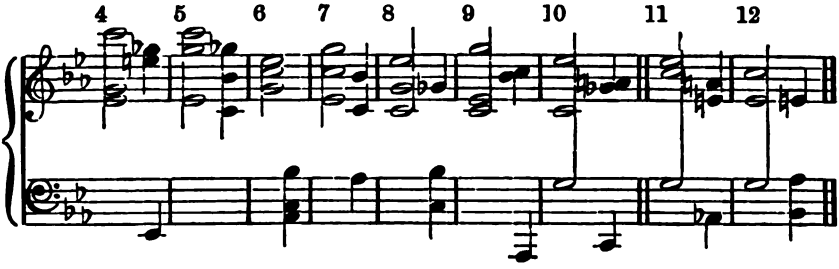
and form a major Third, because in any other position they would produce an unsuitable combinational tone. The fundamental tone and the Fifth must never form an Eleventh, because in that case the resulting combinational tone would make them into a major triad. These conditions can be fulfilled by only three positions of the minor chord, as follows :—

The most Perfect Positions of the Minor Triad.



The remaining positions which do not sound so well are :—

The less Perfect Positions of the Minor Triad.



The positions 4 to 10 each produce two unsuitable combinational tones, one of which necessarily results from the fundamental tone and its third; the other results in 4 from the Eleventh *G C*, and in the rest from the transposed major Third *E*♭ *G*. The two last positions, 11 and 12, are the worst of all, because they give rise to three unsuitable combinational tones.[1]

The influence of the combinational tones may be recognised by comparing the different positions. Thus the position 3, with a

[1] [On the experimental harmonium of Appendix XIX. these chords may be played as written; but on the just concertina there named they should be taken from the minor triad *e g b*, which will allow of their being immediately contrasted with the examples on p. 334, note. They will then be as follows, the first numbers shewing

minor Tenth and major Third, sounds unmistakably better than the position 7, with major Tenth and major Sixth, although the two latter intervals when struck separately sound better than the two first. The inferior effect of chord 7 is consequently solely due to the second unsuitable combinational tone.

This influence of bad combinational tones is also apparent from a comparison with the major chords. On comparing the minor chords 1 to 3, each of which has only one bad combinational tone, with the major chords 11 and 12, each of which has two such tones, those minor chords will be found really pleasanter and smoother than the major. Hence in these two classes of chords it is not the major and minor Third, nor the musical mode, which decides the degree of harmoniousness, it is wholly and solely the combinational tones.

Four Part Chords or Tetrads.

It is easily seen that all consonant tetrads must be either major or minor triads to which the Octave of one of the tones has been added. Every consonant tetrad will be changed into a consonant triad by removing one of its tones. Now this can be done in four ways, so that, for example, the tetrad *C E G c* gives the four following triads :—

C E G, *C E c*, *E G c*, *C G c*.

Any such triad, if it is not merely a dyad, or interval of two tones, with the Octave of one added, must be either a major or a minor triad, because there are no other consonant triads. But the only way of adding a fourth tone to a major or minor triad, on condition that the result should be consonant, is to add the Octave of one of its tones. For every such triad contains two tones, say *C* and *G*, which form either a direct or transposed Fifth. Now the only tones which can be combined with *C* and *G* so as to form a consonance are *E* and *E*♭ ; there are no others at all. But *E* and *E*♭ cannot be both present in the same consonant chord, and hence

the order as in the text, and the bracketed numbers after each chord referring to the corresponding form of the major chord in the note cited.

1)	*g*	*b*	*e′*	[6]	4)	*g*	*b*	*e″*	[11]	7)	*g*	*e′*	*b′*	[10]	10) *b* *e′* *g″* [7]
2)	*e′*	*g′*	*b′*	[4]	5)	*g*	*b′*	*e″*	[12]	8)	*e′*	*b′*	*g″*	[1]	11) *b* *e″* *g″* [8]
3)	*e′*	*g″*	*b″*	[3]	6)	*b*	*e′*	*g′*	[2]	9)	*e′*	*g′*	*b″*	[9]	12) *b* *g′* *e″* [5]

The contrast of the order of harmoniousness is thus seen to be very great.—*Translator.*]

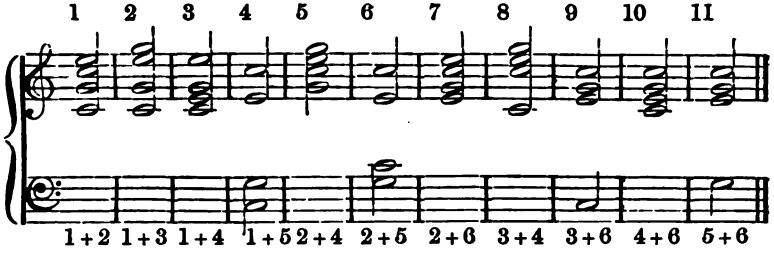
every consonant chord of four or more parts, which contains *C* and *G*, must either contain *E* and the Octaves of *C*, *E*, *G*, or else *E*♭ and the Octaves of *C*, *E*♭, *G*.

Every consonant chord of three or more parts will therefore be either a major or a minor chord, and may be formed from the fundamental position of the major and minor triad, by transposing or adding the Octaves of some or all of its three tones.

To obtain the perfectly harmonious positions of major tetrads, we have again to be careful that no minor Tenths or minor Thirteenths occur. Hence the Fifth may not stand more than a minor Third above, or a Sixth below the Third of the chord; and the fundamental tone must not be more than a Sixth above the Third. When these rules are carried out, the avoidance of the minor Thirteenths is effected by not taking the double Octave of the Third and Fifth. These rules may be briefly enunciated as follows: *Those major chords are most harmonious in which the fundamental tone or the Fifth does not lie more than a Sixth above the Third, or the Fifth does not lie more than a Sixth below it.* The fundamental tone may be as far below the Third as we please.

The corresponding positions of the major tetrads are found by combining any two of the more perfect positions of the major triads which have a common tone, as follows, where the lower figures refer to the positions of the major triads already given.

The most Perfect Positions of Major Tetrads within the Compass of Two Octaves.



We see that *chords of the Third and Sixth* must lie quite close, as No. 7; and that *chords of the Fourth and Sixth* must not have a compass of more than an Eleventh, but may occur in all the three positions (5, 6, 11) in which it can be constructed within

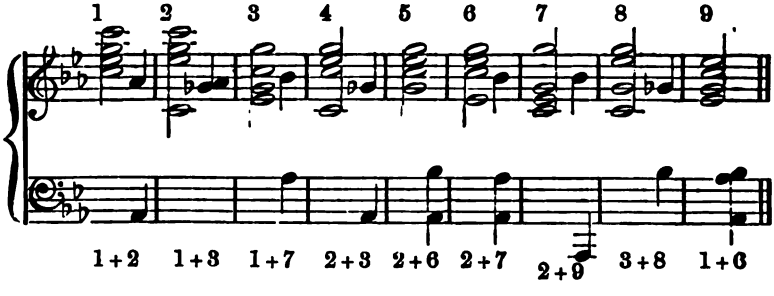
this compass. Chords which have the fundamental tone in the bass can be handled most freely.[1]

It will not be necessary to enumerate the less perfect positions of major tetrads. They cannot have more than two unsuitable combinational tones, as in the 11th position of the major triads, p. 334. The major triads of *C* can only have the false combinational tones marked *b*♭— and *f*× (or very acute *f*), that is, with vibrational numbers bearing to that of *C* the ratios 7 : 1, or 11 : 1.

Minor tetrads, like the corresponding triads, must at least have one false combinational tone. There is only one single position of the minor tetrad which possesses this property; it is marked 1 in the following example, and is compounded of the positions 1 and 2 of the minor triad. There may be as many as 4 false combinational tones, as, for example, on combining positions 10 and 11 of the minor triads.

Here follows a list of the minor tetrads which have not more than two false combinational tones, and which lie within the compass of two Octaves. The false combinational tones only are noted in crotchets, and those which suit the chord are omitted.

Best Positions of Minor Tetrads.



[1] [All these tetrads can be played on the experimental harmonium of Appendix XIX., and also, with the exception of 4 and 9, upon the just concertina there described. Those two forms would have to be taken an Octave higher, as *c′ g′ e″ c‴* and *c′ e″ g″ c‴* respectively. But in order to contrast them with the minor forms, and to shew their composition from the triads as indicated above, it is better, on the just concertina, to make the tetrads from the triads of *e′ g′*♯ *b′*, thus:

1)	*e′*	*b′*	*e″*	*g′*♯	=1+2	7)	*g*♯	*b*	*e′*	*g′*♯	=2+6
2)	*e′*	*b′*	*g″*♯	*b″*	=1+3	8)	*e′*	*e″*	*g″*♯	*b″*	=3+4
3)	*e′*	*g′*♯	*b′*	*g″*♯	=1+4	9)	*e′*	*g′*♯	*b″*	*e‴*	=3+6
4)	*e′*	*b′*	*g″*♯	*e‴*	=1+5	10)	*e′*	*g′*♯	*b′*	*e″*	=4+6
5)	*b*	*e′*	*g′*♯	*b′*	=2+4	11)	*b*	*g′*♯	*b′*	*e″*	=5+6
6)	*b*	*e′*	*g′*♯	*e″*	=2+5						

—*Translator.*]

The chord of the Fourth and Sixth occurs only in its closest position, 5; but that of the Third and Sixth is found in three positions (9, 3, and 6), namely, in all positions where the compass of the chord does not exceed a Tenth; the fundamental chord occurs three times with the Octave of the fundamental note added (1, 2, 4), and twice with the Octave of the Fifth added (7, 8).[1]

In musical theory, as hitherto expounded, very little has been said of the influence of the transposition of chords on harmonious effect. It is usual to give as a rule that close intervals must not be used in the bass, and that the intervals should be tolerably evenly distributed between the extreme tones. And even these rules do not appear as consequences of the theoretical views and laws usually given, according to which a consonant interval remains consonant in whatever part of the scale it is taken, and however it may be transposed or combined with others. They rather appear as practical exceptions from general rules. It was left to the musician himself to obtain some insight into the various effects of the various positions of chords, by mere use and experience. No rule could be given to guide him.

The subject has been treated here at such length in order to shew that a right view of the cause of consonance and dissonance leads to rules for relations which previous theories of harmony could not contain. The propositions we have enunciated agree, however, with the practice of the best composers, of those, I mean, who studied vocal music principally, before the great development of instrumental music necessitated the general introduction of tempered intonation, as anyone may easily convince himself by examining those compositions which aimed at producing an impression of perfect harmony. Mozart is certainly the composer who had the surest instinct for the delicacies of his art. Among his vocal compositions the *Ave verum corpus* is particularly celebrated for its wonderfully pure and smooth harmonies. On examining this little piece as one of the most suitable examples

[1] [All these chords can be played as they stand on the experimental harmonium of Appendix XIX., but on the just concertina there described, it is necessary to form them from the triad *e′ g′ b′*, as follows:

1)	*e′*	*g′*	*b*	*e″*	= 1 + 2
2)	*e′*	*g″*	*b″*	*e‴*	= 1 + 3
3)	*g*	*b*	*e′*	*b′*	= 1 + 7
4)	*e′*	*e″*	*g″*	*b″*	= 2 + 3
5)	*b*	*e′*	*g′*	*b*	= 2 + 6
6)	*g*	*e′*	*g′*	*b′*	= 2 + 7
7)	*e′*	*g′*	*b′*	*b″*	= 2 + 9
8)	*e′*	*b′*	*g″*	*b″*	= 3 + 8
9)	*g*	*b*	*e′*	*g′*	= 1 + 6

—*Translator.*]

for our purpose, we find in its first clause, which has an extremely soft and sweet effect, none but major chords, and chords of the dominant Seventh. All these major chords belong to those which we have noted as having the more perfect positions. Position 2 occurs most frequently, and then 8, 10, 1, and 9. It is not till we come to the final modulation of this first clause that we meet with two minor chords, and a major chord in an unfavourable position. It is very striking, by way of comparison, to find that the second clause of the same piece, which is more veiled, longing, and mystical, and laboriously modulates through bolder transitions and harsher dissonances, has many more minor chords, which, as well as the major chords scattered among them, are for the most part brought into unfavourable positions, until the final chord again restores perfect harmony.

Precisely similar observations may be made on those choral pieces of Palestrina, and of his contemporaries and successors, which have a simple harmonic construction without any involved polyphony. In transforming the Roman church music, which was Palestrina's task, the principal weight was laid on harmonious effect, in contrast to the harsh and unintelligible polyphony of the older Dutch system, and Palestrina and his school have really solved the problem in the most perfect manner. Here also we find an almost uninterrupted flow of consonant chords, with dominant Sevenths, or dissonant passing notes, charily interspersed. Here also the consonant chords wholly, or almost wholly, consist of those major and minor chords which we have noted as being in the more perfect positions. But in the final cadence of a few clauses, on the contrary, in the midst of more powerful and more frequent dissonances, we find a predominance of the unfavourable positions of the major and minor chords. Thus that expression which modern music endeavours to attain by various discords and an abundant introduction of dominant Sevenths, was obtained in the school of Palestrina by the much more delicate shading of various transpositions of consonant chords. This explains the deep and tender expressiveness of the harmony of these compositions, which sound like the songs of angels with hearts affected but undarkened by human grief in their heavenly joy. Of course such pieces of music require fine ears both in singer and hearer, to let the delicate gradation of expression receive its due, now that modern music has accustomed us to modes of expression so much more violent and drastic.

The great majority of major tetrads in Palestrina's *Staba mater* are in the positions 1, 10, 8, 5, 3, 2, 4, 9, and of mino tetrads in the positions 9, 2, 4, 3, 5, 1. For the major chords on might almost think that some theoretical rule led him to avoi the bad intervals of the minor Tenth and the Thirteenth. Bu this rule would have been entirely useless for minor chords Since the existence of combinational tones was not then known we can only conclude that his fine ear led him to this practice and that the judgment of his ear exactly agreed with the rule deduced from our theory.

These authorities may serve to lead musicians to allow th correctness of my arrangement of consonant chords in the orde of their harmoniousness. But anyone can convince himself o their correctness on any justly intoned instrument. The presen system of tempered intonation certainly obliterates somewhat o the more delicate distinctions, without, however, entirely destroyin them.

Having thus concluded that part of our investigations whicl rests upon purely scientific principles, it will be advisable t look back upon the road we have travelled, to review our gains and to examine the position which our results assume with respec to the views of older theoreticians. We started from the acoustica phenomena of upper partial tones, combinational tones and beats These phenomena were long well known both to musicians an acousticians, and the laws of their occurrence were, at least i their essential features, correctly recognised and enunciated. W had only to pursue these phenomena into further detail than ha hitherto been done. We succeeded in finding methods for observ ing upper partial tones, which rendered comparatively easy a observation previously very difficult to make. And with the help o this method we endeavoured to shew that, with few exceptions, th tones of all musical instruments were compounded of partial tones and that, in especial, those qualities of tone which are more parti cularly favourable for musical purposes, possess at least a series o the lower partial tones in tolerable force, while the simple tone like those of stopped organ pipes, have a very unsatisfactor musical effect, although even these tones when loudly sounded a accompanied in the ear itself by some weak harmonic upper partial

On the other hand we found that for the better musical qualities of tone, the higher partial tones, from the Seventh onwards, must be weak, as otherwise the quality, and every combination of tones would be too piercing. In reference to the beats, we had to discover what became of them when they grew quicker and quicker. We found that they then fell into that roughness which is the peculiar character of dissonance. The transition can be effected very gradually, and observed in all its stages, and hence it is apparent to the simplest natural observation that the essence of dissonance consists merely in very rapid beats. The nerves of hearing feel these rapid beats as *rough* and unpleasant, because every intermittent excitement of any nervous apparatus affects us more powerfully than one that lasts unaltered. With this there is possibly associated a psychological cause. The individual pulses of tone in a dissonant combination give us certainly the same impression of separate pulses as slow beats, although we are unable to recognise them separately and count them; hence they form a *tangled* mass of tone, which cannot be analysed into its constituents. The cause of the unpleasantness of dissonance we attribute to this *roughness* and *entanglement*. The meaning of this distinction may be thus briefly stated: *Consonance is a continuous, dissonance an intermittent sensation of tone.* Two consonant tones flow on quietly side by side in an undisturbed stream; dissonant tones cut one another up into separate pulses of tone. This description of the distinction at which we have arrived agrees precisely with Euclid's old definition, 'Consonance is the blending of a higher with a lower tone. Dissonance is incapacity to mix, when two tones cannot blend, but appear rough to the ear.'[1]

After this principle had been once established there was nothing

[1] Euclides, ed. Meibomius, p. 8: Ἔστι δὲ συμφωνία μὲν κρᾶσις δύο φθόγγων, ὀξυτέρου καὶ βαρυτέρου. Διαφωνία δὲ τοὐναντίον δύο φθόγγων ἀμιξία, μὴ οἵων τε κραθῆναι, ἀλλὰ τραχυνθῆναι τὴν ἀκοήν. [In translating this passage in the text, I have endeavoured to make the distinction of μίξις and κρᾶσις; the former is taken to be a mechanical, and the latter a chemical mixture. *Mixing* and *blending* seem to convey the notion. In συμφωνία (which Euclid admitted only for the Octave, Fifth, and Fourth) he felt that the tones *blended*. But the διαφωνία (which he applies to *all* other intervals, for he used Pythagorean major and minor Thirds, which are really dissonant) he found to consist in their not even *mixing*, not even forming a mechanical, much less a chemical unit, so that he goes on to explain that this *non-mixing* of the two tones consisted in *inability to blend*, and resulted in producing a *roughness*, as contradistinguished from a *blending* in the ear. The *tones* are φθόγγοι, properly tones sung, but used even for tones of the lyre.—*Translator*.]

further to do but to inquire in what circumstances, and with what degree of strength, beats would arise in the various combinations of tones through either the partial or the combinational tones. This investigation had hitherto been completed by Scheibler for the combinational tones of two simple tones only. The law of beats being known, it became easy to extend it to compound tones. Every theoretical conclusion on this field can be immediately checked by a proper observation, when the analysis of a mass of tone is facilitated by the use of resonators. All these beats of partial and combinational tones, of which so much has been said in the last chapter, are not inventions of empty theoretical speculation, but rather facts of observation, and can be really heard without difficulty by any practised observer who performs his experiments correctly. The knowledge of the acoustic law facilitates our discovery of the phenomena in question. But all the assertions on which we depend for establishing a theory of consonance and dissonance, such as was given in the last chapters, are founded wholly and solely on a careful analysis of the sensations of hearing, an analysis which a practised ear could have executed without any theoretical assistance, although of course the task is immensely facilitated by the guidance of theory and the assistance of appropriate instruments of observation.

For these reasons the reader is particularly requested to observe that my hypothesis concerning the sympathetic vibration of Corti's organs inside the ear has no immediate connection whatever with the explanation of consonance and dissonance. That explanation depends solely upon observed facts, on the beats of partial tones and the beats of combinational tones. Yet I thought it right not to suppress my hypothesis (which must of course be regarded solely as an hypothesis), because it gathers all the various acoustical phenomena with which we are concerned into one sheaf, and gives a clear, intelligible, and evident explanation of the whole phenomena and their connection.

The last chapters have shewn, that a correct and careful analysis of a mass of sound under the guidance of the principles cited, leads to precisely the same distinctions between consonant and dissonant intervals and chords, as have been established under the old theory of harmony. We have even shewn that these investigations give more particular information concerning individual intervals and chords than was possible with the general rules of the former

theory, and that the correctness of these rules was corroborated both by observation on justly intoned instruments and the practice of the best composers.

Hence I do not hesitate to assert that the preceding investigations, founded upon a more exact analysis of the sensations of tone, and upon purely scientific, as distinct from esthetic principles, exhibit the true and sufficient cause of consonance and dissonance in music.

One circumstance may, perhaps, cause the musician to pause in accepting this assertion. We have found that from the most perfect consonance to the most decided dissonance there is a continuous series of degrees, of combinations of sound, which continually increase in roughness, so that there cannot be any sharp line drawn between consonance and dissonance, and the distinction would therefore seem to be merely arbitrary. Musicians, on the contrary, have been in the habit of drawing a sharp line between consonances and dissonances, allowing of no intermediate links, and Hauptmann advances this as a principal reason against any attempt at deducing the theory of consonance from the relations of rational numbers.[1]

As a matter of fact we have already remarked that the chords of the natural or sub-minor Seventh 4 : 7, and of the sub-minor Tenth 3 : 7 in many qualities of tone sound at least as well as the minor Sixth 5 : 8, and that the sub-minor Tenth really sounds better than the minor Tenth 5 : 12. But we have already noticed a circumstance of great importance for musical practice which gives the minor Sixth an advantage over the intervals formed with the number 7. The inversion of the minor Sixth gives a better interval, the major Third, and its importance as a consonance in modern music is especially due to this very relation to the major Third; it is essentially necessary, and justified, just because it is the inversion of the major Third. On the other hand the inversion or transposition of an interval formed with the number 7

[1] 'Harmonik und Metrik,' p. 4. [At the same time, by accepting equal temperament they accept as consonant a series of tones which really form only *one* consonant interval (the Octave) and only *two* others even approximatively consonant (the Fifth and Fourth), while the commonest intervals on which harmony rests, the Thirds, with their inversions the Sixths, are not merely dissonant but, on the sustained tones of the voice for example, *grossly* dissonant. It is difficult for any ear brought up among these dissonances, to understand the real distinction between consonance and dissonance.—*Translator.*]

leads to intervals worse than itself. Hence the necessity, for the purposes of harmony, of being allowed to transpose the parts at pleasure, would be a reason for drawing the line between the minor Sixth on the one hand, and the intervals characterised by 7 on the other. It is not, however, till we come to construct scales, which we shall have to consider in the next chapter, that we find decisive reasons for making this the boundary. The scales of modern music cannot possibly accept tones determined by the number 7. But in musical harmony we can only deal with chords formed of notes in the scale. Intervals characterised by 5, as the *Thirds* and *Sixths*, occur in the scale, as well as others characterised by 9, as the *major Second* 8 : 9, but there are none characterised by 7, which should form the transition between them. Here then there is a real gap in the series of chords arranged according to the degree of their harmonious effect, and this gap of course determines the boundary between consonance and dissonance.

The decision does not depend, then, on the nature of the intervals themselves but on the construction of the whole tonal system. This is corroborated by the fact that the boundary between consonant and dissonant intervals has not been always the same. It has been already mentioned that the Greeks always represented Thirds as dissonant, and although the original Pythagorean Third 64 : 81, determined by a series of fifths, was not a consonance, yet even when the natural major Third 4 : 5 was afterwards included in the so-called syntono-diatonic mode of Didymus and Ptolemaeus, it was not recognised as a consonance. It has already been mentioned that in the middle ages, first the Thirds and then the Sixths were acknowledged as imperfect consonances, that the Thirds were long omitted from the final chord, and that it was not till later that the major, and quite recently the minor Third was admitted in this position. It is quite a mistake to suppose, with modern musical theorists, that this was merely whimsical and unnatural, or that the older composers allowed themselves to be fettered by blind faith in Greek authority. The last was certainly partly true for writers on musical theory down to the sixteenth century. But we must distinguish carefully between composers and theoreticians. Neither the Greeks, nor the great musical composers of the sixteenth and seventeenth centuries, were people to be blinded by a theory which their ears could upset. The reason for these deviations is to be looked for rather in the difference between the tonal systems

in early and recent times, with which we shall become acquainted in the next part. It will there be seen that our modern system gained the form under which we know it through the influence of a general use of harmonic chords. It was only in this system that a complete regard was paid to all the requisitions of interwoven harmonies. Owing to its strict consistency, we were not only able to allow many licences in the use of the more imperfect consonances and of dissonances, which older systems had to avoid, but we were often required to insert the Thirds in final chords, as a mode of distinguishing with certainty between the major and minor mode, in cases where this distinction was formerly evaded.

But if the boundary between consonance and dissonance has really changed with a change of tonal system, it is manifest that the reason for assigning this boundary does not depend on the intervals and their individual musical effect, but on the whole construction of the tonal system.

The enigma which, 2,500 years ago, Pythagoras proposed to science, 'Why is consonance determined by the ratios of small whole numbers?' has been solved by the discovery that the ear resolves all complex sounds into pendular oscillations, according to the laws of sympathetic vibration. Mathematically expressed, this is done by Fourier's law, which shews how any periodically variable magnitude, whatever be its nature, can be expressed by a sum of the simplest periodic magnitudes.[1] The length of the periods of the simply periodic terms of this sum must be exactly such, that either one or two or three or four, and so on, of their periods are equal to the period of the given magnitude. This, reduced to tones, means that the vibrational numbers of the partial tones must be exactly once, twice, three times, four times, and so on, respectively, as great as that of the prime tone. These are the whole numbers which determine the ratios of the consonances. For, as we have seen, the condition of consonance is that two of the lower partial tones of the notes combined shall be of exactly the same pitch; when they are not, disturbance arises from beats. Ultimately, then, the reason of the rational numerical relations of Pythagoras is to be found in the theorem of Fourier, and in one sense this theorem may be considered as the prime source of the theory of harmony.

[1] Namely magnitudes which vary, as sines and cosines.

The relation of whole numbers to consonance became in ancient times, in the middle ages, and especially among oriental nations, the foundation of extravagant and fanciful speculation. 'Everything is Number and Harmony,' was the characteristic principle of the Pythagorean doctrine. The same numerical ratios which exist between the seven tones of the diatonic scale, were thought to be found again in the distances of the celestial bodies from the central fire. Hence the harmony of the spheres, which was heard by Pythagoras alone among mortal men, as his disciples asserted. The numerical speculations of the Chinese in primitive times reach as far. In the book of Tso-kiu-ming, a friend of Confucius (B.C. 500), the five tones of the old Chinese scale were compared with the five elements of their natural philosophy—water, fire, wood, metal, and earth. The whole numbers 1, 2, 3 and 4 were described as the source of all perfection. At a later time the 12 semitones of the Octave were connected with the 12 months in the year, and so on. Similar references of musical tones to the elements, the temperaments, and the constellations are found abundantly scattered among the musical writings of the Arabs. The harmony of the spheres plays a great part throughout the middle ages. In Athanasius Kircher, not only the macrocosm, but the microcosm is musical. Even Kepler, a man of the deepest scientific spirit, could not keep himself quite free from imaginations of this kind. Nay even in the most recent times natural philosophers may still be found who prefer such dreaming to scientific work.

The celebrated mathematician Leonard Euler,[1] endeavoured in a serious and more scientific manner to found the relations of consonances to whole numbers upon psychological considerations, and his theory may certainly be regarded as the one which found most favour with scientific investigators during the last century, although it perhaps did not entirely satisfy them. Euler[2] begins by explaining that we are pleased with everything in which we can detect a certain amount of perfection. Now the perfection of anything is determined by the co-operation of all its parts towards the attainment of its end. Hence it follows that wherever perfection is to be found there must be order; for order consists in the arrangement of all parts by a certain law from which we can discover why each part lies where it is, rather than in any other

[1] Tentamen novae theoriae Musicae, Petropoli, 1739. [2] *Loc. cit.* chap. 2, § 7.

place. Now in any perfect object such a law of arrangement is determined by the end to be attained which governs all the parts. For this reason order pleases us more than disorder. Now order can be perceived in two ways: either we know the law whence the arrangement is deduced, and compare the deductions from this law with the arrangements observed; or, we observe these arrangements and endeavour to determine the law from them. The latter is the case in music. A combination of tones will please us when we can discover the law of their arrangement. Hence it may well happen that one hearer finds it and that another does not, and that their judgments consequently differ.

The more easily we perceive the order which characterises the objects contemplated, the more simple and more perfect will they appear, and the more easily and joyfully shall we acknowledge them. But an order which costs trouble to discover, though it will indeed also please us, will associate with that pleasure a certain degree of weariness and sadness (*tristitia*).

Now in tones there are two things in which order is displayed, pitch and duration. Pitch is ordered by intervals, duration by rhythm. Force of tone might also be ordered, had we a measure for it. Now in rhythm two or three or four equally long notes of one part may correspond with one or two or three of another, in which the regularity of the arrangement is easily observed, especially when frequently repeated, and gives considerable pleasure. Similarly in intervals we should derive more pleasure from observing that two, three, or four vibrations of one tone coincided with one, two, or three of another, than we could possibly experience if the ratios of the time of vibration were incommensurable with one another, or at least could not be expressed except by very high numbers. Hence it follows that the combination of two tones pleases us the more, the smaller the two numbers by which the ratios of their periods of vibration can be expressed. Euler also remarked that we could better endure more complicated ratios of the periods of vibration, and consequently less perfect consonances, for higher than for deeper tones, because for the former the groups of vibrations which were arranged to occur in equal times, were repeated more frequently than in the latter, and we were consequently better able to recognise the regularity of even a more involved arrangement.

Hereupon Euler develops an arithmetical rule for calcu the degree of harmoniousness of an interval or a chord fr ratios of the periods of the vibrations which characteri intervals. The *Unison* belongs to the first degree, the *Oct* the second, the *Twelfth* and *Double Octave* to the third, the to the fourth, the *Fourth* to the fifth, the major *Tenth* and *El* to the sixth, the *major Sixth* and *major Third* to the sevent *minor Sixth* and *minor Third* to the eighth, the *sub-Seventh* 4 : 7 to the ninth, and so on. To the ninth degree l also the *major triad*, both in its closest position and in the *tion of the Fourth and Sixth*. The *major chord of the Thir Sixth* belongs, however, to the tenth degree. The *minor* both in its closest and in its *position of the Third and* also belongs to the ninth degree, but its *position of the F and Sixth* to the tenth degree. In this arrangement th sequences of Euler's system agree tolerably well with ou results, except that in determining the relation of the ma the minor triad, the influence of combinational tones wa taken into account, but only the kinds of interval. Henc triads in their close position appear to be equally harmo although again the *major chord of the Third and Sixth* the *minor chord of the Fourth and Sixth* are inferior wit as with us.[1]

Euler has not confined these speculations to single conso and chords, but has extended them to their results, to th struction of scales, and to modulations, and brought out

[1] The principle on which Euler calculated the degrees of harmoniousn intervals and chords, is here annexed, because its consequences are very co combinational tones are disregarded. When p is a prime number, the degree All other numbers are products of prime numbers. The number of the degr product of two factors a and b, for which separately the numbers of degree are respectively $= \alpha + \beta - 1$. To find the number of the degree of a chord, which ca pressed by $p : q : r : s$, &c., in smallest whole numbers, Euler finds the least c multiple of p, q, r, s, &c., and the number of its degree is that of the chord. for example:

The number of the degree of 2 is 2, and of 3 is 3,
of $4 = 2.2$, it is $2 + 2 - 1 = 3$,
of $12 = 4.3$, it is $3 + 3 - 1 = 5$,
of $60 = 12.5$, it is $5 + 5 - 1 = 9$.

That of the major triad 4 : 5 : 6 is that of 60, because 60 is the least c multiple of 4, 5, 6, that is, the least number which all of them will divide wi remainder.

surprising specialities correctly. But without taking into account that Euler's system gives no explanation of the reason why a consonance when slightly out of tune sounds almost as well as one justly tuned, and much better than one greatly out of tune, although the numerical ratio for the former is generally much more complicated, it is very evident that the principal difficulty in Euler's theory is that it says nothing at all of the mode in which the mind contrives to perceive the numerical ratios of two combined tones. We must not forget that a man left to himself is scarcely aware that a tone depends upon vibrations. Moreover, immediate and conscious perception by the senses has no means of discovering that the numbers of vibrations performed in the same time are different, greater for high than for low tones, and that determinate intervals have determinate ratios of these numbers. There are certainly many perceptions of the senses in which a person is precisely able to account for the way in which he has attained his knowledge, as when from the resonance of a space he judges of its size and form, or when he reads the character of a man in his features. But in such cases a person has generally had a large experience in such relations, which helps him to form a judgment in analogous circumstances, without having the previous circumstances on which his judgment depends clearly present to his mind. But it is quite different with vibrational numbers. A man that has never made physical experiments has never in the whole course of his life had the slightest opportunity of knowing anything about vibrational numbers or their ratios. And almost everyone who delights in music remains in this state of ignorance from birth to death.

Hence it was certainly necessary to shew how the ratios of vibrational numbers can be perceived by the senses. It has been my endeavour to do this, and hence the results of my investigation may be said, in one sense, to fill up the gap which Euler's left. But the physiological processes which make the difference sensible between consonance and dissonance, or, in Euler's language, orderly and disorderly relations of tone, ultimately bring to light an essential difference between our method of explanation and Euler's. According to the latter, the human mind perceives commensurable ratios of vibrational numbers *as such*; according to our method, it perceives *only the physical effect* of these ratios, namely the continuous or intermittent sensation of

the auditory nerves. The physicist knows, indeed, that the reason why the sensation of a consonance is continuous is that the ratios of its vibrational numbers are commensurable, but when a man, who is unacquainted with physics, hears a piece of music, nothing of the sort occurs to him, nor does the physicist find a chord in any respect more harmonious because he is better acquainted with the cause of its harmoniousness.[1] It is quite different with the order of rhythm. That exactly two crotchets, or three in a triplet, or four quavers go to one minim is perceived by any attentive listener without the least instruction. But while the orderly relation (or commensurable ratio) of the vibrations of two combined tones, on the other hand, undoubtedly affects the ear in a certain way which distinguishes it from any disorderly relation (incommensurable ratio), this difference of consonance and dissonance depends on physical, not psychological grounds.

The considerations advanced by Rameau and d'Alembert[2] on the one side, and Tartini[3] on the other, concerning the cause of consonance agree better with our theory. The latter founded his theory on the existence of combinational tones, the two former on that of upper partial tones. As we see, they had found the proper points of attack, but the acoustical knowledge of last century did not allow of their drawing sufficient consequences from them. According to d'Alembert, Tartini's book was so darkly and obscurely written that he, as well as other well-instructed people, were unable to form a judgment upon it. D'Alembert's book, on the other hand, is an extremely clear and masterly performance, such as was to be expected from a delicate and exact thinker, who was at the same time one of the greatest physicists and mathematicians of his time. Rameau and d'Alembert lay down two facts as the foundation of their system. The first is that every resonant body audibly produces at the same time as the prime (*générateur*) its twelfth and next higher Third, as upper partials (*harmoniques*). The second is that the resemblance between any tone and its Octave is generally apparent. The first fact is used to shew that

[1] [Does a man breathe more easily and aerate his blood better because he knows the constitution of the atmosphere and its relation to his carbonised blood? Does a man feel a weight greater or less, because he knows the laws of gravitation? These are quite similar questions.—*Translator.*]

[2] 'Éléments de Musique, suivant les Principes de M. Rameau,' par M. d'Alembert. Lyon, 1762.

[3] 'Traité de l'Harmonie,' 1754.

the major chord is *the most natural* of all chords, and the second to establish the possibility of lowering the Fifth and the Third by one or two Octaves without altering the nature of the chord, and hence to obtain the major triad in all its different inversions and positions. The minor triad is then found by the condition that all three tones should have the same upper partial or harmonic, namely the Fifth of the chord (in fact *C*, *E*♭, and *G* have all the same upper partial *g′*). Hence although the minor chord is not so perfect and natural as the major, it is nevertheless prescribed *by nature.*

In the middle of the last century, when much suffering arose from an artificial social condition, it might have been enough to shew that a thing was *natural*, in order at the same time to prove that it must be *also* beautiful and desirable. Of course no one who considers the great perfection and suitability of all organic arrangements in the human body, would, even at the present day, deny that when the existence of such natural relations have been proved as Rameau discovered between the tones of the major triad, they ought to be most carefully considered, at least as starting-points for further research. And Rameau had indeed quite correctly conjectured, as we can now perceive, that this fact was the proper basis of a theory of harmony. But that is by no means everything. For in nature we find not only beauty but ugliness, not only help but hurt. Hence the mere proof that anything is natural does not suffice to justify it esthetically. Moreover if Rameau had listened to the effects of striking rods, bells, and membranes, or blowing over hollow chambers, he might have heard many a perfectly dissonant chord, quite unlike those obtained from strings and musical instruments. And yet such chords cannot but be considered equally *natural.*

Again the resemblance of the Octave to its fundamental tone, which was one of Rameau's initial facts, is a musical phenomenon quite as much in need of explanation as consonance itself.

No one knew better than d'Alembert himself the missing links of this system. Hence in the preface to his book he especially guards himself against the expression: 'Demonstration of the Principle of Harmony,' which Rameau had used. He declares that so far as he himself is concerned, he meant only to give a well-connected and consistent account of all the laws of the theory

CHAPTER XIII.

GENERAL VIEW OF THE DIFFERENT PRINCIPLES OF MUSICAL STYLE IN THE DEVELOPMENT OF MUSIC.

UP to this point our investigation has been purely physical. We have analysed the sensations of hearing, and investigated the physical and physiological causes for the phenomena discovered, —partial tones, combinational tones, and beats. In the whole of this research we have dealt solely with natural phenomena, which present themselves mechanically, without any choice, to all living beings whose ears are constructed on the same anatomical plan as our own. In such a field, where necessity is paramount and nothing is arbitrary, science is rightfully called upon to establish constant laws of phenomena, and to demonstrate strictly a strict connection between cause and effect. As there is nothing arbitrary in the phenomena embraced by the theory, so also nothing arbitrary can be admitted into the laws which regulate the phenomena, or into the explanations given for their occurrence. As long as anything arbitrary remains in these laws and explanations, it is the duty of science (a duty which it is generally able to discharge) to exclude it, by continuing the investigations.

But in this third part of our inquiry into the theory of music we have to furnish a satisfactory foundation for the elementary rules of musical composition, and here we tread on new ground, which is no longer subject to physical laws alone, although the knowledge which we have gained of the nature of hearing, will still find numerous applications. We pass on to a problem which by its very nature belongs to the domain of esthetics. When we spoke previously, in the theory of consonance, of agreeable and disagreeable, we referred solely to the immediate impression made on the senses when an isolated combination of sounds strikes the ear, and paid no attention at all to artistic contrasts and means of expression; we thought only of sensuous pleasure, not of esthetic

beauty. The two must be kept strictly apart, although the first is an important means for attaining the second.

The altered nature of the matters now to be treated betrays itself by a purely external characteristic. At every step we encounter historical and national differences of taste. Whether one combination is rougher or smoother than another, depends solely on the anatomical structure of the ear, and has nothing to do with psychological motives. But what degree of roughness a hearer is inclined to endure as a means of musical expression depends on taste and habit; hence the boundary between consonances and dissonances has been frequently changed. Similarly Scales, Modes, and their Modulations have undergone multifarious alterations, not merely among uncultivated or savage people, but even in those periods of the world's history and among those nations where the noblest flowers of human culture have expanded.

Hence it follows,—and the proposition cannot be too vividly present to the minds of our musical theoreticians and historians—*that the system of Scales, Modes, and Harmonic Tissues does not rest solely upon inalterable natural laws, but is at least partly also the result of esthetical principles, which have already changed, and will still further change, with the progressive development of humanity.*

But it does not follow from this that the choice of those elements of musical art, was perfectly arbitrary, and that they do not allow of being derived from some more general law. On the contrary the rules of any style of art form a well-connected system whenever that style has attained a full and perfect development. These rules of art were certainly never developed into a system by the artists themselves with conscious intention and consistency. They are rather the result of tentative exploration or the play of imagination, as the artists think out or execute their plans, and gradually discover by trial, what kind or manner of performance best pleases them. Yet science is capable of discovering the motive forces, whether psychological or technical, which have been at work in this artistic process. Scientific *esthetics* have to deal with the psychological; scientific *physics* with the technical. When the artist's aim in the style he has adopted, and its principal direction, have once been rightly conceived, it can be more or less correctly determined why he was forced to follow this or that rule, or employ this or that technical means. In musical theory

especially, where the peculiar physiological functions of the ear, while not immediately present to conscious self-examination, play an important part, a large and rich field is thrown open for scientific investigation to shew the necessary character of the technical rules for each individual direction in the development of our art.

It does not rest with natural science to characterise the chief problem worked out by each school of art, and the elementary principle of its style. This must be gathered from the results of historical and esthetical inquiry.

The relation we have to treat may be illustrated by a comparison with architecture, which, like music, has pursued essentially different directions at different times. The *Greeks*, in their stone temples, imitated the original wooden constructions; that was the principle of their architectural style. The whole analysis and arrangement of their decorations clearly shew that it was their intention to imitate wooden constructions. The verticality of the supporting columns, the general horizontality of the supported beam, forced them to arrange all the subordinate parts for the great majority of cases in vertical and horizontal lines. The purposes of Grecian worship, which performed its principal functions in the open air, were satisfied by erections, of this kind, in which the internal spaces were necessarily narrowly limited by the length of the stone or wooden beams which could be employed. The old *Italians* (Etruscans), on the other hand, discovered the principle of the arch, composed of wedge-shaped stones. This discovery rendered it possible to cover in much more extensive buildings with arched roofs, than the Greeks could do with their wooden beams. Among these arched buildings the halls of justice (basil'icae) became important, as is well known, for the subsequent development of architecture. The arched roof made the circular arch the chief principle in division and decoration for *Roman* (*Byzantine*) art. The columns, pressed by heavy weights, were transformed into pillars, on which, after the style was fully developed, columns merely appeared in diminished forms, half sunk in the mass of the pillar, as merely decorative articulations, and as the downward continuation of the ribs of the arches which radiated towards the ceiling from the upper end of the pillar.

In the arch the wedge-shaped stones press against each other, but as they all uniformly press inwards, each one prevents the other from falling. The most powerful and most dangerous degree

of pressure is exerted by the stones in the horizontal parts of the arch, where they have either no support or no obliquely placed support, and are prevented from falling solely by the greater thickness of their upper extremities. In very large arches the horizontal middle portion is consequently the most dangerous, and would be precipitated by the slightest yielding of the materials, As, then, medieval ecclesiastical structures assumed continually larger dimensions, the idea occurred of leaving out the middle horizontal part of the arch altogether, and of making the sides ascend with moderate obliquity until they met in a pointed arch. From thenceforward the pointed arch became the dominant principle. The building was divided into sections externally by the projecting buttresses. These, and the omnipresent pointed arch made the outlines hard, and churches became enormously high. But both characters suited the vigorous minds of the northern nations, and perhaps the very hardness of the forms, thoroughly subdued by that marvellous consistency which runs through the varied magnificence of form in a *gothic* cathedral, served to heighten the impression of immensity and power.

We see here, then, how the technical discoveries which were associated with the problems as they rose successively created three entirely distinct principles of style—the horizontal line, the circular arch, the pointed arch—and how at each new change in the main plan of construction, all the subordinate individualities, down to the smallest decorations, were altered accordingly; and hence how the individual rules of construction can only be comprehended from the general principle of construction. Although the gothic style has developed the richest, the most consistent, the mightiest and most imposing of architectural forms, just as modern music among other musical styles, no one would certainly for a moment think of asserting that the pointed arch is nature's original form of all architectural beauty, and must consequently be introduced everywhere. And at the present day it is well known that it is an artistic absurdity to put gothic windows in a Greek building. Conversely anyone can unfortunately convince himself on visiting most of our Gothic cathedrals how detestably unsuitable to the whole effect are those numerous little chapels of the *rénaissance* period built in the Greek or Roman style. Just as little as the Gothic pointed arch, should our diatonic major scale be regarded as a *natural product*, at least such an expression

is quite inapplicable, except in so far as both are necessary and *natural* consequences of the principle of style selected. And just as little as we should use Gothic ornamentation in a Greek temple, should we venture upon improving compositions written in ecclesiastical modes, by providing their notes with marks of transposition in accordance with the scheme of our major and minor harmonies. The feeling for historical artistic conception has certainly made little progress as yet among our musicians, even among those who are at the same time musical historians. They judge old music by the rules of modern harmony, and are inclined to consider every deviation from it as mere unskilfulness in the old composer, or even as barbarous want of taste.[1]

Hence before we proceed to the construction of scales and rules for a tissue of harmony, we must endeavour to characterise the principles of style, at least for the chief phases of the development of musical art. For present purposes we may divide these into three principal periods:—

1. The *Homophonic* or Unison Music of the ancients, to which also belongs the existing music of Oriental and Asiatic nations.

2. The *Polyphonic* Music of the middle ages, with several parts, but without regard to any independent musical significance of the harmonies, extending from the 10th to the 17th century, when it passes into

3. *Harmonic* or *Modern* Music, characterised by the independent significance attributed to the harmonies as such. Its sources date back from the 16th century.

1. Homophonic Music.

Simple melody, sung by a single voice, is the original form of music with all people. It still exists among the Chinese, Indians, Arabs, Turks, and modern Greeks, notwithstanding the greatly developed systems of music possessed by some of these nations. That music in the time of highest Grecian culture, neglecting perhaps individual instrumental ornamentation, cadences, and interludes, was written in one part, or that the voices at most sang in Octaves, can now be considered as established. In the problems

[1] Thus in R. G. Kiesewetter's historico-musical writings, which are otherwise so rich in facts industriously connected, there is evidently an exaggerated zeal to deny everything which will not fit into the modern major and minor modes.

of Aristotle we find the question: 'Why is the consonance of the Octave alone sung? For this and no other consonance is played on the magădis.' This was a harp-shaped instrument [with a bridge dividing the strings at one-third their length]. In another place he remarks that the voices of boys and men form an Octave in singing.[1]

Solo music considered independently and unaccompanied by words, is too poor in forms and changes to develop any of the greater and richer forms of art. Hence purely instrumental music at this stage is necessarily limited to short dances or marches. We really find no more among nations that have no harmonic music. Performers on the flute[2] have certainly repeatedly gained the prize in the Pythian games, but it is possible to perform feats of execution in instrumental music in concise forms of composition, as, for example, in the variations of a short melody. That the principle of varying (*μεταβάλλειν*) a melody with reference to

[1] *Διὰ τί ἡ διὰ πασῶν συμφωνία ἥδεται μόνη; μαγαδίζουσι γὰρ ταύτην, ἄλλην δὲ οὐδεμίαν.* Prob. xix. 18. [Translated in the text.] *Διὰ τί ἥδιόν ἐστι τὸ σύμφωνον τοῦ ὁμοφώνου; Ἢ καὶ τὸ μὲν ἀντίφωνον, σύμφωνόν ἐστι διὰ πασῶν; ἐκ παίδων γὰρ νέων καὶ ἀνδρῶν, γίνεται τὸ ἀντίφωνον· οἱ διεστᾶσι τοῖς τόνοις, ὡς νήτη πρὸς τὴν ὑπάτην.* Prob. xix. 39. ['Why is a consonant union of voices pleasanter than a single voice? Is the singing of voice against voice, a consonant union of voices in Octaves? This singing of voice against voice occurs when young boys and men sing together, and their tones differ as the highest from the lowest of the scale.'] Towards the end of the songs the instrumental accompaniment seems to have separated itself from the voice. Probably this is what is meant by the *krousis* in the passage *τελευτῶσαις δ' εἰς ταὐτὸν, ἓν καὶ κοινὸν τὸ ἔργον συμβαίνει γίνεσθαι, καθάπερ τοῖς ὑπὸ τὴν ᾠδὴν κρούουσι· καὶ γὰρ οὗτοι τὰ ἄλλα οὐ προσαυλοῦντες, ἐὰν εἰς ταὐτὸν καταστρέφωσιν, εὐφραίνουσι μᾶλλον τῷ τέλει, ἢ λυποῦσι ταῖς πρὸ τοῦ τέλους διαφοραῖς, τῷ τὸ ἐκ διαφόρων τὸ κοινὸν, ἥδιστον ἐκ τοῦ διὰ πασῶν γίνεσθαι.* Arist. Prob. xix. 39. ['But when they end in the same, the matter is precisely similar to what occurs *when they play an accompaniment to a song.* For the accompanyists do not follow the rest, but when the singers return to the same, they please more in the end, or displease more in the differences before the end, by which means the common part in what is generally different, pleases more than anything but the Octave.'] See also 'Plutarch de Musica,' xix. xxviii. That the Greeks knew the effect of consonances and did not like it, appears by the following passage from 'Aristotle de Audibilibus,' ed. Bekker, p. 801: 'For this reason we understand a single speaker better than many who are saying the same things at the same time. And so with strings. And much less, when flute and lyre are played at the same time, because the voices are confused by the others. And this is very plain for the consonances. For both tones are concealed, one by the other.' *Διὸ καὶ μᾶλλον ἑνὸς ἀκούοντες συνίεμεν, ἢ πολλῶν ἅμα ταὐτὰ λεγόντων· καθάπερ καὶ ἐπὶ τῶν χορδῶν· καὶ πολὺ ἧττον, ὅταν προσαυλῇ τις ἅμα καὶ κιθαρίζῃ, διὰ τὸ συγχεῖσθαι τὰς φωνὰς ὑπὸ τῶν ἑτέρων. Οὐχ ἥκιστα δὲ τοῦτο ἐπὶ τῶν συμφωνιῶν φανερόν ἐστιν. Ἀμφοτέρους γὰρ ἀποκρύπτεσθαι συμβαίνει τοὺς ἤχους ὑπ' ἀλλήλων.*

[2] The *αὐλοὶ* were perhaps more like our oboes.

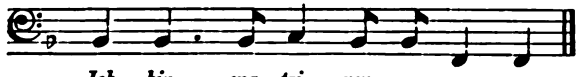
dramatical expression (*μίμησις*), was known to the Greeks, follows also from Aristotle. He describes the matter very plainly, and remarks that choruses must simply repeat the melodies in the antistrophes, because it is easier for one than for several to introduce variations. But public competitors (*ἀγωνισταὶ*) and actors (*ὑποκριταὶ*) are able to grapple with these difficulties.[1]

Extensive works of art, in homophonic music, are only possible in connection with poetry, and this was also the way in which music was applied in classical antiquity. Not only were songs (odes) and religious hymns sung, but even tragedies and long epic poems were performed and recited in some musical manner, and accompanied by the lyre. We are scarcely in a condition to form a conception of how this was done, because modern taste points in precisely the opposite direction, and demands from a great declaimer or public reader that he should produce a dramatic effect true to nature by the speaking voice alone, rating all approach to singing as one of the greatest of faults. Perhaps we have some echoes of the ancient spoken song in the singing tone of Italian

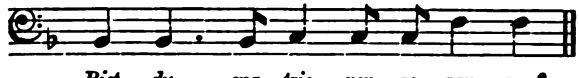
[1] [*Διὰ τί οἱ μὲν νόμοι οὐκ ἐν ἀντιστρόφοις ἐποιοῦντο· αἱ δὲ ἄλλαι ᾠδαὶ αἱ χορικαί; Ἢ ὅτι οἱ μὲν νόμοι, ἀγωνιστῶν ἦσαν, ὃν ἤδη μιμεῖσθαι δυναμένων καὶ διατείνεσθαι, ἡ ᾠδὴ ἐγίνετο μακρὰ καὶ πολυειδής; καθάπερ οὖν καὶ τὰ ῥήματα, καὶ τὰ μέλη τῇ μιμήσει ἠκολούθει ἀεὶ ἕτερα γινόμενα. Μᾶλλον γὰρ τῷ μέλει ἀνάγκη μιμεῖσθαι, ἢ τοῖς ῥήμασι. Δι' ὃ καὶ οἱ διθύραμβοι, ἐπειδὴ μιμητικοὶ ἐγένοντο, οὐκ ἔτι ἔχουσιν ἀντιστρόφους, πρότερον δὲ εἶχον. Αἴτιον δὲ, ὅτι τὸ παλαιὸν οἱ ἐλεύθεροι ἐχόρευον αὐτοί. Πολλοὺς οὖν ἀγωνιστικῶς, ᾄδειν, χαλεπόν ἦν. Ὥστε ἐναρμόνια μᾶλλον μέλη ἐνῇδον. Μεταβάλλειν γὰρ πολλὰς μεταβολὰς τῷ ἑνὶ ῥᾷον, ἢ τοῖς πολλοῖς, καὶ τῷ ἀγωνιστῇ, ἢ τοῖς τὸ ἦθος φυλάττουσι. Δι' ὃ ἁπλούστερα ἐποιοῦντο αὐτοῖς τὰ μέλη. Ἡ δὲ ἀντίστροφος, ἁπλοῦν. Ἀριθμὸς γάρ ἐστι, καὶ ἑνὶ μετρεῖται. Τὸ δ' αὐτὸ αἴτιον καὶ διότι τὰ μὲν ἀπὸ τῆς σκηνῆς οὐκ ἀντίστροφα, τὰ δὲ τοῦ χοροῦ ἀντίστροφα. Ὁ μὲν γὰρ ὑποκριτὴς ἀγωνιστὴς [καὶ μιμητής·] ὁ δὲ χορὸς, ἧττον μιμεῖται.* Arist. Probl. xix. 15. 'Why are themes (*nomoi*) not used in antistrophic singing, while all other choral singing is employed? Is it because themes belong to public performers who are already able to imitate and extend, and hence would make their song long and very figurate? For melodies, like words, follow imitation and change. It is more necessary for melody to imitate, than for words. Wherefore dithyrambic poets also when they became mimetic, disused their previous antistrophic singing. The reason is that formerly gentlemen (*eleutheroi*) used to sing the choruses themselves. It was difficult for many to sing like public performers. So they rather intoned suitable melodies. For it is easier for one to make numerous changes than for many to do so, and for a public performer than for those who retained old usage. Hence the melodies were made simpler for them. Now antistrophic singing is simple, for it depends on number, and is measured by a unit. The same reason shews why the parts of the actors are not antistrophic, but those of the chorus are so. For the actor is a public performer [and a mime], but the chorus does not imitate so well.'—*Translator.*]

declaimers, and the liturgical recitations (intoning) of the Roman Catholic priests.

Indeed, attentive observations on ordinary conversation shews us that regular musical intervals involuntarily recur, although the singing tone of the voice is concealed under the noises which characterise the individual letters, and the pitch is not held firmly, but is frequently allowed to glide up and down. When simple sentences are spoken without being affected by feeling, a certain middle pitch is maintained, and it is only the emphatic words and the conclusions of sentences and clauses which are indicated by change of pitch.[1] The end of an affirmative sentence followed by a pause, is usually marked by the voice falling a Fourth from the middle pitch. An interrogative ending rises, often as much as a Fifth above the middle pitch. For example a bass voice would say:



Ich bin spa - tzie - ren ge - gan - gen.
I *have* been walk - ing this morn - ing.



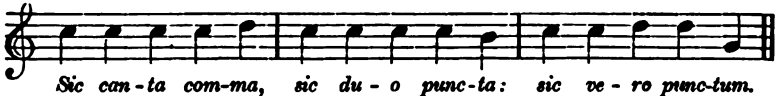
Bist du spa - tzie - ren ge - gan - gen?
Have *you* been walk - ing this morn - ing?

Emphasised words are also rendered prominent by their being spoken about a Tone higher than the rest,[2] and so on. In solemn declamation the alterations of pitch are more numerous and complicated. Modern recitative has arisen from attempting to imitate these alterations of pitch by musical notes. Its inventor, Giacomo Peri, in the preface to his opera of *Eurydice*, published in 1600, distinctly says as much. An attempt was then made to restore the declamation of ancient tragedies by means of recitative. Ancient recitation certainly differed somewhat from modern recitative, by preserving the metre of the poems more exactly, and

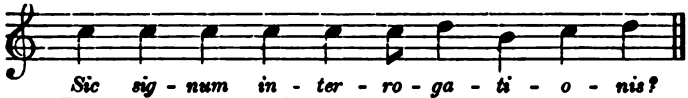
[1] [Prof. Helmholtz's observations on speaking must be read in reference to North German habits only.—*Translator.*]

[2] [By no means uniformly, even in North Germany. The habits of different nations here vary greatly. In Norway and in Sweden the voice is regularly raised on unemphatic syllables. In Scotland the emphasis is often marked by lowering the pitch.—*Translator.*]

by having no accompanying harmonies. Nevertheless our recitative, when well performed, will give us a better conception of the degree in which the expression of the words can be enhanced by musical recitation, than we can obtain from the monotonous repetition of the Roman liturgy, although the latter perhaps is more nearly related in kind to ancient recitation than the former. The settlement of the Roman liturgy by Pope Gregory the Great (A.D. 590 to 604) reaches back to a time in which reminiscences of the ancient art, although faded and deformed, might have been in some degree handed down by tradition, especially if, as we are probably entitled to assume, Gregory really did little more than finally establish the Roman school of singing which had existed from the time of Pope Sylvester (A.D. 314 to 335). The majority of these formulae for lessons, collects, &c., evidently imitate the cadence of ordinary speech. They proceed at an equal height; particular, emphatic, or non-Latin words are somewhat altered in pitch; and for the punctuation certain concluding forms are prescribed, as the following for lessons, according to the customs of Münster.[1]



Sic can-ta com-ma, sic du-o punc-ta: sic ve-ro punc-tum.
Thus sing the com-ma, and thus the co-lon: and thus the full stop.



Sic sig-num in-ter-ro-ga-ti-o-nis?
Thus sing the mark of in-ter-ro-ga-tion?

These and similar final formulae were varied according to the solemnity of the feast, the subject treated, the rank of the priest that sang and that answered, and so on.[1] It is easy to see that they strove to imitate the natural cadences of ordinary speech, and to give them solemnity by eliminating their individual irregu-

[1] Antony, 'Lehrbuch des Gregorianischen Kirchengesanges' [Manual of Gregorian Church-music], Münster, 1829. According to the information collected by Fétis (in his 'Histoire générale de Musique,' Paris, 1869, vol. i. chap. 6), it has become doubtful whether this system of declamation with prescribed cadences, is not rather to be deduced from the Jewish ritual chants. In the oldest manuscripts of the Old Testament, there are 25 different signs employed to denote cadences and melodic phrases of this kind. The fact that the corresponding signs of the Greek Church are Egyptian demotic characters, hints at a still older Egyptian origin for this notation.

larities. Of course in such fixed formula no regard can be paid to the grammatical sense of the clauses, which suffers much in various ways from the intoning. Similarly we may suppose that the ancient tragic poets prescribed the cadences of speech to their actors, and preserved them by a musical accompaniment. And since ancient tragedy kept much further aloof from the realistic than our modern drama, as is shewn by the artificial rhythms, the unusual rolling words, the immovable strange masks, it could admit of a more singing tone for declamation than would, perhaps, please our modern ears. Then we must remember that by emphasising or increasing the loudness of certain words, and by rapidity or slowness of speech, or pantomimic action, much life can be thrown into delivery of this kind, which would certainly be insufferably monotonous if not thus enlivened.

But in any case homophonic music, even when in olden time it had to accompany extensive poems of the highest character, necessarily played an utterly subordinate part. The musical turns must have entirely depended on the changing sense of the words, and could have had no independent artistic value or connection without them. A peculiar melody for singing hexameters throughout an epic, or iambic trimeters throughout a tragedy, would have been insupportable.[1] Those melodies (*νόμοι*) which were allotted

[1] [We must remember that the Greek and Latin so-called *accents* consisted solely in alterations of pitch, and hence to a certain extent determined a melody. See Dionysius of Halicarnassus, *περὶ συνθέσεως ὀνομάτων*, chap. xi., cited at length and translated in my 'Quantitative Pronunciation of Latin,' p. 27, note, where we also find that in his day (first century before Christ) the musical composers transgressed at pleasure the rules of both accent and quantity. But if the written accents in Greek, and the accents as determined by the rules of grammarians in Latin, are carefully examined, it will be found that every line in a Greek or Latin poem had its own distinct melody, the art of the poet being shewn by the great variability of pitch conjoined with a constant quantity or rhythm. This would give the following result for the opening lines of the Iliad. The measures are marked by bars, and half measures by colons; the rise by *r*, fall by *f*, and even pitch (or that of the preceding syllable, whatever it was) by *e*, and an occasional final 'rest' by (—).

| r f : e r | f : e e | r : f | e : e r | f : e e | r f : e — |
μῆ- νιν ἄ- ει-δε θε-ά πη-λη-ΐ-ά- δεω ἀ-χι- λῆ - os,

| e : e r | f : r | f : r f | e : r f | r : f r | f : e — |
οὐ- λο-μέ- νην, ἣ μυ - ρί' ἀ-χαι-οῖς ἄλ-γε' ἔ- θη-κεν,

| e : r | f : r | f : e | r : e f | e : e r | f : e — |
πολ - λάς δ' ἰφ- θί-μους ψυ-χὰς ἄ-ι- δι προ-ΐ- αψ-εν

| e : r | f : e | r : e f | r : f e | r f : e r | f : e — |
ἡ- ρώ- ων, αὐ-τοὺς δὲ ἑ-λώ-ρι-α τεῦ-χε κύ- νεσ-σιν

to odes and tragic choruses, were certainly freer and more independent. For odes there were also well-known melodies (the names of some of them are preserved) to which fresh poems were continually composed.

In the great artistic works just mentioned, then, music must have been entirely subordinate; independently, it could only have formed short pieces. Now this is closely connected with the development of homophonic music as a musical system. Among the nations who possess such music we always find certain degrees of pitch selected for the melodies to move in. These scales are very various in kind, partly, it would seem, very arbitrary, so that many appear to us quite strange and incomprehensible, and yet the best gifted among those nations which possess them, as the Greeks, Arabs, and Indians, have developed them in an extremely subtile and varied manner.

When speaking of these systems of tones, it becomes a ques-

| e : e | r f : r f | r f : e e | r : f e | r : f e | e : r |
ol - ω-νοῖ- σί τε πᾶ-σι, δι-ὸς δ' ἐτε-λεί- ε-το βου-λή

| e : r f | r : f | r f : e e | e : r | f : e r | f : e— |
ἐξ οὗ δὴ τα-πρῶ - τα δι-ασ-τή- την ἐ-ρί- σαν-τε

| e : r | f : e r | f : e | f : r | r f : e e | e : r |
ἀ- τρεί- δης τε ἄ-ναξ ἀν-δρῶν καὶ δῖ - ος ἀ - χιλ-λεύς

Here it will be seen that every line differs from the preceding materially, and it would not be at all difficult for any one with a little musical skill, and the help of a common Greek lyre, to extemporise an effective recitative, especially if the rising and falling intervals were varied; the intervals after a fall, although marked even, might gradually fall further (as is probable), or after a rise might rise further. A similar, but not so varied a melody may also be found in Latin, as in the opening lines of Lucretius:

| e : r f | e : r f | e : r f | e : e | r : f e | r : f
ae-né-a-dum gé-ne-trix hó-mi-num di-vúm-que vo-lúp-tas

| r : f r | f : r | f : e | e : e | r : f e | r : f
ál-ma vé- nus cóe- li sub- ter la -bén-ti- a sig-na

| r : r f | e : r f | r : e | r : f | e : e e | r : f
quae má-re na-ví-ge- rum, quae tér-ras fru-gi-fe-rén-tes

| e : r f | e : e | r : r f | e : r f | r : f e | r : f
con-cé-le- bras; per té quó-ni-am gé-nus óm-ne' a-ni-mán-tum

| e : r f | e : e | r : f | r : f | r : f e | r : f
con-ci-pi- tur vi- sit- que ex-sór- tum lú-mi-na só-lis.

The constant final cadence | r : f e | r : f |, which became almost stereotyped in Virgil and other later poets, must have been very wearisome, notwithstanding the attempt always made to vary the melody of the first four measures of Latin hexameters. No doubt the Greeks looked down with great contempt on the melody of Latin verse. See my paper 'On the Physical Constituents of Accent and Emphasis,' in the 'Transactions of the Philological Society,' for 1873–4, pp. 113–164, and my tract on Latin above cited.—*Translator.*]

tion of essential importance for our present purpose, to inquire whether they are based upon any determinate reference of all the tones in the scale to one single principal and fundamental tone, the *tonic* or *key-note*. Modern music effects a purely musical internal connection among all the tones in a composition, by making their relationship to one tone as perceptible as possible to the ear. This predominance of the tonic, as the link which connects all the tones of a piece, we may, with Fétis, term the principle of *tonality*. This learned musician has properly drawn attention to the fact that tonality is developed in very different degrees and manners in the melodies of different nations. Thus in the songs of the modern Greeks, and chants of the Greek Church, and the Gregorian tones of the Roman Church, they are not developed in a manner which is easy to harmonise, whereas, according to Fétis,[1] it is on the whole easy to add accompanying harmonies to the old melodies of the northern nations of German, Celtic, and Sclavonic origin.

It is indeed remarkable that though the musical writings of the Greeks often treat subtile points at great length, and give the most exact information about all other peculiarities of the scale, they say nothing intelligible about a relation which in our modern system stands first of all, and always makes itself most distinctly sensible. The only hints to be found concerning the existence of the tonic are not in especial musical writings, but as before in the works of Aristotle, who asks :—

'Why is it that if anyone alters the tone on the middle string (*μέση*) after the others have been tuned, and plays, every thing sounds amiss, not merely when he comes to this middle tone, but throughout the whole melody? but if he alters the tone played by the forefinger (*λιχανός*) or any other, the difference is only perceived when that string is struck? Is there a good reason for this? All good melodies often employ the tone of the middle string, and good composers often come upon it, and if they leave it recur to it again; but this is not the case with any other tone.' Then he compares the tone of the middle string with conjunctions in language, such as 'and' and 'then,' without which language could not exist, and proceeds to say: 'In this way the tone of the middle string is a link between tones, especially of the best tones,

[1] Fétis, 'Biographie universelle des Musiciens,' vol. i. p. 126.

because its tone most frequently recurs.'[1] And in another place we find the same question with a slightly different answer, 'Why do the other tones sound badly when the tone of the middle string is altered? but if the tone of the middle string remains, and one of the others is altered, the altered one alone is spoiled? Is it because that all are tuned and have a certain relation to the tone of the middle string, and the order of each is determined by that? The reason of the tuning and connection being removed, then, things no longer appear the same.'[2] In these sentences the esthetic significance of the tonic, under the name of 'the tone of the middle string,' is very accurately described. To this we may add that the Pythagoreans compared the tone of the middle string with the sun, and the other tones in the scale with the planets.[3] It appears as if it had been usual to

[1] Διὰ τί, ἐὰν μέν τις τὴν μέσην κινήσῃ ἡμῶν, ἁρμόσας [δὲ] τὰς ἄλλας χορδὰς, κέχρηται τῷ ὀργάνῳ, οὐ μόνον ὅταν κατὰ τὸν τῆς μέσης γένηται φθόγγον, λυπεῖ, καὶ φαίνεται ἀνάρμοστον, ἀλλὰ καὶ κατὰ τὴν ἄλλην μελῳδίαν· ἐὰν δὲ τὴν λιχανὸν, ἤ τινα ἄλλον φθόγγον, τότε φαίνεται διαφέρειν μόνον, ὅταν κἀκείνῃ τὶς χρῆται; Ἢ εὐλόγως τοῦτο συμβαίνει; πάντα γὰρ τὰ χρηστὰ μέλη, πολλάκις τῇ μέσῃ χρῆται· καὶ πάντες οἱ ἀγαθοὶ ποιηταὶ, πυκνὰ πρὸς τὴν μέσην ἀπαντῶσι· κἂν ἀπέλθωσι ταχὺ ἐπανέρχονται· πρὸς δὲ ἄλλην οὕτως οὐδεμίαν. Καθάπερ ἐκ τῶν λόγων ἐνίων ἐξαιρεθέντων συνδέσμων, οὐκ ἔστιν ὁ λόγος Ἑλληνικός· (οἷον τὸ τὲ, καὶ τὸ τοὶ) καὶ ἔνιοι δὲ οὐθὲν λυποῦσι· διὰ τὸ τοῖς μὲν, ἀναγκαῖον εἶναι χρῆσθαι πολλάκις, ἢ οὐκ ἔσται λόγος Ἑλληνικός· τοῖς δὲ, μή· οὕτω καὶ τῶν φθόγγων ἡ μέση, ὥσπερ σύνδεσμός ἐστι, καὶ μάλιστα τῶν καλῶν, διὰ τὸ πλειστάκις· ἐνυπάρχειν τὸν φθόγγον αὐτῆς. Arist. Prob. xix. 20. [The names of Greek tones were those of the strings on the lyre by which they were played, just as if in English we were to call the tones *g*, *d′*, *a′*, *e″*, the tones of the fourth, third, second, and first strings respectively, because they are produced as the open notes of these strings on the violin, and contracted them to *fourth*, *third*, &c., only omitting the word *string*. As the violin when held sideways in playing, throws the *g* string uppermost, and the *e″* string the lowermost, we might in the same way call *g* the 'uppermost note,' ὑπάτη, although lowest in pitch, and *e″* the 'lowermost note,' νήτη, although highest in pitch. Then *d* might be called the *middle*, μέση, being really the keynote of the violin and one of the two middle strings. This illustrates the Greek names very closely, for the lyre was held with the string sounding the lowest note, uppermost. See the scale on the next page.—*Translator*.]

[2] Διὰ τί, ἐὰν μὲν ἡ μέση κινηθῇ, καὶ αἱ ἄλλαι χορδαὶ ἠχοῦσι φθεγγόμεναι· (one of my colleagues, Prof. Stark, conjectures that in place of φθεγγόμεναι, which makes nonsense, we should read φθειρόμεναι·) ἐὰν δὲ αὖ ἡ μὲν μένῃ, τῶν δ' ἄλλων τὶς κινηθῇ, κινηθεῖσα μόνη φθέγγεται; (for which Prof. Stark again proposes φθείρεται;) Ἢ ὅτι τὸ ἡρμόσθαι ἐστὶν ἀπάσαις, [τὸ δὲ ἔχειν πως πρὸς τὴν μέσην ἀπάσαις], καὶ ἡ τάξις ἡ ἑκάστης, ἤδη δὶ ἐκείνην; ἀρθέντος οὖν τοῦ αἰτίου τοῦ ἡρμόσθαι καὶ τοῦ συνέχοντος, οὐκ ἔτι ὁμοίως φαίνεται ὑπάρχειν. Arist. Probl. xix. 36.

[3] Nicomachus, 'Harmonicē,' lib. i. p. 6, ed. Meibomii. [The following is Nichomachus's arrangement of the comparison, with his reasons:

Saturn *hypatē*, as being highest in position, ὕπατον γὰρ τὸ ἀνώτατον.

begin with the tone of the middle string above-mentioned, for we read in the 33rd problem of Aristotle: 'Why is it more agreeable to proceed from high pitch to low pitch, than from low pitch to high pitch? Can it be that we thus begin at the beginning? for the tone of the middle string is also the leader of the tetrachord and highest in pitch. The second way would be to begin at the end instead of at the beginning. Or can it be that tones of lower pitch sound nobler and more euphoneous after tones of high pitch?'[1] This seems also to shew that it was not the custom to end with the tone of the middle string, which commenced, but with the tone of lowest pitch [produced by the uppermost string or *Hypatē*], of which last tone Aristotle, in his 4th problem, says that, as opposed to its neighbour, the tone of lowest pitch but one, [due to the string of highest position but one, or *Parhypatē*,] it is sung with complete relaxation of all the effort that is felt in the other.[2] These words of Aristotle may certainly be applied to the national Doric scale of the Greeks, which, increased by Pythagoras to eight tones, was as follows:—

Tetrachord of lowest pitch	E Hypătē	[ὑπάτη	uppermost string]
	F Parhypătē	[παρυπάτη	next to uppermost string]
	G Lichănos	λίχανος	forefinger string]
	A Mĕsē (tone of middle string)	[μέση	middle string]
Tetrachord of highest pitch	B Paramĕsē	[παραμέση	next to middle string]
	C Trīte	τρίτη	third string]
	D Paranētē	[παρανήτη	next to lowermost string]
	E Nētē	νήτη	lowermost string]

In modern phraseology the last description cited from Aristotle implies that the Parhypatē was a kind of descending 'leading

Jupiter *parhypatē*, as next highest to Saturn.

Mars *lichanos* or *hypermesē*, as between Jupiter and the sun.

The Sun *mesē*, as lying in the middle, the fourth from either end, middlemost string and planet.

Mercury *paramesē*, as lying between the Sun and Venus.

Venus *paraneatē*, as lying just above the moon.

The Moon *neatē*, as being lowest of all in position and next the earth, *καὶ γὰρ νέατον, τὸ κατώτατον.—Translator.*]

[1] Διὰ τί εὐαρμοστότερον ἀπὸ τοῦ ὀξέος ἐπὶ τὸ βαρύ, ἢ ἀπὸ τοῦ βαρέος ἐπὶ τὸ ὀξύ; Πότερον ὅτι τὸ ἀπὸ τῆς ἀρχῆς γίνεται ἄρχεσθαι; ἡ γὰρ μέση καὶ ἡγεμὼν ὀξυτάτη τοῦ τετραχόρδου. Τὸ δὲ οὐκ ἀπ' ἀρχῆς, ἀλλ' ἀπὸ τελευτῆς. Ἢ ὅτι τὸ βαρὺ ἀπὸ τοῦ ὀξέος γενναιότερον, καὶ εὐφωνότερον; Arist. Prob. xix. 33.

[2] Διὰ τί δὲ ταύτην [τὴν παρυπάτην] χαλεπῶς [ᾄδουσι], τὴν δὲ ὑπάτην ῥᾳδίως· καίτοι δίεσις ἑκατέρας; ἢ ὅτι μετ' ἀνέσεως ἡ ὑπάτη, καὶ ἅμα μετὰ τὴν σύστασιν ἐλαφρὸν τὸ ἄνω βάλλειν; Arist. Prob. xix. 4.

note' to the Hypatē. In the leading tone there is perceptible effort, which ceases on its falling into the fundamental tone.

If then the tone of the middle string answers to the tonic, the Hypatē, which is its Fifth, will answer to the Dominant. For our modern feeling it is far more necessary to close with the tonic than to begin with it, and hence we usually take the final tone of a piece to be its tonic without further inquiry. Modern music however, usually introduces the tonic also in the first beat of the opening bar. The whole mass of tone is developed from the tonic and returns into it. Modern musicians cannot obtain complete repose at the end unless the series of tones converges into its connecting centre.

Ancient Greek music seems, then, to have deviated from ours by ending on the dominant instead of the tonic. And this is in full agreement with the intonation of speech. We have seen that the end of an affirmative sentence is likewise formed on the Fifth next below the principal tone.[1] This peculiarity has also been generally preserved in modern recitative, in which the singer usually ends on the dominant; the accompanying instruments then make this tone part of the chord of the dominant Seventh, leading to the tonic chord, and thus make a close on the tonic in accordance with our present musical feeling. Now since Greek music was cultivated by the recitation of epic hexameters and iambic trimeters, we should not be surprised if the above-mentioned peculiarities of chanting were so predominant in the melodies of odes that Aristotle could regard them as the rule.[2]

From the facts just adduced it follows (and this is what we are chiefly concerned with) that the Greeks, among whom our diatonic scale first arose, were not without a certain esthetic feeling for tonality, but that they had not developed it so decisively as in modern music. Indeed, it does not appear to have even entered into the technical rules for constructing melodies. Hence Aristotle, who treated music esthetically, is the only known writer who mentions it; musical writers proper do not speak of it

[1] [This would be entirely crossed by the Greek system of pitch-accents, just as it now is by a similar system in Norwegian, where the pitch may rise for *both* affirmative *and* interrogative sentences. But see p. 366, note.—*Translator*.]

[2] Among the presumed ancient melodies which have been handed down to us, the fragment of the Homeric Ode to Dēmētēr, which has been published by B. Marcello, shews the above-mentioned peculiarity very distinctly.

at all. And unfortunately the indications furnished by Aristotle are so meagre, that doubt enough still exists. For example, he says nothing about the differences of the various musical modes in reference to their principal tone, so that the most important point of all from which we should wish to regard the construction of the musical scale, is almost entirely obscured.

The reference to a tonic is more distinctly made out in the scales of the old Christian ecclesiastical music. Originally the four so-called *authentic* scales were distinguished, as they had been laid down by Ambrose of Milan (died A.D. 398). Not one of these agrees with any one of our scales. The four *plagal* scales afterwards added by Gregory, are no scales at all in our sense of the word. The four authentic scales of Ambrose are:

1)	*D*	*E*	*F*	*G*	*A*	*B*	*c*	*d*
2)	*E*	*F*	*G*	*A*	*B*	*c*	*d*	*e*
3)	*F*	*G*	*A*	*B*	*c*	*d*	*e*	*f*
4)	*G*	*A*	*B*	*c*	*d*	*e*	*f*	*g*

Perhaps, however, the change of B into B♭ was allowed from the first, and this would make the first scale agree with our descending scale of D minor, and the third scale would become our scale of F major. The old rule was that songs in the first scale should end in D, those in the second in E, those in the third in F, and those in the fourth in G. This marked out these tones as tonics in our sense of the word. But the rule was not strictly observed. The conclusion might fall on other tones of the scale, the so-called *confinal* tones, and at last the confusion became so great that no one was able to say exactly how the scale was to be recognised, all kinds of insufficient rules were formulated, and at last musicians clung to the mechanical expedient of fixing upon certain initial and concluding phrases, called *tropes*, as characterising the scale.

Hence although the rule of tonality had been already remarked in these medieval ecclesiastical scales, the rule was so unsettled and admitted so many exceptions, that the feeling of tonality must have been much less developed than in modern music.

The Indians also hit upon the conception of a tonic, although their music is likewise adapted for one voice only. They called the tonic *Ansa*.[1] Indian melodies as transcribed

[1] Jones, 'On the Music of the Indians, translated by Dalberg,' pp. 36, 37.

by English travellers, seem to be very like modern European melodies. Fétis and Coussemaker[1] have made the same remark respecting the few known remains of old German and Celtic melodies.

Although therefore homophonic music was not entirely without a reference to some tonic, or predominant tone, such a tone was beyond all dispute much more weakly developed than in modern music, where a few consecutive chords suffice to establish the scale in which that portion of the piece is written. The cause of this seems to me traceable to the undeveloped condition and subordinate part which characterises homophonic music. Melodies which move up and down in a few tones which are easily comprehended, and are connected, not by some musical contrivance, but by the words of a poem, do not require the consistent application of any contrivance to combine them. Even in modern recitative tonality is much less firmly established than in other forms of composition. The necessity for a steady connection of masses of tone by purely musical relations, does not dawn distinctly on our feeling, until we have to form into one artistic whole large masses of tone, which have their own independent significance without the cement of poetry.

2. Polyphonic Music.

The second stage of musical development is the polyphonic music of the middle ages. It is usual to cite as the first invented part-music, the so-called *organum* or *diaphony*, as originally described by the Flemish monk Hucbald at the beginning of the tenth century. In this, two voices are said to have proceeded in Fifths or Fourths, with occasional doublings of one or both in Octaves. This would produce intolerable music for modern ears. But according to O. Paul[2] the meaning is not that the two voices sang at the same time, but that there was a responsive repetition of a melody in a transposed condition, in which case Hucbald would have been the inventor of a principle which subsequently became so important in the fugue and sonata.

The first undoubted form of part-music intentionally for several voices, was the so-called *discantus*, which became known at the end of the eleventh century in France and Flanders. The oldest

[1] 'Histoire de l'Harmonie au moyen Age,' Paris, 1852, pp. 5–7.

[2] 'Geschichte des Claviers' [History of the Pianoforte], Leipzig, 1868, p. 49.

specimens of this kind of music which have been preserved are of the following description. Two entirely different melodies—and to all appearance the more different the better—were adapted to one another by slight changes in rhythm or pitch, until they formed a tolerably consonant whole. At first, indeed, there seems to have been an inclination for coupling a liturgical formula with a rather 'slippery' song. The first of such examples could scarcely have been intended for more than musical tricks to amuse social meetings. It was a new and amusing discovery that two totally independent melodies might be sung together and yet sound well.

The principle of discant was fertile, and its nature was suitable for development at that period. Polyphonic music proper was its issue. Different voices, each proceeding independently and singing its own melody, had to be united in such a way as to produce either no dissonances, or merely transient ones which were readily resolved. Consonance was not the object in view, but its opposite, dissonance, was to be avoided. All interest was concentrated on the motion of the voices. To keep the various parts together, time had to be strictly observed, and hence the influence of discant developed a system of musical rhythm, which again contributed to infuse greater power and importance into melodic progression. There was no division of time in the Gregorian *Cantus firmus*. The rhythm of dance music was probably extremely simple. Moreover, melodic movement increased in richness and interest as the parts were multiplied. But the establishment of an artistic connection between the different voices, which, as we have seen, were at first perfectly free, required a new invention, and this, though it cropped up at first in a very humble form, has ended by obtaining predominant importance in the whole art of modern musical composition. This invention consisted in causing a musical phrase which had been sung by one voice to be repeated by another. Thus arose *canonic imitation*, which may be met with sporadically as early as in the twelfth century.[1] This subsequently developed into a highly artistic system, especially among Flemish composers, who, it must be owned, ended by often shewing more calculation than taste in their compositions.

But by this kind of polyphonic music—the repetition of the

[1] Coussemaker, *loc. cit.* Discant: *Custodi nos*, Pl. xxvii., No. iv., translated in p. xxvii., No. xxix.

same melodic phrases in succession by different voices—it first became possible to compose musical pieces on an extensive plan, owing their connection not to any union with another fine art, poetry, but to purely musical contrivances. This kind of music also was especially suited to ecclesiastical songs, in which the chorus had to express the feelings of a whole congregation of worshippers, each with his own peculiar disposition. It was, however, not confined to ecclesiastical compositions, but was also applied to secular songs (madrigals). The sole form of harmonic music yet known, which could be adapted to artistic cultivation, was that founded on canonic repetitions. If this had been rejected, nothing but homophonic music remained. Hence we find a number of little poems set as strict canons or with canonical repetitions, which were entirely unsuited for such a heavy form of composition. Even the oldest examples of instrumental compositions in several parts, the dance music of 1529,[1] are written in the form of madrigals and motets, a character of composition which, more freely treated, lasted down to the suites of S. Bach and Handel's times. Even in the first attempts at musical dramas in the sixteenth century, there was no other way of making the personages express their feelings musically, than by causing a chorus behind or upon the stage to sing over some madrigals in the fugue style. It is scarcely possible for us, from our present point of view, to conceive the condition of an art which was able to build up the most complicated constructions of voice parts in chorus, and was yet incapable of adding a simple accompaniment to the melody of a song or a duet for the purpose of filling up the harmony. And yet when we read how Giacomo Peri's invention of recitative with a simple accompaniment of chorus was applauded and admired, and what contentions arose as to the renown of the invention; what attention Viadana excited when he invented the addition of a *Basso continuo* for songs in one or two parts, as a dependent part serving only to fill up the harmony;[2] it is impossible to doubt that this art of accompanying a melody by chords (as any amateur can now do in the simplest manner possible) was completely unknown to musicians up to the end of the sixteenth century. It was not till the sixteenth century that composers became aware of the meaning possessed by chords as

[1] Winterfeld, 'Johannes Gabrieli und sein Zeitalter,' vol. ii. p. 41.
[2] Winterfeld, *ibid.*, vol. ii. p. 19 and p. 59.

forming an harmonic tissue independently of the progression of parts.

To this condition of the art corresponded the condition of the tonal system. The old ecclesiastical scales were retained in their essentials, the first from *D* to *d*, the second from *E* to *e*, the third from *F* to *f*, and the fourth from *G* to *g*. Of these, the scale from *F* to *f* was useless for harmonic purposes, because it contained the Tritone *F*—*B*, in place of the Fourth *F*—*B*♭. Again, there was no reason for excluding the scales from *C* to *c* and *G* to *g*. And thus the ecclesiastical scales altered under the influence of polyphonic music. But as the old unsuitable names were retained notwithstanding the changes, there arose a terrible confusion in the meaning attached to modes. It was not till nearly the end of this period that a learned theoretician, Glarean, undertook in his *Dodecachordon* (Basle 1547) to put some order into the theory of modes. He distinguished twelve of them, six authentic and six plagal, and assigned them Greek names, which were, however, incorrectly transferred. However, his nomenclature for ecclesiastical modes has been generally followed ever since. The following are Glarean's six authentic ecclesiastical modes, keys, or scales, with the incorrect Greek names he assigned to them.

Ionic . . .	*C*	*D*	*E*	*F*	*G*	*A*	*B*	*c*
Doric . . .	*D*	*E*	*F*	*G*	*A*	*B*	*c*	*d*
Phrygian . .	*E*	*F*	*G*	*A*	*B*	*c*	*d*	*e*
Lydian . .	*F*	*G*	*A*	*B*	*c*	*d*	*e*	*f*
Mixolydian . .	*G*	*A*	*B*	*c*	*d*	*e*	*f*	*g*
Eolic . . .	*A*	*B*	*c*	*d*	*e*	*f*	*g*	*a*

Ionic answers to our major, *Eolic* to our minor system. *Lydian* was scarcely ever used in polyphonic music owing to the false Fourth *F*—*B*, and when it was employed it was altered in many different ways.

Inability to judge of the musical significance of a connected tissue of harmonies again appears in the theory of the keys, by the rule, that the key of a polyphonic composition was determined by considering the separate voices independently. Glarean in certain compositions attributes different keys to the tenor and bass, the soprano and alto. Zarlino assumes the tenor as the chief part for determining the key.

The practical consequences of this neglect of harmony are conspicuous in various ways in musical compositions. The composers confined themselves on the whole to the diatonic scale; 'accidentals,' or signs of transposition were seldom used. The Greeks had introduced the depression of the tone *B* to *B*♭ in a peculiar tetrachord, that of the *synēmmenoi*, and this was retained. Besides this *f*♯, *c*♯ and *g*♯ were used, to introduce leading tones in the cadences. Hence modulation, as we understand it, from the key of one tonic to that of another with a different signature was almost entirely absent. Moreover, the chords used by preference down to the end of the fifteenth century, were formed of the Octave and Fifth without the Third, and such chords now sound very poor, and are avoided as much as possible. To medieval composers who only felt the want of the most perfect consonances, these chords appeared the most agreeable, and none others might be used at the close of a piece. The dissonances which occur are universally those which arise from suspended and passing tones; chords of the dominant Seventh, which, in modern harmony, play such an important part in marking the key, and in connecting and facilitating progressions, were quite unknown.

Great, then, as was the artistic advance in rhythm and the progression of parts, during this period, it did little more for harmony and the tonal system than to accumulate an undigested mass of experiments. Since the involved progression of the parts gave rise to chords in extremely varied transpositions and sequences, the musicians of this period could not but hear these chords and become acquainted with their effects, however little skill they shewed in making use of them. At any rate, the experience of this period prepared the way for harmony proper, and made it possible for musicians to produce it, when external circumstances forced on the discovery.

3. HARMONIC MUSIC.

Modern harmonic music is characterised by the independent significance of its harmonies for the expression and the artistic connection of a musical composition. The external inducements for this transformation of music were of various kinds. First there was the Protestant ecclesiastical chorus. It was a principle of Protestantism that the congregation itself should undertake the singing. But a

which these changes exerted on the constitution of the tonal system. The mode of connecting musical phrases hitherto in vogue —canonic repetitions of similar melodic figures—had necessarily to be abandoned as soon as a simple harmonic accompaniment had to be subordinated to a melody. Hence some new means of artistic connection had to be discovered in the sound of the chords themselves. This was effected, first by making the harmonies refer their tones much more definitely to one predominant tonic than before, and secondly by giving fresh strength to the relations between the chords themselves and between all other chords and the tonic chord. In the course of our investigations we shall see that the distinctive peculiarities of the modern system of tones can be deduced from this principle, and that the principle itself is very strictly carried out in our present music. In reality the mode in which the materials of music are now worked up for artistic use, is in itself a wondrous work of art, at which the experience, ingenuity, and esthetic feeling of European nations has laboured for between two and three thousand years, since the days of Terpander and Pythagoras. But the complete formation of the essential features as we now see it, is scarcely two hundred years old in the practice of musical composers, and theoretical expression was not given to the new principle till the time of Rameau at the beginning of last century. In the historical point of view, therefore, it is wholly the product of modern times, limited nationally to the German, Roman, Celtic, and Sclavonic races.

With this tonal system, which admits great wealth of form with strictly defined artistic consistency, it has become possible to construct works of art, of much greater extent, and much richer in forms and parts, much more energetic in expression, than any producible in past ages; and hence we are by no means inclined to quarrel with modern musicians for esteeming it the best of all, and devoting their attention to it exclusively. But scientifically, when we proceed to explain its construction and display its consensus, we must not forget that our modern system was not developed from a natural necessity, but from a freely chosen principle of style; that beside it, and before it, other tonal systems have been developed from other principles, and that in each such system the highest pitch of artistic beauty has been reached, by the successful solution of more limited problems.

This reference to the history of music was necessitated by our

inability in this case to appeal to observation and experiment for establishing our explanations, because, educated in a modern system of music, we cannot thoroughly throw ourselves back into the condition of our ancestors, who knew nothing about what we have been familiar with from childhood, and who had to find it all out for themselves. The only observations and experiments, therefore, to which we can appeal, are those which mankind themselves have undertaken in the development of music. If our theory of the modern tonal system is correct it must also suffice to furnish the requisite explanation of the former less perfect stages of development.

As the fundamental principle for the development of the European tonal system, we shall assume *that the whole mass of tones and the connection of harmonies must stand in a close and always distinctly perceptible relationship to some arbitrarily selected tonic, and that the mass of tone which forms the whole composition must be developed from this tonic, and must finally return to it.*[1] The ancient world developed this principle in homophonic music, the modern world in harmonic music. But it is evident that this is merely an esthetical principle, not a natural law.

The correctness of this principle cannot be established *à priori.* It must be tested by its results. The origin of such esthetical principles should not be ascribed to a natural necessity. They are the inventions of genius, as we previously endeavoured to illustrate by a reference to the principles of architectural style.

[1] [In Appendix XIX., sections D and E, I have endeavoured to give the stages of this development from a single tonic to the limit of tonic modulation. My object was to determine the tones necessary for musical instruments in just intonation. But the forms and tables thus obtained may be useful in tracing the connections of chords and modulations.—*Translator.*]

CHAPTER XIV.

THE TONALITY OF HOMOPHONIC MUSIC.

MUSIC was forced first to select artistically and then to shape for itself the material on which it works. Painting and sculpture find the fundamental character of their materials, form and colour, in the nature they strive to imitate. Poetry finds its material ready formed in the words of language. Architecture has, indeed, also to create its own forms; but they are partly forced upon it by technical and not by purely artistic considerations. Music alone finds an infinitely rich but totally amorphous plastic material in the tones of the human voice and artificial musical instruments, which must be shaped on purely artistic principles, unfettered by any reference to utility as in architecture, or to the imitation of nature as in the fine arts, or to the existing symbolical meaning of sounds as in poetry. There is a greater and more absolute freedom in the use of the material for music than for any other of the arts. But certainly absolute freedom is more difficult to use properly, than limited freedom where the artist's path is indicated by external landmarks. Hence also the cultivation of the tonal material of music has, as we have seen, proceeded much more slowly than the development of the other arts.

It is now our business to investigate this cultivation.

The first fact that we meet with in the music of all nations, so far as is yet known, is that *alterations of pitch in melodies take place by intervals, and not by continuous transitions.* The psychological reason of this fact would seem to be the same as that which led to rhythmic subdivision periodically repeated. All melodies are motions within extremes of pitch. The incorporeal material of tones is much more adapted for following the musician's intention in the most delicate and pliant manner for every species of motion, than any corporeal material however light. Graceful rapidity, grave procession, quiet advance, wild leaping,

all these different characters of motion and a thousand others in the most varied combinations and degrees, can be represented by successions of tones. And as music expresses these motions, it gives an expression also to those mental conditions which naturally evoke similar motions, whether of the body or the voice, or of the thinking and feeling principle itself. Every motion is an expression of the power which produces it, and we instinctively measure the motive force by the amount of motion which it produces. This holds equally and perhaps more for the motions due to the exertion of power by the human will and human impulses, than for the mechanical motions of external nature. In this way melodic progression can become the expression of the most diverse conditions of human disposition, not precisely of human *feelings*,[1] but at least of that *state of sensitiveness* which is produced by feelings. In English the phrase *out of tune*, *unstrung*, and in German the word *stimmung*, literally *tuning*, are transferred from music to mental states. The words are meant to denote those peculiarities of mental condition which are capable of musical representation. I think we might appropriately define *gemüthsstimmung*, or *mental tune*, as representing that general character temporarily shewn by the motion of our conceptions, and correspondingly impressed on the motions of our body and voice. Our thoughts may move fast or slowly, may wander about restlessly and aimlessly in anxious excitement, or may keep a determinate aim distinctly and energetically in view; they may lounge about without care or effort in pleasant fancies, or, driven back by some sad memories, may return slowly and heavily from the spot with short weak steps. All this may be imitated and expressed by the melodic motion of the tones, and the listener may thus receive a more perfect and impressive image of the 'tune' of another person's mind, than by any other means, except perhaps by a very perfect dramatic representation of the way in which such a person really spoke and acted.

Aristotle also formed a similar conception of the effect of music. In his 29th Problem he says: 'Why do rhythms and melodies, which are composed of sound, resemble the feelings; while this is not the case for tastes, colours, or smells? Can it be

[1] Hanslick seems to me to have the advantage over other æsthetic writers in this point, because music, unassisted by poetry, has no means of clearly characterising the object of feeling.

because they are motions, as actions are also motions? Energy itself belongs to feeling and creates feeling. But tastes and colours do not act in the same way.'[1] And at the end of the 27th Problem he says: 'These motions, *i.e.* rhythms and melodies, are active, and action is the sign of feeling.'[2]

Not merely music but even other kinds of motions may produce similar effects. Water in motion, as in cascades or sea waves, has an effect in some respects similar to music. How long and how often can we sit and look at the waves rolling in to shore! Their rhythmic motion, perpetually varied in detail, produces a peculiar feeling of pleasant repose or weariness, and the impression of a mighty orderly life, finely linked together. When the sea is quiet and smooth we can enjoy its colouring for a while, but this gives no such lasting pleasure as the rolling waves. Small undulations, on the other hand, on small surfaces of water, follow one another too rapidly, and disturb rather than please.

But the motion of tone surpasses all motion of corporeal masses in the delicacy and ease with which it can receive and imitate the most varied descriptions of expression. Hence it arrogates to itself by right the representation of states of mind, which

[1] *Διὰ τί οἱ ῥυθμοὶ καὶ τὰ μέλη φωνὴ οὖσα, ἤθεσιν ἔοικεν· οἱ δὲ χυμοὶ οὔ, ἀλλ' οὐδὲ τὰ χρώματα καὶ αἱ ὀσμαί; ἢ ὅτι κινήσεις εἰσὶν, ὥσπερ καὶ αἱ πράξεις; ἡδὴ δὲ ἡ μὲν ἐνέργεια ἠθικὸν, καὶ ποιεῖ ἦθος· οἱ δὲ χυμοὶ καὶ τὰ χρώματα οὐ ποιοῦσιν ὁμοίως.* Arist. Prob. xix. 29.

[2] [The above words conclude the problem, which it seems best to cite in full. *Διὰ τί τὸ ἀκουστὸν μόνον ἦθος ἔχει τῶν αἰσθητῶν; καὶ γὰρ ἐὰν ᾖ ἄνευ λόγον μέλος, ὅμως ἔχει ἦθος· ἀλλ' οὐ τὸ χρῶμα, οὐδὲ ἡ ὀσμὴ, οὐδὲ ὁ χυμὸς ἔχει. Ἢ ὅτι κίνησιν ἔχει μονονουχὶ, ἣν ὁ ψόφος ἡμᾶς κινεῖ; Τοιαύτη μὲν γὰρ καὶ τοῖς ἄλλοις ὑπάρχει κινεῖ γὰρ καὶ τὸ χρῶμα [καὶ] τὴν ὄψιν· ἀλλὰ τῆς ἑπομένης τῷ τοιούτῳ ψόφῳ αἰσθανόμεθα κινήσεως. Αὕτη δὲ ἔχει ὁμοιότητα, ἔν τε τοῖς ῥυθμοῖς καὶ ἐν τῇ τῶν φθόγγων τάξει τῶν ὀξέων καὶ βαρέων, οὐκ ἐν τῇ μίξει. Ἀλλ' ἡ συμφωνία οὐκ ἔχει ἦθος. Ἐν δὲ τοῖς ἄλλοις αἰσθητοῖς τοῦτο οὐκ ἔστιν. Αἱ δὲ κινήσεις αὗται, πρακτικαί εἰσιν. Αἱ δὲ πράξεις, ἤθους σημασία ἐστί.* Arist. Prob. xix. 27. Which we may perhaps translate thus: 'Why is sound the only sensation which excites the feelings? Even melody without sound has feeling. But this is not the case for colour, or smell, or taste. Is it because they have none of the motion which sound excites in us? For the others excite motion; thus colour moves the eye. But we feel the motion which follows sound. And this is alike, in rhythm, and alteration in pitch, but not in united sounds. Sounding notes together does not excite feeling. This is not the case for other sensations. Now these motions stimulate action, and this action is the sign of feeling.' Aristotle seems to have required motion to excite feeling, and in sounding two notes *together*, there was no motion of one towards the other. It is evident that he had not the slightest inkling of a progression of *harmonies*, and this utter blank in his mind is one of the strongest proofs that the Greeks had never tried harmony.—*Translator.*]

the other arts can only indirectly touch by shewing the situations which caused the emotion, or by giving the resulting words, acts, or outward appearance of the body. The union of music to words is most important, because words can represent the cause of the frame of mind, the object to which it refers, and the feeling which lies at its root, while music expresses the kind of mental transition which is due to the feeling. When different hearers endeavour to describe the impression of instrumental music, they often adduce entirely different situations or feelings which they suppose to have been symbolised by the music. One who knows nothing of the matter is then very apt to ridicule such enthusiasts, and yet they may have been all more or less right, because music does not represent feelings and situations, but only frames of mind, which the hearer is unable to describe except by adducing such outward circumstances as he has himself noticed when experiencing the corresponding mental states. Now different feelings may occur under different circumstances and produce the same states of mind in different individuals, while the same feelings may give rise to different states of mind. Love is a feeling. Direct, as such, it cannot be represented by music. The mental states of a lover may, as we know, shew the extremest variety of change. Now music may perhaps express the dreamy longing for transcendent bliss which love may excite. But precisely the same state of mind might arise from religious enthusiasm. Hence when a piece of music expresses this mental state it is not a contradiction for one hearer to find in it the longing of love, and another the longing of enthusiastic piety. In this sense Vischer's rather paradoxical statement that the mechanics of mental emotion are perhaps best studied in their musical expression, may be not altogether incorrect. We really possess no other means of expressing them so exactly and delicately.

As we have seen, then, melody has to express a motion, in such a manner that the hearer easily, clearly, and certainly appreciates the character of that motion by *immediate perception*. This is only possible when the steps of this motion, their rapidity and amount are also exactly *measurable* by immediate perception. Melodic motion is change of pitch in time. To measure it perfectly, the length of time elapsed, and the distance between the pitches, must be measurable. This is possible for immediate audition only on condition that the alterations both in time and

pitch should proceed by regular and determinate degrees. This is immediately clear for time, for even the scientific, as well as all other measurement of time, depends on the rhythmical recurrence of similar events, the revolution of the earth or moon, or the swings of a pendulum. Thus also the regular alternation of accentuated and unaccentuated sounds in music and poetry gives the measure of time for the composition. But whereas in poetry the construction of the verse serves only to reduce the external accidents of linguistic expression to artistic order; in music, rhythm, as the measure of time, belongs to the inmost nature of expression. Hence also a much more delicate and elaborate development of rhythm was required in music than in verse.

It was also necessary that the alteration of pitch should proceed by intervals, because motion is not measurable by immediate perception unless the amount of space to be measured is divided off into degrees. Even in scientific investigations we are unable to measure the velocity of continuous motion except by comparing the space described with the standard measure, as we compare time with the seconds pendulum.

It may be objected that architecture in its arabesques, which have been justly compared in many respects with musical figures, and which also shew a certain orderly arrangement, constantly employs curved lines and not lines broken into determinate lengths. But in the first place the art of arabesques really began with the Greek meander, which is composed of straight lines set at right angles to each other, following at exactly equal lengths, and cutting one another off in degrees. In the second place, the eye which contemplates arabesques can take in and compare all parts of the curved lines at once, and can glance to and fro, and return to its first contemplation. Hence, notwithstanding the continuous curvature of the lines, their paths are perfectly comprehensible, and it became possible to renounce the strict regularity of the Grecian arabesques in favour of the curvilinear freedom. But whilst freer forms are thus admitted for individual small decorations in architecture, the division of any greater whole, whether it be a series of arabesques or a row of windows or columns &c. throughout a building, is still tied down to the simple arithmetical law of repetition of similar parts at equal intervals.

The individual parts of a melody reach the ear in succession. We cannot perceive them all at once. We cannot observe back-

wards and forwards at pleasure. Hence for a clear and sure measurement of the change of pitch, no means was left but progression by determinate degrees. This series of degrees is laid down in the musical scale. When the wind howls and its pitch rises or falls in insensible gradations without any break, we have nothing to measure the variations of pitch, nothing by which we can compare the later with the earlier sounds, and comprehend the extent of the change. The whole phenomenon produces a confused, unpleasant impression. The musical scale is as it were the divided rod, by which we measure progression in pitch, as rhythm measures progression in time. Hence the analogy between the scale of tones and rhythm naturally occurred to musical theoreticians of ancient as well as modern times.

We consequently find the most complete agreement among all nations that use music at all, from the earliest to the latest times, as to the separation of certain determinate degrees of tone from the possible mass of continuous gradations of sound, all of which are audible, and these degrees form the scale in which the melody moves. But in selecting the particular degrees of pitch deviations of national taste become immediately apparent. The number of scales used by different nations and at different times is by no means small.

Let us inquire, then, what motive there can be for selecting one tone rather than another in its neighbourhood for the step succeeding any given tone? We remember that in sounding two tones together such a relation was observed. We found that under such circumstances certain particular intervals, namely the consonances, were distinguished from all other intervals which were nearly the same, by the absence of beats. Now some of these intervals, the Octave, Fifth, and Fourth, are found in all the musical scales known. Recent theoreticians that have been born and bred in the system of harmonic music, have consequently supposed that they could explain the origin of the scales, by the assumption that all melodies arise from thinking of a harmony to them, and that the scale itself, considered as the melody of the key, arose from resolving the fundamental chords of the key into their separate tones. This view is certainly correct for modern scales; at least these have been modified to suit the requirements of the harmony. But scales existed long before there was any knowledge or experience of harmony at all.

And when we see historically what a long period of time musicians required to learn how to accompany a melody by harmonies, and how awkward their first attempts were, we cannot feel a doubt that ancient composers had no feeling at all for harmonic accompaniment, just as even at the present day many of the more gifted Orientals are opposed to our own harmonic music. We must also not forget that many popular melodies, of older times or foreign origin, scarcely admit of any harmonic accompaniment at all, without injury to their character.

The same remark applies to Rameau's assumption of an 'understood' fundamental bass in the construction of melodies or scales for a single voice. A modern composer would certainly imagine to himself at once the fundamental bass to the melody he invents. But how could that be the case with musicians who had never heard any harmonic music, and had no idea how to compose any? Granted that an artist's genius often unconsciously 'feels out' many relations, we should be imputing too much to it if we asserted that the artist could observe relations of tones which he had never or very rarely heard, and which were destined not to be discovered and employed till many centuries after his time.

It is clear that in the period of homophonic music, the scale could not have been constructed so as to suit the requirements of chordal connections unconsciously supplied. Yet a meaning may be assigned, in a somewhat altered form, to the views and hypotheses of musicians above mentioned, by supposing that the same physical and physiological relations of the tones, which become sensible when they are sounded together and determine the magnitude of the consonant intervals, might also have had an effect in the construction of the scale, although under somewhat different circumstances.

Let us begin with the Octave, in which the relationship to the fundamental tone is most remarkable. Let any melody be executed on any instrument which has a good musical quality of tone, such as a human voice; the hearer must have heard not only the primes of the compound tones, but also their upper Octaves, and, less strongly, the remaining upper partials. When, then, a higher voice afterwards executes the same melody an Octave higher, *we hear again a part of what we heard before*, namely the even partial tones (p. 79) of the former compound tones, and *at the same time we hear nothing that we had not previously heard.* Hence

the repetition of a melody in the higher Octave is a real repetition of what has been previously heard, not of all of it, but of a part.[1] If we allow a low voice to be accompanied by a higher in the Octave above it, the only part music which the Greeks employed, we add nothing new, we merely reinforce the even partials. In this sense, then, the compound tones of an Octave above are really repetitions of the tones of the lower Octaves, or at least of part of their constituents. Hence the first and chief division of our musical scale is that into a series of Octaves. In reference to both melody and harmony, we assume tones of different Octaves which bear the same name, to have the same value, and in the sense intended, and up to a certain point, this assumption is correct. An accompaniment of Octaves gives perfect consonance, but it gives nothing additional; it merely reinforces tones already present. Hence it is musically applicable for increasing the power of a melody which has to be brought out strongly, but it has none of the variety of polyphonic music, and therefore is felt to be *monotonous*, and it is consequently forbidden in polyphonic music.

What is true of the *Octave* is true in a less degree for the *Twelfth*. If a melody is repeated in the Twelfth we again hear only what we had already heard, but the repeated part of what we heard is much weaker, because only the third, sixth, ninth, &c. partial tone is repeated, whereas for repetition in the Octave, instead of the third partial, the much stronger second and weaker fourth partial is heard, and in place of the ninth, the eighth and tenth occur, &c. Hence repetition of a melody in the Twelfth is less complete than repetition in the Octave, because only a smaller part of what had been already heard is repeated. In place of this repetition in the Twelfth, we may substitute one an Octave lower, namely in the *Fifth*. Repetition in the Fifth is not a pure repetition, as that in the Twelfth is. Taking 2 for the vibrational number of the prime tone, the partials are

for the fundamental compound	2	4	6	8		10	12
for the Twelfth . . .			6				12
for the Fifth . . .	3		6		9		12

[1] [Some considerations have been omitted, probably by design. The quality of tone of the voice which sings the Octave above is materially different. The even partials of the lower tone are by no means so powerful as in the higher tone. The upper partials of the higher tone, which are still quite effective, would be inaudible in the lower tone.—*Translator.*]

When we strike the Twelfth we repeat the simple tones 6 and 12, which already existed in the fundamental compound tone. When we strike the Fifth, we continue to repeat the same simple tones, but we also add two others, 3 and 9. Hence for the repetition in the Fifth, only a part of the new sound is identical with a part of what had been heard, but it is nevertheless the most perfect repetition which can be executed at a smaller interval than an Octave. This is clearly the reason why unpractised singers, when they wish to join in the chorus to a song that does not suit the compass of their voice, often take a Fifth to it. This is also a very evident proof that the uncultivated ear regards repetition in the Fifth as natural. Such an accompaniment in the Fifth and Fourth is said to have been systematically developed in the early part of the middle ages. Even in modern music, repetition in the Fifth plays a much more prominent part than repetition in the Octave. In normal fugues the theme, as is well known, is first repeated in the Fifth; in the normal form of instrumental pieces, that of the Sonata, the theme in the first movement is transposed to the Fifth, returning in the second part to the fundamental tone. This kind of imperfect repetition of the impression in the Fifth induced the Greeks also to divide the interval of the Octave into two equivalent sections, namely two *Tetrachords*. Our major scale on being divided in this manner would be:—

$$\underbrace{c \quad d \quad e \quad f}_{\text{I.}} \quad \overbrace{g \quad a \quad b \quad c}^{\text{II.}} \quad \underbrace{d \quad e \quad f}_{\text{III.}}$$

The succession of tones in the second tetrachord is a repetition of that in the first, transposed a Fifth.[1] To pass into the Octave division, the successive tetrachords must be alternately separate and connected. They are said to be *connected* when, as in II. and III., the last tone *c* of the lower becomes the first of the higher tetrachord; and *separate* when, as in I. and II., the last tone of the lower is different from the upper. In the second tetrachord *g* to *c*, every ascending series of tones necessarily leads to *c* as the final tone, and this *c* is also the Octave of the fundamental tone

[1] [This applies to the Pythagorean scale and hence to Greek music, and also to all tempered music. But in just intonation *c* to *d* is a major Tone, and *d* to *e* a minor Tone, whereas *g* to *a* is a minor Tone and *a* to *b* a major tone. These distinctions were of course purposely omitted in the text.—*Translator.*]

of the first tetrachord. Now this *c* is the Fourth of *g*, the fundamental tone of the second tetrachord. To make the succession of tones the same in both tetrachords, the lower tetrachord had to be increased by the tone *f* which answers to *c*. The Fourth *f*, however, would have suggested itself in the same way as the Fifth, independently of this analogy of the tetrachords. The Fifth is a compound tone in which the second partial is the third partial of the fundamental compound tone; the Fourth is a compound tone in which the third partial is the same as the second of the Octave. Hence the limits of the two analogous divisions of the Octave are settled, namely:—

$$c - f, \qquad g - c,$$

but the mode of filling up these gaps remains arbitrary, and different plans for doing so were adopted by the Greeks themselves at different periods, and others again by other nations. But the division of the scale into Octaves, and the Octave into two analogous Tetrachords, occurs everywhere, almost without exception.

Boethius ('De Musica,' lib. i. cap. 20) informs us that according to Nicomachus the most ancient method of tuning the lyre down to the time of Orpheus, consisted of open tetrachords,

$$c - f - g - c,$$

with which certainly it was scarcely possible to construct a melody. But as it contained the chief degrees of the pitch of ordinary speech, a lyre of this kind might possibly have served to accompany declamation.

The relationship of the Fifth, and its inversion the Fourth, to the fundamental tone, is so close that it has been acknowledged in all known systems of music. On the other hand, many variations occur in the choice of the intermediate tones which have to be inserted between the terminal tones of the tetrachord. The interval of a Third is by no means so clearly defined by easily appreciable partial tones, as to have forced itself from the first on the ear of unpractised musicians. We must remember that even if the fifth partial tone existed in the compound tones of the musical instruments employed, it would have had to contend with the much louder prime tone, and would also have been covered by the three adjacent and lower partials. As a matter of fact, the history of

musical systems shews that there was much and long hesitation as to the tuning of the Thirds. And the doubt is even yet felt when Thirds are used in pure melody, unconnected with any harmonies. I must own that on observing isolated intervals of this kind, I cannot come to perfectly certain results, but I do so when I hear them in a well-constructed melody with distinct tonality. The natural major Thirds of 4 : 5 thus seem to me calmer and quieter that the sharper major Thirds of our equally tempered modern instruments, or with the still sharper major Thirds which result from the Pythagorean tuning with perfect Fifths. Both of the latter intervals have a strained effect. Most of our modern musicians, accustomed to the major Thirds of the equal temperament, prefer them to the perfect major Thirds, when melody alone is concerned. But I have convinced myself that artists of the first rank, like Joachim, use the Thirds of 4 : 5 even in melody. For harmony there is no doubt at all. Everyone chooses the natural major Thirds. In Chapter XVI. I shall describe an instrument which will enable anyone to perform experiments of this kind.[1]

Under these circumstances another principle for determining the small intervals of the scale was resorted to during the infancy of music, and seems to be still employed among the less civilised nations. This principle, which has subsequently had to yield to that of tonal relationship, consists in an endeavour to distinguish equal intervals by ear, and thus make the differences of pitch perceptibly uniform.

This attempt has never prevailed over the feeling of tonal relationship for the division of the Fourth, at least in artistically developed music. But in the division of smaller intervals we shall find it applied as an auxiliary in many of the less usual divisions of the Greek tetrachord and in the scales of oriental nations. But arbitrary divisions which are independent of tonal relationship, disappeared everywhere in exact proportion to the higher development of the musical art.

We will now enquire what kind of a scale we should obtain by pursuing to its consequences the natural relationship of the tones. *We shall consider musical tones to be related in the first degree*

[1] [Other instruments with the same intention will be described in Appendix XIX. Section G, of which Nos. 1 and 2 are most simple and inexpensive, and quite sufficient for merely experimental purposes.—*Translator.*]

which have two identical partial tones; and related in the second degree, when they are both related in the first degree to some third musical tone. The louder the coincident in proportion to the non-coincident partials of compound tones related in the first degree, the *closer* is their relationship and the more easily will both singers and hearers feel the common character of both the tones. Hence it follows that the feeling for tonal relationship ought to differ with the qualities of tone; and I believe that this states a fact in nature, because flutes and the soft stops of organs, on which chords are somewhat colourless owing to an absence of upper partials and a consequent incomplete definition of dissonances, retain much of the same colourless character in melodies. This, I think, depends upon the fact, that, for such qualities of tone, the recognition of the natural intervals of the Thirds and Sixths, and perhaps even of the Fourths and Fifths, does not result from the immediate sensation of the hearer, but at most from his recollection. When he knows that on other instruments and in singing he has been able by immediate sensation to recognise the Th irds and Sixths as naturally related tones, he acknowledges them as well-known intervals even when executed by a flute or on the soft stops of an organ. But the mere recollection of an impression cannot possibly have the same freshness and power as the immediate sensation itself.

Since the closeness of relationship depends on the loudness of the coincident upper partial tones, and those having a higher ordinal number are usually weaker than those having a lower one, the relationship of two tones is generally weaker, the greater the ordinal number of the coincident partials. These ordinal numbers, as the reader will recollect from the theory of consonant intervals, also give the ratio of the vibrational numbers of the corresponding notes.

In the following table, the first horizontal line contains the ordinal numbers of the partial tones of the tonic *c*, and the first vertical column those of the corresponding tone in the scale. Where the corresponding vertical columns and horizontal lines intersect, the name of the tone of the scale is given for which this coincidence holds. Only such notes are admitted as are distant from the Tonic by less than an Octave. Below each degree of the scale are placed the two ordinal numbers of the coincident partials,

which will serve as a scale for measuring the closeness of the relationship.

	Partial Tones of the Tonic					
	1	2	3	4	5	6
1	*c* 1 : 1	*c′* 1 : 2				
2	*C* 2 : 1	*c* 2 : 2	*g* 2 : 3	*c′* 2 : 4		
3		*F* 3 : 2	*c* 3 : 3	*f* 3 : 4	*a* 3 : 5	*c′* 3 : 6
4		*C* 4 : 2	*G* 4 : 3	*c* 4 : 4	*e* 4 : 5	*g* 4 : 6
5			*E♭* 5 : 3	*A♭* 5 : 4	*c* 5 : 5	*e♭* 5 : 6
6			*C* 6 : 3	*F* 6 : 4	*A* 6 : 5	*c* 6 : 6

In this systematic comparison we find the following series of notes related to the tonic in the first degree, lying above the fundamental note *c* and related to it in the first degree:

c	*c′*	*g*	*f*	*a*	*e*	*e♭*
1 : 1	1 : 2	2 : 3	3 : 4	3 : 5	4 : 5	5 : 6

and the following series in the Octave below:

c	*C*	*F*	*G*	*E♭*	*A♭*	*A*
1 : 1	2 : 1	3 : 2	4 : 3	5 : 3	5 : 4	6 : 5

The series is discontinued when the resultant intervals become very close. Intervals adapted for practical use must not be too close to be easily taken and distinguished. What is the smallest interval admissible in a scale is a question which different nations have answered differently according to the different direction of their taste, and perhaps also according to the different delicacy of their ear.

It seems that in the first stages of the development of music many nations avoided the use of intervals of less than a Tone, and hence formed scales, which alternated in intervals from a Tone to a Tone and a half. According to examples collected by M. Fétis,[1]

[1] 'Histoire Générale de la Musique,' Paris, 1869, vol. i.

a scale of this kind is found not only among the Chinese but also among the other branches of the Mongol race, among the Malays of Java and Sumatra, the inhabitants of Hudson's Bay, the Papuas of New Guinea, the inhabitants of New Caledonia, and the Fullah negroes. The five-stringed lyre (Kissar) of the inhabitants of North Africa and Abyssinia, which is represented in the bas-reliefs of the Assyrian palaces as an instrument played on by captives, was also, according to Villoteau,[1] tuned by the scale of five degrees:

$$g — a — b — d — e$$

Traces of an ancient scale of this kind are clearly furnished by the five-stringed lyre or lute (κιθάρα) of the Greeks. At least Terpander (circa B.C. 700–650), who played a conspicuous part in the development of ancient Greek music, and who added a seventh string to the former Cithara of six strings, used a scale composed of a tetrachord and a trichord, having the compass of an Octave and tuned thus:—

$$e \smile f — g — a — b \smile — d' — e'$$ [2]

in which there is no c', and the upper tetrachord has no interval of a semitone, although there is an interval of this kind in the lower.[3]

Olympos (circa B.C. 660–620), who introduced Asiatic flute music into Greece and adapted it to Greek tastes, transformed the Greek Doric scale into one of five tones, the *old enharmonic* scale

$$b \smile c — — e \smile f — — a$$

This seems to indicate that he brought a scale of five tones with him from Asia, and merely borrowed the use of the intervals of a semitone from the Greeks. Among the more cultivated nations, the Chinese and the Celts of Scotland and Ireland still retain the

[1] 'Descriptions des Instruments de Musique des Orientaux'; chap. xiii., in the Description de l'Egypte. État Moderne.

[2] Nicomachus makes Philolaus say (edit. Meibomii, p. 17), 'From the Hypatē (e) to the Mesē (a) was a Fourth, from the Mesē (a) to the Nētē (e') a Fifth, from the Nētē (e') to the Tritē (b) a Fourth, from the Tritē (b) to the Hypatē (e) a Fifth.' This shews that c, not b, was the missing note.

[3] [The upper tetrachord was thus reduced to a trichord, while the lower remained a perfect tetrachord.—*Translator.*]

scale of five notes without semitones, although both have also become acquainted with the complete scale of seven notes.

Among the Chinese, a certain prince Tsay-yu is said to have introduced the scale of seven notes amid great opposition from conservative musicians. The division of the Octave into twelve Semitones, and the transposition of scales have also been discovered by this intelligent and skilful nation. But the melodies transcribed by travellers mostly belong to the scale of five notes. The Gaels and Erse have likewise become acquainted with the diatonic scale of seven tones by means of psalmody, and in the present form of their popular melodies the missing tones are sometimes just touched as appoggiature or passing notes. These are, however, in many cases merely modern improvements, as may be seen on comparing the older forms of the melodies, and it is usually possible to omit the notes which do not belong to the scale of five tones without impairing the melody. This is not only true of the older melodies, but of more modern popular airs which were composed during the last two centuries, whether by learned or unlearned musicians. Hence the Gaels as well as the Chinese, notwithstanding their acquaintance with the modern tonal system, hold fast by the old.[1] And it cannot be denied that by avoiding the semitones of the diatonic scale, Scotch airs receive a peculiarly bright and mobile character, although we cannot say as much for the Chinese. Both Gaels and Chinese make up for the small number of tones within the Octave by great compass of voice.

The scale of five tones admits of a certain variety in its construction. Assume *c* as the tonic and add to it the nearest related notes in the ascending Octave, till you come to a semitone. This gives

$$c - c' - g - f - a.$$

The next note *e* would form a semitone with *f*. In the descending Octave we find in the same way

$$c - C - F - G - E\flat.$$

[1] Chinese Melodies, in Ambrosch's 'Geschichte der Musik,' vol. i. pp. 30, 34, 35. Of Scotch melodies there is a fine collection with reference to the authorities and the older forms in G. F. Graham's 'Songs of Scotland,' 3 vols. Edinburgh, 1859. The modern pianoforte accompaniment which has been added, is often ill enough suited to the character of the airs.

The great gaps in the scales between c and f in the first, and between G and c in the second are filled up by tones related in the second degree. Since the tones related to the Octave can only be repetitions of those directly related to the tonic, the next tones to be considered are those related to the upper Fifth g, and lower Fifth F, and these are d (the Fifth above the upper Fifth g) and $B\flat$ (the Fifth below the lower Fifth F). We thus obtain the scales

1) Ascending $\quad c - d - \smile f - g - a - \smile c'$

$$1 \quad \tfrac{9}{8} \quad \tfrac{4}{3} \quad \tfrac{3}{2} \quad \tfrac{5}{3} \quad 2$$

2) Descending $\quad C - \smile E\flat - F - G - \smile B\flat - c$

$$1 \quad \tfrac{6}{5} \quad \tfrac{4}{3} \quad \tfrac{3}{2} \quad \tfrac{16}{9} \quad 2$$

But in place of the tones more distantly related to the tonic in the first degree, both systems of tones related in the second degree might be used, and this would give a scale resulting from a simple progression by Fifths, as

3) $\quad c - d - \smile f - g - \smile b\flat - c'$

$$1 \quad \tfrac{9}{8} \quad \tfrac{4}{3} \quad \tfrac{3}{2} \quad \tfrac{16}{9} \quad 2$$

Then there are also some more irregular forms of this scale of five tones, in which the major Third e replaces the Fourth f, which is more nearly related to the tonic c. This transformation is probably due to the modern preference for the major mode, and it has made its appearance in very many Scotch melodies. The scale is then

4) $\quad c - d - e - \smile g - a - \smile c'$

$$1 \quad \tfrac{9}{8} \quad \tfrac{5}{4} \quad \tfrac{3}{2} \quad \tfrac{5}{3} \quad 2$$

The examples of a similar exchange of the Fifth g for the minor Sixth $a\flat$ are doubtful. This would give the scale

5) $\quad C - \smile E\flat - F - \smile A\flat - B\flat - c$

$$1 \quad \tfrac{6}{5} \quad \tfrac{4}{3} \quad \tfrac{8}{5} \quad \tfrac{16}{9} \quad 2$$

The scale $\quad c - \smile e\flat - f - g - a - \smile c$

$$1 \quad \tfrac{}{5} \quad \tfrac{4}{3} \quad \tfrac{5}{4} \quad \tfrac{5}{3} \quad 2$$

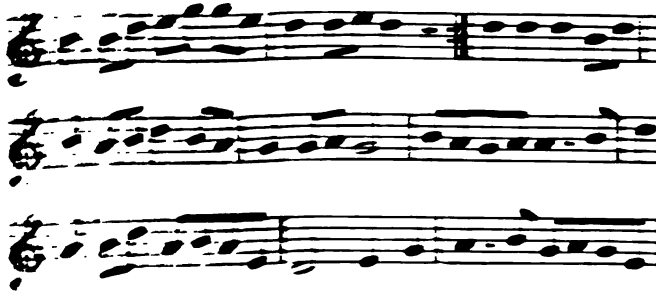
in which all the notes are related in the first degree, but for which the nearest notes to the tonic, either way, are a Tone and a Semitone distant from it, has not yet been discovered in actual use.

The above five forms of the scale of five tones can all be so

[illegible] that they [illegible] on the black notes without touching the lower ones. This is the well-[illegible] rule for composing Scotch melodies. Any one of the notes may then be used as a tonic, but the [illegible] or [illegible] Fifth [illegible] among the black notes [illegible] a very [illegible] as a tonic.

The following are examples of the use of these [illegible] of five tones:—

1. The First Scale without Third or Seventh. [illegible] John Barrow.[1]



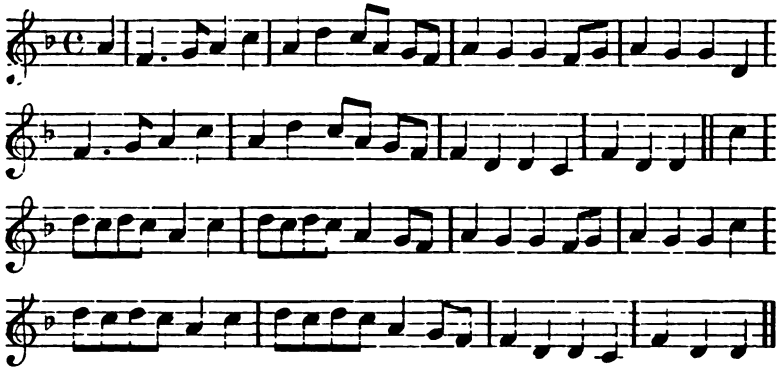
[1] In the following way, the numbers referring to the schemes in the

1, [illegible]
2, [illegible]
3, [illegible]
4, [illegible]
5, [illegible]

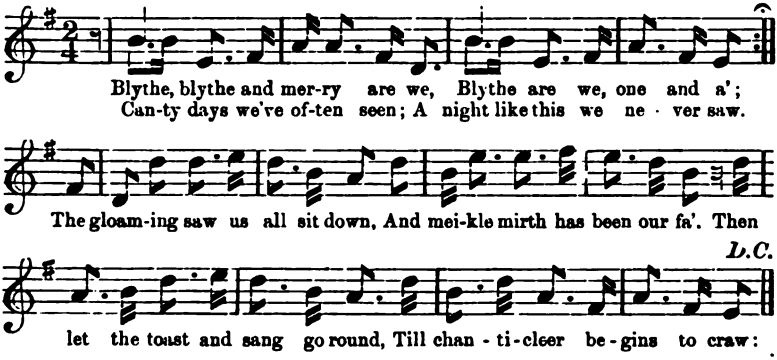
And this shews that all five are formed by a simple succession of Fifth[illegible] black notes arranged in order of Fifths are [illegible] piano being tuned in equal temperament gives very nearly perfect Fif[illegible] very well imitates a succession of five notes thus tuned. If, however [illegible] correct view, then in Nos. 2, 4, and 5 the Pythagorean and not the just or [illegible] Third, minor Third, and minor Sixth are to be used. The scale of five [illegible] treated on this assumption in Appendix XIX., at the end of Section D[illegible] II. A., where names are proposed for the different varieties or modes, [illegible] exceed five, as there are but five tones to commence with. The scale [illegible] the text as not found, would be in the notation of Appendix XIX., c — [illegible] a — c, and could not be arranged as a series of Fifths. If the Pyth[illegible] e♭ and f♯ were chosen, still b♭ and d would be wanted to complete a se[illegible] and if they were introduced we should have the Greek Phrygian m[illegible] of the minor Seventh, described in a few pages further on, with th[illegible] Seventh of the scale omitted. The black digitals of the just or expe[illegible] monium described in App. XIX., Sect. G, No. 2, will allow of playing [illegible] five tones arranged in perfect Fifths, in just intonation.—*Translator.*]

[1] [Scale d — e ⌣ — g — a — b ⌣ — d′. That is, no f and no c.—

2. *To the Second Scale*, without Second or Sixth, belong most Scotch airs which have a Minor character. In the modern forms of these airs one or other of the missing tones is often transiently touched. Here follows an older form of the air called *Cockle Shells.*[1]



3. *For the Third Scale*, without Third and Sixth. Gaelic. Probably an old bagpipe tune.[2]

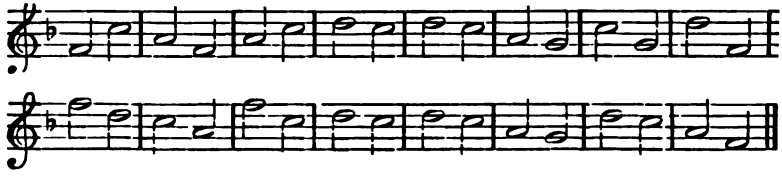


4. *To the Fourth Scale*, without Fourth or Seventh, belong most Scotch airs which have the character of a major mode.

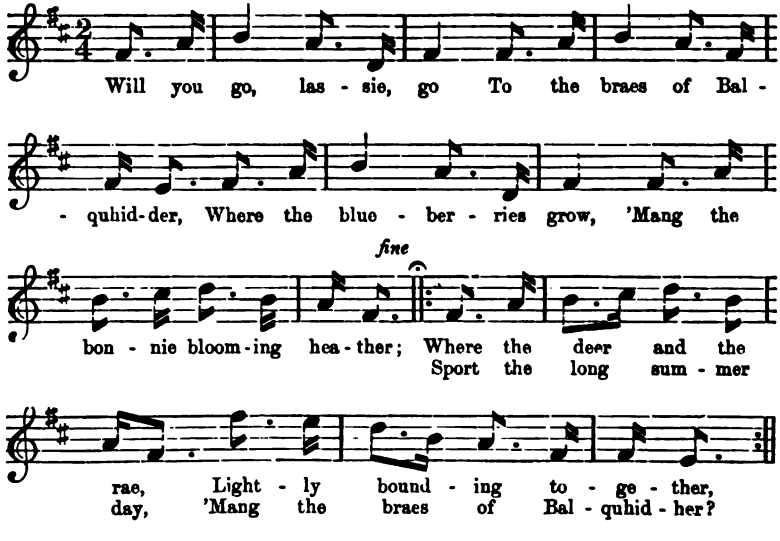
[1] Playford's 'Dancing-master,' ed. 1721. The first edition appeared in 1657. 'Songs of Scotland,' vol. iii. p. 170. [Scale *d* – ⌣ *f* – *g* – *a* ⌣ – *c'* – *d'*, without *e* or *b*♭.—*Translator.*]

[2] There is a Chinese tone of the same kind in Ambrosch, *loc. cit.* vol. i. p. 34, second piece. Another, with a single occurrence of the Sixth, 'My Peggie is a young thing,' may be seen in 'Songs of Scotland,' vol. iii. p. 10. [Scale, *e* – *f*♯ ⌣ – *a* – *b* ⌣ – *d'* – *e'*; without *g* or *c*.—*Translator.*]

Since dozens of Scotch tunes of this kind are to be found in every collection, and are perfectly well known, I give here a Chinese temple hymn, after Bitschurin,[1] as an example:



5. *For the Fifth Scale*, without Second and Fifth, I have found no perfectly pure examples. But there are melodies with either only the Fifth or else with a mere transient use of both Second and Fifth. In the latter case the *minor* Second is used, giving it the character of the ecclesiastical Phrygian tone, for example in the very beautiful air, 'Auld Robin Gray.' I give an example with the tonic *f*♯, in which the Second (*g*♯ or *g*) is altogether absent, and the fifth *c*♯ is only once transiently touched, so that it might just as well have been omitted.



[1] Ambrosch, *loc. cit.* vol. i. p. 30. To the same class belongs the first piece on p. 35 after Barrow and Amiot. [Scale, *f* – *g* – *a* ‿ – *c'* – *d'* – ‿ *f'*, without *b*♭ or *e*.—*Translator*.]

We might also in this example assume *b* as the tonic, and regard the conclusions as formed upon the dominant and subdominant in the old-fashioned way.[1] In these scales of five tones the determination of the tonic is much more doubtful than in the scales of seven tones.

The rule usually given for the Gaelic and Chinese scales, to omit the Fourth and Seventh, applies therefore only to the fourth of the above scales, which corresponds to our major scale.[2] True this scale is often used in the usual Scotch airs of the present day, and is probably due to the reaction of our modern tonal system. But the examples here adduced shew that every possible position may be assumed by the tonic in the scale of five tones, if indeed we allow these scales to have a tonic at all. In Scotch melodies the omissions in both major and minor scales are so contrived as to avoid the intervals of a Semitone, and substitute for them intervals of a Tone and a half. Among the Chinese airs, however, I have found one which belongs rather to the old Greek enharmonic system, to be considered presently, and it will be explained at the same time.

We now proceed to the construction of *scales with seven degrees*. The first form was developed in Greece under the influence of the tetrachordal divisions. The ancient Greek melodies had a small compass and few degrees, a peculiarity especially emphasised even by later authors, as Plutarch, but it is also found among most nations in the early stages of their musical cultivation. Hence the scale was at first formed within a less compass than an Octave, namely within the tetrachord. On looking within this compass for the tones nearest related to the limiting tonic (μέση), we find only the Thirds. Thus if we assume *e* (the last tone in the the tetrachord, *b*—*e*) as a tonic, its next related tone within the compass of that tetrachord is *c*, the major Third below *e*. This gives :—

[1] [Taking *f*♯ as the Tonic, the scale would be No. 5, without Second and Fifth, thus:

f♯ — ◡ *a* — *b* — ◡ *d* — *e* — *f*♯

but taking *b* as the tonic the scale would be No. 2, without Second and Sixth, as

b — ◡ *d* — *e* — *f*♯ — ◡ *a* — *b*

which is altogether different.—*Translator.*]

[2] [Simply because we temper the tone, and hence make the Third a tempered major Third, which we have learned to endure in harmony, instead of a Pythagorean major Third, which would occur on the theory of perfect Fifths, and which is too dissonant for any ears.—*Translator.*]

be tuned by means of intervals of a Fifth, giving:

$$\begin{array}{ccccccc} b & \smile & c & - & d & - & e \\ \frac{4}{3} & & \frac{81}{64} & & \frac{9}{8} & & 1 \end{array}$$

$$\underbrace{\qquad\qquad}_{\frac{243}{256}}\ \underbrace{\qquad}_{\frac{9}{8}}\ \underbrace{\qquad}_{\frac{9}{8}}$$

The tetrachord thus obtained is the Greek *Doric*, which is considered as normal, and made the basis of all considerations on other scales. Accordingly those tones which formed the lower notes of the semitonic intervals of the scale, were, at least theoretically, considered as the immovable limiting tones of the tetrachord while the intermediate tones might change their position. Practically the intonation of even these fixed tones was a little changed, as Plutarch tells us, which may mean that in the *Lydian*, and *Phrygian* modes, &c., the tonic is not selected from one of these so-called fixed tones of the tetrachords. Thus we shall see further on, that when *d* is the tonic, the *b* in the natural intonation of such a scale does not form a perfect Fifth with *e*.

The tetrachord could, however, be differently completed by inserting tones which formed major or minor Thirds with either of the extreme tones.

Two minor Thirds give the *Phrygian* tetrachord—

$$\begin{array}{ccccccc} d & - & e & \smile & f & - & g \\ \frac{3}{4} & & \frac{5}{6} & & \frac{9}{10} & & 1 \end{array}$$

If a major Third were taken upwards from the lower extreme tone, and a minor Third downwards from the upper extreme, we obtain the *Lydian tetrachord*—

$$\begin{array}{ccccccc} c & - & d & - & e & \smile & f \\ \frac{3}{4} & & \frac{5}{6} & & \frac{15}{16} & & 1 \end{array}$$

Two major Thirds would form a variety of the chromatic scale: $b \smile c - d\sharp \smile e$, which does not seem to have been used, or at any rate not to have been distinguished from the chromatic form.[1]

These are all the normal sub-divisions of the tetrachord that

[1] [Adopting the notation of App. XIX., Sect. A., the pitch of the tones in these tetrachords may be accurately written thus:

1. Olympos	$b \smile c - \quad - e$
2. Old Chromatic	$b \smile c \smile \ddagger c\sharp - \smile e$
3. Diatonic	$b \smile c - d - e$
4. Didymus	$b \smile c - \ddagger d - e$
5. Pythagoras (Doric, normal)	$b \smile \ddagger c - \ddagger d - e$
6. Phrygian	$\begin{cases} d - e \smile \dagger f - g \text{ or} \\ b - \ddagger c\sharp - d - e \end{cases}$
7. Lydian	$\begin{cases} c - \ddagger d - e \smile f \text{ or} \\ b - \ddagger c\sharp - \ddagger d\sharp \smile e \end{cases}$
8. Unused	$b \smile c - \ddagger d\sharp \smile e$

have been used. But other subdivisions occur which the Greeks themselves termed *irrational* (ἄλογα), and we do not know with certainty how far they were practically used. One of them, the *soft diatonic mode*, makes use of the interval 6 : 7, which is at any rate very near to a natural consonance, being that between the Fifth and the subminor Seventh of the fundamental note, an interval occasionally used in harmonic music when unaccompanied singers take the minor Seventh of the chord of the dominant Seventh. The intervals are:

$$\overbrace{\underbrace{\tfrac{21}{20} \qquad \tfrac{10}{9}}_{6:7} \qquad \tfrac{8}{7}}^{3:4}$$

By lowering the Lichanos the Parhypatē is also flattened. However, the small interval $\frac{21}{20}$ is very nearly the Pythagorean Semitone, which expressed approximately is $\frac{20}{19}$.[1]

The equal diatonic mode of Ptolemy, which was divided thus:

$$\overbrace{\underbrace{\tfrac{12}{11} \qquad \tfrac{11}{10}}_{5:6} \qquad \tfrac{10}{9}}^{3:4}$$

contained a perfect minor Third divided as evenly as possible.[2]

The second forms of No. 6 and 7 have been given to facilitate comparison with the other tetrachords by using the same extreme tones. All these tetrachords from *b* to *e* (except No. 5) can be played in just intonation on the just concertina, App. XIX. Sect. G, No. 1, and No. 5 may be played as †*a* ⌣ *b*♭ – *c* – *d*. On the just harmonium, No. 5 cannot be played, but the others may be played thus: 1. *d* †*e*♭ *g*, 2. *d* †*e*♭ *e g*, 3. *d* †*e*♭ †*f g*, 4. *d* †*e*♭ *f g*, 6. *d e* †*f g*, 7. *g a b c*, 8. *g* †*a*♭ *b c*.—*Translator*.]

[1] Using the notation of App. XIX. Sect. A, and Table II., ii. for septimal intervals, this tetrachord may be written thus:

$$\overbrace{\underbrace{\underbrace{\tfrac{4}{3}\times\tfrac{63}{64} \quad \tfrac{5}{4}}_{\frac{21}{20}} \quad \underbrace{\tfrac{9}{8}}_{\frac{10}{9}} \quad \underbrace{\tfrac{63}{64}\times 1}_{\frac{8}{7}}}_{6:7}}^{3:4}$$

Translator.]

[2] [In App. XIX. Sect. A, Table II., I have provided for septimal, but not for undecimal or tredecimal intervals. These must be expressed by writing the fractions against the nearest tone. Thus $\frac{11}{9}\,c = \frac{11}{9} \times \frac{4}{5}\,e = \frac{44}{45}\,e$, which will therefore be the proper way of writing the tone. Then this tetrachord may be written—

There is a similar succession of tones, in an inverse order, in the modern Arabic scale as measured by the Syrian, Michael Meshakah.[1] In this case the Octave is divided into twenty-four Quarter tones; the tetrachord has ten of them, its lowest interval four, and each of the upper intervals three. Under these circumstances the two upper intervals together form very nearly a minor Third, which, as in the *equal diatonic* scale of the Greeks, is divided into two equal intervals, without paying regard to any sensible relationship of the intermediate tone thus produced.[2]

The closer the interval, the more easy and certain is its division into two intervals, by the mere feeling for difference of pitch. This is, in particular, possible for intervals which approach to the limits at which differences of pitch are distinguishable by the ear. The distinctness with which the yet sensible difference can be felt then furnishes a measure of its magnitude. In this sense we have probably to explain the possibility of the later *enharmonic mode* of the Greeks, which, however, had already fallen into disuse in the time of Aristoxenus, and was perhaps hunted up again by later writers as an antiquarian curiosity. In this mode the Semitone of the ancient enharmonic mode already mentioned (No. 1, p. 404) was again subdivided into two Quarter Tones, so that a tetrachord was produced like the chromatic one, but with closer intervals

$$\overbrace{f \smile \tfrac{44}{45}\, e - \ddagger d - c}^{3:4}$$

$$\underbrace{\tfrac{4}{3} \quad \tfrac{11}{9}}_{\frac{12}{11}} \quad \underbrace{\tfrac{11}{9} \quad \tfrac{10}{9}}_{\frac{11}{10}} \quad \underbrace{\tfrac{10}{9} \quad 1}_{\frac{10}{9}}$$

$$\underbrace{\qquad\qquad\qquad}_{5:6}$$

Translator.]

[1] 'Journal of the American Oriental Society,' vol. i. p. 173, 1847.

[2] [As the equally tempered scale is divided into 12 Semitones, this Quarter Tone will be half an equal Semitone, or $\sqrt[24]{2} = 1{\cdot}029302$, or nearly $\frac{35}{34}$. Hence using || as in Appendix XIX. Table II., iii., for denoting the notes in the equal temperament, the tetrachord of the text might be written:

$$\begin{array}{ccccccc} \| f & - & \| e\flat & - & \frac{35}{34} & \| c\sharp & - c \\ \sqrt[24]{2}^{10} & - & \underbrace{\sqrt[24]{2}^{6} \quad - \quad \sqrt[24]{2}^{3}}_{\sqrt[12]{2}^{3}:1} & & & & - 1 \end{array}$$

If such Quarter Tones were in use it would be necessary to have a special sign as ||| to represent them, so that the series would begin $\|c - \|\|c - \|c\sharp - \|\|c\sharp - \|d - \|\|d - \|d\sharp - \| d\sharp$, &c., and the additional bar means, 'multiply by $\sqrt[24]{2}$.' —*Translator.*]

between the adjacent tones. The division of this *enharmonic tetrachord* [1] was

$$\overbrace{\frac{32}{31} \qquad \frac{31}{30} \qquad \frac{5}{4}}^{3\,:\,4}$$

This Quarter Tone can only be considered as a transition in the melodic movement towards the lowest extreme of the tetrachord. A similar interval occurs in this way in existing Oriental music. A distinguished musician whom I requested to pay attention to it on a visit to Cairo, wrote to me as follows: 'This evening I have been listening attentively to the song on the minarets, to try to appreciate the Quarter Tones, which I had not supposed to exist, as I had thought that the Arabs sang *out of tune*. But to-day as I was with the dervishes I became certain that such Quarter Tones existed, and for the following reasons. Many passages in litanies of this kind end with a tone which was at first the Quarter Tone and then ended in the pure tone. As the passage was frequently repeated, I was able to observe this every time, and I found the intonation invariable.' The Greek writers on music themselves say that it is difficult to distinguish the enharmonic Quarter Tones.

The later interpreters of Greek musical theory have mostly advanced the opinion that the above-mentioned differences, which the Greeks called *colourings* (χρόαι), were merely speculative and never came into practical use.[2] They consider that these distinctions were too delicate to produce any esthetic effect except on an incredibly well cultivated ear. But it seems to me that this opinion could never have been entertained or advanced by modern theorists, if any of them had practically attempted to form these various tonal modes and to compare them by ear. On an harmonium which will shortly be described I am able to compare natural

[1] [If we use δ for $\frac{31}{30}$, this tetrachord may then be written

$$\overbrace{f \smile \delta e \smile e - c}^{3\,:\,4}$$

$$\frac{4}{3} \qquad \frac{31}{24} \qquad \frac{5}{4} \qquad 1$$

$$\underbrace{\frac{32}{31} \qquad \frac{31}{30}}_{15\,:\,16} \qquad \frac{5}{4}$$

Translator.]

[2] Even Bellerman is of this opinion ('Touleitern der Griechen,' p. 27). Westphal, in his 'Fragmenten der Griechischen Rhythmiker,' p. 209, has collected passages from Greek writers proving the real practical use of these intervals. According to Plutarch ('De Musica,' pp. 38 and 39), the later Greeks had even a preference for these surviving intervals.

intonation with Pythagorean, and to play the diatonic mode at one time after the method of Didymus and at another after that of Ptolemy, and also to make other deviations. It is not at all difficult to distinguish the difference of a comma $\frac{81}{80}$ in the intonation of the different degrees of the scale, when well-known melodies are performed in different 'colourings,' and every musician with whom I have made the experiment has immediately heard the difference. Melodic passages with Pythagorean Thirds have a strained and restless effect, while the natural Thirds make them quiet and soft, although our ears are habituated to the Thirds of the equal temperament, which are nearer to the Pythagorean than to the natural intervals. Of course where delicacy in any artistic observations made with the senses, comes into consideration, moderns must look upon the Greeks in general as unsurpassed masters. And in this particular case they had very good reason and abundance of opportunity for cultivating their ear better than ours. From youth upwards we are accustomed to accommodate our ears to the inaccuracies of equal temperament, and the whole of the former variety of tonal modes, with their different expression, has reduced itself to such an easily apprehended difference as that between major and minor. But the varied gradations of expressions which moderns attain by harmony and modulation, had to be effected by the Greeks and other nations that use homophonic music, by a more delicate and varied gradation of the tonal modes. Can we be surprised, then, if their ear became much more finely cultivated for differences of this kind than it is possible for ours to be?

The Greek scale was soon extended to an Octave. Pythagoras is said to have been the first to establish the eight complete degrees of the diatonic scale. At first two tetrachords were connected in such a way as to have a common tone, the μέση:

$$\underbrace{e \smile f - g - a}\ \underbrace{\smile b\flat - c - d}$$

which produced a scale of seven degrees. Then this scale was changed into the following form:

$$\underbrace{e \smile f - g - a} - \underbrace{b \smile - d - e}$$

and thus made to consist of a tetrachord and a trichord, of which mention has already been made (p. 397). Finally Lichaon of

Samos, (according to Boethius,) or Pythagoras, (according to Nicomachus,) completed the trichord into a tetrachord, and thus established a scale consisting of two disjunct tetrachords.

The diatonic scale thus obtained could be continued either way at pleasure by adding higher and lower octaves, and then produced a regularly alternating series of Tones and Semitones. But for each piece of music a portion only of this unlimited diatonic scale was employed, and the tonal systems were distinguished by the character of the portions selected.

These sectional scales might be produced in very different ways. The first practical object which necessarily forces itself on attention, as soon as an instrument with a limited number of strings, like the Greek lyre, is used for executing a piece of music, is, of course, that there should be a string for every musical tone required. This prescribes a certain series of tones which must be provided and tuned on the instrument. Now as a rule when a certain series of tones is thus prescribed as a scale for the tuning of a lyre, no question is raised as to whether a tonic is to be distinguished or not, or if so which it should be. A tolerable number of melodies may be found in which the lowest tone is the tonic: others in which an interval below the tonic is touched; and others, again, in which the Fifth or Fourth above the Octave below the tonic is used. This is the kind of difference between the *authentic* and *plagal* scales of the middle ages. In the authentic scales the deepest tone of the scale, in the plagal its Fifth, was the tonic; thus:—

FIRST AUTHENTIC ECCLESIASTICAL SCALE, tonic *d*.

$$\underbrace{d - e - f - g - a}\ \overbrace{- b - c - d}$$

FOURTH PLAGAL SCALE, tonic *g*.

$$\overbrace{d - e - f - g}\ \underbrace{- a - b - c - d}$$

The scales were looked upon as composed of a Fifth and a Fourth, as the brackets shew. In the authentic tone the Fifth lay below; in plagal, above. Now if we have nothing else before us but a scale of this kind, which marks out the accidental compass of a series of melodies, we can collect but little respecting the key. Such scales themselves may be fittingly termed *acci-*

dental. They comprise, among others, the medieval plagal scales. On the other hand, those scales which, like the modern, are bounded at each extremity by the tonic, may be termed *essential.* Now practical needs clearly lead in the first place to accidental scales alone. When a lyre had to be tuned to accompany the human voice in unison, it was indispensably necessary that all the tones required should be present. There was no immediate practical need for marking the tonic of a song sung in unison, or even to become fully aware that it had a tonic at all. In modern music, where the structure of the harmony essentially depends on the tonic, the case is entirely different. Theoretical considerations on the structure of melody could alone lead to distinguishing one tone as tonic. It has been already mentioned in the preceding chapter, that Aristotle, as a writer on esthetics, has left a few notices indicating such a conception, but that the authors who have especially written on music say nothing about it.

In the best times of Greece, song was usually accompanied by an eight-stringed lyre, tuned so as to embrace an Octave of tones selected from the diatonic scale. These were the following:—

1. Lydian . . . $\underbrace{c - d - e - f} - \overbrace{g - a - b - c}$
2. Phrygian . . . $\underbrace{d - e - f - g} - \overbrace{a - b - c - d}$
3. Doric . . . . $\underbrace{e - f - g - a} - \overbrace{b - c - d - e}$
4. Hypo-lydian . . $f - \underbrace{g - a - b -} \overbrace{c - d - e - f}$
5. Hypo-phrygian (Ionic) . . $\Big\}$ $g - \underbrace{a - b - c -} \overbrace{d - e - f - g}$
6. Hypo-doric (Eolic or Locrian) . . $\Big\}$ $a - \underbrace{b - c - d -} \overbrace{e - f - g - a}$
7. Mixo-lydian . . $b - \underbrace{c - d - e - f} - \overbrace{g - a - b - (c)}$

Hence any one of the tones in the diatonic scale could be used as the initial or final extremity of such a tonal mode. The Lydian and Hypolydian scales contain Lydian, the Phrygian and Hypophrygian contain Phrygian, and the Doric and Hypodoric contain Doric tetrachords. In the Mixolydian two Lydian tetrachords seem to have been assumed, one of which was divided, as shewn by the ties above.

The scales or *tropes* of the best Greek period have hitherto been considered as essential, that is, the lowest tone or *hypatē* has been considered as the tonic. But I cannot find any definite ground for this assumption. What Aristotle says, as we have seen, makes the middle tone or *mesē*, function as the tonic, but yet it cannot be denied that other attributes of our tonic belong to the *hypatē*.[1] Whatever may have been the real state of the case, whether the *mesē* or *hypatē* be regarded as the tonic, whether the scale be considered as all authentic or all plagal, it is extremely probable that the Greeks, among whom we first find the complete diatonic scale, took the liberty of using every tone of this scale as a tonic, just as we have seen that every one of the five tones forming the scales of the Chinese and Gaels occasionally functions as a tonic. The same scales are also found, probably handed down immediately by ancient tradition, in the ancient Christian ecclesiastical music.

Hence if we disregard the chromatic and enharmonic scales, and the apparently arbitrary scales of the Asiatics, none of which have shewn themselves capable of further development, homophonic vocal music developed seven diatonic scales, which differ from one another in about the same way as our major and minor scales. These differences will be better appreciated by making them all begin with the same tonic *c*.

[1] R. Westphal, in his 'Geschichte der alten und mittelalterlichen Music,' Breslau, 1864, which is unfortunately still incomplete, uses the above citations from Aristotle, to frame an hypothesis on the tonic and final cadence of the above scales. But he applies the remarks of Aristotle only to the Doric, Phrygian, Lydian, Mixolydian and Locrian scales, and not to the Eolic and Ionic, which were also known at that time, although the ground for their exclusion is not apparent. In the first four of these he takes the *mesē* as tonic and the *hypatē* as the terminal tone. In those scales distinguished by the prefix *Hypo-*, the *hypatē* was both tonic and terminal; but in those having the prefix *Syntono-*, the *hypatē* was both terminal and the Third of the tonic, and the same was the case perhaps for the *Boeotian* scale, which is only mentioned once. Hence it follows that the minor scale of *A* occurs as Doric with the terminal *e*, as Hypodoric with the terminal *a*, as Boeotian with the terminal *c*. Moreover the Mixolydian would be a minor scale of *E*, with a minor Second, and a terminal in *b*; the Locrian a minor scale of *D* with a major Sixth, and a terminal in *a*; the Phrygian, Hypophrygian or Iastic, and the Syntono-iastic, major scales of *G*, with a minor Seventh, the terminals being *d*, *g*, and *b* respectively. Finally the Lydian, Hypolydian and Syntono-lydian would be major scales of F, with superfluous Fourth, and with the terminals *c*, *f*, and *a* respectively. But according to Westphal the normal major scale was entirely absent. If the Ionic were interpreted according to the words of Aristotle, it would yield a correct major scale. The tonic *F* with *B* (instead of *B♭*) as its Fourth, has a totally impossible appearance to modern musical feeling.

Ancient Greek Names	Scales beginning with *c*	Glarean's Ecclesiastical Names	Proposed new Names
			Mode of the
1. Lydian .	*c*–*d* –*e* –*f* –*g* –*a* –*b* –*c'*	Ionic	First (*major*)
2. Ionic . .	*c*–*d* –*e* –*f* –*g* –*a* –*b*♭ –*c'*	Mixo-lydian	Fourth
3. Phrygian .	*c*–*d* –*e*♭ –*f* –*g* –*a* –*b*♭ –*c'*	Doric	minor Seventh
4. Eolic . .	*c*–*d* –*e*♭ –*f* –*g* –*a*♭ –*b*♭ –*c'*	Eolic	minor Third (*minor*)
5. Doric . .	*c*–*d* –*e*♭ –*f* –*g* –*a*♭ –*b*♭ –*c'*	Phrygian	minor Sixth
6. Mixo-lydian.	*c*–*d*♭ –*e*♭ – *f* –*g*♭ –*a*♭ –*b*♭ –*c'*	Lydian	minor Second
7. Syntono-lydian .	*c*–*d* –*e* –*f*♯ –*g* –*a* –*b* –*c'*		Fifth

To assist the reader I have added the names assigned to the ecclesiastical modes by Glarean, which were wrongly distributed among the scales owing to his confusing the older tonal modes with the later, transposed, minor Greek scales, but which are more known among musicians than the proper Greek names. But I shall not use Glarean's names without expressly mentioning that they refer to an ecclesiastical mode. It would be really better to forget them altogether. The old numerical notation of Ambrose was much more suitable, but as his figures have been altered again and do not suffice for all modes, I have ventured to propose a new nomenclature in the above table, which will save the reader the trouble of memorising the systems of Greek names, of which Glarean's are certainly wrong, and the others are also perhaps not quite correctly applied. The principle of the new nomenclature is this. By 'the mode of Fourth of *C*,' is meant a mode of which *C* is the tonic, but which has the same *signature* (or additional ♯ and ♭ signs) as the major scale formed on the Fourth of the diatonic scale beginning with *C*; that is on *F*. The minor Seventh, minor Third, minor Sixth, and minor Second must always be understood as the intervals intended in this case.[1] If the major intervals were selected the tonic would not occur in their scales. Thus 'the mode of the minor Third of *C*' is the scale with the tonic *C*, having the signature of *E*♭ major (that is *B*♭, *E*♭, *A*♭), because *E*♭ is the minor Third of *C*; this is therefore *C* minor, at least as it is played

[1] [The qualification *minor* will therefore be always used in this translation, and has been inserted in the above table.—*Translator.*]

in the descending scale. I hope the reader will have no difficulty in understanding what is meant by this notation.[1]

This was the tonal system in the best times of Greek art, up to the Macedonian empire. Airs were at first limited to a tetrachord, as is still often the case in the Roman Catholic liturgy. They were afterwards extended to an Octave. Longer scales were not necessary for singing, as the Greeks refused to employ the straining upper notes and unmetallic deep notes of the human voice. Modern Greek songs, of which Weitzmann has made a collection,[2] have also a surprisingly small compass. If Phrynis (victor in the Panathenaic competitions, B.C. 457) added a ninth string to his cithara, the chief advantage of the arrangement was to allow of passing from one kind of scale to another.

The later Greek scale, which first occurs in Euclid's works of the third century B.C., embraces two Octaves, thus arranged:

A	added Tone	*Proslambanomenos*	
B	lowest Tetrachord	*Tetra. hypatōn*	
c			
d			
e	middle Tetrachord	*Tetra. mesōn*	
f			
g			
a		*a*	conjunct Tetr. *T. synēmmenōn.*
b	disjunct Tetra. *T. diezeugmenōn*	*b♭*	
c′		*c′*	
d′		*d′*	
e′	superfluous Tetr. *T. hyperbolaiōn.*		
f′			
g′			
a′			

[1] [In Appendix XIX. Section D, I have endeavoured to deduce scales for harmonic use, from a general theory of harmony which determines the precise value of each tone as part of a chord, and I have given precise names for them, which are exemplified in Table III. In that table I have also included the Greek scales, on the assumption of Pythagorean intonation, and exhibited the difference between them and those in just intonation. This harmonic deduction of scales is quite independent of the historical melodic deduction in the text. In the preceding table the precise intonation of each scale is not given. That is considered afterwards. In the Table III. of Appendix XIX., I have referred back to Prof. Helmholtz's names, in relation to the harmonies with which he subsequently connects them, and not in relation to the present identification with the Greek scales, which were excluded from a purely harmonic investigation.—*Translator.*]

[2] 'Geschichte der Griecischen Musik,' Berlin, 1855.

This scheme gives first the Hypodoric scale (No. 6 of p. 411) for two Octaves, and then an added tetrachord which introduces a *b*♭ in addition to the *b*, and thus in modern language, allows of modulation from the principal scale into that of the subdominant.[1]

This scale, essentially of a minor character, was transposed, and thus a new series of scales were generated that correspond with the (descending) minor scales of modern music. To these were applied the old names of the tonal modes, by giving originally to each minor mode the name belonging to that tonal mode which was formed by the section of the minor scale which lay between the extreme tones of the Hypodoric[2] scale. According to the Greek method of representing the notes, these extreme tones would have to be written *f* — *f*. Their actual pitch was probably a Third lower. Thus the minor scale of *D* was called Lydian, because in this scale—

$$d-e- \mid f-g-a-b\flat-c-d-e-f \mid g-a-b\flat-c-d$$

the section of the scale lying between the extreme tones *f* and *f* belonged to the Lydian tonal mode. In this way the old names of the *tonal modes* altered their meaning into those of *tonal keys*. The following table shews the correspondence of these names :—

1) Hypo-doric	= *F*	minor	8) Phrygian	= *C*	minor
2) Hypo-ionic (deeper Hypo-phrygian)	= *F*♯	minor	9) Eolic (deeper Lydian)	= *C*♯	minor
3) Hypo-phrygian	= *G*	minor	10) Lydian	= *D*	minor
4) Hypo-eolic (deeper Hypo-lydian)	= *G*♯	minor	11) Hyper-doric (Mixo-lydian)	= *E*♭	minor
5) Hypo-lydian	= *A*	minor	12) Hyper-ionic (higher Mixo-lydian)	= *E*	minor
6) Doric	= *B*♭	minor	13) Hyper-phrygian (Hyper-mixo-lydian)	= *f*	minor } later additions
7) Ionic (deeper Phrygian)	= *B*	minor	14) Hyper-eolic	= *f*♯	minor } later additions
			15) Hyper-lydian	= *g*	minor } later additions

Within each of these scales each of the previously mentioned tonal modes might be formed, by using the corresponding part of the scale. Besides this it was possible to pass into the conjunct tetrachord and thus modulate into the tonal key of the subdominant.

[1] Singularly enough this species of musical scale has been preserved in the Zillerthal in Tyrol, for the wood-harmonica. This scale has two rows of sticks. One forms a regular diatonic scale with the disjunct tetrachord. The other, which lies deep. has the conjunct tetrachord in its upper part.

[2] [This seems to be an error for Hypolydian, No. 4 of p. 411.—*Translator.*]

The experiments on transposition which formed the basis of these scales shewed that the Octave might be considered as composed approximatively of twelve Semitones. Even Aristoxenus knew that by taking a series of twelve Fifths we reached a tone that was at least very near to a higher Octave of the initial tone. Thus in the series

$$f-c-g-d-a-e-b-f\sharp-c\sharp-g\sharp-d\sharp-a\sharp-e\sharp$$

he identified $e\sharp$ with f, and by thus closing the series of tones he obtained a circle of Fifths. Mathematicians denied the fact, and with reason, because if the Fifths are taken perfectly true, $e\sharp$ is a little sharper than f. For practical purposes, however, the error was quite insensible, and might be justly neglected in homophonic music in particular.[1]

This closes the development of the Greek tonal system. Complete as is our acquaintance with its outward form, we know but little of its real nature, because the examples of melodies which we possess are not only few in number, but very doubtful in origin.

Whatever may have been the nature of tonality in Greek scales, and however numerous may be the questions about it that are still unresolved, yet so far as the theory of the general historical development of tonal modes is concerned we learn all we want from the laws of the earliest Christian ecclesiastical music, which at its commencement touched upon the ancient construction as it died out. In the fourth century of our era, Bishop Ambrose, of Milan, established four scales for ecclesiastical song, which in the untransposed diatonic scale were:

[1] It is by no means an unimportant fact, for our appreciation of the Greek scale, that a flute was found in the royal tombs at Thebes in Egypt (now in the Florentine Museum, No. 2,688), which, according to M. Fétis, who examined it, gave an almost perfect scale of Semitones for about an Octave and a half; namely,

Series of primes	a	$b\flat$	b	c'	$c'\sharp$	d'
First upper partial tones	a'	$b'\flat$	b'	c''	$c''\sharp$	d''
Second upper partial tones	e''	f''	$f''\sharp$	g''	$g''\sharp$	a''
Third upper partial tones	a''	$b''\flat$	b''	c'''	$c'''\sharp$	d'''

Representations of such flutes are found in the very oldest Egyptian monuments. They are very long, the holes are all near the end, and hence the arms must have been greatly stretched, giving the player a characteristic position. The Greeks can scarcely have been ignorant of this scale of Semitones. That it was not introduced into their theory till after the time of Alexander, clearly shews the preference they gave to the diatonic scale.

First key: *d* — *e* — *f* — *g* — *a* — *b* — *c* — *d* { mode of the minor Seventh.

Second key: *e* — *f* — *g* — *a* — *b* — *c* — *d* — *e* { mode of the minor Sixth.

Third key: *f* — *g* — *a* — *b* — *c* — *d* — *e* — *f* { mode of the Fifth (unmelodic).

Fourth key: *g* — *a* — *b* — *c* — *d* — *e* — *f* — *g* { mode of the Fourth.

The variable character of the tone *b*, which was transmutable into *b*♭ in the later Greek scales, remained, and produced the following scales :—

First: *d* — *e* — *f* — *g* — *a* — *b*♭ — *c* — *d* { mode of the minor Third.

Second: *e* — *f* — *g* — *a* — *b*♭ — *c* — *d* — *e* { mode of the minor Second (unmelodic).

Third: *f* — *g* — *a* — *b*♭ — *c* — *d* — *e* — *f* { mode of the First (*major*).

Fourth: *g* — *a* — *b*♭ — *c* — *d* — *e* — *f* — *g* { mode of the minor Seventh.

There can be no doubt that these Ambrosian scales are to be regarded as essential (see p. 411), for the old rule is that melodies in the first are to end in *d*, those in the second in *e*, those in the third in *f*, and those in the fourth in *g*, and this marks the initial tones of the scale as tonics. We may certainly assume that this arrangement was made by Ambrose for his choristers as a practical simplification of the old musical theory, which was overladened with an inconsistent nomenclature, and this leads us to conclude that we were right in conjecturing that the similar older Greek scales could have been really used as different essential scales.

Pope Gregory the Great inserted between the Ambrosian essential scales the same number of accidental scales (pp. 410–11), called *plagal*, proceeding from the Fifth to the Fifth of the tonic. The Ambrosian scales were, then, called *authentic* for distinction. The existence of these plagal ecclesiastical scales helped to increase the confusion which broke over the ecclesiastical scales towards the end of the middle ages, as composers began to neglect the rules which fixed the terminal tones, and this confusion assisted in

favouring a freer development of the tonal system. This confusion also shewed, as we remarked in the last chapter, that no feeling for the thorough predominance of the tonic was much developed in the middle ages. But a step, at least, was made in advance of the Greeks, by recognising as a rule that the piece should close on the tonic, although this rule was not always observed.

Glarean endeavoured in 1547 to reduce the theory of the scales to order again, in his *Dodecachordon*. He shewed by an examination of the musical compositions of his contemporaries, that *six*, and not *four*, authentic scales should be distinguished, and adorned them with the Greek names in the table on p. 413. Then he assumed six plagal scales, and hence on the whole distinguished twelve tonal keys, whence the name of his book. Hence down to the sixteenth century essential and accidental scales were reckoned as parts of one series. Among Glarean's scales one is unmelodic, namely the mode of the Fifth, which he calls the Lydian. There are no examples of these to be found, as we know from a careful examination of medieval compositions made by Winterfeld,[1] and this confirms Plato's opinion of the Mixolydian and Hypolydian modes.

Hence there remain the following five melodic tonal modes applicable strictly for homophonic and polyphonic vocal music, namely:

	In our Nomenclature	Ancient Greek	Glarean's Names	Scale
1	Major Mode . . .	Lydian	Ionic	*C* – *c*
2	Mode of the Fourth .	Ionic	Mixolydian	*G* – *g*
3	Mode of the minor Seventh	Phrygian	Doric	*D* – *d*
4	Mode of the minor Third	Eolic	Eolic	*A* – *a*
5	Mode of the minor Sixth .	Doric	Phrygian	*E* – *e*

The *rational construction* of these scales when extended to the Octave or beyond the Octave results from the principle of tonal relationship already explained. The limits of the extent to which tones related in the first degree should be used, are determined by the necessity of avoiding intervals too close to be distinguished with certainty. The larger gaps thus left have to be filled with the tones most nearly related in the second degree.

[1] von Winterfeld's 'Johannes Gabrieli und sein Zeitalter,' Berlin, 1834, vol. i. pp. 73 to 108.

The Chinese and Gaels made the whole Tone $\frac{10}{9}$ the smallest interval. The Orientals, as we have seen, still retain Quarter Tones. The Greeks experimented with them, but soon gave them up and kept to the Semitone $\frac{16}{15}$ as the smallest.

European nations have followed Greek habits, and retained the Semitone $\frac{16}{15}$ as the limit. The interval between $E\flat$ ($\frac{6}{5}$) and E ($\frac{5}{4}$), and between $A\flat$ ($\frac{8}{5}$) and A ($\frac{5}{3}$) in the natural scale is smaller, being $\frac{25}{24}$, and we consequently avoid using both $E\flat$ and E, or both $A\flat$ and A in the same scale. We thus obtain the following two series of intervals between the most nearly related tones for ascending and descending scales:

$$\begin{array}{lccccccccccc}
\text{Ascending:} & c & — & — & e & — & f & — & g & — & a & — & — & c' \\
 & & \frac{5}{4} & & & \frac{16}{15} & & \frac{9}{8} & & \frac{10}{9} & & \frac{6}{5} & & \\
\text{Descending:} & c & — & — & A\flat & — & G & — & F & — & E\flat & — & — & C \\
 & & \frac{5}{4} & & & \frac{16}{15} & & \frac{9}{8} & & \frac{10}{9} & & \frac{6}{5} & &
\end{array}$$

The numbers below the series shew the intervals between the two tones between which they are placed.

It is at once seen that the intervals from and to the tonic are too large, and might be further divided. But as we have come to the limits of relationship in the first degree, we have to fill these gaps by tones related in the second degree.

The closest relationship in the second degree is necessarily furnished by the tones most nearly related to the tonic. Among these the Octave stands first. The tones related to the Octave of the tonic are of course the same as those related to the tonic itself; but by passing to the Octave of the tonic we obtain the descending in place of the ascending scale, and conversely.

Thus, ascending from c we found the following degrees of our major scale—

$$c — — e — f — g — a — — c'$$

But taking the tones related to c', we obtain—

$$c — — e\flat — f — g — a\flat — — c'.$$

Hence the second degree of relationship to the tonic gives an ascending minor scale. In this scale $e\flat$ is given as the major Sixth below c'. But it has also the weak relationship to c marked by 5 : 6. Now we found that the sixth partial of a compound tone was clearly audible in many qualities of tone for which

the seventh or eighth could not be heard; for example, on the pianoforte, the narrower organ pipes, and the mixture stops of the organ. Hence the relationship expressed by 5 : 6 may often become evident as a natural relationship in the first degree. This, however, could scarcely be the case for the relationship c — $a\flat$ or 5 : 8. Hence it is more natural to change e into $e\flat$ than a into $a\flat$ in the ascending scale. The latter, $a\flat$, can only be related to the tonic in the second degree. Hence the three ascending scales in order of intelligibility are—

$$c - - e - f - g - a - - c'$$
$$c - - e\flat - f - g - a - - c'$$
$$c - - e\flat - f - g - a\flat - - c'$$

These distinctions based on a relationship in the second degree, through the medium of the Octave, are certainly very slight, but they make themselves felt in the well-known transformation of the ascending minor scale, to which these distinctions clearly refer.

Descending from c, instead of the relations in the first degree, given in

$$c - - A\flat - G - F - E\flat - - C$$

we may assume relations in the second degree, that is of the deeper C, and obtain

$$c - - A - G - F - E - - C$$

In the latter, A is connected with the initial tone by the distant relationship in the first degree, 5 : 6, and E only by a relationship in the second degree. Hence the third descending scale

$$c - - A - G - F - E\flat - - C$$

which we also found as an ascending scale. For descending scales we have therefore the following series

$$c - - A\flat - G - F - E\flat - - C$$
$$c - - A - G - F - E\flat - - C$$
$$c - - A - G - F - E - - C$$

Generally, since all Octaves of the tonic, distant or near, higher or lower, are so closely related that they can be almost identified with it, all higher and lower Octaves of the individual degrees of the scale are almost as closely related to the tonic, as those of the next adjacent tonic of the same name.

Next to the relations of the Octave c' of c, follow those of g, the Fifth above, and F the Fifth below c. We must therefore proceed to study their effect in the construction of the scale. Let us begin with the relations of g, the Fifth above the tonic.

ASCENDING SCALES.

Related to c:	c	—	—	e	—	f	—	g —	a — — c'
Related to g:	c	d	$e\flat$	—		—	g	— —	b — c'

Uniting the two, we have—

1) *The Major Scale* (Lydian mode of the ancient Greeks):

c —	d —	e —	f —	g —	a —	b —	c'
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

The change of e into $e\flat$ is here facilitated by its second relationship to g. This gives—

2) *The Ascending Minor Scale:*

c —	d —	$e\flat$ —	f —	g —	a —	b —	c'
1	$\frac{9}{8}$	$\frac{6}{5}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

DESCENDING SCALES.

Related to c:	c	—	—	$A\flat$ —	G —	F —	$E\flat$	— —	C
Related to g:	c	$B\flat$	—	— —	G —	— —	$E\flat$ —	D —	C

giving:—

3) *The Descending Minor Scale* (Hypodoric or Eolic mode of the ancient Greeks—our mode of the minor Third):

c —	$B\flat$ —	$A\flat$ —	G —	F —	$E\flat$ —	D —	C
2	$\frac{9}{5}$	$\frac{8}{5}$	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{6}{5}$	$\frac{9}{8}$	1

or in the mixed scale, changing $A\flat$ into A,

4) *Mode of the minor Seventh* (ancient Greek Phrygian):

c —	$B\flat$ —	A —	G —	F —	$E\flat$ —	D —	C
2	$\frac{9}{5}$	$\frac{5}{3}$	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{6}{5}$	$\frac{9}{8}$	1

On examining the relations of F, the Fifth below the tonic c, the following scales result:

ASCENDING SCALES.

Related to c:	c	—	—	— e	— f	— g	— a	— —	— c'
Related to F:	c	— d	—	—	— f	— —	— a	— $b\flat$	— c'

This gives—

5) *The Mode of the Fourth* (ancient Greek Hypophrygian or Ionic):

$$\begin{array}{ccccccccc} c & - & d & - & e & - & f & - & g & - & a & - & - & b\flat & - & c' \\ 1 & & \frac{10}{9} & & \frac{5}{4} & & \frac{4}{3} & & \frac{3}{2} & & \frac{5}{3} & & & \frac{16}{9} & & 2 \end{array}$$

By changing e into $e\flat$, we again obtain—

6) *The mode of the minor Seventh*, but with a different determination of the intercalary tones d and $b\flat$, from those in No. 4:

$$\begin{array}{ccccccccccccccc} c & - & d & - & e\flat & - & f & - & g & - & a & - & b\flat & - & c' \\ 1 & & \frac{10}{9} & & \frac{6}{5} & & \frac{4}{3} & & \frac{3}{2} & & \frac{5}{3} & & \frac{16}{9} & & 2. \end{array}$$

DESCENDING SCALES.

Related to c: $c - - - - A\flat - G - F - E\flat - - - C$

Related to F: $c - B\flat - A - - - - F - - - D\flat - C$

giving:—

7) *The mode of the minor Sixth* (ancient Greek Doric):

$$\begin{array}{ccccccccccccccc} c & - & B\flat & - & A\flat & - & G & - & F & - & E\flat & - & D\flat & - & C \\ 2 & & \frac{16}{9} & & \frac{8}{5} & & \frac{3}{2} & & \frac{4}{3} & & \frac{6}{5} & & \frac{16}{15} & & 1 \end{array}$$

In this way the melodic tonal modes of the ancient Greeks and Christian Church have all been rediscovered by a consistent method of derivation. As long as homophonic vocal music is alone considered, all these tonal modes are equally justified in their construction.[1]

[1] [As the exact pitch of each tone in these scales is given by figures, they can be written in my notation, and hence compared with the scales I have given in Appendix XIX. Table III., C. The agreement is as follows:

No. in Text	Name in Text	No. in Table III.	Name in Table III.
1	major Scale	3	mapáma
2	ascending minor Scale . .	17	mapíma
3	descending minor Scale . .	52	mipími
4	mode of the minor Seventh .	45	mapími
5	mode of the Fourth . . .	5	mamapá
6	mode of the minor Seventh . (with intercalary tones different from those in No. 4 of text) .	33	mamapí
7	mode of the minor Sixth . .	54	mimipí

By comparing these numbers in the table it will be seen that they are the harmonic representatives of the Greek scales assigned to them in the text, but, assuming Pythagorean intonation for the Greek, different from them materially in the intervals, several tones being raised or depressed by a comma.—*Translator.*]

The scales have been given above in the order in which they are most naturally deduced. But, as we have seen, each of the three scales

$$c - - e - f - g - a - - c'$$
$$c - - e\flat - f - g - a - - c'$$
$$c - - e\flat - f - g - a\flat - - c'$$

can be played either upwards or downwards, although the first is best suited to ascending and the last to descending progression, and hence the gaps of any one of them may be filled up with either the relations of F or the relations of g, or even one gap with those of F and the other with those of g.

The vibrational number or pitches of the tones directly related to the tonic are of course fixed [1] and unchangeable, because they are given by the condition that the tones should form consonances with the tonic, and are thus more strictly determined than by any more distant connection. On the other hand, the intercalary tones related in the second degree are by no means so precisely fixed.

Taking $c=1$, we have for the Second—

1) the d derived from $g = \frac{9}{8}$.
2) the d derived from $f = \frac{10}{9} = \frac{80}{81} \times \frac{9}{8}$.
3) the $d\flat$ derived from $f = \frac{16}{15}$.

and for the Seventh—

1) the b derived from $g = \frac{15}{8}$.
2) the $b\flat$ derived from $g = \frac{9}{8}$.
3) the $b\flat$ derived from $f = \frac{16}{9} = \frac{80}{81} \times \frac{9}{5}$.

Hence while b and $d\flat$ are given with certainty, $b\flat$ and d are uncertain. Either of them may be distant from the tonic by the major Tone $\frac{9}{8}$ or the minor Tone $\frac{10}{9}$.

In order henceforth to mark this difference of intonation with certainty and without ambiguity, we will introduce a method of distinguishing the tones determined by a progression of Fifths, from those given by the relationship of a Third to the tonic. We have already seen that these two methods of determining the tones lead to somewhat different pitches, and hence in accurate theo-

[1] Thus I cannot agree with Hauptmann, in allowing a Pythagorean a, the Fifth above d, in the ascending minor scale of c. d'Alembert wants to introduce the same tone even in the major scale, by passing from g to b through the fundamental bass d. But this would indicate a distinct modulation into G major, which is not required when the natural relations of the tones to the tonic are preserved. See Hauptmann, 'Harmonik und Metrik,' p. 60.

retical researches both kinds of tones must be kept distinct, although in modern music they are practically confused.

The idea of this notation belongs to Hauptmann, but as the capital and small letter which he uses, and which I also, in consequence, employed in the first edition of this book, have a different meaning in our method of writing tones, I now introduce a slight modification of his notation.

Let C be the initial tone, and write[1] its Fifth G, the Fifth of this Fifth D, and so on. In the same way let the Fourth of C be F, the Fourth of this Fourth $B\flat$, and so on. In this way we have a series of Tones, here written with simple capitals, all distant from each other by a perfect Fifth or a perfect Fourth:

$$B\flat - F - C - G - D - A - E, \text{ \&c.}$$

The pitch of every tone in the whole series is, therefore, known when that of any one is known.

The major Third of C, on the other hand, will be expressed by $\underline{E}$, that of F by $\underline{A}$, and so on. Hence the series of tones

$$B - \underline{D} - F - \underline{A} - C - \underline{E} - G - \underline{B} - D - \underline{F}\sharp - A, \text{ \&c.,}$$

is a series of alternate major and minor Thirds. It is therefore clear that the Tones

$$\underline{D} - \underline{A} - \underline{E} - \underline{B} - \underline{F}\sharp, \text{ \&c.,}$$

also form a series of perfect Fifths.

We have already found that the tone $\underline{D}$, that is the minor Third below or major Sixth above F, is lower in pitch than the tone D, which would be reached by a series of Fifths from F, and that the difference of pitch is that known as a *comma*, the numerical value of which is $\frac{81}{80}$, or musically about the tenth part of a whole Tone.[2] Since, then, $D - A$ and $\underline{D} - \underline{A}$ are both perfect Fifths, A must be also a comma higher that $\underline{A}$, and so also every letter with a stroke *below* it will represent a tone which is a comma *lower* in pitch than that represented by the same letter with *no* stroke below it, as is easily seen by carrying on the series.

[1] 'Die Natur der Harmonik und Metrik,' Leipzig, 1853, pp. 26 and following. I cannot but join with C. E. Naumann in expressing my regret that so many delicate musical apperceptions as this work contains, should have been needlessly buried under the abstruse terminology of Hegelian dialectics, and hence have been rendered inaccessible to any large circle of readers.

[2] [The logarithm of an equally tempered tone is ·050 1717, of a major Tone or $\frac{9}{8}$ is ·051 1525, of a minor Tone or $\frac{10}{9}$ is ·045 7575, and of a comma $\frac{81}{80}$ is ·005 3950, so that a comma is more nearly the *ninth* part of a tempered Tone.—*Translator.*]

A major triad will therefore be written thus:

$$C-\underset{\cdot}{E}-G$$

and a minor triad

$$\underset{\cdot}{A}-C-\underset{\cdot}{E} \text{ or } \underset{\cdot}{C}-E\flat-\underset{\cdot}{G}.$$

Now if we lay it down as a rule that every stroke *below* a letter *depresses* its tone by the interval $\frac{81}{80}$, and every stroke *above* a letter *raises* its tone by the same interval, we may write the major triad as

$$c-\underset{\cdot}{e}\ -g \text{ or } \bar{c}-e-\bar{g}$$

and the minor triad as

$$c-\bar{e}\flat-g \text{ or } \underset{\cdot}{c}-e\flat-g,$$

or even

$$\bar{c}-\bar{e}\flat-\bar{g} \text{ or } \underset{\cdot}{c}-\underset{\cdot}{e}\flat-\underset{\cdot}{g}.^{1}$$

[1] In the first edition of this book, as in Hauptmann's, the small letters were supposed to be a comma lower than the capital letters, and a stroke above or below the letters was only occasionally used for raising or depressing the pitch by two commas. Hence a major triad was written $C - e - G$ or $\bar{c} - E - \bar{g}$; a minor triad $a - C - e$, or $\underset{\cdot}{A} - c - \underset{\cdot}{E}$, &c. The notation used here and also in the French translation is due to Herr A. v. Oettingen, and is much more readily comprehended.

[In accordance with Prof. Helmholtz's wish, I retain this notation in the present translation. But before I became acquainted with it, I had for my own purposes invented another which I employed in my paper: 'On the Conditions, Extent, and Realisation of a perfect Musical Scale on Instruments with Fixed Tones,' read before the Royal Society January 21, 1864, and printed in the 'Proceedings,' vol. xiii. p. 93. Although Herr v. Oettingen's notation obviates the inconvenience of the large and small letters and an occasional use of a stroke in addition, yet it remains troublesome to the printer and inapplicable to the staff notation of music. My notation, of which Prof. Helmholtz so far approved as to explain it in the last Appendix to his second edition, and in Appendix XVIII. to his third edition, obviates these difficulties. It consists in allowing the simple letters to be used as on p. 26, and in expressing elevation by a comma by a prefixed † read *high*, and depression by a comma by a prefixed ‡ read *low*, so that *D* to †*A* and ‡*D* to *A* will be perfect Fifths, just as *B* to *F*♯ and *B*♭ to *F* are perfect Fifths. This notation is fully explained in Appendix XIX., Section A, where also the signs to be employed in staff notation are furnished. A complete comparison of the two notations results from supposing that in Appendix XIX., Table V., my prefixed † and ‡ are omitted, and column III., then remaining as written, *one* line is written *under* each note in column IV., *two* in column V., and *three* in column VI., while *one* line is written *over* each note in column II., and *two* in column I. Hence we have a simple rule for referring Herr v. Oettingen's notation to mine. The number of lines above and below indicates the column, and the letters and ♯ and ♭ signs being the same, the proper † and ‡ are found in the column indicated. I shall, however, generally give the corresponding signs in foot-notes. Thus the following is the principal series of Fifths, in Herr v. Oettingen's and my notation compared:—

V. Oettingen. *F*♭ *C*♭ *G*♭ *D*♭ *A*♭ *E*♭ *B*♭ *F C G D A E B F*♯ *C*♯ *G*♯
Ellis. ‡*F*♭ ‡*C*♭ ‡*G*♭ ‡*D*♭ *A*♭ *E*♭ *B*♭ *F C G D* †*A* †*E* †*B* †*F*♯ †*C*♯ †*G*♯

V. Oettingen. *A*♭ *E*♭ *B*♭ *F C G D A E B F*♯ *C*♯ *G*♯ *D*♯ *A*♯ *E*♯ *B*♯
Ellis. ‡*A*♭ ‡*E*♭ ‡*B*♭ ‡*F* ‡*C* ‡*G* ‡*D A E B F*♯ *C*♯ *G*♯ *D*♯ †*A*♯ †*E*♯ †*B*♯

Other signs for accurately marking finer distinctions of pitch, when required, are also given in the same place.—*Translator.*]

The three series of Tones directly related to *C* are consequently to be written thus:

$$C - - \underline{E} - F - G - \underline{A} - - c$$
$$C - - \bar{E}\flat - F - G - \underline{A} - - c$$
$$C - - \bar{E}\flat - F - G - \bar{A}\flat - - c$$

and the intercalary tones are—

Between the tonic and Third, *D*, $\underline{D}$, or $\bar{D}\flat$

Between the Sixth and Octave, $\underline{B}$ and $B\flat$ or $\bar{B}\flat$.[1]

Consequently the melodic tonal modes of the ancient Greeks and old Christian Church are,

1) MAJOR MODE,

$$C - D - \underline{E} - F - G - \underline{A} - \underline{B} - c$$
$$\underline{D}$$

2) MODE OF THE FOURTH,

$$C - D - \underline{E} - F - G - \underline{A} - B\flat - c$$
$$\underline{D} \qquad\qquad \bar{B}\flat$$

3) MODE OF THE MINOR SEVENTH,

$$C - D - \bar{E}\flat - F - G - \underline{A} - B\flat - c$$
$$\underline{D} \qquad\qquad \bar{B}\flat$$

4) MODE OF THE MINOR THIRD,

$$C - D - \bar{E}\flat - F - G - \bar{A}\flat - B\flat - c$$
$$\underline{D} \qquad\qquad \bar{B}\flat$$

5) MODE OF THE MINOR SIXTH,

$$C - \bar{D}\flat - \bar{E}\flat - F - G - A\flat - B\flat - c$$
$$\underline{A} \quad \bar{B}\flat$$[2]

[1] [In the notation of Appendix XIX.,

Scales $C - - E - F - G - A - - c$
$C - - \dagger E\flat - F - G - A - - c$
$C - - \dagger E\flat - F - G - \dagger A\flat - - c$

Intercalary tones, D, ‡D, D♭, and B, B♭, †B♭.—*Translator.*]

[2] [In the notation of Appendix XIX.,

1) $C - D - E - F - G - A - B - c$
$\ddagger D$
2) $C - D - E - F - G - A - B\flat - c$
$\ddagger D \qquad \dagger B\flat$
3) $C - D - \dagger E\flat - F - G - A - B\flat - c$
$\ddagger D \qquad \dagger B\flat$
4) $C - D - \dagger E\flat - F - G - \dagger A\flat - B\flat - c$
$\ddagger D \qquad \dagger B\flat$
5) $C - D\flat - \dagger E\flat - F - G - \dagger A\flat - B\flat - c$
$A - \dagger B\flat$

In the German text only $\underline{A}$ = my *A*, is given in No. 5, but by referring to No. 7 on p. 422, it will be seen that $\bar{A}\flat$ = my $\dagger A\flat$, corresponds with $B\flat$. Hence I have

By this notation, then, the intonation is always exactly expressed, and the kind of consonance which each tone makes with the tonic, or the tones related to it is clearly shewn.

In the ancient Greek Pythagorean intonation these scales would have to be written:

Major mode— $C-D-E-F-G-A-B-C.$

and the others in a similar manner, all with letters of the same kind, belonging to the same series of Fifths.[1]

In the formulae here given for the diatonic tonal modes, the intonation of the Second and Seventh is partly undetermined. In these cases I have given D the preference over $\underline{D}$, and $B\flat$ the preference over $\bar{B}\flat$, because the relationship of the Fifth is closer than that of the Third; but $B\flat$ and D stand in the relation of the Fifth respectively to F, G, the tones nearest related to the tonic, while $\underline{D}$ and $\bar{B}\flat$ are only in the relation of the major Third. But this reason is certainly not sufficient entirely to exclude the two last tones in homophonic vocal music. For if in a melodic phrase, the Second of the tonic came into close connection with tones related to F—for instance, if it fell between F and $\underline{A}$, or followed them—an accurate singer would certainly find it more natural to use the $\underline{D}$ which is directly related to F and $\underline{A}$, than the D which is related to them only in the third degree. The slightly closer relationship of the latter to the Tonic could scarcely give the decision in its favour in such a case.[2]

given both forms. The scales as here given will receive several forms by varying the intercalary tones, and the tones related to F and G. Of course it is understood that both forms of such notes are not used in ascending and both in descending, but that occasionally one is used and occasionally another. This, however, amounts to a change of mode. Thus if in (1), taking my notation throughout, D is used, we have C mapáma, No. 3 of App. XIX., Table III. But if $\ddagger D$ is used, we have A mitími, No. 53 of that table. Similarly in (2), D and $B\flat$ would not give a harmonisable scale, but D and $\dagger B\flat$ give C mapámi, No. 31; and $\ddagger D$ and $B\flat$ give F mamapá, No. 5. In (3) again D and $B\flat$ give nothing harmonisable. But D and $\dagger B\flat$ give C mapími, No. 45; and $\ddagger D$ and $B\flat$ give F mamapí, No. 33. For (4) also D and $B\flat$ cannot come together, but D and $\dagger B\flat$ give C mipimi, No. 52; and $\ddagger D$ and $B\flat$ give F mamipí, No. 47. For (5) $\dagger A\flat$ and $B\flat$ give F mimipi, No. 54; A and $B\flat$ give F mimapí, No. 40; but neither $\dagger A\flat$ and $\dagger B\flat$ nor A and $\dagger B\flat$ give anything harmonisable. By 'harmonisable' I mean forming a major or minor triad with other notes of the scale.—*Translator.*]

[1] [In Appendix XIX., Table III., B., all the seven Pythagorean modes are written out in my notation. The five in the text are 7. C. 2, 3, 4, 5, and 6 respectively.—*Translator.*]

[2] [In the notation of Appendix XIX., then, the author prefers $B\flat$ and D to $\dagger B\flat$ and $\ddagger D$, but thinks that the latter would not be entirely excluded; for example, $F \ddagger D\ A$, or $F\ A \ddagger D$ would be a more natural progression than $F\ D\ A$ or $F\ A\ D$.—*Translator.*]

This ambiguity in the intercalary tones cannot, I think, be considered as a fault in the tonal system, since in our modern minor mode, the Sixth and Seventh of the tonic, are often altered not merely by a comma, but by a whole Semitone, according to the direction of the melodic progression. We shall find, however, more decisive reasons for the use of *D* in place *D̲* in the next chapter, when we pass from homophonic music to the influence of harmonic music on the scales.

The account here given of the rational construction of scales and the corresponding intonation of intervals, deviates essentially from that given to the Greeks by Pythagoras, which has thence descended to the latest musical theories, and even now serves as the basis of our system of musical notation. Pythagoras constructed the whole diatonic scale from the following series of Fifths :—

$$F-C-G-D-A-E-B^1,$$

and calculated the intervals from it as they have been given above.[2] In his diatonic scale there are but two kinds of small intervals, the *whole Tone* $\frac{9}{8}$, and the *Limma* $\frac{256}{243}$.

In this series if *C* be taken as the tonic, *A* would be related to the tonic in the Third degree, *E* in the Fourth, and *B* in the Fifth. Such a relationship would be absolutely insensible to any ear that has no guide but direct sensation.

A series of Fifths may certainly be tuned on any instrument, and continued as far as we please; but neither singer nor hearer could possibly discover in passing from *c* to *e* that the latter is the fourth from the former in the series of Fifths. Even in a relation of the second degree through Fifths, as of *c* to *d*, it is doubtful whether a hearer can discover the relation of the two tones. But in this case when we pass from one tone to the other we can

[1] [In the notation of Appendix XIX., $F - C - G - D - \dagger A - \dagger E - \dagger B$, where the introduction of † before the last three tones draws attention to the great difference between this and the modern harmonisable scale.—*Translator.*]

[2] [In order of Fifths, but reduced to the compass of an Octave, they are—

$$\begin{array}{ccccccc} F - & C - & G - & D - & \dagger A - & \dagger E - & \dagger B \\ \frac{4}{3} & 1 & \frac{3}{2} & \frac{9}{8} & \frac{27}{16} & \frac{81}{64} & \frac{243}{128} \end{array}$$

of which the three last are $\frac{81}{80}$ times the $A = \frac{5}{3}$, $E = \frac{5}{4}$, $B = \frac{15}{8}$ found on p. 421, No. 1. Hence the Pythagorean Lydian scale (Appendix XIX. Table III., B., 7.C.2) is—

$$\begin{array}{ccccccccc} C - & D - & \dagger E - & F - & G - & \dagger A - & \dagger B - & c \\ 1 & \frac{9}{8} & \frac{81}{64} & \frac{4}{3} & \frac{3}{2} & \frac{27}{16} & \frac{243}{128} & 2 \\ \underbrace{\quad}_{\frac{9}{8}} & \underbrace{\quad}_{\frac{9}{8}} & \underbrace{\quad}_{\frac{256}{243}} & \underbrace{\quad}_{\frac{9}{8}} & \underbrace{\quad}_{\frac{9}{8}} & \underbrace{\quad}_{\frac{9}{8}} & \underbrace{\quad}_{\frac{256}{243}} & \end{array}$$

—*Translator.*]

imagine the insertion of 'a silent *g*,' so to speak, forming the Fourth below *c*, and the Fifth below *d*, and thus establish a connection, for the mind's ear at least, if not for the body's. This is probably the meaning to be attached to Rameau's and d'Alembert's explanation that a singer effects the passage from *c* to *d* by means of the fundamental bass *g*. If the singer does not hear the bass note *g* at the same time as *d*, he cannot possibly bring his *d* into consonance with that *g*; but the melodic progression may certainly be facilitated by conceiving the existence of such a tone. This is a well-known means for striking the more difficult intervals, and is often applied with advantage. But of course it completely fails when the transition has to be made between tones widely separated in the series of Fifths.[1]

[1] [One of the practical results of the Tonic Solfa system of teaching to sing the diatonic major scale as marked on p. 421, No. 1, in just intonation, (see Appendix XVIII.,) has been the discovery that it is not so easy to learn to strike the proper tone by a knowledge of the *interval* between two adjacent tones in a melodic passage, as by a knowledge of the *mental effect* produced by each tone of the scale in relation to the tonic. These mental effects are perhaps not very clearly characterised by the mere names given to them in the Tonic Solfa books, but the teacher soon makes his class understand them, and then finds them the most valuable instrument which he possesses for inspiring a feeling for just intonation. On these characters of each tone in the (just) diatonic scale, a system of manual signs has been formed, by which classes are constantly led. Particulars are given in 'The Standard Course of Lessons and Exercises in the Tonic Solfa Method of Teaching Music (founded on Miss Glover's "Scheme for rendering Psalmody Congregational," A.D. 1835), with additional exercises, by John Curwen, new edition, re-written, A.D. 1872,' pp. 160, 96, 16 and 4, total 276. But it may be convenient to mention the characters and manual signs in this place (*ib.* p. iv).

I. First step.

Do, Tonic, 'the STRONG or *firm* tone,' fist closed, horizontal, thumb down.

So, Fifth, 'the GRAND or *bright* tone,' the fingers extended and horizontal, hand with little finger below and thumb above, so that the palm of the hand is vertical.

Mi, Major Third, 'the STEADY or *calm* tone,' fingers extended and horizontal, palm of hand horizontal and undermost.

II. Second step.

Re, Second, 'the ROUSING or *hopeful* tone,' fingers extended, hand forming half a right angle with ground pointing upwards, palm downwards.

Ti, Seventh, 'the PIERCING or *sensitive* tone,' only the forefinger extended and pointing up, the other fingers and thumb closed, hand forming half a right angle with ground, *back of hand* downwards.

III. Third step.

Fa, Fourth, 'the DESOLATE or *awe-inspiring* tone,' only the forefinger extended and pointing down, at half a right angle with the ground, the back of hand upwards.

La, major Sixth, 'the SAD or *weeping* tone,' fingers fully extended, whole hand pointing down with a weak fall, back of hand upwards.

It is thus seen that the order of teaching takes the tonic chord first, then the dominant, and lastly the sub-dominant. The doubtful Second thus comes early on.

Finally there is no perceptible reason in the series of Fifths why they should not be carried further, after the gaps in the diatonic scale have been supplied. Why do we not go on till we reach the chromatic scale of Semitones? To what purpose do we conclude our diatonic scale with the following singularly unequal arrangement of intervals—

$$1,\ 1,\ \tfrac{1}{2},\ 1,\ 1,\ 1,\ \tfrac{1}{2}\ ?$$

The new tones introduced by continuing the series of Fifths would lead to no closer intervals than those which already exist. The old scale of five tones appears to have avoided Semitones as being too close. But when such two intervals already appear in the scale, why not introduce more?

The *Arabic and Persian musical system*, so far as its nature is shewn in the writings of the elder theorists, also knew no method of tuning but by Fifths. But this system, which seems to have developed its peculiarities in the Persian dynasty of the Sassanides (A.D. 226–651) before the Arabian conquest, shews an essential advance on the Pythagorean system of Fifths.

'The teacher first sings the exercise to [the names of] consecutive figures, telling his pupils that he is about to introduce a new tone (that is one *not* DO, MI, or SO), and asking them to tell him on which figure it falls. When they have distinguished the new tone, he sings the exercise again—laa-ing it [this is calling each note *la*]—and asks them to tell him how that tone "makes them feel." Those who can *describe* the feeling hold up their hands, and the teacher asks one for a description. But others, who are not satisfied with words, may also perceive and feel. The teacher can tell by their eyes whether they have done so. He multiplies examples until *all* the class have their attention fully awakened to the effect of the new tone. This done he tells his pupils the Sol-fa name and the manual sign for the new tone, and guides them by the signs to Sol-fa the exercise and themselves produce the proper effect. The signs are better in this case than the notation, because with them the teacher can best command the attention of every eye and ear and voice, and at the first introduction of a tone, attention should be acute' (*ibid.* p. 15). This passage, the result of practice with hundreds of thousands of children, shews that a totally new principle of understanding the relation of the tones in a scale to the tonic has not only been introduced, but worked out on a large scale practically, and, as I myself know, successfully. See Prof. Helmholtz's own impression of the success, ten years ago, in Appendix XVIII. Since that time great experience has been gained and many methods improved. But the object of introducing this notice here is to shew that *proper training* (such as the ancient Greeks certainly had) could produce the corresponding *feeling* for the effect of any tone in any scale anyhow divided, independently of the relation of consonances, and that this consideration may help to explain the persistence of many scales which are harmonically inexplicable. No doubt Pythagorean singers hit the degrees of their scale quite correctly, and no doubt the 'mental effects' of their †*A*, †*E*, †*B*, were very different from those of the harmonisable *A*, *E*, *B*. We can partially judge of them by the effects of equal temperament, which melodically cannot differ much, although they certainly differ sensibly, from those of Pythagorean intonation.—*Translator.*]

In order to judge of this system of music, which has been hitherto completely misunderstood, the following relation has to be known. By tuning four Fifths upwards from C

$$C - G - D - A - E$$

we come to a tone, E, which is $\frac{81}{80}$ or a comma higher than the natural major Third of C, which we write $\underline{E}$. The former E forms the major Third in the Pythagorean scale. But if we tune eight Fifths downwards from C, thus—

$$C - F - B\flat - E\flat - A\flat - D\flat - G\flat - C\flat - F\flat$$

we come to a tone, $F\flat$, which is almost exactly the same as the natural $\underline{E}$. The interval of C to $F\flat$ is expressed by

$$\frac{8192}{6561} = \frac{5}{4} \times \frac{32768}{32805}, \text{ or nearly } \frac{5}{4} \times \frac{885}{886}$$

Hence the tone $F\flat$ is lower than the natural major Third $\underline{E}$ by the extremely small interval $\frac{886}{885}$, which is about the eleventh part of a comma. This interval between $F\flat$ and $\underline{E}$ is practically scarcely perceptible, or at most only perceptible by the extremely slow beats produced by the chord $C - F\flat - G$ upon an instrument most exactly tuned. Practically, then, we may without hesitation assume that the two tones $F\flat$ and $\underline{E}$ are identical, and of course that their Fifths are also identical, or

$$F\flat = \underline{E},\ C\flat = \underline{B},\ G\flat = \underline{F}\sharp, \text{ \&c.}$$ [1]

Now in the Arabic and Persian scale the Octave is divided into 16 intervals, but in our equal temperament it is divided into 6 whole Tones. The modern interpreters of the Arabic and Persian system of music have hence been misled into the conclusion that each of the 17 degrees of the scale corresponded to about the third of a Tone in our music. In that case the intonation of the degrees in the Arabic and Persian scale would not be executable on our instruments. But in Kiesewetter's work on the Music of the Arabs,[2] which was written with the assistance of the celebrated Orientalist

[1] [On this substitution, which amounts to a temperament with perfect Fifths, and major Thirds too flat by a skhisma, or nearly the eleventh of a comma, and which I therefore call skhismic temperament, see Appendix XIX., section C. The two series of Fifths in the text are in the notation of Appendix XIX., $C - G - D - \dagger A - \dagger E$ upwards, and $C - F - B\flat - E\flat - A\flat - \ddagger D\flat - \ddagger G\flat - \ddagger C\flat - \ddagger F\flat$.—*Translator.*]

[2] R. G. Kiesewetter, 'Die Musik der Araber nach Originalquellen dargestellt, mit einem Vorworte von dem Freiherrn von Hammer-Purgstall.' Leipzig, 1842. pp. 32, 33. The directions given in an anonymous manuscript of the 666th year of the Hegira, A.D. 1267, in the possession of Prof. Salisbury, are essentially the same. See 'Journal of the American Oriental Society,' vol. 1, pp. 204–209.

von Hammer-Purgstall, there is given a translation of the directions for the division of the monochord laid down by Abdul Kadir, a celebrated Persian theorist of the fourteenth century of our era, that lived at the courts of Timur and Bajazet. These directions enable us to calculate the intonation of the Oriental scale with perfect certainty. These directions also agree in essentials both with those of the much older Farabi,[1] who died in A.D. 950, and of his contemporary, Mahmud Shirazi,[2] who died in 1315, for dividing the fingerboard of lutes. According to the directions of Abdul Kadir all the tonal degrees of the Arabic scale are obtained by a series of 16 Fifths, and if we call the lowest degree *C*, and arrange them in order of pitch within the compass of an Octave, they will be the following, as expressed in our notation :

1) *C*— 2) *D*♭— 3) *D* ⌣ 4) *D*— 5) *E*♭— 6) *E* ⌣
7) *E*— 8) *F* — 9) *G*♭—10) *G*⌣11) *G* —12) *A*♭—
13) *A* ⌣14) *A* —15) *B*♭—16) *B*—17) *c* ⌣18) *c*.

where the line — between two tones indicates the interval of a Pythagorean Limma $\frac{256}{243}$ (which is nearly $\frac{20}{19}$), and the sign ⌣ a comma $\frac{81}{80}$. The Limma is about $\frac{4}{5}$ and the comma about $\frac{1}{5}$ of the natural Semitone $\frac{16}{15}$.[3]

[1] J. G. L. Kosegarten, 'Alii Ispahanensis Liber Cantilenarum,' pp. 76–86.

[2] Kiesewetter, 'Die Musik der Araber nach Originalquellen darg.,' p. 33.

[3] [In the preceding account of the Arabic tonal degrees, Prof. Helmholtz, without especially saying so, has introduced that practical *skhismic* temperament explained on last page, note 1. Having calculated the intervals independently from Abdul Kadir's instructions as given by Kiesewetter, I find that in order of Fifths they give precisely in the notation of Appendix XIX.,—

4 Fifths up . . *C* — *G* — *D* — †*A* — †E

12 Fifths down . *C* — *F* — *B*♭ — *E*♭ — *A*♭ — ‡*D*♭ — ‡*G*♭ — ‡*C*♭ — ‡*F*♭ — ‡*B*♭♭ — ‡*E*♭♭ — ‡*A*♭♭ — ‡‡*D*♭♭.

Now if these be written in one series from †*E* to ‡‡*D*♭♭, it is readily seen that they give the Pythagorean scales beginning with *F*, *B*♭, *E*♭, *A*♭, ‡*D*♭, ‡*G*♭, ‡*C*♭, ‡*F*♭, ‡*B*♭♭, ‡*E*♭♭, and ‡*A*♭♭ respectively; and, as will be most readily found by referring to Appendix XIX., Table V., that the last five might be identified with the Pythagorean scales of *B*, *E*, *A*, ‡*D*, and ‡*G*. Again, write the tones as follows with the pitches calculated for *C* = 264, and with a third column giving the just major Thirds to the tones in the first column.

				Just Major Thirds.	
†*E*	334·125				
†*A*	445·500	‡*D*♭	278·123	*C*♯	278·438
D	297·000	‡*G*♭	370·831	*F*♯	371·250
G	396·000	‡*C*♭	494·442	*B*	495·000
C	264·000	‡*F*♭	329·628	*E*	330·000
F	352·000	‡*B*♭♭	439·504	*A*	440·000
B♭	469·333	‡*E*♭♭	293·002	‡*D*	293·333
E♭	312·889	‡*A*♭♭	390·670	‡*G*	391·111
A♭	417·185	‡‡*D*♭♭	520·893	‡*C*	521·462

Abdul Kadir assigns the following intonation to the three first of the 12 principal tonal modes or *Makamat*:—

Arabic										Ancient Greek
1. Uschak	C	D	E	F	G	A	$B\flat$	c		Hypophrygian or Ionian.
2. Newa	C	D	$E\flat$	F	G	$A\flat$	$B\flat$	c		Hypodorian or Eolic.
3. Buselik	C	$D\flat$	$E\flat$	F	$G\flat$	$A\flat$	$B\flat$	c		Mixolydian, all on p. 413.

These three are therefore completely identical with the ancient Greek scales in Pythagorean intonation.[1] Since the Arabic theoreticians divide these scales into the Fourth $C-F$ and the Fifth $F-c$, and since C, F and $B\flat$ are considered to be invariable tones, and the others to be variable, it is probable that F must be regarded as the Tonic. In this case

1. *Uschak* would be $=F$ major.
2. *Newa* would be $=$the mode of the minor Seventh of F.[2]
3. *Buselik* would be $=$the mode of the minor Sixth of F.

all three in Pythagorean intonation. The Persian school also considers the scale to be related.

The next group consists of five tonal modes having just or natural intonation, namely:

We then see that the tones in the second column are generally less than those in the third, by one vibration in two seconds. In melody, therefore, no error would be felt if we used the second column for the third. In harmony the major Third C to $\ddagger F\flat$ would beat 1·488 per second, c and $\ddagger f\flat$ double that or 2·977, nearly 3 times a second, which would be very perceptible, and c' and $\ddagger f'\flat$ beat nearly 6 times in a second. Hence, although much better than the equally tempered major Thirds, these would be felt as rough in the higher Octaves. This, however, could not be perceived by the Arabs who used melody only. Hence this arrangement gave the tempered but nearly just major scales of D, G, C, F, $B\flat$, and $E\flat$, as well as the Pythagorean major scales above noted.—*Translator.*]

[1] [These scales in the notation of App. XIX. are—

1. Uschak.	C	D	$\dagger E$	F	G	$\dagger A$	$B\flat$	c
2. Newa .	C	D	$E\flat$	F	G	$A\flat$	$B\flat$	c
3. Buselik.	C	$\ddagger D\flat$	$E\flat$	F	$\ddagger G\flat$	$A\flat$	$B\flat$	c

See App. XIX., Table III., B, 7. C. 3, 5, 7, respectively.—*Translator.*]

[2] [In the German text. *Quartengeschlecht*, or the mode of the Fourth of F. The tones in the mode of the Fourth of F are those in the Pythagorean scale of $B\flat$, or, in order of Fifths, $E\flat - B\flat - F - C - G - D - \dagger A$, and the tones of the mode of the minor Seventh of F are those in the Pythagorean scale of $E\flat$, or, in order of Fifths, $A\flat - E\flat - B\flat - F - C - G - D$. The correction is therefore evident. —*Translator.*]

4. Rast	C —	$\underline{D}$ —	$\underline{E}$ —	F —	G —	$\underline{A}$ —	$B\flat$ —	c
5. Husseini	C —	$\underline{D}$ —	$E\flat$ —	F —	$\underline{G}$ —	$A\flat$ —	$B\flat$ —	c
6. Hidschaf	C —	$\underline{D}$ —	$E\flat$ —	F —	$\underline{G}$ —	$\underline{A}$ —	$B\flat$ —	c
7. Rahewi	C —	$\underline{D}$ —	$\underline{E}$ —	F —	$\underline{G}$ —	$A\flat$ —	$B\flat$ —	c
8. Sengule	C —	D —	$\underline{E}$ —	F —	$\underline{G}$ —	$\underline{A}$ —	$B\flat$ —	c[1]

Rast may be regarded as the mode of the Fourth of C; *Hidschaf* as the mode of the Fourth of F; *Husseini* as the mode of the Fourth of $B\flat$; as such they would have perfectly natural intonation. In *Rahewi*, if we refer it to the tonic F, the minor Third $A\flat$ is in Pythagorean, not natural, intonation. It might be regarded as the mode of the minor Seventh of F in which the major Seventh $\underline{E}$ is used as the leading note in place of the minor Seventh, as in our own minor mode. The natural intonation of such a tonal mode cannot, indeed, be properly represented by the existing 17 tonal degrees. It becomes necessary to take either Pythagorean minor Thirds and natural major Thirds or conversely. *Husseini* may be regarded as the same tonal mode with *Rahewi*, having the same false minor Third, but a minor Seventh. Finally *Sengule* may be regarded as F major with a Pythagorean Sixth. *Rast* may be conceived in the same way; they are merely distinguished by the different values of the Seconds G or $\underline{G}$.

The four last *Makamat* have each 8 tones, new intercalary tones being employed. Two of them resemble the modes *Rast* and *Sengule*, and between $B\flat$ and C there is an intercalary tone $\underline{c}$ introduced; named

9. Irak	C —	$\underline{D}$ —	$\underline{E}$ —	F —	G —	$\underline{A}$ —	$B\flat$ —	$\underline{c}$ —	c
10. Iszfahan	C —	D —	$\underline{E}$ —	F —	$\underline{G}$ —	$\underline{A}$ —	$B\flat$ —	$\underline{c}$ —	c

The last transposed a Fourth gives

11. Büsürg	C —	D —	$\underline{E}$ —	F —	$\underline{G}$ —	G —	A —	$\underline{B}$ —	c

[1] [In the notation of App. XIX.

4. Rast	C —	$\ddagger D$ —	E —	F —	G —	A —	$B\flat$ —	c
5. Husseini	C —	$\ddagger D$ —	$E\flat$ —	F —	$\ddagger G$ —	$A\flat$ —	$B\flat$ —	c
6. Hidschaf	C —	$\ddagger D$ —	$E\flat$ —	F —	$\ddagger G$ —	A —	$B\flat$ —	c
7. Rahewi	C —	$\ddagger D$ —	E —	F —	$\ddagger G$ —	$A\flat$ —	$B\flat$ —	c
8. Sengule	C —	D —	E —	F —	$\ddagger G$ —	A —	$B\flat$ —	c

4. Rast, according to Kiesewetter, is

$$C — D — \dagger E — F — G — \ddagger B\flat\flat — B\flat — c$$

where the true $\ddagger B\flat\flat$ is substituted for the approximate A. The D, $\dagger E$ for $\ddagger D$, E essentially alter the scale.—*Translator.*]

The last tonal mode is

12. Zirefkend $C - D - E\flat - F - G - A\flat - A - B - c^1$

which certainly, if rightly reported, is a very singular creation. It

[1] [In the notation of App. XIX.—

9.	C	—	‡D	—	E	—	F	—	G	—	A	—	$B\flat$	—	‡c	—	c
10.	C	—	D	—	E	—	F	—	‡G	—	A	—	$B\flat$	—	‡c	—	c
11.	C	—	D	—	E	—	F	—	‡G	—	G	—	A	—	B	—	c
12.	C	—	‡D	—	$E\flat$	—	F	—	‡G	—	$A\flat$	—	A	—	B	—	c

The systems selected from Kiesewetter, with the exception of Rast, are those which he calls the 'Keys of the Arabo-Persian School after Abdul Kadir.' Kiesewetter, however, also gives 12 keys with the same names, with (occasionally) different intonations, which he calls 'Makamat, after the Arabic system.' To compare these systems, I here annex both in the notation of App. XIX., with the perfect Fifths of Abdul Kadir's measurements (p. 432, note) according to Kiesewetter's numbers. The capitals are the Arabo-Persian system of the text, the small letters the Arabic system; the order is that in the text. In the Arabic system, Husseini and Buselik are identical, perhaps by an error of Kiesewetter's:—

1. Uschak .	C	—	D	—	†E	—	F	—	G	—	†A	—	$B\flat$	—	c
	c	—	d	—	†e	—	f	—	g	—	†a	—	$b\flat$	—	c
2. Newa . .	C	—	D	—	$E\flat$	—	F	—	G	—	$A\flat$	—	$B\flat$	—	c
	c	—	d	—	$e\flat$	—	f	—	g	—	$a\flat$	—	$b\flat$	—	c'
3. Buselik .	C	—	‡$D\flat$	—	$E\flat$	—	F	—	‡$G\flat$	—	$A\flat$	—	$B\flat$	—	c
	c	—	‡$d\flat$	—	$e\flat$	—	f	—	‡$g\flat$	—	$a\flat$	—	$b\flat$	—	c'
4. Rast . .	C	—	D	—	†E	—	F	—	G	—	‡$B\flat\flat$	—	$B\flat$	—	c
	c	—	d	—	‡$f\flat$	—	f	—	g	—	‡$b\flat\flat$	—	$b\flat$	—	c'
5. Husseini .	C	—	‡$E\flat\flat$	—	$E\flat$	—	F	—	‡$A\flat\flat$	—	$A\flat$	—	$B\flat$	—	c
	c	—	‡$d\flat$	—	$e\flat$	—	f	—	‡$g\flat$	—	$a\flat$	—	$b\flat$	—	c'
6. Hidschaf	C	—	‡$E\flat\flat$	—	$E\flat$	—	F	—	‡$A\flat\flat$	—	‡$B\flat\flat$	—	$B\flat$	—	c
	c	—	‡$d\flat$	—	$e\flat$	—	f	—	‡$g\flat$	—	‡$b\flat\flat$	—	$b\flat$	—	c'
7. Rahewi .	C	—	‡$E\flat\flat$	—	‡$F\flat$	—	F	—	‡$A\flat\flat$	—	$A\flat$	—	$B\flat$	—	c
	c	—	‡$d\flat$	—	‡$f\flat$	—	f	—	‡$g\flat$	—	$a\flat$	—	$b\flat$	—	c'
8. Sengule .	C	—	D	—	‡$F\flat$	—	F	—	‡$A\flat\flat$	—	‡$B\flat\flat$	—	$B\flat$	—	c
	c	—	d	—	‡$f\flat$	—	f	—	‡$g\flat$	—	‡$b\flat\flat$	—	$b\flat$	—	c'
9. Irak . .	C	—	‡$E\flat\flat$	—	‡$F\flat$	—	F	—	‡$A\flat\flat$	—	‡$B\flat\flat$	—	$B\flat$	—	‡‡$D\flat\flat$	—	c
	c	—	‡$e\flat\flat$	—	‡$f\flat$	—	f	—	‡$a\flat\flat$	—	$a\flat$	—	‡$b\flat\flat$	—	‡$c\flat$	—	c'
10. Iszfahan .	C	—	D	—	‡$F\flat$	—	F	—	G	—	‡$B\flat\flat$	—	$B\flat$	—	‡‡$D\flat\flat$	—	c
	c	—	‡$d\flat\flat$	—	‡$f\flat$	—	f	—	‡$a\flat\flat$	—	g	—	$b\flat$	—	c'
11. Büsürg .	C	—	‡$E\flat\flat$	—	‡$F\flat$	—	F	—	‡$A\flat\flat$	—	G	—	†A	—	‡$c\flat$	—	c
	c	—	d	—	‡$f\flat$	—	f	—	‡$g\flat$	—	g	—	†a	—	‡$c'\flat$	—	c'
12. Zirefkend	C	—	‡$E\flat\flat$	—	$E\flat$	—	F	—	‡$A\flat\flat$	—	$A\flat$	—	‡$B\flat\flat$	—	‡$c\flat$	—	c
	c	—	‡$e\flat\flat$	—	‡$f\flat$	—	f	—	‡$a\flat\flat$	—	$a\flat$	—	$b\flat\flat$	—	c'		

The orthography of the names is that of the text, except as regards 10, *Iszfahan*, which is misprinted *Ifzfahan*, here corrected after Kiesewetter, whose orthography is, however, not consistent, and he has unfortunately not given the words in Arabic letters.—*Translator.*]

might be looked upon as a minor scale with a major Seventh, and both a major and minor Sixth, but then the Fifth *G* is wrong. On the other hand, if *F* is taken as the tonic, it has no Fourth, for which certainly there is some analogy in the Mixolydian and Hypolydian scales. The instructions for scales of eight notes are very contradictory, to judge by the different authorities cited by Kiesewetter.

The following four are distinguished as the principal modes of the *Makamat*:—

1. Uschak = Pythagorean *F* major
2. Rast = Natural mode of the Fourth of *C*, or natural *F* major with acute Sixth.
3. Husseini = Natural mode of the minor Seventh of *F*.
4. Hidschaf = Natural mode of the Fourth of *F*.[1]

We find then a decided predominance of scales with a perfectly correct natural intonation, which has been attained by a skilful use of a continued series of Fifths. This makes the Arabic and Persian tonal system very noteworthy in the history of the development of music. Moreover, in some of these scales we find ascending leading notes, which are perfectly foreign to the Greek scales. Thus in Rahewi, *E* is the leading note to *F*, although the minor Third *A*♭ stands above *F*, while no Greek scale could have allowed this without at the same time changing *E* into *E*♭. Similarly in Zirefkend the *B* is used as a leading note to *C*, although the minor Third *E*♭ is used above *C*.

At a little later period a new musical system was developed in Persia with twelve Semitones to the Octave, analogous to the modern European system. Kiesewetter here hazards the very unlikely hypothesis that this scale was introduced into Persia by Christian missionaries. But it is clear that the system of 17 tonal degrees which had been previously in popular use, merely required the feeling for the finer distinctions to grow dull so that intervals

[1] [These four Arabic modes on the left are those previously marked 1, in note to p. 433, and 4, 5, 6 in note to p. 434, in the notation of App. XIX. But the four scales named on right in the text, would be in the same notation—

1.	*F*	—	*G*	—	†*A*	—	*B*♭	—	*c*	—	*d*	—	†*e*	—	*f*
2.	*C*	—	‡*D*	—	*E*	—	*F*	—	*G*	—	†*A*	—	*B*♭	—	*c*
3.	*F*	—	‡*G*	—	*A*♭	—	*B*♭	—	‡*c*	—	‡*d*	—	*e*♭	—	*f*
4.	*F*	—	‡*G*	—	*A*	—	*B*♭	—	*c*	—	‡*d*	—	*e*♭	—	*f*

And on comparing, it is seen that (1) and (4) agree entirely with Uschak and Hidschaf, but (2) differs from Rast as before laid down, by having †*A* for *A*, and again (3) differs from Husseini in having ‡*c* for *c*.—*Translator.*]

which differed only by a comma should be confused, in order to generate the system of twelve Semitones.[1] No foreign influence was necessary here. Moreover, the Greek system of music had long been taught to the Arabs and Persians by Farabi. Again, the European theory of music had not made any essential advance in the fourteenth and fifteenth centuries, if we except the study of harmony, which never found favour with the Orientals. Hence the Europeans of those days could teach the Orientals nothing that they did not already know better themselves, except some imperfect rudiments of harmony which they did not want. There is much more reason, I think, for asking whether the imperfect fragments of the natural system which we find among the Alexandrine Greeks, do not depend on Persian traditions, and also, whether the Europeans in the time of the Crusades did not learn much music from the Orientals. It is very probable that they brought the lute-shaped instruments with fingerboards and the bowed instruments from the East. In the construction of tonal modes we might especially instance the use of the leading note, which we have here found existing in the East, and which at that time also began to figure in the Western music.

The use of the major Seventh of the scale as a leading note to the tonic marks a new conception, which admitted of being used for the further development of the tonal degrees of a scale, even within the domain of purely homophonic music. The tone *B* in the major scale of *C* has the most distant relationship of all the tones to the tonic *C*, because as the major Third of the dominant *G*, it has a less close connection with it than its Fifth *D*. We may perhaps assume this to be the reason why, when a Sixth tone was introduced into some Gaelic airs, the Seventh was usually omitted. But, on the other hand, the major Seventh *B* developed a peculiar relation to the tonic, which in modern music is indicated by calling it the *leading note*. The major Seventh *B* differs from the Octave *c* of the tonic by the smallest interval in the scale, namely a Semitone, and this proximity to the tonic allows the Seventh to be struck easily and pretty surely, even when starting from tones in the scale which are not at all related to *B*. The leap *F*—*B*, for example, is difficult, because there is no relation-

[1] [Referring to the table in note to p. 432, it will be seen that with these commatic confusions the scale would be C — ‡D♭ —(‡D = D) — E♭ —(E = †E) — F — ‡G♭ —(‡G = G)— A♭ —(A = †A) — B♭ — B — (‡c = c), so that if the new scale agreed in any respect with equal temperament we must suppose the intonation of every note to have been somewhat changed.—*Translator.*]

ship at all between the tones. But when a singer has to perform the passage *F*—*B*—*c*, he conceives the interval *F*—*c*, which he can easily execute, but does not force his voice up sufficiently high to reach *c* at first, and thus strikes *B* on the way. Thus *B* assumes the appearance of a preparation for *c*, and this view alone justifies it to the ears of a listener, by whom the transition into *c* is, therefore, expected. Hence it has been said that *B* leads to *c*; or that *B* is the *leading note* to *c*. In this sense it also becomes easy to sharpen *B* somewhat, making it *B*, for example, to bring it near to *c*, and mark its reference to that tone more distinctly.

According to my own feeling, the leading effect of the tone *B* is much more marked in such passages as *F*—*B*—*c* or *F*—*A*—*B*—*c*, in which *B* is not related to the preceding tones, than in such a passage as *G*—*B*—*c*. But as I have found nothing on this point in musical writings, I do not know whether musicians are likely to agree with me in this opinion. For the other Semitone of the scale, *E*—*F*, the *E* does not seem to lead to *F*, if the tonality of the melody is well preserved, because in this case *E* has its own independent relation to the tonic, and hence is musically quite determinate. The hearer, then, has no occasion to regard *E* as a mere preparation for *F*. Similarly for the interval *G*—*A*♭ in the minor mode. The *G* is more nearly related to the tonic *C* than *A*♭ is. On the other hand, Hauptmann is probably right in considering the interval *D*—*E* in the minor mode, as one in which *D* leads to *E*♭, because *D* has only a relationship of the second degree to the tonic *C*, although its relationship is certainly closer than that of *B* to *C*.

But the relation of *D*♭ in descending passages of the mode of the minor sixth of *C* (the old Greek Doric) is perfectly similar to the effect of *B* in the ascending scale of *C* major. It really forms a kind of descending leading note, and since in the best period of Greek music descending passages were felt as nobler and more harmonious than ascending ones,[1] this peculiarity of the Doric mode may have been of special importance and have been a reason for the preference given to this scale. The cadence with the chord of the superfluous Sixth—

D♭ — *F* — *G* — *B*
C — *E*♭ — *G* — *c*

[1] Aristotle, Problems xix. 33. [The passage has already been cited at full, p. 370, note.—*Translator.*]

is almost the only isolated and misunderstood remnant of the ancient tonal modes. This is a (Greek) Doric cadence, in which *D*♭ and *B* are both used at the same time as leading notes to *C*.[1]

The relation of the second or *parhypatē* of the Greek Doric scale, to the lowest tone or *hypatē*, seems also to have been perfectly well felt by the Greeks themselves, to judge by Aristotle's remarks in the third and fourth of his problems on harmony. I cannot abstain from adducing them here because they admirably and delicately characterise the relation. Aristotle inquires why the singer feels his voice more taxed in taking the *parhypatē* than in taking the *hypatē*, although they are separated by so small an interval. The *hypatē* is sung with a remission of effort. And then he adds that in order to reach an aim easily it is necessary that in addition to the motive which determines the will, the kind of volitional effort should be quite familiar and easy to the mind.[2] The effort felt in singing the leading note does not lie in the larynx, but in the difficulty we feel in fixing the voice upon it

[1] [This cadence is a *union* of the ancient Doric beginning with *c*, rendered harmonisable as the mode of the minor Sixth, (*c* — *d*♭ — †*e*♭ — *f* — *g* — †*a*♭ — *b*♭ — *c*′,) with the modern minor, beginning with *c*, (*c* — *d* — †*e*♭ — *f* — *g* — †*a*♭ — *b* — *c*′,) and will be more particularly considered in the next chapter. See also App. XIX., Table III., D, VIII , mimipi-ma.—*Translator*.]

[2] This periphrasis seems to me to render correctly the last clause in the following citation: *Διὰ τί τὴν παρυπάτην ᾄδοντες μάλιστα ἀποῤῥήγνυνται, οὐχ ἧττον ἢ τὴν νήτην καὶ τὰ ἄνω, μετὰ δὲ διαστάσεως πλείονος; Ἢ ὅτι χαλεπώτατα ταύτην ᾄδουσι, καὶ αὕτη ἡ ἀρχή; τὸ δὲ χαλεπὸν, διὰ τὴν ἐπίτασιν [καὶ πίεσιν] τῆς φωνῆς; ἐν τούτοις δὲ πόνος· πονοῦντα δὲ μᾶλλον διαφθείρεται. Διὰ τί δὲ ταύτην χαλεπῶς, τὴν δὲ ὑπάτην ῥᾳδίως· καίτοι δίεσις ἑκατέρας; Ἢ ὅτι μετ' ἀνέσεως ἡ ὑπάτη, καὶ ἅμα μετὰ τὴν σύστασιν ἐλαφρὸν τὸ ἄνω βάλλειν; Διὰ ταὐτὸ δὲ ἔοικε πρὸς μίαν ᾄδεσθαι τὰ πρὸς ταύτην ἢ παρανήτην ᾀδόμενα· δεῖ γὰρ μετὰ συννοίας καὶ καταστάσεως οἰκειοτάτης τῷ ἤθει πρὸς τὴν βούλησιν.* Arist. Probl. xix. 3, 4. [The whole passage may perhaps be translated thus: 'Why do those who sing the *parhypatē* break down not less than those who sing the *nētē* and higher tones, though with a greater disagreement (*διάστασις*)? Is it because they sing this with the greatest difficulty, even when this is the beginning? Does not difficulty arise from straining [and forcing] the voice? This occasions effort, and things done with effort are most apt to fail. But why do they sing the *parhypatē* with difficulty, and yet take the *hypatē* easily, although there is only a *diesis* (Semitone or Quartertone) between them? Is it because the hypatē is sung with a remission of effort, and at the same time it is easy to go upwards after getting oneself together for the effort (*σύστασις*)? For the same reason it is easy to sing what leads up to any note, or the *paranētē*. For the will requires not only conscious thought (*σύνοια*) but an inclination (*κατάστασις*) which is perfectly familiar to the habit of mind (*ἦθος*).' The passage is very difficult, and there was clearly a connection in the writer's mind between *διάστασις*, *σύστασις*, and *κατάστασις*, which influenced his reasoning, but evaporates in translation.—*Translator*.]

by mere volition while another tone is already in our mind, to which we desire to pass, and which, by its proximity conducted us to the leading note. It is not till we reach the final tone that we feel ourselves at home and at rest, and this final tone is sung without any strain on the will.

Proximity in the scale then gives a new point of connection between two tones, which is not merely active in the case of the leading note, just considered, but also, as already mentioned, in interpolating tones between two others in the chromatic and enharmonic modes. Intervals of pitch are in this respect analogous to measurements of distance. When we have the means of determining one point (the tonic) with great exactness and certainty, we are able by its means to determine other points with certainty, when they are at a known small distance from it (the interval of a Semitone), although perhaps we could not have determined them with so much certainty independently. Thus the astronomer employs his fundamental fixed stars, of which the positions have been determined with the greatest possible accuracy, for accurately determining the positions of other stars in their neighbourhood.

We may also remark that the interval of a Semitone plays a peculiar part as the introduction (*appoggiatura*) to another note. As an appoggiatura in a melody any tone can be used, even when not in its scale, provided it makes the interval of a Semitone with the note in the scale which it introduces; but a foreign tone which makes the interval of a whole Tone with that note in the scale, cannot be so used. The only justification of this use of the Semitone is certainly its existence as a well-known interval in the diatonic scale, which the voice can sing correctly and the ear can readily appreciate, even when the relations on which its magnitude depends are not clearly sensible in the passage where it is used. Hence also no arbitrarily chosen small interval can be thus employed. Although slight changes in the interval of the leading note may be introduced by practical musicians to give a stronger expression to its tendency towards the tonic, they must never go so far as to make those changes clearly felt.

Hence the major Seventh in its character of leading note to the tonic acquires a new and closer relationship to it, unattainable by the minor Seventh. And in this way the note which is most distantly related to the tonic becomes peculiarly valuable in the scale. This circumstance has continually grown in importance in

modern music, which aims at referring every tone to the tonic in the clearest possible manner; and hence, in ascending passages going to the tonic, the major Seventh has been preferred in all modern keys, even in those to which it did not properly belong. This transformation appears to have begun in Europe during the period of polyphonic music, but not in part songs only, for we find it also in the homophonic *Cantus firmus* of the Roman Catholic Church. It was blamed in an edict of Pope John XXII., in 1322, and in consequence the sharpening of the leading note was omitted in writing, but was supplied by the singers, a practice which Winterfeld believes to have been followed by Protestant musical composers even down to the sixteenth and seventeenth centuries, because it had once come into use. And this makes it impossible to determine exactly what were the steps by which this change in the old tonal modes was effected.[1]

Even to the present day, according to A. v. Oettingen's report,[2] the Esthonians struggle against singing the leading note in minor scales, although it may be clearly struck on the organ.

Among the ancient tonal modes, the Greek Lydian (major mode), and the unmelodic Hypolydian (mode of the Fifth) had the major Seventh as the leading note to the tonic, and hence the first was developed into the principal tonal mode of modern music, the major mode. The Greek Ionic (mode of the Fourth) differed from it only in having a minor Seventh. On simply altering this into a major Seventh, this mode also became major. On giving a major Seventh to each of the other three, they gradually converged to our present minor mode during the seventeenth century. From the Greek Phrygian (mode of the minor Seventh) by changing *B♭* into *B̲* we obtain

THE ASCENDING MINOR SCALE.

C—D—E♭—F—G—A̲—B̲—c

as we had already found from a simple consideration of the relationship of tones. The Greek Hypodoric or Eolic (mode of the

[1] 'Der evangelische Kirchengesang.' Leipzig, 1843, vol. 1, introduction.

[2] 'Das Harmoniesystem in dualer Entwickelung.' Dorpat und Leipzig, 1866, p. 113. [Mr. Curwen in reference to the ecclesiastical Doric, or mode of the minor Seventh, says: 'In Wales, both North and South, this mode is much preferred to the *Lah* mode,' the modern minor mode, 'and *popular tunes printed in one mode are sung in the other.*'

minor Third), which answers to our descending minor scale, gives on changing $B\flat$ into $\underline{B}$,

THE INSTRUMENTAL MINOR SCALE.

$$C - D - E\flat - F - G - A\flat - \underline{B} - c$$

which is difficult for singers to execute, on account of the interval $A\flat - \underline{B}$, but frequently occurs in modern music both ascending and descending.[1]

The Greek Doric (mode of the Sixth) with a major instead of a minor Seventh, is still discoverable in the final cadence mentioned on p. 438.

The general introduction of the leading tone represents, therefore, a continually increasing consistency in the development of a feeling for the predominance of the tonic in a scale. By this change, not only is the variety of character in the ancient tonal modes seriously injured, and the wealth of previous means of expression essentially diminished, but even the links of the chain of tones in the scale were disrupted or disturbed. We have seen that the most ancient theory made tonal systems consist of series of Fifths, and that each system had at first four and afterwards six intervals of a Fifth. The predominance of a tonic as the single focus of the system was not yet indicated, at least externally; it became apparent at most by a limitation of the number of Fifths to contain those tones only which occurred in the natural scale.

He then cites '*Bangor*, a tune of the ancient British Church, as it may now be heard in the churches of Wales and Scotland whenever sung, without book or instrument,' with the cadences $d'' - c'' - b' - a'$, and $a' - g' - a'$, and says that 'when musical men altered the melody to suit the requirements of modern harmony, and printed these altered melodies, the consequence was that the people either ceased to use the tune or continued to sing it differently from the printed copy.' And then in reference to the tune *Windsor* or *Dundee*, he mentions that some copies used the cadences $a' - \sharp g' \natural$ twice, and $a' - \sharp g' \natural - a'$ once, in place of the original $a' - g'$ and $a' - g' - a'$, in which form it appeared in Este's 'Whole Book of Psalms,' A.D. 1592, but so strong was the resistance of the popular ear to such an alteration of the melody that forty years later the tune appears in John Knox's Psalter with the first and second [g'] unaltered, and only the last made into [$\sharp g' \natural$]. 'Standard Course,' pp. 84, 85.—*Translator.*]

[1] [In the notation of App. XIX., the ascending minor scale is

$$C - D - \dagger E\flat - F - G - A - B - c,$$

and the instrumental minor scale is

$$C - D - \dagger E\flat - F - G - \dagger A\flat - B - c.$$ —*Translator.*]

All Greek tonal modes may be formed from the tones in the series of Fifths—

$$F - C - G - D - A - E - B.$$[1]

Directly we proceed to the natural intonation of Thirds, the series of Fifths is interrupted by an imperfect Fifth, as in

$$F - C - G - D - \underline{A} - \underline{E} - \underline{B}$$

where the Fifth $D - \underline{A}$ is imperfect. And when finally the sharp leading note is introduced, as by the use of $\underline{G}\sharp$ for G in $\underline{A}$ minor, the series is entirely interrupted.

In the gradual development of the diatonic system, therefore, the various links of the chain which bound the tones together were sacrificed successively to the desire of connecting all the tones in a scale with one central tone, the tonic. And in exact proportion to the degree with which this was carried out, the conception of tonality consciously developed itself in the minds of musicians.

The further development of the European tonal system is due to the cultivation of harmony, which will occupy us in the next chapter.

But before leaving our present subject, some doubtful points have still to be considered. In the preceding chapter I have shewn that the melodic relationship of tones can be made to depend upon their upper partials, precisely in the same way as their consonance was shewn to be determined in Chapter X. Now this method of explanation may in a certain sense be considered to agree with the favourite assertion that '*melody is resolved harmony*,' on which musicians do not hesitate to form musical systems without staying to enquire how harmonies could have been resolved into melodies at times and places where harmonies had either never been heard, or were, after hearing, repudiated. According to our explanation, at least, the same physical peculiarities in the composition of musical tones, which determined consonances for tones struck simultaneously, would also determine melodic relations for tones struck in succession. The former then would not be the reason for the latter, as the above phrase suggests, but both would have a common cause in the natural formation of musical tones.

[1] [In the notation of App. XIX.,

$$F - C - G - D - \dagger A - \dagger E - \dagger B.$$—*Translator.*]

Again, in consonance we found other peculiar relations, due to combinational tones, which become effective even when simple tones, or tones with few and faint upper partials, are struck simultaneously. I have already shewn that combinational tones very imperfectly replace the effect of upper partial tones in a consonance, and that consequently a chord formed of simple tones is wanting in brightness and character, the distinctions between consonance and dissonance being only very imperfectly developed.

In melodic passages, however, combinational tones have no effect at all, and hence the question arises as to how far a melodic effect could be produced by a succession of simple tones. There is no doubt that we can recognise melodies which we have already heard, when they are executed on the stopped pipes of an organ, or are whistled with the mouth, or struck out on a glass or wood or steel harmonica, as a musical box, or are played on bells. But there is also no doubt that all these instruments, which generate simple tones, either alone or accompanied by weak and remote inharmonic secondary tones, are incapable of producing any effective melodic impression without an accompaniment of musical instruments proper. They may be often extremely effective for performing single parts when accompanied by the organ, or the orchestra, or a pianoforte, but by themselves they produce very poor music indeed, which degenerates into absolute unpleasantness when the inharmonic secondary tones are somewhat too loud.

We are bound, however, to give some reason why any impression of melody at all can be produced by such instruments.

Now we must first remember that, as shewn at the end of Chapter VII., the construction of the ear itself favours the generation of weak harmonic upper partials within its chambers when powerful but objectively simple tones are sounded. Hence it is at most very weak objectively simple tones which can be regarded as also subjectively simple.

Next, there is an effect of memory to be brought into account. Supposing that I have been used to hear Fifths taken at all possible pitches, and have recognised them by aural sensation as having a very close melodic relationship, I should know the magnitude of this interval by experience for every tone in the scale, and should retain the knowledge thus acquired by the action of a man's memory

of sensations, even of those for which he has no verbal expression.

When, then, I hear such an interval executed on tuning-forks, I am able to recognise it as an interval I have often heard, although its tones have either none, or only some faint remnants of those upper partials which formerly gave it a right to be considered as a favourite interval of close melodic relationship. And just in the same way I shall be able to recognise, as previously known, other melodic passages or whole melodies which are executed in simple tones, and even if I hear a melody for the first time in this way, whistled with the mouth or chimed by a clock, or struck on a glass harmonica, I should be able to complete it by imagining how it would sound if executed on a real musical instrument, as the voice or a violin.

A practised musician is able to form a conception of a melody by merely reading the notes. If we give the prime tones of these notes on a glass harmonica, we give a firmer basis to the conception by really exciting a large portion of the impression on the senses which the melody would have produced if sung. Simple tones, however, merely exhibit an outline of the melody. All that gives the melody its charm is absent. We know, indeed, the individual intervals which it contains, but we have no immediate impression on our senses which serves to distinguish those which are distantly from those which are closely related, or the related from the totally unrelated. Observe the great difference between merely whistling a melody or playing on a violin; between striking it on a glass harmonica or on the piano! The difference is somewhat of the same kind as that between viewing a single photograph of a landscape, and seeing two corresponding photographs of it through a stereoscope. The first enables me by means of my memory, to form a conception of the relative distances of its parts, and this conception may be often very satisfactory. But the stereoscopic fusion of the two figures gives me the real impression on the senses which the relative distances of the parts of the landscape would have themselves produced, and which I am obliged in the case of a single image to supply by experience and memory. Hence the stereoscopic picture is more lively than the simple perspective view, exactly in the same way as immediate impressions on our senses are more lively than our recollections.

CHAPTER XV.

THE CONSONANT CHORDS OF THE TONAL MODES.

POLYPHONY was the form in which music for several voices first attained a certain degree of artistic perfection. The peculiar characteristic of this style of music was that several voices were singing each its own independent melody at the same time, which might be a repetition of the melodies already sung by the other voices, or else quite a different one. Under these circumstances each voice had to obey the general law of tonality common to the construction of all melodies, and, moreover, in this peculiar case each melody had to be referred to the same tonic. Hence each voice had to commence separately on the tonic or some tone closely related to it, and to close in the same way. In practice each voice was made to begin with the tonic or its Octave. This fulfilled the law of tonality, but necessitated the closing of a polyphonic piece by a unison.

The reason why higher Octaves might accompany the tonic at the close, lies, as we saw in the last chapter, in the fact that higher Octaves are merely repetitions of portions of the fundamental tone. Hence by adding its Octave to the tonic at the close, we merely reinforce part of its compound tone; no new compound or simple tone is added, and the union of all the tones contains only the constituents of the tonic itself.

The same is true for all the other partial tones which are contained in the tonic. The next step in the development of the final chord was to add the Twelfth of the tonic. Now the chord $c-c'-g'$ contains no element which is not also a constituent of the compound tone c when sounded alone, and consequently, being a mere representative of the single musical tone c, it is suitable for the termination of a piece of music having the tonic c.

Nay even the chord $c'-g'-c''$ might be so used, for when it is struck we hear, weakly indeed, but still sensibly, the combi-

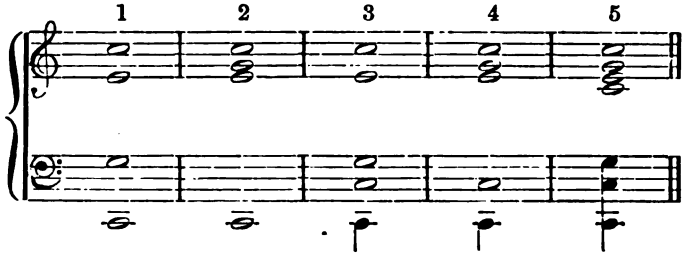
national tone *c*, so that the whole mass of tone again contains nothing more than the constituents of the tone *c*. It must be owned, however, that this combination would answer to a rather unusual quality of tone, with a proportionably weak prime partial.

On the other hand, it was not possible to use the chords $c-c'-f'$ or $c'-f'-c''$ to end a piece having the tonic *c*, although these chords are consonances as well as the preceding, because *f* is not an element of the compound tone *c*, and hence the closing chord would contain something which was not the tonic at all. It is here probably that we have to look for the reason why some medieval theoreticians wished to reckon the Fourth among the dissonances.[1] But perfect consonance was not sufficient to make an interval available for the final chord. There was a second condition which the theoreticians did not clearly understand. The tones of the final chord had to be constituents of the compound tone of the tonic. This was the only case in which they could be employed.

The Sixth of the tonic is as ill suited as the Fourth for use in the final chord. But the major Third can be used, because it occurs as the Fifth partial tone of the tonic. Since the qualities of tone which are fit for music generally allow the fifth and sixth partial tone to be audible, but make the higher partials either entirely inaudible or at least very faint, and since, moreover, the seventh partial is dissonant with the fifth, sixth, and eighth, and is not used in the scale, the series of tones available for the closing chord terminates with the Third. Thus we actually find down to the beginning of the eighteenth century, that the final chord has either no Third, or only a major Third, even in tonal modes which contain only the minor and not the major Third of the tonic. To attain fulness, it was preferred to do violence to the scale by using the major Third in the closing chord. The minor Third of the tonic could never be a constituent of its compound tone. Hence it was originally as much forbidden as the Fourth and Sixth of the tonic. Before a minor chord could be used to close a piece of music the feeling for harmony had to be cultivated in a new direction.

[1] [Of course the Eleventh $c-f'$ in $c-c'-f'$ is very harsh and almost dissonant; but this does not apply to $c'-f'-c''$, which, however, has the tones of *f*, as $f-c'-f'-c''$, and not of *c*. See also p. 299, note 2.—*Translator.*]

The ear is the more satisfied with a closing major chord, the more closely the order of the tones used imitates the arrangement of the partial tones in a compound. Since in modern music the upper voice is most conspicuous, and hence has the principal melody, this voice must usually finish with the tonic. Bearing this in mind, we can use any of the following chords for the close (combinational tones are added as crotchets):—



In the chords 1 and 2 all the notes coincide with partials of *C*, and they therefore most closely resemble the compound tone *C* itself. And then closer positions of the chord can be substituted, provided they resemble the first by having *C* for the fundamental tone as in 3, 4, 5. They still retain sufficient resemblance to the compound tone of *C* to be used in its place. Moreover, the combinational tones, written as crotchets in 3, 4 and 5, assist in the effect by making the deeper partials of the compound *C*, at least faintly, audible. But the first two positions always give the most satisfactory close. The tendency towards a deep final tone in harmonic music is very characteristic, and I believe that the above is its proper explanation. There is nothing of the kind in the construction of homophonic melodies. It is peculiar to the bass of part music.

Precisely in the same way that the tonic, when used as the bass of its major chord at the close, gives it a resemblance to its own compound tone, and is hence felt as the essential tone of the chord, all major chords sound best when the lowest tone of their closest triad position (No. 4, p. 333) is made the bass. The other major chords in the scale are those on its Fourth and Fifth, and hence for the scale of *C* major, are $F-\underline{A}-C$ and $G-\underline{B}-D$. Hence if we make the harmony of a piece of music to consist of these major chords only, each having its fundamental tone in the bass, the effect is almost that of a compound tonic in different qualities of

tone passing into its two nearest related compound tones, the Fourth and Fifth. This makes the harmonisation transparent and definite, but it would be too uniform for long pieces. Modern popular tunes, songs and dances, are however, as is well known, constructed in this manner. The people, and generally persons of small musical cultivation, can be pleased only by extremely simple and intelligible musical relations. Now the relations of the tones are generally much easier to feel with distinctness in harmonised than in homophonic music. In the latter the feeling of relationship of tone depends solely on the sameness of pitch of two partials in two consecutive musical tones. But when we hear the second compound tone we can at most remember the first, and hence we are driven to complete the comparison by an act of memory. The consonance, on the other hand, gives the relation by an immediate act of sensation; we are no longer driven to have recourse to memory; we hear beats, or there is a roughness in the combined sound, when the proper relations are not preserved. Again, when two chords having a common note occur in succession, our recognition of their relationship does not depend upon weak upper partials, but upon the comparison of two independent notes, having the same force as the other notes of the corresponding chord.

When, for example, I ascend from *C* to its Sixth *A*, I recognise their mutual relationship in an unaccompanied melody, by the fact that *e'*, the fifth partial of *C*, which is already very weak, is identical with the third of *A*. But if I accompany the *A* with the chord *F*—*A*—*c*, I hear the former *c* of the chord continue to sound powerfully, and know by immediate sensation, that *A* and *C* are consonant, and both of them constituents of the compound tone *F*.

When I pass melodically from *C* to *B* or *D*, I am obliged to imagine a kind of mute *G* between them, in order to recognise their relationship, which is of the second degree. But if I audibly sustain the note *G* while the others are sounded, their common relationship becomes really sensible to my ear.

Habituation to the tonal relations so evidently displayed in harmonic music, has had an indisputable influence on modern musical taste. Unaccompanied songs no longer please us; they seem poor and incomplete. But if merely the twanging of a guitar adds the fundamental chords of the key, and indicates the

harmonic relations of the tones, we are satisfied. Again, we cannot fail to see that the clearer perception of tonal relationship in harmonic music has greatly increased the practicable variety in the relations of tones, by allowing those which are less marked to be freely used, and has also rendered possible the construction of long musical pieces which require powerful links to connect their parts into one whole.

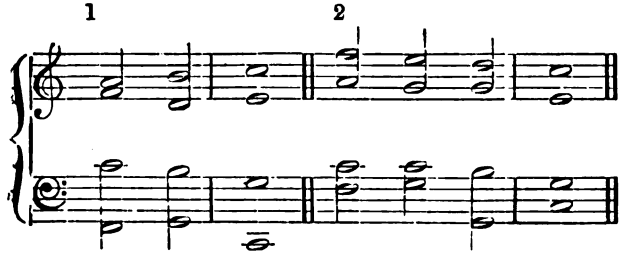
The closest and simplest relation of the tones is reached in the major mode, when all the tones of a melody are treated as constituents of the compound tone of the tonic, or of the Fifth above or the Fifth below it. By this means all the relations of tones are reduced to the simplest and closest relation existing in any musical system—that of the Fifth.

The chord of the tonic *C* is somewhat differently related to the chord of *G*, the Fifth above it, and to the chord of *F*, the Fifth below it. When we pass from *C*—*E*—*G* to *G*—*B*—*d* we use a compound tone, *G*, which is already contained in the first chord, and is consequently properly introduced, while at the same time such a step leads us to those degrees of the scale which are most distant from the tonic, and have only an indirect relationship with it. Hence this passage forms a distinct progress in the harmony, which is at once well assured and properly based. It is quite different with the passage from *C*—*E*—*G* to *F*—*A*—*c*. The compound tone *F* is not prepared in the first chord, and it has therefore to be discovered and struck. The justification of this passage, then, is not complete on the ground of close relationship between the chords, until it is felt that the chord of *F* contains no tones which are not closely related to the tonic *C*. Hence in this passage from the chord of *C* to that of *F* we miss that distinct and well-assured progression which marked the passage from the chord of *C* to that of *G*. But as a compensation, the progression from the chord of *C* to that of *F* has a softer and calmer kind of beauty, due, perhaps, to its keeping within tones directly related to the tonic *C*. Popular music, however, favours the other passage from the tonic to the Fifth above (hence called the *dominant* of the key), and many of the simpler popular songs and dances consist merely of an interchange of tonic and dominant chords. Hence also the common harmonica (accordion, German concertina), which is arranged for them, gives the tonic chord on opening the bellows, and the dominant chord on closing them. The Fifth below the tonic is

called the *sub*dominant of the key. Its chord is seldom introduced at all into usual popular melodies, except, perhaps, once near the close, to restore the equilibrium of the harmony, which had chiefly inclined towards the dominant.

When a section of a piece of music terminates with the passage of the dominant into the tonic chord, musicians call the close a *complete cadence.* We thus return from the tones most distantly related to the tonic, to the tonic itself, and, as befits a close, make a distinct passage from the remotest parts of the scale to the centre of the system itself. If, on the other hand, we close by passing from the subdominant to the tonic chord, the result is called an *imperfect* or *plagal cadence.* The tones of the subdominant triad are all directly related to the tonic, so that we are already close upon the tonic before we pass over to it. Hence the imperfect cadence corresponds to a much quieter return of the music to the tonic chord, and the progression is much less distinct than before.

In the complete cadence the chord of the tonic follows that of the dominant, but to preserve the equilibrium of the system in relation to the subdominant, its chord is made to precede that of the dominant as in 1 or 2.



This succession really forms the *complete close*, by bringing all the tones of the whole scale together, and thus in conclusion collecting and fixing every part of the key.

The major mode, as we have seen, permits the requisitions of tonality to be most easily and completely united with harmonic completeness. Every tone of its scale can be employed as a constituent of the musical tone of the tonic, the dominant, or the subdominant, because these fundamental tones of the mode are also fundamental tones of major chords. This is not equally the case in the other ancient tonal modes.

1. MAJOR MODE: $\underbrace{f - \underline{a} - c}_{\text{major}} \overbrace{- \underline{e} - g}^{\text{major}} \underbrace{- \underline{b} -}_{\text{major}}$

2. MODE OF THE FOURTH: $\underbrace{f - \underline{a} - c}_{\text{major}} \overbrace{- \underline{e} - g}^{\text{major}} \underbrace{- \bar{b}\flat - d}_{\text{minor}}$

3. MODE OF THE MINOR SEVENTH: $\underbrace{f - \underline{a} - c}_{\text{major}} \overbrace{- \bar{e}\flat - g}^{\text{minor}} \underbrace{- b\flat - d}_{\text{minor}}$

4. MODE OF THE MINOR THIRD (*minor mode*): $\underbrace{f - \bar{a}\flat - c}_{\text{minor}} \overbrace{- \bar{e}\flat - g}^{\text{minor}} \underbrace{- \bar{b}\flat - \bar{d}}_{\text{minor}}$

5. MODE OF THE MINOR SIXTH: $\underbrace{b\flat - \bar{d}\flat - f}_{\text{minor}} \overbrace{- \bar{a}\flat - c}^{\text{minor}} \underbrace{- \bar{e}\flat - g}_{\text{minor}}$ [1]

In the minor chords, the Third does not belong to the compound tone of its fundamental note, and hence cannot appear as a constituent of its quality; so that the relation of all the parts of a minor chord to the fundamental note is not so immediate as that for the major chord, and this is a source of difficulty in the final chord. For this reason we find almost all popular dance and song music written in the major mode; indeed, the minor mode forms a rare exception. The people must have the clearest and simplest intelligibility in their music, and this can only be furnished by the

[1] [In the notation of Appendix XIX., Sections A and B, where the use of (+) and (—) between notes, to shew that their interval is a major or minor Third respectively, sufficiently indicates the nature of the chords, these five scales will be:

1. $f + a - c + e - g + b - d$
2. $f + a - c + e - g - \dagger b\flat + d$
3. $f + a - c - \dagger e\flat + g - \dagger b\flat + d$
4. $f - \dagger a\flat + c - \dagger e\flat + g - \dagger b\flat + d$
5. $b\flat - d\flat + f - \dagger a\flat + c - \dagger e\flat + g$.—*Translator.*]

major mode. But there was nothing like this predominance of the major key in homophonic music. For the same reason the harmonic accompaniment of chorales in major keys was developed with tolerable completeness as early as the sixteenth century, so that many of them correspond with the cultivated musical taste of the present day; but the harmonic treatment of the minor and the other ecclesiastical modes was still in a very unsettled condition, and the various harmonies then used strike modern ears as very strange.

In a major chord $c-\underline{e}-g$, we may regard both g and $\underline{e}$ as constituents of the compound tone of c, but neither c nor g as constituents of the compound tone of $\underline{e}$, and neither c nor $\underline{e}$ as constituents of the compound tone of g.[1] Hence the major chord $c-\underline{e}-g$ is completely unambiguous, and can be compared only with the compound tone of c, and consequently c is the predominant tone in the chord, its *root*, or, in Rameau's language, its *fundamental bass*; and neither of the other two tones in the chord has the slightest claim to be so considered.

In the minor chord $c-\bar{e}\flat-g$, the g is a constituent of the compound tones of both c and $\bar{e}\flat$. Neither $\bar{e}\flat$ nor c occurs in either of the other two compound tones.[2] Hence it is clear that g at least is a dependent tone. But, on the other hand, this minor chord can be regarded either as a compound tone of c with an added $\bar{e}\flat$ or as a compound tone of $\bar{e}\flat$ with an added c. Both views are entertained at different times, but the first usually prevails. If we regard the chord as the compound tone of c, we find g for its third partial, and while the foreign tone $\bar{e}\flat$ occupies the place of the weak fifth partial $\underline{e}$. But if we regarded the chord as a compound tone of $\bar{e}\flat$, although the weak fifth partial g would be properly represented, the stronger third partial, which ought to be $\bar{b}\flat$ is replaced by the foreign tone c. Hence in modern music we usually find the minor chord $c-\bar{e}\flat-g$ treated as if its root or fundamental bass were c, so that the chord appears as a somewhat

[1] [Taking only seven partials, and using the notation of Appendix XIX., we have for—

Compound Tones	Simple Partial Tones						
	1	2	3	4	5	6	7
C	C	c	g	c'	e'	g'	$\underline{e}$ $b'\flat$
E	E	e	b	e'	$\ddagger g'\natural$	b'	$\underline{e}\ddagger$ d''
G	G	g	d'	g'	b'	d''	$\underline{e}$ f''
$\dagger E\flat$	$\dagger E\flat$	$\dagger e\flat$	$\dagger b\flat$	$\dagger e'\flat$	g'	$\dagger b\flat$	$\underline{e}$ $d''\flat$.—*Translator.*]

[2] [In the notation of Appendix XIX., the minor chord is $c-\dagger e\flat+g$. The upper partials of $\dagger E\flat$ are given in the last note.—*Translator.*]

altered and obscured compound tone of *c*. But the chord also occurs in the position $\bar{e}\flat - g - c$ (or better still as $\bar{e}\flat - g - \bar{c}$) even in the key of $\bar{B}\flat$ major, as a substitute for the chord of the subdominant $\bar{e}\flat$. Rameau then calls it the chord of the major Sixth, and, more correctly than most modern theoreticians, regards $\bar{e}\flat$ as its fundamental bass.[1]

When it is important to guide the ear in selecting one or other of these two meanings of the minor chord, the root intended may be emphasised by giving it a low position or by throwing several voices upon it. The low position of the root allows such other tones as could be fitted into its compound tone, to be considered directly as its partials, whereas the low compound tone itself cannot be considered as the partial of any other and much higher tone. In the first half of last century when the minor chord was first used as a close, composers endeavoured to give prominence to the tonic by increasing the loudness of the tonic note in comparison with its minor Third. Thus in Handel's oratorios, when he concludes with a minor chord, most of the conspicuous vocal and instrumental parts are concentrated on the tonic, while the minor Third is either touched by one voice alone, or merely by the accompanying pianoforte or organ. The cases are much rarer where in minor keys he gives only two voices to the tonic in the closing chord, and one to its Fifth and another to its Third, which is his rule in major modes.

When the minor chord appears in its second subordinate signification, as $\bar{e}\flat - g - c$ with the root $\bar{e}\flat$, this fact is shewn partly by the position of $\bar{e}\flat$ in the bass, and partly by its close relationship to the tonic $\bar{b}\flat$. Modern music even makes this interpretation of the chord still clearer by adding $\bar{b}\flat$ as the Fifth of $\bar{e}\flat$, so that the chord becomes dissonant in the form $\bar{e}\flat - g - \bar{b}\flat - \bar{c}$.[2]

[1] [The scale of $\dagger B\flat$ major, using the notation of Appendix XIX., has the chords, $\dagger e\flat + g - \dagger b\flat + d - \dagger f + \dagger a - \dagger c$; hence, regarding the chord as made up of the notes of this scale, it would be $\dagger c \mid \dagger e\flat + g$, (where $\mid$ indicates a Pythagorean minor Third,) which is not a minor chord at all, like $c - \dagger e\flat + g$, as it has a Pythagorean in place of a natural minor Third. It was only tempered intonation which confused the two cases. Attention will be hereafter drawn to this important distinction.—*Translator.*]

[2] [In the notation of Appendix XIX.,

$$\dagger c \mid \dagger e\flat + g - \dagger b\flat$$

which is not a minor chord at all, but a major chord, the subdominant in the key of $\dagger B\flat$, rendered dissonant by introducing $\dagger c$, the Second of the key, or the Sixth above the subdominant $\dagger e\flat$. Observe that it is $\dagger c$ which is now introduced in the text, in place of *c*. If *c* is retained, thus $c - \dagger e\flat + g - \dagger b\flat$, the chord is one of those chords of the Seventh considered in Chapter XVI.—*Translator.*]

The disinclination of older composers to close with a minor chord, may be explained partly by the obscuration of its consonance from false combinational tones, and partly because, as already mentioned, it does not give a mere quality of the tonic tone, but mixes foreign constituents with it. But in addition to the minor Third, which does not fit into the compound tone of the tonic, the combinational tones of a minor chord are equally foreign to it. As long as the feeling of tonality required a definite single compound tone for the connecting centre of the key, it was impossible to form a satisfactory close except by a reproduction of the pure compound tone of the tonic with no foreign admixture. It was not till a further development of musical feeling had given the chords of the mode an independent significance, that the minor chord, notwithstanding its possession of constituents foreign to the compound tone of the tonic, could be justified in its use as a close.

Hauptmann[1] gives a different reason for avoiding the minor chord at the close. He asserts that before the chord of the dominant Seventh came into use, there was no voice suitable for falling into the minor Third of the tonic. Thus if the final cadence consisted of the chords $G-\underline{B}-D$, $C-\bar{E}\flat-G$, the D of the first chord was the only one which could pass melodiously in $\bar{E}\flat$, but this would have appeared like the passage of the leading note D in the key of $\bar{E}\flat$ major into its tonic $\bar{E}\flat$, and hence have called up the feeling of $\bar{E}\flat$ major in lieu of C minor. We may admit that this relation of the leading note would have drawn the hearer's special attention to the two tones in question, and to a certain extent disturbed his recognition of the key, but yet it is clear that even without the help of this chord of the dominant Seventh, there were several ways for the voices to pass through dissonances into the minor Third of the closing chord, if composers had felt any wish to do so. Thus in the plagal cadence

$$\begin{array}{l} c-\bar{e}\flat-g\ \ -c' \\ F-f\ \ -\bar{a}\flat-c' \\ C-\bar{e}\flat-g\ \ -c' \end{array}$$

which is so often used on other occasions, the Fourth f could be made to descend to the minor Third $\bar{e}\flat$ without any inconvenience. Indeed, we find that when the chord of the dominant Seventh had actually come into use, and the Seventh F of the chord $G-\underline{B}-D-F$ ought by every right to have descended into the

[1] 'Harmonik und Metrik,' Leipzig, 1853, p. 216.

minor Third *E*♭ of the closing chord, musical pieces of the fifteenth century[1] avoid this progression, and make this Seventh *F* either ascend to the Fifth *G*, or descend to the major Third *E* of the final chord, instead of to *E*♭, its minor Third. This custom prevailed down to Bach's time.

In Chapter XIII. we characterised modern harmonic music, as contrasted with medieval polyphony by its development of a feeling for the independent significance of chords. In Palestrina, Gabrieli, and still more in Monteverde and the first composers of operas, we find the various degrees of harmoniousness in chords carefully used for the purposes of expression. But these masters are almost entirely without any feeling for the mutual relation of consecutive chords. Their chords often follow one another by entirely unconnected leaps, and the only bond of union is the scale, to which all their notes belong.

The transformation which took place from the sixteenth to the beginning of the eighteenth century, may, I think, be characterised by the development of a feeling for the independent relationship of chords one to the other, and by the establishment of a central core, the *tonic chord*, round which were grouped the whole of the consonant chords that could be formed out of the notes of the scale. For these chords there was a repetition of the same effort which was formerly shewn in the construction of the scale, where interrelations of the tones were first grounded on a chain of intervals, and afterwards on a reference of each note to a central compound tone, the tonic.

Two chords which have one or more tones in common will here be termed *directly related.*

Chords which are directly related to the same chord will be here said to be *related in the second degree.*

Thus $c-\underline{e}-g$ and $g-\underline{b}-d$ are directly related, and so are $c-\underline{e}-g$ and $\underline{a}-c-\underline{e}$; but $g-\underline{b}-d$ and $\underline{a}-c-\underline{e}$ are related only in the second degree.

When two chords have two tones in common they are more closely related than when they have only one tone in common. Thus $c-\underline{e}-g$ and $\underline{a}-c-\underline{e}$ are more closely related than $c-\underline{e}-g$ and $g-\underline{b}-d$.

[1] See an example in Anton Brumel, in Forkel's 'Geschichte der Musik,' vol. ii. p. 647. Another, with a plagal cadence by Josquin, will be found, *ibid.* p. 550, where the voices might have easily been led to the minor Third.

The tonic chord of any tonal mode can of course only be one which more or less perfectly represents the compound tone of the tonic, that is, that major or minor chord of which the tonic is the root. The tonic note, as the connecting core of all the tones in a regularly constructed melody, must be heard on the first accented part of a bar, and also at the close, so that the melody starts from it and returns to it; the same is true for the tonic chord in a succession of chords. In both of these positions in the scale we require to hear the tonic note, accompanied not by any arbitrary chord, but by the tonic chord itself, having the tonic note itself as its root. This was not the case even as late as the sixteenth century, as is seen by the example on p. 379 taken from Palestrina.

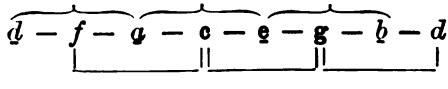
When the tonic chord is major, the domination of all the tones by the tonic note is readily reconciled with the domination of all the chords by the tonic chord, for as the piece begins and ends with the tonic chord, it also begins and ends at the same time with the pure unmixed compound tone of the tonic note. But when the tonic chord is minor, all these conditions cannot be so perfectly satisfied. We are obliged to sacrifice somewhat of the strictness of the tonality in order to admit the minor Third into the tonic chord at the beginning and end. At the commencement of the eighteenth century we find Sebastian Bach using minor chords at the end of his preludes, because these were merely introductory pieces, but not at the end of fugues, chorales, and at other complete closes. In Handel and even in the ecclesiastical pieces of Mozart, the close in a minor chord is used alternately with the close in a chord without any Third, or with the major Third. And the two last composers cannot be accused of external imitation of old habits, for we find that in these usages they always observe the expression of the piece. When at the close of a composition in the minor mode, a major chord is introduced, it has the effect of a sudden and unexpected brightening up of the sadness of the minor key; producing a cheering, enlightening, and reconciling effect after the sorrow, grief, or unquiet of the minor. Thus a close in the major suits the prayer for the peace of the departed in the words, 'et lux perpetua luceat eis,' or the conclusion of the *Confutatis maledictis*, which runs thus:—

Oro supplex et acclinis
Cor contritum quasi cinis
Gere curam mei finis.

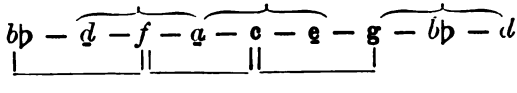
But such a closing major chord is certainly somewhat startling for our present musical feeling, even though its introduction may, at one time, add wondrous beauty and solemnity, or, at another, dart like a beam of hope into the gloom of deepest despair. If the unquiet remains to the last, as in the *Dies irae* of Mozart's 'Requiem,' the minor chord, in which an unresolved disturbance exists, forms a fitting close. Mozart was wont to terminate ecclesiastical pieces of a less decided character with a chord that had no Third. There are many similar examples in Handel. Hence although both masters stood on the very same platform as modern musical feeling, and themselves gave, as it were, the finishing touch to the construction of the modern tonal system, they were not altogether strangers to the feeling which had prevented older musicians from using the minor third of the tonic in the final chord. They followed no strict rule, however, but acted according to the expression and character of the piece and the sense of the words with which they had to close.

Those tonal modes which furnish the greatest number of consonant chords related to one another or to the common chord, are best adapted for artistically connected harmonies. Since all consonant chords, when reduced to their closest position and simplest form, are triads consisting of a major and a minor Third, all the consonant chords of any key can be found by simply arranging them in order of Thirds, as in the following tables. The braces above and below connect the chords together. The ordinary round braces, which are placed above point out minor chords; the square braces below indicate major chords. The tonic chord is printed in thick letters.

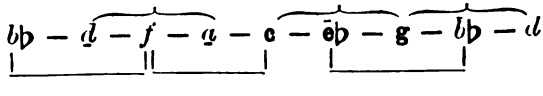
1) MAJOR MODE



2) MODE OF THE FOURTH



3) MODE OF THE MINOR SEVENTH



4) MODE OF THE MINOR THIRD

$$b\flat - d - f - \bar{a}\flat - c - \bar{e}\flat - g - \bar{b}\flat - d$$

5) MODE OF THE MINOR SIXTH

$$b\flat - \bar{d}\flat - f - \bar{a}\flat - c - \bar{e}\flat - g - \bar{b}\flat$$

In this arrangement I have introduced the different intonations of the Second and Seventh of the key, which we found in the construction of the scales for homophonic music.[1] But we observe that the chords directly related to the tonic chord contain every tone in the scale, excepting in the mode of the minor Sixth. The Second and Seventh of the tonic occur first in the chord of *G*, which is directly related to the tonic chord, and next in the chord of *F*, which is not directly connected with the tonic chord. The supplementary tones of the scale which are related to the dominant thus acquire in harmonic music an important preponderance over those related to the subdominant. We must necessarily prefer

[1] [These scales differ from those transcribed in p. 453 note, only in the addition of the secondary forms of intercalary tones, ‡*d*, *b*♭, or †*b*♭, which, in fact, imply modulations into adjacent modes, or else give a double and ambiguous character to each mode. Thus (1) has ‡*d*, and hence A mipími, No. 52 (of App. XIX., Table III., C), in addition to its own mode; (2, 3, 4) have *b*♭, ‡*d*, and hence (2) has the complete scale of *F* major, as well as the mode of the Fourth of *C*; (3) has F mapámi, No. 31 (*ibid*), as well as its own mode, and (4) has F mapími No. 45 (*ibid*), as well as its own mode; (5) by the addition of †*b*♭ has †*A*♭ major in addition to its own mode. This *double modality* is shewn as follows, the upper braces referring to the names in the text:—

1) $\ddagger d - \overbrace{\underbrace{f + a - c + e - g + b}_{A \text{ mipími}} - d}$

2) $\underbrace{b\flat + \ddagger d - \overbrace{f + a - c + e - g}}_{F \text{ mapáma, or major}} - \overbrace{\dagger b\flat + d}$

3) $\underbrace{b\flat + \ddagger d - \overbrace{f + a - c - \dagger e\flat + g}}_{F \text{ mapámi}} - \overbrace{\dagger b\flat + d}$

4) $\underbrace{b\flat + \ddagger d - \overbrace{f - \dagger a\flat + c - \dagger e\flat + g}}_{F \text{ mapími}} - \overbrace{\dagger b\flat + d}$

5) $\overbrace{b\flat - \underbrace{d\flat + f - \dagger a\flat + c - \dagger e\flat + g}_{\dagger A\flat \text{ mapáma, or major.}} - \dagger b\flat}$ —*Translator.*]

direct to indirect relations for determining scalar degrees. Hence by confining ourselves to the chords which are directly related to the tonic chord, we obtain the following arrangement of the tonal modes[1]:—

1) MAJOR MODE

f — *a* — c — e — g — *b* — *d*

2) MODE OF THE FOURTH

f — *a* — c — e — g — *b*♭ — *d*

3) MODE OF THE MINOR SEVENTH

f — *a* — c — e♭ — g — *b*♭ — *d*

4) MODE OF THE MINOR THIRD

f — *a*♭ — c — e♭ — g — *b*♭ — *d*

5) MODE OF THE MINOR SIXTH

d♭ — *f* — *a*♭ — c — e♭ — g — *b*♭.

A glance at this table shews that the *major mode* and *mode of the minor Third* (*minor mode*) possess the most complete and connected series of chords, so that these two are decidedly superior to the rest for harmonic purposes. This is also the reason which led to the preference given to them in modern music.

[1] [The transcription in p. 453, note, will suffice for the first four of these cases also. The settlement in the text avoids the *double modality* alluded to on the last page, note, and fixes the modes in the meanings of

(1) *C* mapáma, No. 3 of Table III., C. App. XIX.
(2) *C* mapámi, No. 31 " "
(3) *C* mapimi, No. 45 " "
(4) *C* mipimi, No. 52 " "
(5) †*A*♭ matáma, No. 4 " "

The last is *d*♭ + *f* — †*a*♭ + *c* — †*e*♭ + *g* — †*b*♭, but begins with *e*. The proper and peculiar form of the mode of the minor Sixth of *C*, on the other hand, seems assumed by the composers to be *F* mimipí, No. 54, or *b*♭ — *d*♭ + *f* — †*a*♭ + *c* — †*e*♭ + *g*, beginning, therefore, with *C*. But where all notation has hitherto been tempered, so that *b*♭ and †*b*♭ are not distinguished, great ambiguity necessarily prevails.—*Translator.*]

And in this way we obtain a final settlement of the proper intonation of the supplementary tones of the scale for the first four modes. Hauptmann, with whom I agree, considers the tone *D* alone to be the essential constituent of both the major and minor modes of *C*. This *D* forms an imperfect (Pythagorean) minor Third with *F*, so that the chord *D* — *F* — *A* must be considered as dissonant. This chord thus intoned is in reality most decidedly dissonant to the ear. On the other hand, Hauptmann admits a major mode which inclines to the subdominant, and uses *D* in place of *D*. I consider this conception to be a very happy expression of the real state of things. When the consonant chord *D* — *F* — *A* occurs in any composition it is impossible to return immediately, without any transitional tone, to the tonic chord *C* — *E* — *G*. The result would be felt as an harmonic leap without adequate notice. Hence it is a correct expression of the state of affairs to look upon the use of this chord as the beginning of a modulation beyond the boundaries of the key of *C* major, that is, beyond the limits of direct relationship to its tonic chord. In the minor mode this would correspond to a modulation into the chord of *D*♭ — *F* — *A*♭. Of course in the modern tempered intonation the consonant chord *D* — *F* — *A* is not distinguished from the dissonant *D* — *F* — *A*, and hence the feeling of musicians has not been sufficiently cultivated to make them appreciate this difference on which Hauptmann insists.[1]

As regards the other supplementary tone *b*♭, which may occur in the chords *e*♭ — *g* — *b*♭ and *g* — *b*♭ — *d'*, I have already shewn in the last chapter that even in homophonic music it is almost always replaced by *b*. Harmonic considerations likewise favour the use of *b*, independently of melodic progression. It has been already shewn that when the two tones of the scale which are but distantly related to the tonic, make their appearance as constituents of the dominant, they enter into close relation to the tonic. Now this can only be the case with the compound tones of the major chord *g* — *b* — *d*, and not with those of the minor chord *g* — *b*♭ — *d*. Considered independently, the tones *b*♭ and *d* are quite as closely related to *c* as the tones *b* and *d*. But by regarding the two latter as constituents of the compound tone *g*, we connect them with *c* by the same closeness of relationship as *g* is itself connected with *c*.

[1] [This was referred to in p. 455, note 1.—*Translator.*]

Hence, in all modern music, wherever $\bar{b}\flat$ might occur as a const tuent of the dominant chord of the key of *c* minor, or of son dissonant chord replacing the dominant chord, it is usual t change it into $\underline{b}$, and otherwise to use either $\bar{b}\flat$ or $\underline{b}$ according t the melodic progression, but more frequently the latter, as I hav already remarked when treating of the construction of min scales. It is this systematic use of the major Seventh $\underline{b}$ in pla of the minor Seventh $\bar{b}\flat$ of the key which now distinguishes th modern minor mode from the ancient Hypodoric, or the mode the minor Third. Here again, then, some part of the consisten of the scale is sacrificed to make the harmony more consistent.

The chain of consonant chords in the mode of the min Third is certainly impaired when that mode is transformed int our minor by the introduction of $\underline{b}$. In place of the chain

$$\overbrace{f-\bar{a}\flat-c}\overbrace{-\bar{e}\flat-g}\overbrace{-\bar{b}\flat-d}$$

our minor furnishes only

$$\overbrace{f-\bar{a}\flat-c}\overbrace{-\bar{e}\flat-g}-\underline{b}-d$$

which has one triad less. But this is compensated by the fre alternation of both tones.

The introduction of the leading note $\underline{b}$ into the key of *c* minc generated a new difficulty in the complete closing cadence of th key. When the chord $g-\underline{b}-d$ is followed by the chord $c-\bar{e}\flat-$ the first being a perfectly harmonious major chord, and the latt an obscurely harmonious minor chord, the defect in the ha moniousness of the latter is made much more evident by th contrast. But it is precisely in the final chord that perfect con sonance is essential to satisfy the feeling of the hearer. Hen this close could not become satisfactory until the chord of th dominant Seventh had been invented, which changed the dominan consonance into a dissonance.

The preceding explanation shews that when we try to institut a close connection among all the chords peculiar to a mode of th same kind as the close connection among the tones of the scal (that is, when we require all the consonant triads in the harmoni tissue to be related to one of their number, the tonic triad, in manner analogous to that in which the notes of the scale a

related to one of their number, the tonic tone), there are only *two tonal modes*, the *major* and *minor*, which properly satisfy such conditions of related tones and related chords.

The *major mode* fulfils the two conditions of chordal relationship and tonal relationship in the most perfect manner. It has five triads which are immediately related to the tonic chord

$$f - a - c - e - g - b - d$$

Its harmonisation can be so conducted, (indeed, in popular pieces which must be readily intelligible, it is so conducted,) that all tones appear as constituents of the three major chords of the system, those of the tonic, dominant, and subdominant. These major chords, when their roots lie low, appear to the ear as reinforcements of the compound tones of the tonic, dominant, and subdominant, which tones are themselves connected by the closest possible relationship of Fifths. Hence in this mode everything can be reduced to the closest musical relationship in existence. And since the tonic chord in this case represents the compound tone of the tonic immediately and completely, the two conditions —predominance of the tonic tone and of the tonic chord—go hand in hand, without the possibility of any contradiction, or the necessity of making any changes in the scale.

The major mode has, therefore, the character of possessing the most complete melodic and harmonic consistency, combined with the greatest simplicity and clearness in all its relations. Moreover, its predominant chords being major, are distinguished by full, unobscured harmoniousness, when such positions are selected for them as do not introduce inappropriate combinational tones.

The major scale is purely diatonic, and possesses the ascending leading note of the major Seventh, whence it results that the tone most distantly related to the tonic is brought into closest melodic connection with it.

The three predominant major chords furnish tones sufficient to produce two minor chords, which are closely related to them, and can be employed to diversify the succession of minor chords.

The *minor mode* is in many respects inferior to the major. The chain of chords for its modern form is—

$$f - \bar{a}\flat - c - \bar{e}\flat - g - \underline{b} - d$$

Minor chords do not represent the compound tone of their root as well as the major chords; their Third, indeed, does not form any part of this compound tone. The dominant chord is the only major chord,[1] and it contains the two supplementary tones of the scale. Hence when these appear as constituents of the dominant triad, and therefore of the compound tone of the dominant, they are connected with the tonic by the close relationship of Fifths. On the other hand, the tonic and subdominant triads do not simply represent the compound tones of the tonic and subdominant notes, but are accompanied by Thirds which cannot be reduced to the close relationship of Fifths. The tones of the minor scale can therefore not be harmonised in such a way as to link them with the tonic note by so close a relationship as in the major mode.

The conditions of tonality cannot be so simply reconciled with the predominance of the tonic chord as in the major mode. When a piece concludes with a minor chord, we hear, in addition to the compound tone of the tonic note, a second compound tone which is not a constituent of the first. This accounts for the long hesitation of musical composers respecting the admissibility of a minor chord in the close.

The predominant minor chords have not the clearness and unobscured harmoniousness of the major chords, because they are accompanied by combinational tones which do not fit into the chord.

The minor scale contains an interval $\bar{a}\flat - \underline{b}$, which exceeds a whole tone in the diatonic scale, and answers to the numerical ratio 75 : 64.[2] To make the minor scale melodic it must have a

[1] [That is, among the characteristic chords. The two minor chords, as is shewn in the text, contain the tones of one major chord, $\dagger a\flat + c - \dagger e\flat$.—*Translator.*]

[2] [The interval is so strange, when unaccompanied, that if it had to be taken merely as an interval, $\dagger a\flat - b$, a singer would probably fail. But the $\dagger a\flat$ is taken as the minor Third of f with ease, and the b is taken as the leading note to c', with equal ease, so that the perfectly unmelodic and inharmonic interval $\dagger a\flat - b$ never comes into consideration at all. To get rid of it, the subdominant is often taken major, producing the chords of C mapima, No. 17, Appendix XIX., Table III., C, which makes the scale $c - d - \dagger e\flat - f - g - a - b - c'$, and this differs from the major only by having $\dagger e\flat$ in place of e. In many pianoforte instruction books this is given as the only form of the ascending minor. Mr. Curwen ('Standard Course,' p. 86) says that this major Sixth 'ascending is very difficult to sing,' and 'has a hard and by no

different form in descending from what it has in ascending, as mentioned in the last chapter.

The minor mode, therefore, has no such simple, clear, intelligible consistency as the major mode; it has arisen, as it were, from a compromise between the different conditions of the laws of tonality and the interlinking of harmonies. Hence it is also much more variable, much more inclined to modulating into other modes.

This assertion that the minor system is much less consistent than the major, will be combated by many modern musicians, just as they have contested the assertion already made by me, and by other physicists before me, that minor triads are generally inferior in harmoniousness to major triads. There are many eager assurances of the contrary in recent books on the theory of harmony. But the history of music, the extremely slow and careful development of the minor system in the sixteenth and seventeenth centuries, the guarded use of the minor close by Handel, the partial avoidance of a minor close even by Mozart,—all these seem to leave no doubt that the artistic feeling of the great composers agreed with our conclusions. To this must be added the varied use of the major and minor Seventh, and the major and minor Sixth of the scale, the modulations rapidly introduced and rapidly changing, and finally, but very decisively, popular custom. Popular melodies can contain none but clear transparent relations. Look through collections of songs now preferred by those classes among the western nations which have often an opportunity of hearing harmonic music, as students, soldiers, artisans. There are scarcely two or three per cent. in minor keys, and those are mostly old popular songs which have descended from the times of homophonic music. It is also characteristic that, as I have been assured by an experienced teacher of singing, pupils of only moderate musical talent have much more difficulty in hitting the minor than the major Third.

But I am by no means of opinion that this character depreciates the minor system. The major mode is well suited for all frames of mind which are completely formed and clearly understood, for strong resolve, and for soft and gentle or even for

means pleasant effect,' and points out that it leads singers to forget the key, and in such a phrase as $g - a - b - c' - d' - \dagger e'\flat$, the pupils will sing e' instead of $\dagger e'\flat$; and even in singing such a passage as $g - a - b - c' - g$, instead of falling upon the same note with which they began, will take e', the major third of c'. Hence the difficulty is not avoided but increased by introducing the ambiguity of the major key, into which this is a real modulation from g onwards.—*Translator.*]

sorrowing feelings, when the sorrow has passed into the condition of dreamy and yielding regret. But it is quite unsuited for indistinct, obscure, unformed frames of mind, or for the expression of the dismal, the dreary, the enigmatic, the mysterious, the rude, and whatever offends against artistic beauty;—and it is precisely for these that we require the minor mode, with its veiled harmoniousness, its changeable scale, its ready modulation, and less intelligible basis of construction. The major mode would be an unsuitable form for such purposes, and hence the minor mode has its own proper artistic justification as a separate system.

The harmonic peculiarities of the modern keys are best seen by comparing them with the harmonisation of the other ancient tonal modes.

MAJOR MODE.

Among the melodic tonal modes the *Lydian* of the Greeks (the ecclesiastical *Ionic*), in agreement with our major, is the only one which has an ascending leading note in the form of a major Seventh. The four others had originally and naturally only minor Sevenths, which even in the later periods of the middle ages began to give place to major Sevenths, in order that the Seventh of the scale, which was in itself so loosely connected with the tonic, might be more closely united to it by becoming the leading note to the tonic at the close.

MODE OF THE FOURTH.

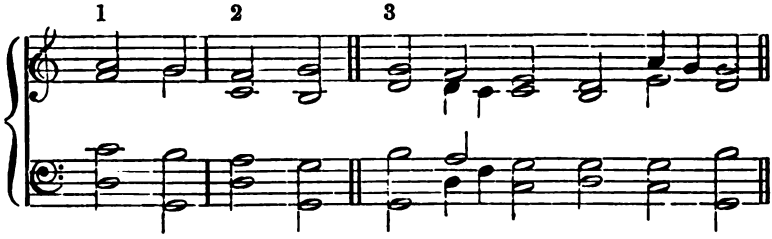
The *mode of the Fourth* (the Greek *Ionic*, and ecclesiastical *Mixolydian*) is principally distinguished from the major mode by its minor Seventh. By merely changing this into the major we obliterate the difference between them. When the tonic is *g*, the tonic chord being major must be *g* — *b̲* — *d*, and hence the chain of chords in the unaltered mode must be

$$c - \underline{e} - \overbrace{\mathbf{g} - \underline{\mathbf{b}}} - \overbrace{\mathbf{d} - \bar{f} - a}^{1}$$

If we attempt to form a complete cadence in this mode, as in the following examples 1 and 2, they will sound dull from want of the

[1] [In the notation of Appendix XIX., $c + e - g + b - d - \dagger f + \dagger a$.—*Translator.*]

leading note, even when the dominant chord is extended to a chord of the Seventh.



The second example, in which the leading note *f* lies uppermost, is even duller than the first example, in which that note *f* is more concealed. The *f* in these examples has a very uncertain sound. It is not closely enough related to the tonic, it is not part of the compound tone of the dominant note *d*, it is not sufficiently close in pitch to serve as a leading note to the tonic, and it has no tendency to push on to the tonic. Hence when the older composers wished to distinguish pieces written in the mode of the Fourth from those in the major mode, by their close, they employed the imperfect or plagal cadence, as in example 3. And as such a cadence wants the decisive progression required for a close, the sluggishness previously caused by the absence of a leading tone ceases to be striking.[1]

In the course of a piece written in this tonal mode, the leading note may of course be used in ascending passages, provided the minor Seventh is employed often enough in descending passages. But the effect of the mode is destroyed when an essential tone of the scale is changed at the close. Hence pieces in the mode of the Fourth sound like pieces in a major mode which have a decided inclination to modulate into the major mode of the subdominant.[2] For reasons already given, transition to the subdominant appears to be less active than transition to the dominant. This tonal mode has also no decided progression at the close, and the major chords, of which the tonic is one, predominate with their more complete harmoniousness. The mode of the Fourth is consequently

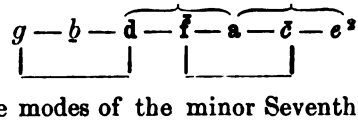
[1] [On the experimental harmonium, Appendix XIX., Sect. G, No. 2, it is necessary to take *f* + *a* — *c* + *e* — *g* + †*b*b — *d* as an example, and hence transpose the examples by playing them a Fourth higher.—*Translator.*]

[2] [This inclination seems to arise from the tempered confusion of †*f*, †*a* with *f*, *a*, so that the scale *c* + *e* — *g* + *b* — *d* — †*f* + †*a* becomes confused with its subdominant *f* + *a* — *c* + *e* — *g* + *b* — *d*.—*Translator.*]

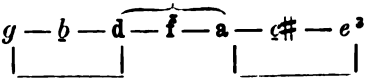
as soft and harmonious as the major mode, but it wants the powerful forward impetus of major movement. This agrees with the character assigned to it by Winterfeld.[1] He describes the ecclesiastical *Ionic* (major) mode, as a scale which, 'strictly self-contained and founded on the clear and bright major triad—a naturally harmonious and satisfactory fusion of different tones,—also bears the stamp of bright and cheerful satisfaction.' On the other hand, the ecclesiastical *Mixolydian* (mode of the Fourth) is a scale 'in which every part by sound and movement hastens to the source of its fundamental tone' (that is, to the major mode of its subdominant), 'and this gives it a yearning character in addition to the former cheerful satisfaction, not unlike to the Christian yearning for regeneration and redemption, and a return to primitive innocency, though softened by the bliss of love and faith.'

MODE OF THE MINOR SEVENTH.

The *mode of the minor Seventh* (Greek *Phrygian*, ecclesiastical *Doric*) has a minor chord on *d* as the tonic, and originally another on *a* as the dominant, while it has a major chord on its subdominant *g*, and this last chord distinguishes the mode from the *mode of the minor Third* (Eolic); thus



Both of these modes of the minor Seventh and minor Third may without destroying their character change the minor Seventh into a leading note, and our minor mode is a fusion of both. The ascending minor scale belongs to the mode of the minor Seventh, in which the leading note is used, and the descending to the mode of the minor Third. But when the mode of the minor Seventh admits the leading note, its chain of chords reduces to the three essential chords of the scale



This tonal mode has all the character of a minor, but the transition

[1] 'Johannes Gabrieli und sein Zeitalter,' vol. i. p. 87.

[2] [In the notation of App. XIX., $g + b - d - \dagger f + \dagger a - \dagger c + \dagger e$.—*Translator.*]

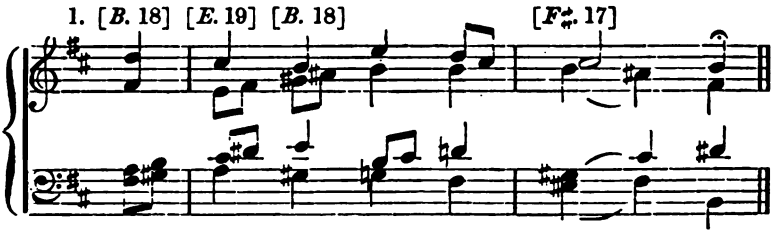
[3] [In the notation of App. XIX., $g + b - d - \dagger f + \dagger a + c\sharp - \dagger e$.—*Translator.*]

to the chord of the subdominant has a brighter effect than in the normal minor, where the subdominant chord is also minor. On forming the complete cadence both dominant and subdominant chords are major, while the tonic remains minor. This has of course an unpleasant effect in the close, because it makes the final chord obscurer than either of the other two principal chords. Hence it is necessary to introduce strong dissonances into these two chords, to restore the balance. But if we follow the old composers and make the final chord major, we give the closing cadence of this mode an unmistakably major character. As in ecclesiastical modes it is always allowed to change *B* into *B*♭ or *B*♭, which would change the subdominant chord of the mode of the minor Seventh into a minor chord, we can protect the mode of the minor Seventh from confusion with the major mode in its final cadence, but then again it will entirely coincide with the old minor cadence.

Sebastian Bach introduces the major Sixth of the tonic, which is peculiar to this tonal mode, into other chords for the closing cadence, and thus avoids the major triad on the subdominant. He very usually employs the major Sixth as the Fifth of the chord of the Seventh on the Second of the scale,[1] as in the following examples. No. 1 is the conclusion of the chorale: *Was mein Gott will, das gescheh, allzeit*, in the St. Matthew-Passion. No. 2 is the conclusion of the hymn *Veni redemptor gentium*, at the end of the cantata: *Schwingt freudig Euch empor zu den erhabenen Sternen.* In both the tonic is *b*, the major Sixth *g*♯.[2]

[1] [Suppose, as in the text, that the scale is $g + b - d - \dagger f + \dagger a - \dagger c + \dagger e$. Then *b* is the major Sixth, and †*e* is the Second of the scale. It is evident, therefore, that when S. Bach makes *b* the Fifth of †*e* he is thinking in tempered music, and that in just intonation he must immediately raise *b* to †*b*, and consequently *g* to †*g*, thus modulating into $d - \dagger f + \dagger a - \dagger c + \dagger e - \dagger g + \dagger b$, or into the dominant of the original mode. But, on the other hand, if he make the Second †*e* a Fifth below *b* the major Sixth, he would have to change †*e* into *e*, and also †*c* into *c*, that is to modulate into $c + e - g + b - d - \dagger f + \dagger a$, which is a subdominant scale. In fact, then, the ambiguities of a temperament which ignores the difference of a comma result in treating the *one tempered* scale $g + b - d - f + a - c + e$, as the *three just* scales which can be constructed from $c + e - g + b - d - \dagger f + \dagger a - \dagger c + \dagger e - \dagger g + \dagger b$, as in No. 3, p. 460, note. Hence the confusion of a comma amounts to an indeterminacy of modulation, which may be into dominant or subdominant, and at times may restlessly oscillate between the two in a manner very distressing to singers who are not *mis*led by a tempered accompaniment. On a justly intoned instrument with fixed tones the same difficulty would be felt.—*Translator.*]

[2] [In the notation of Appendix XIX. the tonic is †*b*, and the major Sixth is *g*♯. The notes in staff notation are the usual tempered notes; the bracketed letters and numbers



introduced above are sufficient to show the precise just intonation of every note according to the system of Appendix XIX., Section G, Nos. 6 and 7. These letters or numbers are called the *duodenal*, and mark the *duodene* in which all the notes are contained from the chord over which it is written to the chord over which the next letter or number is written, according to the Appendix XIX., Table VII., and hence, in justly intoned instruments, shew the *manual* on which they have to be played, as marked in Table VIII. In any particular case it is convenient to extract the duodenes, thus—

F♯. 17.	†E	G♯	B♯
B. 18.	†A	C♯	E♯
E. 19.	D	F♯	A♯
	G	B	‡D♯
	C	E	‡G♯
	F	A	‡C♯

Each duodene contains the four lines commencing with the line against which its letter or *root*, and number in Tables VII. and VIII., are placed. The four lines of *B.* 18 are separated by horizontal lines. For *F♯*. 17, the dominant, the last of the four lines of *B.* 18 has to be omitted and the line above added. For *E.* 19, the subdominant, the first of the four lines of *B.* 18 has to be omitted and the line below added. The following analysis shews the exact distribution of the chords.

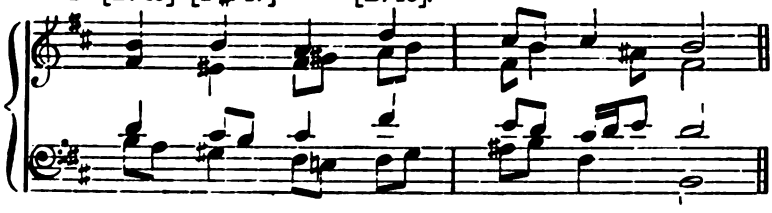
(1)

B. 18.	*f♯*	†*a*	*f′♯*	*d″*
	‡*g♯*	*b*	*f′♯*	*d″*
E. 19.	*a*	‡*c♯*	*e′*	‡*c″♯*
	a	‡*d′♯*	*f′♯*	‡*c″♯*
B. 18.	‡*g♯*	*e′*	‡*g′♯*	*b′*
	‡*g♯*	*e′*	*a′♯*	*b′*
	g	*b*	*b′*	*e″*
	g	*c♯*	*b′*	*e″*
	f♯	*d′*	*b′*	*d″*
	f♯	*d′*	*b′*	*c″♯*
F♯. 17.	*e♯*	*g♯*	*b′*	*c″♯*
	f♯	*c′♯*	*a′♯*	*c″♯*
	B	‡*d′♯*	*f′♯*	*b′*

(2)

B. 18.	*b*	*d′*	*f′♯*	*b′*
	†*a*	*d′*	*f′♯*	*b′*
F♯. 17.	*g♯*	*c′♯*	*e′♯*	*b′*
	g♯	*b*	*e′♯*	*b′*
	f♯	*c′♯*	*f′♯*	†*a′*
	†*e*	*c′♯*	*g′♯*	†*a′*
B. 18.	*f♯*	*f′♯*	†*a′*	*d″*
	g	*f′♯*	*b′*	*d″*
	a♯	*e′*	*f′♯*	*c″♯*
	b	*d′*	*b′*	*b′*
	f♯	*c′♯*	*b′*	*c″♯*
	f♯	*d′*	*b′*	*c″♯*
	f′♯	*e′*	*a′♯*	*c′♯*
	B	*d′*	*f′♯*	*b′*

2 [*B.* 18] [*F*♯ 17] [*B.* 18].



There are many similar examples. He evidently evades a regular close.

MINOR-MAJOR MODE.

Modern composers, when they wish to insert a tonal mode which lies between Major and Minor, to be used for a few phrases or cadences, have generally preferred giving the minor chord of the mode to the subdominant and not to the tonic. Hauptmann calls this the minor-major mode (*Moll-Durtonart*). Its chain of chords is—

$$\overbrace{f—\bar{a}\flat—c}—\overbrace{\underline{e}—g—\underline{b}}—d$$ [1]

This gives a leading note in the dominant chord, and a complete final cadence in the major chord of the tonic, while the minor relation of the subdominant chord remains undisturbed. This minor-major mode is at all events much more suitable for harmonisation than the old mode of the minor Seventh. But it is unsuitable for homophonic singing, unless in the ascending scale $\bar{a}\flat$ is changed into $\underline{a}$, because the voice would otherwise have to make the complicated step $\bar{a}\flat - \underline{b}$. The old modes were derived from homophonic singing, for which the mode of the minor

When it is known that in each duodene the notes in the same horizontal line are major Thirds, and in the same column Fifths, and in the same oblique line from right to left upwards minor Thirds to each other, a glance at the duodene perfectly determines the nature of the chords and the progressions of the harmony. Taking the three last chords in the first example, it is clearly seen that the first of them is $C\sharp + E\sharp - G\sharp \mid B$ or the dominant Seventh, resolving on $F\sharp + A\sharp - C\sharp$, the dominant of the major chord $B + \ddagger D\sharp - F\sharp$, with which the piece concludes, almost exactly as a piece in a major scale would have done.—*Translator.*]

[1] [In the notation of Appendix XIX., $f - \dagger a\flat + c + e - g + b - d$.—*Translator.*]

Seventh is perfectly well fitted, as we know from its being still used as our ascending minor scale.[1]

MODE OF THE MINOR SIXTH.

While the *mode of the minor Seventh* oscillates indeterminately between major and minor without admitting of any consistent treatment, *the mode of the minor Sixth* (Greek *Doric*, ecclesiastical *Phrygian*), with its minor Second, has a much more peculiar character, which distinguishes it altogether from all other modes. This minor Second stands in the same melodic connection to the tonic as a leading note would do, but it requires a descending progression. Hence for descending passages this mode possesses the same melodic advantages as the major mode does for ascending passages. The minor Second has the most distant relationship with the tonic, due entirely to the subdominant. The mode cannot form a dominant chord without exceeding its limits. If we call *e* the tonic, the chain of chords is

$$d - \bar{f} - a - \bar{c} - \mathrm{e} - \bar{\mathrm{g}} - \mathrm{b} - d^{2} \quad .$$

In this case the chords $d - \bar{f} - a$ and $\bar{f} - a - \bar{c}$ are not directly related to the tonic. The tone $\bar{f}$ cannot enter into any consonant chord which is directly related to the tonic. But since $\bar{f}$ is the characteristic minor Second of the mode, such chords cannot well be avoided, not even in the cadence. Although, then, there is a close relationship between the consecutive links of the chain of chords, some of its indispensable terms are only distantly related to the tonic. Moreover, in the course of a piece in this mode, it will always be necessary to form the dominant chord $b - d\# - f\#$,[3]

[1] [After the introduction of the leading note to form a major dominant chord.—*Translator.*]

[2] [In the notation of Appendix XIX., $d - \dagger f + \dagger a - \dagger c + \dagger e - \dagger g + \dagger b - \dagger d$. Observe that in the text $\dagger e$ (and not e, the major Third of c) is taken as the tonic, and observe that both d and $\dagger d$ are introduced.—*Translator.*]

[3] [In the notation of Appendix XIX., $\dagger b + d\sharp - \dagger f\sharp$. The introduction of this chord shews that the composer is writing in the key of $\dagger e$, but has a prevailing tendency to modulate into the subdominant, from which d, $\dagger f$ are chosen. When $\dagger d$ is used for d, or $\dagger f\sharp$ for $\dagger f$, the modulation into the subdominant does not take place. The major chord $\dagger G + \dagger B - \dagger D$ is entirely adventitious. If it is used in ascending, thus, in the notation of Appendix XIX., $\dagger e \,.\, \dagger f \ldots \dagger g \,..\, \dagger a \ldots \dagger b \,.\, \dagger c' \ldots \dagger d' \,..\, \dagger e'$ the result is the scale of $\dagger C$ matáma, No. 4, of Table III., C, in that Appendix.—*Translator.*]

although it contains two tones foreign to the original mode, as otherwise we could not prevent the prevalence of the impression that *a* is the tonic and *a* — *c̄* — *e* the tonic chord. It follows, therefore, that the *mode of the minor Sixth* must be still less consistent in its harmonisation and still more loosely connected than the minor mode, although it admits of very consistent melodic treatment. It contains three essential minor chords, namely the tonic *e* — *ḡ* — *b*, the subdominant *a* — *c̄* — *e*, and the chord which contains the two tones slightly related to the tonic *d* — *f̄* — *a*. It is exactly the reverse of the major mode, for whereas that mode proceeds towards the dominant, this mode proceeds towards the subdominant.

Major:		*f* —	*a* —	c —	e —	g —	*b* — *d*
Mode of minor Sixth:	*b*♭ — *d*♭ —	*f* —	*a*♭ —	c —	e♭ —	g[1]	

For harmonisation the difference of the two cases is, first, that the related tones introduced into the scale by the subdominant *f*, namely *b*♭ and *d*♭, are not partials to the compound tone of the subdominant, whereas tones *b* and *d*, which are introduced by the dominant, are some of the partials of the tonic; and, secondly, that the tonic chord always lies on the dominant side of the tonic tone. Hence in the harmonic connection, the tones *b*♭ and *d*♭ cannot be so closely united with either the tonic tone or the tonic chord, as is the case with the supplementary tones introduced by the dominant. This gives the mode of the minor Sixth, when harmonised, a kind of exaggerated minor character. Its tones and chords are certainly connected, but much less clearly and intelligibly than those of the minor system. The chords which can be brought together in this key, without obscuring reference to *e* as the tonic, are *d* minor and *f̄* major on the one hand and *b* major on the other, chords which in the major system could not

[1] [In the notation of Appendix XIX.,

f + *a* − *c* + *e* − *g* + *b* − *d*
b♭ − *d*♭ + *f* − †*a*♭ + *c* − †*e*♭ + *g*.

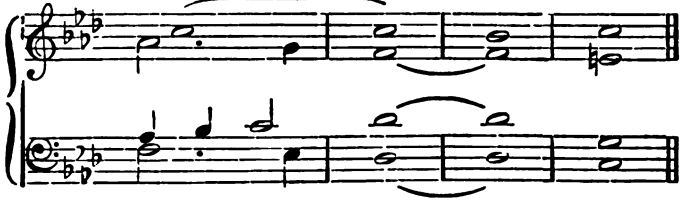
Observe here that †*b*♭, which should form the last term according to the former scheme, is omitted. See No. 5, p. 461. In the major mode there is formed one chord a Fifth above, and another a Fifth below the tonic. Whereas in the mode of the minor Sixth, no chord is formed on the Fifth above. A more complete analogy is obtained by comparing *G* pámama (of Appendix XIX., Table III., C, No. 1) with *F* mimipí (*ibid.* No. 54) thus—

c + *e* − *g* + *b* − *d* + *f*♯ − †*a*
b♭ − *d*♭ + *f* − †*a*♭ + *c* − †*e*♭ + *g*. *Translator.*]

be brought together without extraordinary modulational appliances.[1] The esthetical character of the mode of the minor Sixth corresponds with this fact. It is well suited for the expression of dark mystery, or of deepest depression, and an utter lapse into melancholy, in which it is impossible to collect one's thoughts. On the other hand, as its descending leading note gives it a certain amount of energy in descent, it is able to express earnest and majestic solemnity, to which the concurrence of those major chords which are so strangely connected gives a kind of peculiar magnificence and wondrous richness.

Notwithstanding that the mode of the minor Sixth has been rejected from modern musical theory, much more distinct traces of its existence have been left in musical practice than of any other ancient mode; for the mode of the Fourth has been fused into the major, and the mode of the minor Seventh into the minor. Certainly a mode like that we have described is not suitable for frequent use; it is not closely enough connected for long pieces, but its peculiar power of expression cannot be replaced by that of any other mode. Its occurrence is generally marked by its peculiar final cadence which starts from the minor Second in the root. In Handel the natural cadence of this system is used with great effect. Thus in the *Messiah*, the magnificent fugue *And with his stripes we are healed*, which has the signature of F minor, but by its frequent use of the harmony of the dominant Seventh on *G*, shews that *C* is the real tonic, introduces the pure Doric cadence as follows:[2]

[*F.* 12]

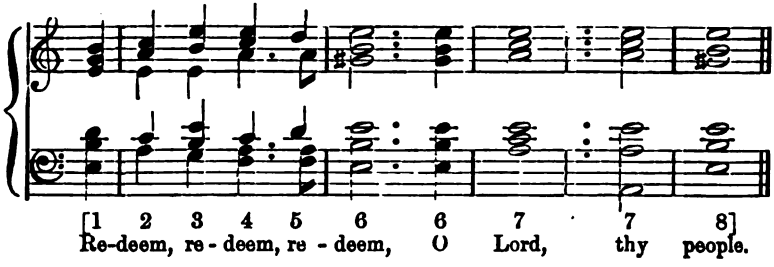


[1] [This, in fact, lengthens the original chain of chords into *d* − †*f* + †*a* − †*c* + †*e* − †*g* + †*b* + *d*♯ − †*f*♯, and leads to the treatment of the mode as merely †*E* minor, with a tendency to modulate into †*A* minor. The †*E* minor is, however, the modern minor †*E* mipima (Appendix XIX, Table III., C, No. 24), and the †*A* minor is †*A* mipimi (*ibid.*, No. 52), which is much more gloomy.—*Translator.*]

[2] [The cadence is produced by passing from the minor subdominant *B*♭ − *D*♭ + *F* to the major dominant, *C* + *E* − *G*, in the key of *F* minor. This is the concluding

Similarly in *Samson*, the chorus, *Hear, Jacob's God*, which, written in the Doric mode of *E*, finely characterises the earnest prayer of the anxious Israelites as contrasted with the noisy sacrificial songs of the Philistines in *G* major, which immediately follow. The cadence here also is purely Doric.[1]

[*A*. 20]



The chorus of Israelites which introduces the third part: *In Thunder come, O God, from heaven!* and is chiefly in *A* minor, has likely an intermediate Doric section.

Sebastian Bach also, in the chorales which he has harmonised,

cadence of the whole fugue, and for this reason apparently, the signature in Novello's edition is that of *C* minor, not of *F* minor, and the *d*♭ is marked as an accidental throughout. That is, Novello takes the key to be *C* minor with a constant tendency to modulate into the key of the subdominant, from which it borrows the chord *B*♭ – *D*♭ + *F*. But the fugue begins with *F* — *f* in the bass, and the opening subject, in the treble, is *c*″, †*a*′♭, *d*″♭, *e*′, *f*′, *g*′, †*a*′♭, *b*♭, *c*, which is clearly in the scale of *F* minor, with the chordal system *b*♭ – *d*♭ + *f* – †*a*♭ + *c* + *e* – *g*, of which it contains every note. To shew the different harmonic views taken, I may mention that in the Tonic Solfa edition, the key is given as *A*♭, (meaning †*A*♭, the major key of which *F* is the relative minor,) and on the last *c* mentioned above the key is changed to *E*♭ (meaning †*E*♭, the major key of which *C* is the relative minor), between which keys the music is made to change constantly, but that the conclusion is placed in †*A*♭ major (that is, *F* minor), the last chord being the major dominant. It is evident that to modern ears the tonality is not fully marked. In the text the bracketed [*F*. 12] shews the duodene in Appendix XIX., Tables VII. and VIII., which contains all the tones in the passage. The whole fugue oscillates between the duodenes *C*. 11 and *F*. 12.—*Translator*.]

[1] [The duodene is *A*. 20, in Appendix XIX., Tables VII. and VIII. The succession of chords, each reduced to the simplest form, as referred to by the bracketed figures below the notes, is 1. *e* – *g* + *b*, 2. *a* – *c* + *e*, 3. *e* – *g* + *b*, 4. *f* + *a* – *c*, 5. ‡*d* – *f* + *a*, 6. *e* + ‡*g*♯ – *b*, 7. *a* – *c* + *e*, 8. *e* + *g*♯ – *b*. Hence, assuming the scale to have the chordal system ‡*d* – *f* + *a* – *c* + *e* – *g* + *b*, with *e* – *g* + *b* as the tonic chord, taken major as *e* + ‡*g*♯ – *b* in the close, we have the 'Doric cadence' between chords 5 & 6, which is then lengthened by introducing the remaining tones of the key in 7, the whole closing as in 8. It would be most probably received as in *A* minor, closing in the dominant.—*Translator*.]

has left them in the mode of the minor Sixth, to which they melodically belonged, whenever the text requires a deeply sorrowful expression, as in the *De Profundis* or the *Aus tiefer Noth schrei' ich zu dir*, and again in Paul Gerhardt's song, *Wenn ich einmal soll scheiden, so scheide nicht von mir.* But he has harmonised the same melody arranged for other texts, as *Befiehl Du deine Wege*, and *O Haupt von Blut und Wunden*, &c., as major or minor, in which case the melody ends on the Third or Fifth of the key, instead of on the Doric tonic.

Fortlage[1] had already observed that Mozart had applied the Doric mode in Pamina's air in the second act of *Il Flauto Magico.* One of the finest examples for the contrast between this and the major mode occurs in the same composer's *Don Giovanni*, in the Sestette of the second act (No. 21), where Ottavio and Donna Anna enter. Ottavio sings the comforting words—

Tergi il ciglio, o vita mia,
E dà calma al tuo dolore

in *D* major, which, however, is peculiarly coloured by a preponderating, although not uninterrupted, inclination to the subdominant, as in the mode of the Fourth. Then Anna, who is plunged in grief, begins in perfectly similar melodical phrases, and with a similar accompaniment, and after a short modulation through *D* minor, establishes herself in the mode of the minor Sixth for *C*, with the words—

Sol la morte, o mio tesoro,
Il mio pianto può finir.

The contrast between gentle emotion and crushing grief is here represented with a most wonderfully beautiful effect, principally by the change of mode. The dying Commandant, in the introduction to *Don Giovanni*, ends with a Doric cadence.[2] Similarly the *Agnus Dei* of Mozart's *Requiem*—although, of course, we are not quite certain how much of this was written by himself.

Among Beethoven's compositions we may notice the first movement of the Sonata, No. 90, in *E* minor, for the pianoforte, as an

[1] Examples of Instrumental Music mentioned by Ekert in his 'Habilitationsschrift.' 'The Principles of Modulation and Musical Ideas,' Heidelberg, 1860, p. 12.

[2] [No. 1 of the opera. Representing major chords by capitals and minor by small letters, the final chords of the vocal music are *F*, *D*♭, *G*♭, *f*, *C*, *f*, so that all the tones will lie in the scheme *g*♭ + *b*♭ — *d*♭ + *f* — †*a*♭ + *c* + *e* — *g*. The tonic is *F*.—*Translator*.]

Radical Tone or *Root*, as distinguished from its *bass*, that is, the tone which belongs to the lowest part. The major triad has the same *root* whatever be its inversion or position. In the chords $c - \underline{e} - g$, or $g - c - \underline{e}$, the root is still c. The minor chord $d - \bar{f} - a$ has also as a rule only d as its root in all its inversions, but in the chord of the major Sixth $f - a - d$ we may also consider $\bar{f}$ as the root, and it is in this sense that it occurs in the cadence of $\bar{c}$ major.[1] Rameau's successors have partly given up this last distinction; but it is one in which Rameau's fine artistic feeling fully corresponded with the facts in nature. The minor chord really admits of this double interpretation, as we have already shewn (p. 454.)

The essential difference between the old and new tonal modes is this: the old have their minor chords on the dominant, the new on the subdominant side.

The reasons for the following construction have been already investigated.[2]

	In the	The chord of the Subdominant is	The chord of the Tonic is	The chord of the Dominant is
Old	Mode of the minor Third .	Minor	Minor	Minor
Old	Mode of the minor Seventh .	Major	Minor	Minor
Old	Mode of the Fourth . . .	Major	Major	Minor
	Major Mode	Major	Major	Major
New	Minor-Major mode . . .	Minor	Major	Major
New	Minor mode	Minor	Minor	Major

[1] [In the notation of Appendix XIX. 'the minor chord $d - \dagger f + \dagger a$ has the root d in all positions, but in the position $\dagger f + \dagger a \times_1 d$ we may also consider $\dagger f$ as the root, and it is in this sense that it occurs in the cadence of $\dagger C$ major.'—*Translator.*]

[2] [It will be seen that this arrangement does not include the mode of the minor Sixth. If we for the moment cease to regard historical derivation, and commence by a purely modern harmonic notion of chord relationship, agreeing to consider a minor triad as equally valuable with the major triad for harmonic purposes, we may derive scales of seven notes by means of six chords, with common Fifths, but three with major and three with minor Thirds, in the manner shewn in Appendix XIX., Section D and Table III., in which the principle of the table on this page is more completely carried out in practice.—*Translator.*]

CHAPTER XVI.

THE SYSTEM OF KEYS.

THERE is nothing in the nature of music itself to determine the pitch of the tonic of any composition. If different melodies and musical pieces have to be executed by musical instruments or singing voices of definite compass, the tonic must be chosen of a suitable pitch, differing when the melody rises far above the tonic and when it sinks much below it. In short, the pitch of the tonic must be chosen so as to bring the compass of the tones of the piece within the compass of the executants. This inevitable practical necessity entails the condition of being able to give any required pitch to the tonic.

Moreover, in the longer pieces of music it is necessary to be able to make a temporary change of tonic, that is, to *modulate*, in order to avoid uniformity and to utilise the musical effects resulting from changing and then returning to the original key. Just as consonances are made more prominent and effective by means of dissonances, the feeling for the predominant tonality and the satisfaction which arises from it, is heightened by previous deviations into adjacent keys. The variety in musical turns produced by modulational connection has become all the more necessary for modern music, because we have been obliged entirely to renounce, or at any rate materially to circumscribe the old principle of altering expression by means of the various tonal modes. The Greeks had a free choice among seven different tonal modes, the middle ages among five or six, but we can choose between two only, major and minor. Those old tonal modes presented a series of different degrees of tonal character, out of which two only remain suitable for harmonic music. But the clearer and firmer construction of an harmonic piece gives modern composers greater freedom in modulational deviations from the original key, and places at their command new sources of musical wealth, which were scarcely accessible to the ancients.

Finally I must just touch on the question, so much discussed, whether each different key has a peculiar character of its own.

It is quite clear that, within the course of a single piece of music, modulational deviations into the more or less distantly related keys on the dominant or subdominant side produce very different effects. But this arises simply from the contrast they offer to the original principal key. This would be merely a *relative character*. But the question here mooted is, whether individual keys have an *absolute character* of their own, independently of their relation to any other key.

This is often asserted, but it is difficult to determine how much truth the assertion contains, or even what it precisely means, because probably a variety of different things are included under the term *character*, and perhaps the amount of effect due to the particular instrument employed has not been allowed for. If an instrument of fixed tones is completely and uniformly tuned according to the equal (hemitonic) temperament, so that all semitones throughout the scale have precisely the same magnitude, and if also the musical quality of all the tones is precisely the same, there seems to be no ground for understanding how each different key should have a different character. Musicians fully capable of forming a judgment have also admitted to me, that no difference in the character of the keys can be observed on the organ, for example. And Hauptmann,[1] I think, is right when he makes the same assertion for singing voices with or without an organ accompaniment. A great change in the pitch of the tonic can at most cause all the higher notes to be strained or all the lower ones obscured.

On the other hand, there is a decidedly different character in different keys on pianofortes and bowed instruments. *C* major and the adjacent *D*♭ major have different effects. That this difference is not caused by difference of absolute pitch, can be readily determined by comparing two different instruments tuned to different pitches. The *D*♭ of the one instrument may be as high as the *C* of the other, and yet on both the *C* major retains its brighter and stronger character, and the *D*♭ its soft and veiled harmonious effect. It is scarcely possible to think of any other reason than that the method of striking the short narrow black keys of the piano must produce a somewhat different quality of tone, and that

[1] 'Harmonic und Metrik,' p. 188.

difference of character arises from the different distribution of the stronger and gentler quality of tone among the different degrees of the scale. The difference made in the tuning of those Fifths which the tuner keeps to the last, and on which are crowded the whole of the errors in tuning the other Fifths in the circle of Fifths, may possibly be regular, and may contribute to this effect, but of this I have no personal experience.

In bowed instruments the more powerful quality of tone in the open strings is conspicuous, and there are also probably differences in the quality of strings which are stopped at short and long lengths, and these may alter the character of the key according to the degree of the scale on which they fall. This assumption is confirmed by the inquiries I have made of musicians respecting the mode in which they recognise keys under certain conditions. The inequality of intonation will add to this effect. The Fifths of the open strings are perfect Fifths. But it is impossible that all the other Fifths should be perfect if in playing in different keys each note has the same sound throughout, as appears at least to be the intention of elementary instruction on the violin. In this way the scales of the various keys will differ in intonation, and this will necessarily have a much more important influence on the character of the melody.[1]

[1] [The want of fixed tones both on the bowed instruments and the human voice, and the extreme ease with which pitch can be almost involuntarily and unconsciously altered to suit the feeling or circumstances of the moment, while forming of course the great point of perfection which distinguishes these musical instruments from all others, yet occasion great difficulties in the way of scientific investigation. It is impossible to depend with certainty on hearing the same intonation twice. The difficulty in singing is well known, and this renders the examination of dissonant chords very difficult. Inquiries of violin-players I find lead to very small results. The strings are not stopped by a hard point, but by a soft pad; the different degrees of pressure of this pad, the conscious or unconscious forward or backward direction of the pressure, all influence the result. Few violinists know the meaning of just intonation. They learn their stopping by habit of ear, by sheer empiricism. And that this produces different results for each is shewn by the frequently unsatisfactory results of quartett playing by four excellent soloists unaccustomed to each other's 'playing,' including intonation among other matters. But as regards the main question of the character of keys in the human voice, a slight change of the pitch of the tonic, as from *C* to *B* or to *D*♭, does not sensibly affect the result, except when parts of the melody reach the breaks of the register, or the extreme limits of the voice. The difference in the place where the register breaks will most materially affect the character of the song. Hence singers constantly transpose slightly. But this can hardly be called a difference in the character of keys; it is a difference in the character of voices. On the other hand, when the changes are great, as from *C* to *G*, or *C* to *F*, the compass of the

The differences in quality of tone of different notes on wind instruments are still more striking.

If this view is correct, the character of the keys would be very different on different instruments, and I believe this to be the case. But it is a matter to be decided by a musician with delicate ears, who directs his attention to the points here raised.

It is, however, not impossible that by a peculiarity of the human ear, already touched upon in p. 167, certain common features may enter into the character of keys, independent of the difference of musical instruments, and dependent solely on the absolute pitch of the tonic. Since *g''''* is a proper tone of the human ear, it sounds peculiarly shrill under ordinary circumstances, and somewhat of this shrillness is common to *f''''*♯ and *a''''*♭. To a somewhat less extent those musical tones of which *g''''* is an upper partial, as *g'''*, *c'''*, and *g''* have a brighter and more piercing tone than their neighbours. It is possible, then, that it is not indifferent for pieces in *C* major to have its high Fifth *g''* and high tonic *c'''* thus distinguished in brightness from other tones, but these differences must in all cases be very slight, and for the present I must leave it undecided whether they have any weight at all.

All or some of these reasons, then, made it necessary for musicians to have free command over the pitch of the tonic, and hence even the later Greeks transposed their scales on to all degrees of the chromatic scale. For singers these transpositions offer no

instrument for which the melody is written affects the relative proportions of the degrees used above and below the tonic, and reproduces one of the effects of the Greek modes. This is very conspicuous for vocal music. On the violin when the difference of pitch of the tonic throws different degrees of the scale on to different strings, the extreme difference in the quality of their tones must mark the difference of key. But in what that difference consists, accurate measurable estimates are wanting. A violin is tuned in the keys of ♯*D* (major or minor, as the Thirds may vary), if we take its *A* string to be tuned to the *A* in *C* major, so that the four open strings are ♯*g*, ♯*d'*, *a'*, *e''*. Hence we should expect the keys of ♯*D* to be the 'best' keys on the instrument, and it will be found that these keys are frequent in violin music. But violinists consider *E* major to be their most brilliant key, although here the fact of the existence of a brilliant high harmonic *e'''* as well as the open *e''* string, may have something to do with the result. By altering the strings on which he takes the notes, from those naturally selected, to those possible, a violinist would most probably materially change the character of the same key. Thus if in the key of *E* major, he played the *a'* on the ♯*d'* string, and relegated the notes usually played on the *e''* string to the *a'* string, as far as possible—occasioning, of course, such difficulties of fingering that the experiment could only be tried for 'themes,' such as Mayseder's well-known air with variations in *E* major— he would probably no longer find *E* major a brilliant key absolutely on the violin.—*Translator.*]

difficulties. They can begin with any required pitch, and find in their vocal instrument all such of the corresponding degrees as lie within the extreme limits of their voice. But the matter becomes much more difficult for musical instruments, especially for such as only possess tones of certain definite degrees of pitch. The difficulty is not entirely removed even on bowed instruments. It is true that these can produce every required degree of pitch; but players are unable to hit the pitch as correctly as the ear desires without acquiring a certain mechanical use of their fingers, which can only result from an immense amount of practice.

The Greek system was not accompanied with great difficulties, even for instruments, so long as no deviations into remote keys were permitted, and hence but few marks of sharps and flats had to be used. Up to the beginning of the seventeenth century musicians were content with two signs of depression for the notes *B*♭ and *E*♭, and with the sign ♯ for *F*♯, *C*♯, *G*♯, in order to have the leading tones for the tonics *G*, *D*, and *A*. They took care to avoid the enharmonically equivalent tones *A*♯ for *B*♭, *D*♯ for *E*♭, *G*♭ for *F*♯, *D*♭ for *C*♯, and *A*♭ for *G*♯. By help of *B*♭ for *B*[1] every tonal mode could be transposed to the key of its subdominant, and no other transposition was made.

In the Pythagorean system, which maintained its predominance over theory to the time of Zarlino in the sixteenth century, tuning proceeded by ascending Fifths, thus—

C *G* *D* *A* *E* *B* *F*♯ *C*♯ *G*♯ *D*♯ *A*♯ *E*♯ *B*♯[2]

Now if we tune two Fifths upwards and an Octave downwards, we make a step having the ratio $\frac{3}{2} \times \frac{3}{2} \times \frac{1}{2} = \frac{9}{8}$, which is a major second. This gives for the pitch of every second tone in the last list—

C	*D*	*E*	*F*♯	*G*♯	*A*♯	*B*♯.
1	$\frac{9}{8}$	$(\frac{9}{8})^2$	$(\frac{9}{8})^3$	$(\frac{9}{8})^4$	$(\frac{9}{8})^5$	$(\frac{9}{8})^6$

[1] [The letter *B* was used, as it still is in Germany, for our *B*♭, and the new tone introduced was written *H*, our *B*. The old cursive writing for *h*, the second stroke descending as much as the first ascends, gave rise to our sign ♮, and this doubled, so that the first stroke of the second ♮ cut the two cross strokes of the first in half, seems to have given rise to our ♯, which implied a double elevation by ♮, or twice the change from *B* (*i.e.* *B*♭) to *H* (that is, *B*♮). The old use of *B* for *B*♭ is perpetuated in the form of the flat sign ♭, which is only a medieval form of *b*.—*Translator*.]

[2] [In the notation of Appendix XIX. this succession is –

C *G* *D* †*A* †*E* †*B* †*F*♯ †*C*♯ †*G*♯ †*D*♯ ††*A*♯ ††*E*♯ ††*B*♯

in which the increment of a comma must be especially noticed.—*Translator*.]

Now if we proceed *down*wards by Fifths from *C* we obtain the series—

C F B♭ E♭ A♭ D♭ G♭ C♭ F♭ B♭♭ E♭♭ A♭♭ D♭♭.[1]

If we descend two Fifths and rise an Octave, we may obtain the tones—

C	*B♭*	*A♭*	*G♭*	*F♭*	*E♭♭*	*D♭♭*
1	$\frac{8}{9}$	$(\frac{8}{9})^2$	$(\frac{8}{9})^3$	$(\frac{8}{9})^4$	$(\frac{8}{9})^5$	$(\frac{8}{9})^6$

Now the interval $(\frac{8}{9})^6 = \frac{262144}{531441} = \frac{1}{2} \times \frac{524288}{531441}$

or, approximately $(\frac{8}{9})^6 = \frac{1}{2} \times \frac{73}{74}$

$(\frac{9}{8})^6 = 2 \times \frac{74}{73}$.[2]

Hence the tone *B♯♯* is higher than the Octave of *C* by the small interval $\frac{74}{73}$, and the tone *D♭♭* is lower than the Octave below *C* by the same interval.[3] If we ascend by perfect Fifths from *C* and *D♭♭*, we shall find the same constant difference between

C G D A E B F♯ C♯ G♯ D♯ A♯ E♯ B♯ and
D♭♭ A♭♭ E♭♭ B♭♭ F♭ C♭ G♭ D♭ A♭ E♭ B♭ F C.[4]

The tones in the upper line are all higher than those in the lower by the small interval $\frac{74}{73}$. Our staff notation had its principles settled before the development of the modern musical system, and has consequently preserved these differences of pitch. But for practice on instruments with fixed tones the distinction between degrees of tone which lie so near to each other, was inconvenient, and attempts were made to fuse them together. This led to many imperfect attempts, in which individual intervals were more or less altered in order to keep the rest true, producing the so-called *unequal temperaments*, and finally to the system of

[1] [In the notation of Appendix XIX.,

C F B♭ E♭ A♭ ‡D♭ ‡G♭ ‡C♭ ‡F♭ ‡B♭♭ ‡E♭♭ ‡A♭♭ ‡‡D♭♭

where, again, particular attention must be directed to the depression by a comma.—*Translator.*]

[2] [The approximation $\frac{73}{74}$ is readily obtained by continued fractions. Since $\frac{531441}{524288} = \frac{81}{80} \times \frac{32805}{32768}$, and in Appendix XIX. $\frac{81}{80}$ is represented by †, and $\frac{80}{81}$ by ‡, it becomes convenient to represent $\frac{32805}{32768}$ by ¶ and its inverse by ♭, so that the interval in the text, or 'Pythagorean comma,' is ¶†, from which name the notation ¶ was derived. On the whole subject of notation of pitch and these relations, see Appendix XIX.—*Translator.*]

[3] [In the notation of Appendix XIX., then, ††*B♯♯* = †¶*c*, and ‡‡*d♭♭* = ‡♭*c*.—*Translator.*]

[4] [These relations are well shewn in Appendix XIX., Table V.—*Translator.*]

equal temperament, in which the Octave was divided into twelve precisely equal degrees of tone.[1] We have seen that we can ascend from C by 12 perfect Fifths to $B\sharp\sharp$, which differs from c by about $\frac{1}{5}$ of a semitone, namely by the interval $\frac{74}{73}$.[2] In the same way we can descend by 12 perfect Fifths to $D\flat\flat$, which is as much lower than C, as $B\sharp\sharp$ is higher. If then we put $C = B\sharp\sharp = D\flat\flat$, and distribute this little deviation of $\frac{74}{73}$ equally among all the 12 Fifths of the circle, each Fifth will be erroneous by about $\frac{1}{60}$ of a semitone [or $\frac{1}{11}$ of a comma], which is certainly a very small interval. By this means all varieties of tonal degrees within an Octave are reduced to twelve, as on our modern keyed instruments.

The Fifth in the system of equal temperament, is, when expressed approximately in the smallest possible numbers, $= \frac{3}{2} \times \frac{885}{886}$.[3] It is very seldom than any difficulty could result from its use in place of the perfect Fifth. The root struck with its Fifth makes one beat in the time that the Fifth makes $442\frac{1}{2}$ complete vibrations. Now since a' makes 440 vibrations in a second, it follows that the tempered Fifth $d'-a'$ will produce exactly *one* beat in a second. In long-sustained tones this would, indeed, be perceptible, but by no means disturbing, and for quick passages it would have no time to occur. The beats are still less disturbing in lower positions, where they decrease in rapidity with the vibrational numbers of the tones. In higher positions they certainly become more striking; $d'''-a'''$ gives four, and $a'''-e'''$ six beats in a second; but chords very seldom occur with such high notes in slow passages. The Fourths of the equal temperament are $\frac{4}{3} \times \frac{886}{885}$. There is one beat for every $221\frac{1}{4}$ vibrations of the lower tone of the Fourth. Hence the Fourth $a-d'$ makes one beat in a second, the same as the Fifth $d'-a'$. The pure consonances retained in the Pythagorean system are therefore not injured to any extent worth notice by equal temperament. In melodic progression of tones the interval $\frac{885}{886}$ borders on the very limits of distinguishable differences of pitch, which according to Weber's experiments lie at $\frac{1000}{1001}$ for practised violin players. But when two tones are sounded at the same time still finer differences are of course perceptible.

[1] [The general relations on which the schemes of temperament depend, and the mode of calculating temperaments, are given in Appendix XIX., Section C.—*Translator.*]

[2] [Or, more accurately, twelve-elevenths of a comma. See Appendix XIX., Section C.—*Translator.*]

[3] [More accurately it is $\frac{3}{2} \times \frac{1}{1}$, or $\frac{3}{2} \times (1 - \cdot 001\ 1286)$, and quite accurately $^{12}\sqrt{2^7}$.—*Translator.*]

The Thirds and Sixths of the equal temperament are nearer the perfect intervals than are the Pythagorean.

Intervals	Perfect	Equally Tempered	Pythagorean
Major Third . . .	$\frac{5}{4}$	$\frac{5}{4} \times \frac{127}{128}$	$\frac{5}{4} \times \frac{81}{80}$
Minor Sixth . . .	$\frac{8}{5}$	$\frac{8}{5} \times \frac{126}{127}$	$\frac{8}{5} \times \frac{80}{81}$
Minor Third . . .	$\frac{6}{5}$	$\frac{6}{5} \times \frac{121}{122}$	$\frac{6}{5} \times \frac{80}{81}$
Major Sixth . . .	$\frac{5}{3}$	$\frac{5}{3} \times \frac{122}{121}$	$\frac{5}{3} \times \frac{81}{80}$
Semitone . . .	$\frac{16}{15}$	$\frac{16}{17}$ or $\frac{16}{15} \times \frac{147}{148}$	$\frac{21}{20}$ or $\frac{16}{15} \times \frac{80}{81}$ [1]

The dissonances occasioned by the upper partial tones are consequently somewhat milder than those due to Pythagorean intervals, but the combinational tones are much more disagreeable. For the Pythagorean Thirds $c' - e'$ and $e' - g'$ the combinational tones are $C\sharp$ and $B_{,}$ both differing by a semitone [2] from the combinational tone C, which would result from the perfect intervals in both cases. For the Pythagorean minor chord $e - g' - b'$ the combinational tones are $B_{,}$ and $G\sharp$. The first, $B_{,}$ is very suitable, better even than the combinational tone C which results from perfect intonation. But the second, $G\sharp$, belongs to the major and not to the minor chord of E. However, as in perfect intonation one of the two combinational tones C and G is false, the Pythagorean minor chord can hardly be considered inferior in this respect. But the combinational tones of the equally tempered Thirds lie between those of the perfect and Pythagorean Thirds, and are less than a semitone different from those of just intonation. Hence they correspond to no possible modulation, no tone of the chromatic scale, no dissonance that could possibly be introduced by the progression of the melody; they simply sound out of tune and wrong.[3]

[1] [See Appendix XIX., Section C.—*Translator.*]

[2] [The accurate results are $\frac{17}{16}$ C (a very little flatter than the $D\flat$ of Appendix XIX.; this note being exactly $\frac{16}{15}$ C, and the $C\sharp$ of the text an approximative not an exact notation) and $\frac{15}{16}$ C (or B exactly). And similarly in other cases.—*Translator.*]

[3] [This may be seen more clearly by calculating the vibrational numbers, assuming c' to be 264 as on p. 26. Then—

Notes	Just, Difference	Pythagorean, Difference	Tempered, Difference
c'	264	264	264
	$66 = C$	70·12	68·61
e'	330	334·12	332·61
	$66 = C$	$61·88 = B_{,}$	62·94
g'	396	396	395·55
	$99 = G$	105·19	102·81
b'	495	501·19	498·36

These bad combinational tones have always been to me the most annoying part of equally tempered harmonies. When moderately slow passages in Thirds at rather a high pitch are played, they form a horrible bass to them, which is all the more disagreeable for coming tolerably near to the correct bass, and hence sounding as if they were played on some other instrument which was dreadfully out of tune. They are heard most distinctly on the harmonium and violin. Here every professional and even every amateur musician observes them immediately, when their attention is properly directed. And when the ear has once become accustomed to note them, it can even discover them on the piano. In the Pythagorean intonation the combinational tones sound rather as if some one were playing dissonances on purpose. Which of these two evils is the worse, I will not venture to decide. In lower positions where the very low combinational tones can be scarcely, if at all, heard, the equally tempered Thirds have the advantage over the Greek, because they are not so rough, and produce fewer beats. In higher positions the latter advantage is perhaps destroyed by their abominable combinational tones. However the equally tempered system is suitable for effecting everything that can be done by the Pythagorean, and with less expenditure of means.

C. E. Naumann,[1] who has lately defended the Pythagorean as opposed to the equally tempered system, grounds his reasons chiefly on the fact that the Semitones which separate the ascending leading tone from the tonic, and the descending minor Seventh from the Third of the chord on which it has to be resolved, are smaller in the Pythagorean (where they are about $\frac{21}{20}$; as appears in the table on p. 488) than in the equally tempered, where they are about $\frac{18}{17}$; while they are greatest of all in just intonation, viz. $\frac{16}{15}$. Now in the equally tempered scale there is only one tone between *f* and *g*, which is accepted at one time as *f*♯ to be a leading note to *g*, and at another as *g*♭ to act as a Seventh resolving upon *f*; but in the Pythagorean there are two tones, *f*♯ and *g*♭, of which the latter is the flatter. Hence the Semitone always approaches the tone on to which it would fall in regular resolution, and the height of the

The 'differences' give the vibrational numbers of the combinational tones. Now we have by p. 26, $C = 66$, $G = 29$, $B_{\prime} = 61{\cdot}88$, but the others correspond to no precise tones. The nearest *tempered* intervals $C\sharp = 69{\cdot}93$, and $G\sharp = 104{\cdot}76$. See also Appendix XIX., Table VI., D.—*Translator.*]

[1] 'Ueber die verschiedenen Bestimmungen der Tonverhältnisse.' [On the various determinations of the ratios of tones.] Leipzig, 1858.

pitch determines the direction of resolution. But although the leading tone plays an important part in modulations, it is perfectly clear that we are not justified in changing its pitch at will in order to bring it nearer to the note on which it has to be resolved. There would otherwise be no limit to our making it come nearer and nearer to that tone, as in the ancient Greek enharmonic mode.[1] Suppose we replaced the Pythagorean semitone, which is about $\frac{4}{5}$ of the natural semitone, by another still smaller, about $\frac{3}{5}$ of the natural one, say $\frac{16}{15} \times \frac{80}{81} \times \frac{80}{81}$; the result would be perfectly unnatural as a leading note. We have already seen that the character of the leading note essentially depends upon its being that tone in the scale which is most distantly related to the tonic, and hence most uncertain and alterable [melodically]. Hence we are perfectly unjustified in deducing from such a tone the principle of construction for the whole scale.

The principal fault of our present tempered intonation, therefore, does not lie in the Fifths; for their imperfection is really not worth speaking of, and is scarcely perceptible in chords. The fault rather lies in the Thirds, and this error is not due to forming the Thirds by means of a series of imperfect Fifths, but it is the old Pythagorean error of forming the Thirds by means of an ascending series of four Fifths. Perfect Fifths in this case give even a worse result than flat Fifths. The natural relation of the major Third to the tonic both melodically and harmonically, depends on the ratio $\frac{5}{4}$ of the vibrational numbers. Any other Third is only a more or less unsatisfactory substitute for the natural major Third. The only correct system of tones is that in which, as Hauptmann proposed, the system of tones generated by Fifths should be separated from those generated by major Thirds. Now as it is important for the solution of many theoretical questions to be able to make experiments on tones which really form with each other the natural intervals required by theory, to prevent the ear from being deceived by the imperfections of the equal temperament, I have endeavoured to have an instrument constructed by which I could modulate by perfect intervals into all keys.

[1] [However justifiable in unaccompanied melody, such alterations are destructive of harmony, and hence do not belong to harmonic music proper. Of all the older temperaments, the mesotonic is most harmonious, but this makes the leading tones still further from the tone on which they are resolved, than even in just intonation. No diminution of the just semitone can be made without injury to the major Thirds.—*Translator.*]

If we were really obliged to produce in all its completeness the system of tones distinguished by Hauptmann, in order to obtain perfect intervals in all keys, it would certainly be scarcely possible to overcome the difficulties of the problem. Fortunately it is possible to introduce a great and essential simplification by means of the artifice originally invented by the Arabic and Persian musicians, and previously mentioned on p. 431.

We have already seen that the tones of Hauptmann's system which are generated by Fifths, and are marked by letters without any subscribed or superscribed lines, as $c - g - d - a -$ &c., are one comma or $\frac{81}{80}$ higher than the notes which bear the same names, when generated by major Thirds, and which are here distinguished by a subscribed line as $\underline{c} - \underline{g} - \underline{d} - \underline{a} -$ &c. We have further seen that if we descend from b by a series of 12 Fifths, down to $c\flat$, the last tone, reduced to the proper Octave, is lower than $b\flat$ by about $\frac{74}{73}$.[1] Hence we have—

$$b : \underline{b} = 81 : 80$$
$$b : c\flat = 74 : 73$$

Now these two intervals are very nearly alike; $\underline{b}$ is rather higher than $c\flat$, but only in the proportion—

$$c\flat : \underline{b} = 5913 : 5920$$

or, using the approximation obtained by continued fractions—

$$c\flat : \underline{b} = 885 : 886.$$

The interval between $c\flat$ and $\underline{b}$ is consequently about the same as that between a perfect and an equally tempered Fifth.[2]

Now $\underline{b}$ is the true major Third of g, and if we descend 8 Fifths from g we arrive at $c\flat$ thus:

$$g - c - f - b\flat - e\flat - a\flat - d\flat - g\flat - c\flat$$

[1] [Beginning at $\dagger b$ and descending twelve Fifths, we have, in the notation of Appendix XIX., $\dagger b - \dagger e - \dagger a - d - g - c - f - b\flat - e\flat - a\flat - \ddagger d\flat - \ddagger g\flat - \ddagger c\flat$. And $\dagger b : b = \dagger$, but $\dagger b : \ddagger c\flat = \dagger\P$. The accurate ratio of the two intervals $\dagger\P : \dagger$ is $\P$; see p. 486, note 2.—*Translator.*]

[2] [On account of the approximate character of the calculation, the extreme closeness of the result is not well shewn. Taking the accurate numbers, the ratio

$\dagger b \div \ddagger c\flat = \frac{32805}{32768}$, of which the log. = ·000 4901 071

Perfect Fifth ÷ tempered Fifth = $\frac{3}{2} \div \sqrt[12]{2^7}$; of which the log. = ·000 4904 228

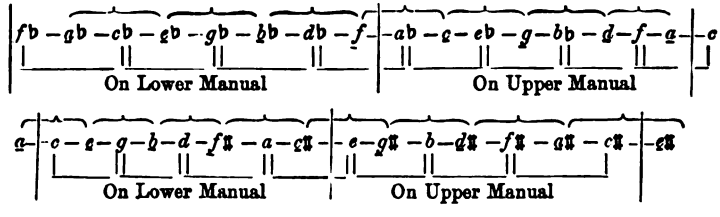
Human ears, however much assisted by human contrivances, could never hear the difference.—*Translator.*]

Now, as *c♭* is flatter than *b*, if we diminished[1] all the Fifths by $\frac{1}{8}$ of the small interval $\frac{886}{885}$ we should arrive at *b* instead of *c♭*.

Now, since the interval $\frac{886}{885}$ is itself on the limits of sensible difference of pitch, the eighth part of this interval cannot be taken into account at all, and we may consequently identify the following tones of Hauptmann's system, by proceeding in a series of Fifths from *c♭* = *b*, that is, the upper line with the lower, or—

$$\begin{array}{l} f\flat - c\flat - g\flat - d\flat - a\flat - e\flat - b\flat \\ = e \;\; - b \;\; - f\sharp - c\sharp - g\sharp - d\sharp - a\sharp \end{array}$$ [2]

Among musical instruments, the harmonium, on account of its uniformly sustained tone, the piercing character of its quality of tone, and its tolerably distinct combinational tones, is particularly sensitive to inaccuracies of intonation. And as its vibrators also admit of a delicate and durable tuning, it appeared to me peculiarly suitable for experiments on a more perfect system of tones. I therefore selected an harmonium of the larger kind,[3] with two manuals, and a set of vibrators for each, and had it so tuned that by using the tones of the two manuals I could play all the major chords from *F♭* major to *F♯* major. The tones are thus distributed:



This instrument therefore furnishes 15 major chords and as many minor chords, with perfectly pure Thirds but with Fifths too flat by $\frac{1}{8}$ of the interval by which an equally tempered Fifth

[1] [Accidentally misprinted 'increased,' that is, 'zu gross,' instead of 'zu klein' in all the three German editions.—*Translator.*]

[2] [In the notation of Appendix XIX. the two series are—

$$\begin{array}{l} \ddagger f\flat - \ddagger c\flat - \ddagger g\flat - \ddagger d\flat - a\flat - e\flat - b\flat \\ = e \;\; - b \;\; - f\sharp - c\sharp - g\sharp - d\sharp - \dagger a\sharp \end{array}$$

of which the second series is a skhisma, or ¶, sharper than the first. That is, the real equation is $\ddagger c\flat = \|b$. This will be fully elucidated in Appendix XIX., Section C, for the proposal really amounts to a temperament, which is somewhat different from the Arabic artifice that suggested it.—*Translator.*]

[3] Made by Messrs. J. and P. Schiedmayer, in Stuttgart.

is too flat.[1] On the Lower Manual we have the complete scales of *C*♭ major and *G* major and in the upper the complete scales of *E*♭ major and *B* major inclusive complete. All the major scales exist from *C*♭ major to *B* major, and they can all be played in

[1] The tuning of this instrument was easily managed. Herr Schiedmayer succeeded at the first attempt by the following direction. Starting from *a* on the lower manual, tune the Fifths *d* — *a*, *g* — *d*, *c* — *g* perfectly just, and thus obtain *c*, *g*, *d*. Then tune the major chords *c* — *e* — *g*, *g* — *b* — *d*, *d* — *f*♯ — *a*, giving the three tones *e*, *b*, *f*♯, and finally the Fifth, *f*♯ — *c*♯, to obtain *c*♯. Then putting *e* = *f*♭, *b* = *c*♭, *f*♯ = *g*♭, *c*♯ = *d*♭, tune the major chords *f*♭ — *a*♭ — *c*♭, *c*♭ — *e*♭ — *g*♭, *g*♭ — *b*♭ — *d*♭ with pure Thirds giving no beats, thus obtaining *a*♭, *e*♭, *b*♭, and finally the Fifth *b*♭ — *f*, giving *f*. This completes the tuning of the notes on the lower manual. For the upper manual first tune *e* as the perfect Fifth of the *a* in the lower manual, and then the three major chords *e* — *g*♯ — *b*, *b* — *d*♯ — *f*♯, *f*♯ — *a*♯ — *c*♯, and the Fifth *a*♯ — *e*♯, giving *b*, *f*♯, *c*♯, and then *g*♯, *d*♯, *a*♯ and also *e*♯. Then put *g*♯ = *a*♭, *d*♯ = *e*♭, *a*♯ = *b*♭, *e*♯ = *f*, and tune the Thirds in the major chords *a*♭ — *c* — *e*♭, *e*♭ — *g* — *b*♭, *b*♭ — *d* — *f*, and the Fifth *d* — *a*. This gives *c*, *g*, *d*, and *a*, and completes the whole tuning, which is much easier than for a series of equally tempered tones.

[The theoretical flattening of all the Fifths by $\frac{1}{8}$ of a skhisma is here neglected, as it would be impossible by ear only, and in all probability many other errors in tuning were committed, which could not be detected. The result is that the two manuals were tuned to the following tones, retaining the notation of the text, and using capital letters to represent the large or white digitals, and the small letters the small or black digitals. The Roman letters below shew the secondary meaning attached to the letters above them for the tuning of the notes marked with a * above them.

Upper Manual.

*C	*c*♯	*D	*d*♯	*E*	*E*♯	*f*♯	*G	*g*♯	*A	*a*♯	*B*
			e♭		f			a♭		b♭	

Lower Manual.

C	*c*♯	*D*	*c*♭	*E*	*F*	*f*♯	*G*	*g*♭	*A*	*b*♭	*B*
	d♭			F♭		g♭					C♭

In the following transcription into the notation of Appendix XIX., some other useful synonyms are written above:—

Upper Manual (Translator's Notation).

B♯	d♭	c♯♯				g♭	f♯♯		g♯♯		
‡*C*	†*c*♯	‡*D*	‡*d*♯	†*E*	†*E*♯	†*f*♯	‡*G*	*g*♯	*A*	†*a*♯	†*B*
			e♭		f			a♭		b♭	

Lower Manual (Translator's Notation).

			‡d♯		e♯			‡g♯		a♯	
C	*c*♯	*D*	‡*c*♭	*E*	‡*F*	*f*♯	*G*	‡*a*♭	†*A*	‡*b*♭	*B*
	‡d♭			‡F♭		‡g♭					‡C♭

The whole theory of constructing instruments in just intonation is given in Appendix XIX., and many forms are suggested in Section G, where this is No. 3, of which No. 8 is an improved form.—*Translator.*]

perfect purity in the natural intonation. But to modulate beyond *B* major on the one side and *C*♭ major on the other, it is necessary to make a really enharmonic interchange between *B* and *C*♭, which perceptibly alters the pitch (by a comma $\frac{81}{80}$). The minor modes on the lower manual are *B* minor or *C*♭ minor complete, on the upper manual *D*♯ minor or *E*♭ minor.

For the minor keys this series of tones is not quite so satisfactory as for the major keys. The dominant of a minor key is the Fifth of a minor triad and the root of a major triad. But as the minor chord has to be written as *a* — *c* — *e*, and the major chord as *f*♭ — *a*♭ — *c*♭, the corresponding dominant must be written in the first chord with an *underlined* letter, and in the second with a letter that has no line beneath it; that is, they must be tones of the kind which we have identified by means of the assumptions here made, as in the present case where *e* is identified with *f*♭. Hence the instrument furnishes the following perfectly just minor scales [where the letters in brackets indicate those which are not written in the account of the manuals in the text of p. 492]:—

1) *a* or *b*♭♭ minor: *d* — *f* — *a* — *c* — *e* — [*g*♯ — *b*]
f♭ — *a*♭ — *c*♭

2) *e* or *f*♭ minor: *a* — *c* — *e* — *g* — *b* — [*d*♯ — *f*♯]
c♭ — *e*♭ — *g*♭

3) *b* or *c*♭ minor: *e* — *g* — *b* — *d* — *f*♯ — [*a*♯ — *c*♯]
g♭ — *b*♭ — *d*♭

4) *f*♯ or *g*♭ minor: *b* — *d* — *f*♯ — *a* — *c*♯ — [*e*♯ — *g*♯
d♭ — *f* — *a*♭

5) *c*♯ or *d*♭ minor: *f*♯ — *a* — *c*♯ — *e* — *g*♯ — [*b*♯ — *d*♯]
a♭ — *e* — *e*♭

6) *g*♯ or *a*♭ minor: *c*♯ — *e* — *g*♯ — *b* — *d*♯ — [*f*♯♯ — *a*♯]
e♭ — *g* — *b*♭

7) *d*♯ or *e*♭ minor: *g*♯ — *b* — *d*♯ — *f*♯ — *a*♯ — [*c*♯♯ — *e*♯]
b♭ — *d* — *f*

8) *a*♯ or *b*♭ minor: *d*♯ — *f*♯ — *a*♯ — *c*♯ — *e*♯ — [*g*♯♯ — *b*♯]
f — *a* — *c*

Of these, the six last tonics from *C*♭ to *B*♭ are also provided with major scales. Hence there are complete minor scales on all degrees of the scales of *B* major and *E* major; and complete

minor and major scales on all degrees of the scale of *B* major, with the exception of *E*.

After previous experiments on another harmonium, where I had at command only the two sets of tones on the Octave common to two stops with one manual, I had expected that it would be scarcely observed if either the other minor keys had a somewhat too sharp Pythagorean Seventh, or if minor chords, which are themselves rather obscurely harmonious, were executed in Pythagorean intonation. When isolated minor chords are struck the difference is, indeed, not much observed. But when long series of justly-intoned chords have been employed, and the ear has grown accustomed to their effect, it becomes so sensitive to any intermixture of chords in imperfect intonation, that the disturbance is very appreciable.[1]

The least disturbance is caused by taking the Pythagorean Seventh, because this leading tone is in modern compositions scarcely ever used but in the chord of the dominant Seventh, or other dissonances. In a pure major triad its effect is certainly very harsh. But in a discord it has a less disturbing effect, because by its sharpness it brings out the character of the leading note more distinctly. On the other hand, I have found minor chords with Pythagorean Thirds, absolutely intolerable when coming between justly-intoned major and minor chords.[1] By allowing, then, a Pythagorean Seventh in the scale, or a Pythagorean major Third in the chord of the dominant Seventh, we may form the following minor scales:—

9) *d* minor: *g* — *b*♭ — *d* — *f* — *a* — *c*♯ — *e*
10) *g* minor: *c* — *e*♭ — *g* — *b*♭ — *d* — *f*♯ — *a*
11) *c* minor: *f* — *a*♭ — *c* — *e*♭ — *g* — *b* — *d*
12) *f* minor: *b*♭ — *e*♭ — *f* — *a*♭ — *c* — *e* — *g*
13) *b*♭ minor: *e*♭ — *g*♭ — *b*♭ — *d*♭ — *f* — *a* — *c*
14) *e*♭ minor: *a*♭ — *c*♭ — *e*♭ — *g*♭ — *b*♭ — *d* — *f*

In the former series, Nos. 8 and 7, we had already *b*♭ minor and *e*♭ minor, which are a comma sharper than Nos. 13 and 14.

[1] [My own experience is that the minor chords even more than the major shew the vast superiority of the just intonation over the equal temperament; and that the occasional introduction of Pythagorean among justly-intoned chords, major or minor, is comparable only to the 'wolves' on the 'bad keys,' as *E*♭, or *E*, of the old organ tuning. My instruments enable me to compare these effects readily, and both arise from similar, though not the same causes.—*Translator*.]

Hence the series of minor keys is also completed by the fusion of their extremities through enharmonic interchange.

In most cases it is possible to transpose the music to be played on such instruments, so as to avoid the necessity of making these enharmonic interchanges, provided the modulations do not extend too far into different keys. But if it is not possible to avoid enharmonic interchanges, they must be introduced where two unrelated[1] chords follow each other. This is best done between dissonant chords. Naturally this enharmonic change is always necessary when a piece of music modulates through the whole circle of Fifths—from *C* major to *B*♯ major, for example.[2] But Hauptmann is certainly right when he characterises such circular modulation as unnatural artificiality, which could only be rendered possible by the imperfections of our modern system of temperament. Such a process must certainly destroy the hearer's feeling for the unity of the tonic. For although *B*♯ has very nearly the same pitch as *C*, or can be even improperly identified with it, the hearer can only restore his feeling for the former tonic by going back on the same path that he advanced. He cannot possibly retain his recollection of the absolute pitch of the first tonic *C* after his long modulations up to *B*♯, with such a degree of exactness as to be able to recognise that they are identical. For any fine artistic feeling *B*♯ must remain a tonic far removed from *C* on the dominant side; or, more probably, after such distant modulations, the hearer's whole feeling for tonality will have become confused, and it will then be perfectly indifferent to him in what key the piece ends. Generally speaking, an immoderate use of striking modulations is a suitable and easy instrument in the hands of modern composers, to make their pieces piquant and highly coloured. But a man cannot live upon spice, and the consequence of restless modulation is almost always the obliteration of artistic connection. It must not be forgotten that modulations should be only a means of giving prominence to the tonic by contrasting it with another and then returning into it, or of attaining isolated and peculiar effects of expression.

Since harmoniums with two manuals have usually two sets of vibrators for each manual of which the above system of tuning only uses one, I have had the two others (an 8-foot and a 16-foot stop)

[1] [That is, chords not having a common tone.—*Translator.*]

[2] [In the notation of Appendix XIX., from *C* to ††*B*♯.—*Translator.*]

tuned in the usual equal temperament, which renders it very easy to compare the effect of this tuning with just intonation, as I have merely to pull out or push in a stop to make the difference.[1]

As regards musical effect, the difference between the just and the equally-tempered or the just and the Pythagorean intonations are very remarkable. The justly-intoned chords, in favourable positions, notwithstanding the rather piercing quality of the tone of the vibrators, possess a full and as it were saturated harmoniousness; they flow on, with a full stream, calm and smooth, without tremor or beat. Equally-tempered or Pythagorean chords sound beside them rough, dull, trembling, restless. The difference is so marked that everyone, whether he is musically cultivated or not, observes it at once. Chords of the dominant Seventh in just intonation have nearly the same degree of roughness as a common major chord of the same pitch in tempered intonation. The difference between natural and tempered intonation is greatest and most unpleasant in the higher Octaves of the scale, because here the false combinational tones of the tempered intonation are more observable, and the number of beats for equal differences of pitch becomes larger, and hence the roughness greater.

A second circumstance of essential importance is, that the differences of effect between major and minor chords, between different inversions and positions of chords of the same kind, and between consonances and dissonances are much more decided and conspicuous, than in the equal temperament. Hence modulations become much more expressive. Many fine distinctions are sensible, which otherwise almost disappear, as, for instance, those which depend on the different inversions and positions of chords, while, on the other hand, the intensity of the harsher dissonances is much increased by their contrast with perfect chords. The chord of the diminished Seventh, for example, which is so much used in modern music, borders upon the insupportable, when the other chords are tuned justly.[2]

Modern musicians who, with rare exceptions, have never heard

[1] Proposals for making the series of tones in this system of intonation more complete and at the same time greatly facilitating fingering by the use of a single manual, will be found in the Appendix XVII. [And another method, for which the requisite mechanism exists, is given in Appendix XIX., Section G, No. 8.—*Translator.*]

[2] [I can fully confirm this by my own experience on my justly-intoned concertina, Appendix XIX., Section G, No. 1. The last-named chord, although as ♯*g*♮ — *b* | ♯*d* — *f* it has two perfect minor Thirds. ♯*g*♮ — *b* and ♯*d* — *f*, which sound far superior to

any music executed except in equal temperament, mostly make light of the inexactness of tempered intonation. The errors of the Fifths are very small. There is no doubt of that. And it is usual to say that the Thirds are much less perfect consonances than the Fifths, and consequently also less sensitive to errors of intonation. The last assertion is also correct, so long as homophonic music is considered, in which the Thirds occur only as melodic intervals and not in harmonic combinations. In a consonant triad every tone is equally sensitive to false intonation, as theory and experience alike testify, and the bad effect of the tempered triads depends especially on the imperfect Thirds.[1]

There can be no question that the simplicity of tempered intonation is extremely advantageous for instrumental music, that any other intonation requires an extraordinarily greater complication in the mechanism of the instrument, and would materially increase the difficulties of manipulation, and that consequently the high development of modern instrumental music would not have been possible without tempered intonation. But it must not be imagined that the difference between tempered and just intonation is a mere mathematical subtilty without any practical value. That this difference is really very striking even to unmusical ears, is shewn immediately by actual experiments with properly tuned instruments. And that the early musicians, who were still accustomed to the perfect intervals of vocal music, which was then most carefully practised, felt the same, is immediately seen by a glance at the musical writings of the latter half of the seventeenth and the earlier part of the eighteenth centuries, at which time there was much discussion about the introduction of different kinds of temperament, and one new method after another was invented and rejected for escaping the difficulties, and the most ingenious forms

their equally tempered substitutes, and one Pythagorean minor Third, *b* | ♯*d*, which is fully as good as the equally-tempered minor Third (p. 488), is a mere piece of noise, of a worse kind than the noise of the equally-tempered imitation of the same chord in the same quality of tone. In just intonation the chord of the diminished Seventh can therefore be used only with mild qualities of tone.—*Translator.*]

[1] [A triad in which the major Third is perfect, but the Fifth and minor Third both too small by a quarter of a comma (as in the mean tone temperament, in which I have a concertina tuned), has a very much better effect than the equally-tempered triad, where the Fifth is only one-eleventh of a comma too flat, and the major Third is seven-elevenths of a comma too sharp, and hence the minor Third is eight-elevenths of a comma too flat. The effect is much more strongly felt in playing passages than isolated chords.—*Translator.*]

of instrument were designed for practically executing the enharmonic differences of the tones. Praetorius[1] mentions a universal cymbalum, which he saw at the house of the court-organist of the Emperor Rudolph II. in Prague, and which had 77 digitals in 4 Octaves, or 19 to the Octave, the black digitals being doubled, and others inserted between those for *e* and *f*, and between those for *b* and *c*. In the older directions for tuning, several tones are usually tuned by Fifths which beat slightly, and then others as perfect major Thirds. The intervals on which the errors accumulated were called *wolves*. Praetorius says: 'It is best for the wolf to remain in the wood with his abominable howling, and not disturb our *harmonicas concordantias*.' Rameau, too, who at a later period contributed greatly to the introduction of equal temperament, in 1726[2] still defended a different style of tuning, in which the Thirds of the more usual keys were kept perfect at the expense of the Fifths and of the unusual keys. Thus he tuned up from *C*, in Fifths so much diminished, that the fourth Fifth, instead of being *E*, became the perfect Third of *C*, namely $\underline{E}=F\flat$. Then again four Fifths more to $\underline{A}\flat$, the perfect Third of $F\flat$, instead of to $A\flat$. But then the four Fifths between this $\underline{A}\flat$ and *C* had necessarily[3] to be made too large, because it is not $\underline{A}\flat$ but $A\flat$ which is four perfect Fifths distant from *C*. This plan of tuning gives the perfect major Thirds, $C-\underline{E}$, $G-\underline{B}$, $D-\underline{F}\sharp$, $E-\underline{G}\sharp$, but when we proceed further from *E* on the dominant side, or from

[1] 'Syntagma musicum,' II., Chap. XI., p. 63.

[2] 'Nouveau Système de Musique,' Chap. XXIV.

[3] [That is, if only twelve digitals might be used, so that the temperament became unequal. But this style of tuning, which at first was the mean tone temperament, where the Fifths are made a quarter of a comma too flat, should be carried out through twenty-six Fifths, requiring twenty-seven tones as under, to be really effective, and if any fewer are employed no attempt should be made to modulate into keys not provided with proper tones. See Appendix XIX., Section C, and Table VI. It is a temperament with which I am practically familiar. It is far superior to the equally-tempered, and is even endurable on the concertina, which used to be always so tuned, but having fourteen digitals, extends from *A*♭ to *D*♯. The twelve digitals could play then only in *B*♭, *F*, *C*, *G*, *D*, and *A* major, and in *G*, *D*, and *A* minor, which of course failed to satisfy the requirements of modulation. Hence players sought to identify *D*♯, *A*♯, *E*♯, *F*𝄪 *C*𝄪 *G*𝄪 with *E*♭, *B*♭, *F*, *G*, *D*, *A* on the one hand, and *D*♭, *G*♭, *C*♭, *F*♭, *B*♭♭, *E*♭♭, *A*♭♭ with *C*♯, *F*♯, *B*, *E*, *A*, *D*, *G* on the other. Hence came the wolves. And a system of tuning was blamed for not doing what it never professed to do. As long as twelve digitals only are insisted on, the equal temperament, by dividing the Octave into twelve equal Semitones, is a necessity, unless a series of such manuals can be furnished as proposed in Appendix XIX., Sect. G, Nos. 13 and 14.—*Translator*.]

C on the subdominant side, we find Thirds which become worse and worse. The error in the Fifths is about three times that in the equal temperament. Even in 1762, this system could be characterised by d'Alembert as that commonly used in France, in opposition to the equal temperament which Rameau subsequently proposed. Marpurg [1] has collected a long series of other systems of tuning. Since players found themselves compelled by using only 12 digitals to the Octave, to put up with a series of false intervals, and to let their ears become accustomed to them, it was certainly better to make up their minds to give up their few perfect major Thirds still remaining in the scale, and to make all the major Thirds equally erroneous. It necessarily produces more disturbance to hear very falsely tuned Thirds amidst correct intervals, than to hear intervals which are all equally out of tune and are not contrasted with others in perfect intonation. Hence as long as it is necessary practically to limit the number of separate tones within the Octave to 12, there can be no question at all as to the superiority of the equal temperament with its 12 equal semitones, over all others, and, as a natural consequence, this has become the sole acknowledged method of tuning. It is only bowed instruments, with [including the *Tenor*] their four perfect Fifths *C* — *G* — *D* — *A* — *E*, which still deviate from it.

The equal temperament came into use in Germany before it was introduced into France. In the second volume of Matheson's *Critica Musica*, which appeared in 1752, he mentions *Neidhard* and *Werckmeister* as the inventors of this temperament.[2] Sebastian Bach had already used it for the pianoforte (*clavier*), as we must conclude from Marpurg's report of Kirnberger's assertion, that when he was a pupil of the elder Bach he had been made to tune all the major Thirds too sharp. Sebastian's son, Emanuel, who was a celebrated pianist, and published in 1753 a work of great authority in its day 'on the true art of playing the pianoforte,' requires this instrument to be always tuned in the equal temperament.

The old attempts to introduce more than 12 degrees into the scale have led to nothing practical, because they did not start from

[1] 'Versuch über die musikalische Temperatur,' Breslau, 1776.

[2] *Op. cit.* p. 162. The following works of these two authors are cited by Forkel: *Neidhard* (Royal Prussian Band-conductor) 'Die beste und leichteste Temperatur des Monochordi' (the best and easiest temperament of the monochord), Jena, 1706; 'Sectio canonis harmonici,' Königsberg, 1724. *Werckmeister* (organist at Quedlinburg, born 1645,) 'Musikalische Temperatur,' Frankfurt and Leipzig, 1691.

any right principle. They always attached themselves to the Greek system of Pythagoras, and imagined only that it was necessary to make a difference between *c*♯ and *d*♭, or between *f*♯ and *g*♭, and so on. But that is not by any means sufficient, and is not even always correct. According to our system of notation we may identify *c*♯ with *d*♭[1] but we must distinguish the *c*♯ found from the relation of Fifths, from the *c*♯ found from the relation of Thirds.[2] Hence the attempts to construct instruments with complex arrangements of manuals and digitals, have led to no result, which was at all commensurate with the trouble bestowed upon them, and the increased difficulties of fingering which they occasioned. The only instrument of the kind which is still used is the pedal harp *à double mouvement*, on which the intonation can be changed by the foot.

Not only habitual use, and the absence of any power to compare its effects with those of just intonation, but some other circumstances are favourable to equal temperament.

First, it should be observed that the disturbances due to beats in the tempered scale, are the less observable the swifter the motion and the shorter the duration of the single notes. When the note is so short that but very few beats can possibly occur while it lasts, the ear has no time to remark their presence. The beats produced by a tempered triad are the following[3]:

1. Beats of the tempered Fifth. Suppose we take the number of vibrations of *c′* to be 264, the tempered Fifth *c′* — *g′* would produce 9 beats in 10 seconds, partly by the upper partials, and partly by the combinational tones. These beats are always quite audible.

2. Beats of the two first combinational tones of *c′* — *e′* and *e′* — *g′* in tempered intonation; $5\frac{2}{3}$ in the second. These are plainly audible in all qualities of tones, if the tones themselves are not too weak.

3. Beats of the major Third *c′* — *e′* alone, $10\frac{1}{2}$ in the second, which, however, are not plainly audible unless the qualities of tone employed have high upper partials.

[1] [That is, in the notation of Appendix XIX., we may identify †*c*♯ with *d*♭, which is really one skhisma flatter.—*Translator.*]

[2] [That is, in the notation of Appendix XIX., †*c*♯ found from *c* — *g* — *d* — †*a* — †*e* — †*b* — †*f*♯ — †*c*♯, from *c*♯ found from first the major Third *c* — *e* and then the Fifths *e* — *b* — *f*♯ — *c*♯.—*Translator.*]

[3] [See also Appendix XIX., Table VI., D.—*Translator.*]

4. Beats of the minor Third $e' - g'$, 18 in the second, mostly much weaker than those of the major Third, and also heard only in qualities of tone having high upper partials.

All these beats occur twice as fast when the chord lies an Octave higher, and half as fast when it occurs an Octave lower.

Of these beats, the first, arising from the tempered Fifths, have the least injurious influence on the harmoniousness of the chord. They are so slow that they can be heard only for very slow notes in the middle parts of the scale, and then they produce a slow undulation of the chord which may occasionally have a good effect. Beats of the second kind are most striking for the softer quality of tone. In an *Allegro*, of four crotchets in a bar, two bars occupy about three seconds. If then the tempered chord $c' - e' - g'$ is played on a crotchet in this bar, $2\frac{1}{8}$ of these beats will be heard, so that if the tone begins soft, it will swell, decrease, swell again, decrease and then finish. It would be certainly worse if this chord were played an Octave or two higher, so that $4\frac{1}{4}$ or $8\frac{1}{2}$ beats could be heard, because these could not fail to strike the ear as a marked roughness.

For the same reason the beats of the third and fourth kinds, arising from the Thirds, which are clearly audible on harsher qualities of tone (as on the harmonium), are also decidedly disturbing in the middle positions, even in quick time, and essentially injure the calmness of the triad, because they are twice and thrice as fast as the others. It is only in soft qualities of tone that they are but little observed, or, when observed, are so covered by stronger unbroken tones as to be very slightly marked.

Hence in rapid passages, with a soft quality and moderate intensity of tone, the evils of tempered intonation are but little apparent. Now, almost all instrumental music is designed for rapid movement, and this forms its essential advantage over vocal music. We might, indeed, raise the question whether instrumental music had not rather been forced into rapidity of movement by this very tempered intonation, which did not allow us to feel the full harmoniousness of slow chords to the same extent as is possible from well-trained singers, and that instruments had consequently been forced to renounce this branch of music.

Tempered intonation was first and especially developed on the pianoforte, and hence gradually transferred to other instruments. Now, on the pianoforte circumstances really favour the concealment

of the imperfections due to temperament. The tones of a pianoforte are very loud only at the moment of striking, and their loudness rapidly diminishes. This, as I have already had occasion to mention, causes their combinational tones to be audible at the first moment only, and hence makes them very difficult to hear. Beats from that source must therefore be left out of consideration. The beats which depend on the upper partials have been eliminated in modern pianofortes (especially in the higher Octaves, where they would have done most harm), owing to the mode in which upper partials are greatly weakened and the quality of tone much softened by regulating the striking place, as I have explained in Chap. V. (p. 123). Hence on a pianoforte the deficiencies of the intonation are less marked than on any instrument with sustained tones, and yet they are not quite absent. When I go from my justly-intoned harmonium to a grand pianoforte, every note of the latter sounds false and disturbing, especially when I strike isolated successions of chords. In rapid melodic figures and passages, and in arpeggio chords, the effect is less disagreeable. Hence older musicians especially recommended the equal temperament for the pianoforte alone. Matheson, in doing so, acknowledges that for organs Silberman's unequal temperament, in which the usual keys were kept pure,[1] is more advantageous. Emanuel Bach says that a correctly tuned pianoforte has the *most perfect intonation of all instruments*, which in the above sense is correct. The great diffusion and convenience of pianofortes made it subsequently the chief instrument for the study of music and its intonation the pattern for that of all other instruments.

On the other hand, for the harsher stops on the organ, as the mixture and reed stops, the deficiencies of equal temperament are extremely striking. It is considered inevitable that when the mixture stops are played with full chords an awful din (*höllenlärm*) must ensue, and organists have submitted to their fate. Now this is mainly due to equal temperament, because if the Fifths and Thirds in the pipes for each digital of the mixture stops were not tuned justly, every single note would produce beats by itself. But when the Fifths and Thirds between the notes belonging to the different digitals are tuned in equal temperament, every chord furnishes at once tempered and just Fifths and Thirds, and the result is a restless

[1] [Probably this was the mean tone temperament explained on p. 499, note 3.—*Translator.*]

blurred confusion of sounds. And yet it is precisely on the organ it would be so easy by a few stops to regulate the action for each key so as to produce harmonious chords.[1]

Whoever has heard the difference between justly-intoned and tempered chords, can feel no doubt that it would be the greatest possible gain for a large organ to omit half its stops, which are mostly mere toys, and double the number of tones in the Octave, so as to be able, by means of suitable stops, to play correctly in all keys.[2]

The case is the same for the harmonium as for these stops on the organ. Its powerful false combinational tones and its gritty trembling chords, both due to tempered intonation, are certainly the reason why many musicians pronounce this instrument to be out of tune, and dismiss it at once as too trying to the nerves.

[1] From Zamminer's book, I see that in Silliman's 'American Journal of Science' for 1850 there is a description of an organ by Poole, which is tuned justly for all keys by means of stops. [See Appendices XVIII. and XIX.]

[2] [That is, as correctly as on the author's justly-intoned harmonium, but that is far too deficient in power of modulation into minor keys, to make it worth while to construct it on a great organ. In Appendix XIX. it will be shewn that at least forty-eight degrees of tone are required for this purpose, which would require the omission of *three-quarters* instead of one-half of the stops. In the same Appendix it will be shewn that it is possible at present by simple tuning to play perfectly by means of four performers on four ordinary organs, harmoniums, or pianofortes, and that therefore the difficulty reduces itself in the first place to determining what notes should be played. This is not always a slight difficulty, because temperaments from the first (whether the mean tone or the equal temperament) proceeded on the principle of ignoring the difference of a comma, as explained in Appendix XIX., Section C, and consequently the composers never allowed for this difference. If, then, we were always to pursue the natural course of the harmony we should be led far away from the original key, and if we were to keep to the original key, we should have to make enharmonic changes in the harmony. The difficulty lies in seeing how to introduce these. The voice and bowed instruments easily glide over this difficulty, but the theorist has to reduce it to paper, and the player of instruments with fixed tones feels and hears it strongly. Such modulations as those in the passages from *Don Giovanni* cited on p. 477, present great difficulties in this respect. We see, then, that before just intonation can become established in the highest walks of music, theoreticians must come to understand its meaning. In Appendix XIX., Sections D, E, and F, I have endeavoured to lay down some elementary principles which will be of assistance in this respect. The use of the *duodenal* as exemplified in the staff notation on pp. 471, 472, 475 and 476, will render the written expression of the value of the notes comparatively easy, without interfering in the slightest degree with the habits of musical notation. The difficulty is consequently reduced to determining the *duodene* in which any chord occurs, and that difficulty results from the habits of composers in systematically ignoring the difference of a comma between the Second of any major Scale and the Sixth of the major Scale of its subdominant. It is very probable that great changes in the conduct of modulation would result from the acceptance of just intonation by composers.—*Translator.*]

Orchestral instruments can generally alter their pitch slightly. Bowed instruments are perfectly unfettered as to intonation; wind instruments can be made a little sharper or flatter by blowing with more or less force. They are, indeed, all adapted for equal temperament, but good players have the means of indulging the ear to some extent. Hence, passages in Thirds for wind instruments, when executed by indifferent players, often sound desperately false (*verzweifelt falsch*), whereas good performers, with delicate ears, make them sound perfectly well.

The bowed instruments are peculiar. From the first they have retained their tuning in perfect Fifths. The violins themselves have the perfect Fifths, $G - D - A - E$. The tenor and violoncello give the Fifth $C - G$ in addition. Now, every scale has its own peculiar fingering, and hence every pupil could be easily practised in playing each scale in its proper intonation, and then, of course, tones of the same name but in different keys could not be stopped in the same way, and even the major Third of the major scale of C, when the C of the tenor is taken as the tonic, must not be played on the E string of the violin, because this gives E and not $\underline{E}$.[1] Nevertheless, the modern school of violin-playing since the time of Spohr, aims especially at producing equally-tempered intonation, although this cannot be completely

[1] [This supposes the violin and tenor to be tuned by the Fifths $C - G - D - \dagger A - \dagger E$, in the key of D, and hence the first string would give $\dagger E$, a comma sharper than E, the major Third of C. But the violin is really always tuned by a', by which orchestral tuning is always guided, and is hence in the key of $\ddagger d$. Now if we take $a' = 440$, this would give for the open strings of the tenor and violin, the perfect Fifths $\ddagger c - \ddagger g - \ddagger d' - a' - e''$. If we compare these with the corresponding tempered Fifths we have the following result for the vibrational numbers :—

	Just		Equal
e''	660·000	e''	657·739
a'	440·000	a'	440·000
$\ddagger d'$	293·333	d'	293·665
$\ddagger g$	195·555	g	195·998
$\ddagger c$	130·370	c	130·813

Where it is seen that below a' the Fifths are sharper, and as the pitch is lower the beats would be fewer and hence less distinctly heard, but for the higher e'' the 2·26 beats in a second would be distinct. Of course in the key of tempered c the orchestra would be throughout sharper than the tenor and violoncello, if the open string were taken as the standard.—*Translator.*]

attained, owing to the perfect Fifths of the open strings.[1] At any rate, the acknowledged intention of present violin-players is to produce only 12 degrees in the Octave. The sole exception which they allow is for double-stop passages, in which the notes have to be somewhat differently stopped from what they are when played alone. But this exception is decisive. In double-stop passages the individual player feels himself responsible for the harmoniousness of the interval, and it lies completely within his power to make it good or bad. Any violin-player will easily be able to verify the following fact. Tune the strings of the violin in the perfect Fifths $G - D - A - E$, and find where the finger must be pressed on the A string to produce the B, which will give a perfect Fourth $B - E$. Now, let him, without moving his finger, strike this same B with the open D string. The interval D—B would, according to the usual view, be a major Sixth, but it would be a Pythagorean one. In order to obtain the consonant Sixth $D - B$, the finger would have to be drawn back for about $1\frac{3}{8}$ Paris lines (nearly $\frac{3}{20}$ inch), a distance quite appreciable in stoppings, and sufficient to alter the pitch and the beauty of the consonance most perceptibly.

But it is clear that if individual players feel themselves obliged to distinguish the different values of the notes in the different consonances, there is no reason why the bad Thirds of the Pythagorean series of Fifths should be retained in quartett playing. Chords of several parts, executed by several performers in a quartett, often sound very ill, even when each single one of the performers is an excellent solo-player; and, on the other hand, when quartetts are played by finely-cultivated artists, it is impossible to detect any false consonances. To my mind the only assignable reason for these results is that practised violinists with a delicate sense of harmony, know how to stop the tones they want to hear, and hence do not submit to the rules of an imperfect school. That performers of the highest rank do really play in just intonation, has been directly proved by the very interesting and exact results of Delezenne.[2] This observer determined the

[1] [Hence, perhaps, the rule of avoiding open strings. For the ♯g string of the violin and ♯c string of the tenor the difference is, however, too slight to be of consequence, and these are the only open strings which cannot be avoided.—*Translator.*]

[2] 'Recueil des Travaux de la Société des Sciences, de l'Agriculture, et des Arts de Lille, 1826 et premier semestre 1827; Mémoire sur les Valeurs numérique des Notes de la Gamme,' par M. Delezenne. For observations on corresponding circumstances in singing, see below, Appendix XVIII.

individual notes of the major scale, as it was played by distinguished violinists and violoncellists, by means of an accurately gauged string, and found that these players produced correctly perfect Thirds and Sixths, and neither equally tempered nor Pythagorean Thirds or Sixths. I was fortunate enough to have an opportunity of making similar observations by means of my harmonium on Herr Joachim. He tuned his violin exactly with the $g - d - a - e$ of my instrument. I then requested him to play the scale, and immediately he had played the Third or Sixth, I gave the corresponding note on the harmonium. By means of beats it was easy to determine that this distinguished musician used $\underset{\smile}{b}$ and not b as the major Third to g, and $\underset{\smile}{e}$ not e as the Sixth.[1] But if the best players who are thoroughly acquainted with what they are playing are able to overcome the defects of their school and of the tempered system, it would certainly wonderfully smooth the path of performers of the second order, in their attempts to attain a perfect *ensemble*, if they had been accustomed from the first to play the scales by natural intervals. The greater trouble attending the first attempts would be amply repaid by the result when the ear has once become accustomed to hear perfect consonances. It is really much easier to apprehend the differences between notes of the same name in just intonation than people usually imagine. A confusion between $\underset{\smile}{a}$ and a in a consonant chord on my harmonium strikes me with the same readiness and certainty as a confusion between A and $A\flat$ on a pianoforte.

I am, however, too little acquainted with the technicalities of violin-playing, to attempt making any proposals for a definite regulation of the tonal system of bowed instruments. This must be left to masters of this instrument who at the same time possess

[1] Messrs. Cornu and Mercadier have lately published contradictory observations. (Comptes Rendus de l'Acad. des Sc. de Paris, 8 et 22 Février, 1869.) They let a musician play the Third of a major chord first in melodic succession, and then in harmonious consonance. In the latter case it was always 4 : 5. But in melodic succession the performer selected a somewhat sharper Third. I am bound to reply, that in melodic succession the major Third is not a very characteristically determined interval, and that all living musicians have been accustomed to sharp Thirds on the pianoforte. In the simple succession $c - e - g$, when isolated from the rest of the scale, I find it difficult to distinguish between the just and the Pythagorean major Third. But when I play on my harmonium the complete melody of some well-known air without harmonies the Pythagorean Third always feels to me strained, the perfect Third calm and soft. It is only in the leading note, perhaps, that the sharper Third is more expressive.

the powers of a composer. Such men will readily convince themselves by the testimony of their ears, that the facts here adduced are correct, and perceive that far from being useless mathematical speculations, they are practical questions of very great importance.

The case is precisely similar for our present singers. For singing, intonation is perfectly free, whereas on bowed instruments, the five tones of the open strings at least have an unalterable pitch. In singing the pitch can be made most easily and perfectly to follow the wishes of a fine musical ear. Hence all music began with singing, and singing will always remain the true and natural school of all music. The only intervals which singers can strike with certainty and perfection, are such as they can comprehend with certainty and perfection, and what the singer easily and naturally sings the hearer will also easily and naturally hear.

Down to the seventeenth century singers were practised by the monochord for which Zarlino in the middle of the sixteenth century reintroduced the correct natural intonation. Singers were then practised with a degree of care of which we have at present no conception. We can even now see from the Italian music of the fifteenth and sixteenth centuries that they were calculated for most perfect intonation of the chords, and that their whole effect is destroyed as soon as this intonation is insufficiently perfect.

But it is impossible not to acknowledge that at the present day few even of our opera singers are able to execute a little piece for several voices, when either totally unaccompanied, or at most accompanied by occasional chords, (as, for example, the trio for the three masks, *Protegga il giusto cielo*, from the finale to the first act of Mozart's *Don Giovanni*,) in a manner suited to give the hearer a full enjoyment of its perfect harmony. The chords almost always sound a little sharp or uncertain, so that they disturb a musical hearer. But where are our singers to learn just intonation and make their ears sensitive for perfect chords? They are from the first taught to sing to the equally-tempered pianoforte. If a major chord is struck as an accompaniment, they may sing a perfect consonance with its root, its Fifth, or its Third. This gives them about the fifth part of a semitone for their voices to choose from without decidedly singing out of harmony, and even if they sing a little sharper than consonance with the sharp Third requires, or a little flatter than consonance with the flat Fifth requires, the

harmoniousness of the chord will not be really much more damaged. The singer who practises to a tempered instrument has no principle at all for exactly and certainly determining the pitch of his voice.[1]

On the other hand, we often hear four musical amateurs who have practised much together, singing quartetts in perfectly just intonation. Indeed, my own experience leads me almost to affirm that quartetts are more frequently heard with just intonation when sung by young men who scarcely sing anything else and often and regularly practise them, than when sung by instructed solo singers who are accustomed to the accompaniment of the pianoforte or the orchestra. But correct intonation in singing is so far above all others the first condition of beauty, that a song when sung in correct intonation even by a weak and unpractised voice always sounds agreeable, whereas the richest and most practised voice offends the hearer when it sings false, or sharpens.

The case is the same as for bowed instruments. The instruction of our present singers by means of tempered instruments is unsatisfactory, but those who possess good musical talents are ultimately able by their own practice to strike out the right path for themselves, and overcome the error of their original instruction. They even succeed the earlier, perhaps, the sooner they quit school, although, of course, I do not mean to deny that fluency in singing, and the disuse of all kinds of bad ways can only be acquired in school.

It is clearly not necessary to temper the instruments to which the singer practises. A single key suffices for these exercises, and that can be correctly tuned. We do not require to use the same piano for the teaching to sing and for playing sonatas. Of course it would be better to practise the singer to a justly-intoned organ or harmonium in which by means of two manuals all keys may be used.[2] Sustained tones are preferable as an accompaniment be-

[1] See Appendix XVIII.

[2] [Voices differ so much that the same pitch for the tonic, that is the same key, would not suit. The Tonic-Solfaists teach without any accompaniment, not even that of the teacher's voice, but rapidly introduce part music. But such instruments as the *hepht harmonium* or *hexharmonium* (App. XIX., Sect. G, No. 10), which offer no mechanical difficulties that have not been already overcome, and which admit of a great variety of modulation, and several varieties of tonic, would probably be of great service to singers for home practice. In classes the *trioni* or *quartoni* arrangements for three or four performers (*ibid.*, Nos. 6 and 7) would give everything that could possibly be

cause the singer himself can immediately hear the beats between the instrument and his voice when he alters the pitch slightly. Draw his attention to these beats, and he will then have a means of checking himself in the most decisive manner. This is very easy on my justly-intoned harmonium, as I know by experience. It is only after the singer has learned to hear every slight deviation from correctness announced by a striking incident, that it becomes possible for him to regulate the motions of his larynx and the tension of his vocal chords with sufficient delicacy to produce the tone which his ear desires. When we require a delicate use of the muscles of any part of the human body, as, in this case, of the larynx, there must be some sure means of ascertaining whether success has been attained. Now the presence or absence of beats gives such a means of detecting success or failure when a voice is accompanied by sustained chords in just intonation. But tempered chords which produce beats of their own are necessarily quite unsuited for such a purpose.

Finally, we cannot, I think, fail to recognise the influence of tempered intonation upon the style of composition. The first effect of this influence was favourable. It allowed composers as well as players to move freely and easily into all keys, and thus opened up a new wealth of modulation. On the other hand, we also cannot fail to recognise that the alteration of intonation compelled composers to have recourse to some such wealth of modulation. For when the intonation of consonant chords ceased to be perfect, and the differences between their various inversions and transpositions were, as a consequence, nearly obliterated, it was necessary to use more powerful means, to have recourse to a frequent employment of harsh dissonances, and to endeavour by less usual modulations to replace the characteristic expression, which the harmonies proper to the key itself had ceased to possess. Hence in many modern compositions dissonant chords of the dominant Seventh form the majority, and consonant chords the minority, yet no one can doubt that this is the reverse of what ought to be the case; and continual bold modulational leaps threaten entirely to destroy the feeling for tonality. These are

required for the most complicated pieces of music; while for practice on a small scale the experimental harmonium (*ibid.*, No. 2) or just concertina (*ibid.*, No. 1) offers great facilities, as they are solo instruments and can be prepared by simple tuning.—*Translator.*]

unpleasant symptoms for the further development of art. The mechanism of instruments and attention to their convenience, threaten to lord it over the natural requirements of the ear, and to destroy once more the principle upon which modern musical art is founded, the steady predominance of the tonic tone and tonic chord. Our last greatest composers, Mozart and Beethoven, were yet at the commencement of the reign of equal temperament. Mozart had still an opportunity of making extensive studies in the composition of song. He is master of the sweetest possible harmoniousness, where he desires it, but he is almost the last of such masters. Beethoven eagerly and boldly seized the wealth offered by instrumental music, and in his powerful hands it became the appropriate and ready tool for producing effects which none had hitherto attempted. But he used the human voice as a mere handmaid, and consequently she has also not lavished on him the highest magic of her beauty.

And after all, I do not know that it was so necessary to sacrifice purity of intonation to the convenience of musical instruments. As soon as violinists have resolved to play every scale in just intonation, which can scarcely occasion any difficulty, the other orchestral instruments will have to suit themselves to the correcter intonation of the violins. Horns and trumpets have already naturally just intonation.

Moreover, we must observe that when just intonation is made the groundwork of modulations, even comparatively simple modulational excursions will occasion enharmonic confusions (amounting to a comma) which do not appear as such in the tempered system.[1]

To me it seems necessary that the new tonic into which we modulate should be related to the tonic in which we are playing; the nearer the relationship, the more striking the transition. Again, it is not advisable to remain long in a key which is not related to the principal tonic of the piece. With these principles the rules for modulation usually given coincide. The easiest and most usual transitions are into the key of the dominant or subdominant, these tones being, as is well known, the nearest relations of the first tonic. Hence if the original key is *C*, we can pass immediately into *G* major, and thus change the tones *F* and *A* of the scale of *C* major into *F*♯ and *A*. Or we can pass into *F*

[1] [See p. 504, note 2, and page 527, note.—*Translator.*]

major by exchanging $\underline{B}$ and D for $B\flat$ and $\underline{D}$. After this step has been made, the music will often pass into a key with a tonic related to C in the second degree only, as from G to D or from F to $B\flat$. By proceeding in this way we should come to keys as A and $E\flat$, of which the relation to the original tonic C would be very obscure and in which it would certainly not be advisable to remain long for fear of too much weakening the feeling for the original tonic.[1]

Again, we may also modulate from the principal tonic C to its Thirds and Sixths, to $\underline{E}$ and $\underline{A}$ or $\bar{E}\flat$ and $\bar{A}\flat$. In tempered intonation these changes seem to be the same as from G and D to A and E, or from F and $B\flat$ to $E\flat$ and $A\flat$. But they differ in the pitch, as shewn by the different marks A and $\underline{A}$, &c. In the tempered intonation it seems allowable to go by a Sixth from c to the key a, and then by Fifths back, to d, g, and then c again. But in reality we thus reach a different c from that with which we began. By such a transition, which is certainly not quite natural, we should be obliged to make an enharmonic exchange [alteration of pitch by a comma], and this would be best done while in the key of d, since both d and $\underline{d}$ are related to c in the second degree.[2] In the complicated modulations of modern composers such enharmonic changes will of course have to be often made. A cultivated

[1] [In the notation of Appendix XIX. we proceed to G major by introducing $F\natural$ and $\dagger A$ for F and A, and to F major by introducing $B\flat$ and $\ddagger D$ in place of B and D. For each new major key we therefore change two notes. Hence when we have made the transition to D major or $B\flat$ major, four notes are changed, and when we have made the transition to $\dagger A$ major or $E\flat$ major, six notes are changed, and only D will remain in the first case, and only F in the second, out of the original scale. For the whole subject of modulation in just intonation see Appendix XIX., Sections D, E and F.—*Translator.*]

[2] [It is interesting to observe how it happens that such a modulation would return to the original c in tempered intonation. The sixth a is already $\frac{8}{11}$ of a comma too sharp (see p. 487), and hence a change almost amounting to the required comma has been effected within the key itself. Then each fifth being $\frac{1}{11}$ of a comma too flat, the fall to d makes d in this case $\frac{9}{11}$ of a comma too sharp, the fall to g makes g $\frac{10}{11}$ of a comma too sharp, and the fall to c makes c $\frac{11}{11}$, that is a whole comma, too sharp for just intonation; and consequently the $\ddagger c$ of just intonation has been raised to c. This could therefore be best imitated by using $\dagger A$ for A in the first change. But there would be this difference, that the sharp A of tempered intonation was used in the original key, and no ear used to just intonation would endure $\dagger A$ in the original key. Hence there must be a jump. It is precisely here, in the modulation through the major Sixth, that those who study just intonation on paper or on instruments without fixed tones (as the voice and violin) differ from those who study it on instruments with fixed tones. The former find no difficulty in the leap. The latter find it unnatural and impossible. And so it appears to me.—*Translator.*]

taste will have to judge in each individual case how they are to be introduced, but it will be probably advisable to retain the rules already mentioned, and to choose the intonation of the new tonics introduced by modulation in such a manner as will keep them as closely related to the principal tonic as possible. Enharmonic changes are least observed when they are made immediately before or after strongly dissonant chords, as those of the diminished Seventh. Such enharmonic changes of pitch are already sometimes clearly and intentionally made by violinists, and where they are suitable even produce a very good effect.[1]

[1] See examples in C. E. Naumann's 'Bestimmungen der Tonverhältnisse' (Determinations of the Tonal Ratios), Leipzig, 1858, pp. 48, sqq.

CHAPTER XVII.

OF DISCORDS.

WHEN voices move forward melodically in part music, the general rule is that they must form consonances which each other. For it is only as long as they are consonant, that there is an uninterrupted fusion of the corresponding auditory sensations. As soon as they are dissonant the individual parts mutually disturb each other, and each is a hindrance to the free motion of the other. To this esthetic reason must be added the purely physical consideration, that consonances cause an agreeable kind of gentle and uniform excitement to the ear which is distinguished by its greater variety from that produced by a single compound tone, whereas the sensation caused by intermittent dissonances is distressing and exhausting.

However, the rule that the various parts should make consonances with each other, is not without exception. The esthetic reason for this rule is not opposed to an occasional and temporary dissonance among the various parts, provided the motion of the parts is so contrived as to make the directions of the different voices perfectly easy to follow by the ear. Hence, in addition to the general laws of scale and key, to which the direction of every part is subject, there are particular rules for the progression of voices through discords. Again, dissonances cannot be entirely excluded because consonances are physically more agreeable. Though the physically agreeable is an important adjunct and support to esthetic beauty, it is certainly not identical with it. On the contrary, in all arts we frequently employ its opposite, the physically disagreeable, partly to bring the beauty of the first into relief, by contrast, and partly to gain a more powerful means for the expression of passion. Dissonances are used for similar purposes in music. They are partly means of contrast, to give prominence to the impression made by consonances, and partly means of expression, not merely for peculiar and isolated emotional disturbances, but generally to

heighten the impression of musical progress and impetus, be when the ear has been distressed by dissonances it longs to r to the calm current of pure consonances. It is for this last r that dissonances are prominently employed immediately befor conclusion of a piece, where they were regularly introduced by the old masters of medieval polyphony. But to effect object in using them, the motion of the parts must be so ducted that the hearer can feel throughout that the part pressed forward through the dissonance to a following conson and, although this may be delayed or frustrated, the anticip of its approach is the only motive which justifies the exis of the dissonances.

Since any relation of pitch which cannot be expressed in numbers is dissonant, and it is only the number of the conson which is limited, the number of possible dissonances woul infinite were it not that the individual parts composing a di in music must necessarily obey the laws of melodic motion, is, must lie within the scale. Consonances have an indepe right to exist. Our modern scales have been formed upon t But dissonances are allowable only as transitions between sonances. They have no independent right of existence, and parts composing them are consequently obliged to move withi degree of the scales, by the same laws that were establishe favour of the consonances.

On proceeding to a detailed consideration of the sep dissonant intervals, it should be remembered that in theore music the normal position of discords is taken to be that arranges their tones as a series of Thirds. This, for exampl the rule for the chord of the dominant Seventh, which consi the root, its Third, Fifth, and Seventh. The Fifth forms a T with the Third, and the Seventh forms a Third with the F Hence we can consider a Fifth to be composed of two, a Seventh of three Thirds. By inverting Thirds we obtain Si by inverting Fifths we obtain Fourths, and by inverting Seve we obtain Seconds. In this way all the intervals in the scal reproduced.

Using the present modification of Hauptmann's notation, easily seen how different intervals of the same name must c from each other in magnitude. We have only to remember $\bar{c}$ is a comma higher than c, and $\underline{c}$ two commas lower than $\bar{c}$

one comma lower than *c*, and that the comma is about the fifth part of a Semitone.

To obtain a general view of both the magnitude and roughness of the dissonant intervals, I have constructed fig. 61 (p. 520), in which the curve of roughness is copied from fig. 60 A (p. 292). The base line X Y signifies the interval of an Octave, upon which the individual consonant and dissonant intervals are set off from X, according to their magnitude on this scale.[1] On the lower side of the base are marked the twelve equal Semitones of the equally-tempered scale, and on the upper side the consonant and dissonant intervals which occur in justly-intoned scales. The magnitude of the interval is always to be measured on the base line from X to the corresponding vertical note.[2] The vertical lines corresponding to the consonances have been produced to the upper margins of the diagram, and those for the dissonances have been made shorter.[3] The length of the verticals intercepted between the base and the curve of roughness shews the comparative degree of roughness probably possessed by the interval when played in a violin quality of tone.[4]

[1] [That is, assuming X Y to represent log. 2 = ·30103, the distance from X of any line shewing the interval, gives the log. of the ratio of that interval.—*Translator.*]

[2] [But the interval is not reckoned from *C* to another tone, but between the two tones where it usually occurs, except in the equal intonation below.—*Translator.*]

[3] [They have been placed in two rows in recutting the diagram for this translation, merely for the purpose of clearness, to prevent the letters from coming too close to each other, but without attaching any meaning to the difference of row; the other differences described in the text have been retained.—*Translator.*]

[4] [The following tabular expression of the diagram will be often found convenient. The degree of roughness was determined by measuring the lengths of the verticals in the diagram in hundredths of an inch. The names of the notes are given in the notation of the text, and then also as referred to the same base *c* in notation of Appendix XIX. The sign ‖ means 'equally tempered,' and * 'consonance.' The cross lines group the just intervals represented by a single tempered interval.

Intervals	No.	Helmholtz's Notation, as in Diagram	Ellis's Notation of Intervals reckoned from *c*	Ratio	Logarithm	Roughness
Unison	1	*c* — *c*	*c* — *c*	1 : 1	0·00000	0
Minor	2	‖*c* — *c*♯	‖*c* — *c*♯	¹³⁄₂	‖ ·02509	‖ 76
Seconds	3	*b̂* — *c′*	*c* — *d*♭	16 : 15	·02803	70
	4	*d* — *e̱*	*c* —‡*d*	10 : 9	·04576	38
Major	5	‖*c* — *d*	‖*c* — *d*	¹³⁄₄	‖ ·05017	‖ 25
Seconds	6	*c* — *d*	*c* — *d*	9 : 8	·05115	32
	7	*b̂* — *d̄*♭	*c* —†*e*♭♭	256 : 225	·05606	30

Intervals	No.	Helmholtz's Notation, as in Diagram	Ellis's Notation of Intervals reckoned from *c*	Ratio	Logarithm	Roughness
Minor Thirds	8	*ā*b — *b̲*	*c* —‡*d*♯	75 : 64	·06888	24
	9	*d* — *f*	*c* — *e*b	32 : 27	·07379	26
	10	‖*c* — *d*♯	‖*c* — *d*♯	12/8	‖ ·07526	‖ 24
	11	**c* — *ē*b	**c* —†*e*b	*6 : 5	* ·07918	* 20
Major Thirds	12	**c* — *e̲*	**c* — *e*	*5 : 4	* ·09691	* 8
	13	‖*c* — *e̲*	‖*c* — *e*	12/16	‖ ·10034	‖ 18
	14	*b̲* — *ē*b	*c* —‡*f*b	32 : 25	·10721	25
Fourths	15	**c* — *f*	**c* — *f*	*4 : 3	* ·12494	* 2
	16	‖*c* — *f*	‖*c* — *f*	12/32	‖ ·12543	‖ 3
	17	*g̲* — *d′*	*c* —‡*f*	27 : 20	·13033	27
Sharp Fourths or Flat Fifths	18	*b̌*b — *g′*	*c* —‡*f*♯	25 : 18	·14267	32
	19	*f* — *b̲*	*c* — *f*♯	45 : 32	·14806	20
	20	‖*c* — *f*♯	‖*c* — *f*♯	13/64	‖ ·15052	‖ 18
	21	*b̲* — *f′*	*c* — *g*b	64 : 45	·15297	28
	22	*g̲* — *b̌*b	*c* —†*g*b	36 : 25	·15836	35
Fifths	23	*d* — *g̲*	*c* —‡*g*	40 : 27	·17070	44
	24	‖*c* — *g*	‖*c* — *g*	13/128	‖ ·17560	‖ 1
	25	**c* — *g*	**c* — *g*	*3 : 2	* ·17609	* 0
Minor Sixths	26	*ē*b — *b̲*	*c* —‡*g*♯	25 : 16	·19382	39
	27	‖*c* — *g*♯	‖*c* — *g*♯	12/256	‖ ·20069	‖ 22
	28	**c* — *ā*b	**c* —†*a*b	*8 : 5	* ·20412	* 20
Major Sixths	29	**c* — *g̲*	**c* — *a*	*5 : 3	* ·22185	* 3
	30	‖*c* — *a*	‖*c* — *a*	12/512	‖ ·22577	‖ 22
	31	*f* — *d′*	*c* —†*a*	27 : 16	·22724	24
	32	*b̲* — *ā*b	*c* —†*b*bb	128 : 75	·23215	24
Minor Sevenths	33	*ď*b — *b̲*	*c* — *a*♯	225 : 128	·24497	15
	34	*d* — *c′*	*c* — *b*b	16 : 9	·24988	23
	35	‖*c* — *b*b	‖*c* — *b*b	12/1024	‖ ·25086	‖ 24
	36	*g̲* — *ď′*	*c* —†*b*b	9 : 5	·25527	25
Major Sevenths	37	*c* — *b̲*	*c* — *b*	15 : 8	·27300	42
	38	‖*c* — *b*	‖*c* — *b*	12/2048	‖ ·27594	‖ 48
Octave	39	**c* — *c′*	**c* — *c′*	*2 : 1	* ·30103	* 0

The following is a comparative arrangement of these intervals in order of roughness, the consonances being marked *, and the tempered intervals ‖. The number in

The various Thirds, Fifths, and Sevenths of the scale are found by arranging it in Thirds thus:—

a parenthesis is that of the interval when it is contained in the preceding table. The name given to each interval in App. XIX., Table II., is annexed, following by its *roughness*, marked 'ro.'

Roughness

* 0 — (25) just Fifth

‖ 1 — (24) tempered Fifth representing (25) just Fifth, ro. 0, and (23) low Fifth, ro. 44.

* 2 — (15) just Fourth.

‖ 3 — (16) tempered Fourth, representing (15) just Fourth, ro. 2, and (17) high Fourth, ro. 27.

* 3 — (29) just major Sixth.

* 8 — (12) just major Third.

15 — (33) high superfluous Sixth.

‖ 18 — (13) tempered major Third, representing (12) just major Third, ro. 8, and (14) diminished Fourth, ro. 25,—and also the Pythagorean major Third, if required, ro. 19.

‖ 18 — (20) tempered sharp Fourth or flat Fifth, representing (19) false Fourth or Tritone, ro. 20, (21) diminished Fifth, ro. 28, (18) superfluous Fourth, ro. 32, and (22) high diminished Fifth, ro. 35.

19 — Pythagorean major Third $\frac{81}{64}$.

* 20 — (28) just minor Sixth.

* 20 — (11) just minor Third.

20 — (19) false Fourth or Tritone.

‖ 22 — (30) tempered major Sixth, representing (29) just major Sixth, ro. 3, (31) Pythagorean major Sixth, ro. 24, and (32) diminished Seventh, ro. 24.

‖ 22 — (27) tempered minor Sixth, representing (28) just minor Sixth, ro. 20, and (26) superfluous Fifth, ro. 39.

23 — (34) minor Seventh.

‖ 24 — (35) tempered minor Seventh, representing (33) the high superfluous Sixth, ro. 15; (34) the minor Seventh, ro. 23, and (36) the high minor Seventh, ro. 25.

‖ 24 — (10) tempered minor Third, representing (11) just minor Third, ro. 20; (8) augmented Tone, ro. 24, and (9) Pythagorean minor Third, ro. 26.

24 — (31) Pythagorean major Sixth.

24 — Pythagorean minor Sixth, $\frac{128}{81}$.

24 — (32) diminished Seventh.

24 — (8) augmented Tone.

‖ 25 — (5) tempered major Second or whole Tone, representing (7) diminished Third, ro. 30, (6) major Tone, ro. 32, and (4) minor Tone, ro. 38.

25 — (14) diminished Fourth.

25 — (36) high minor Seventh.

26 — (9) Pythagorean minor Third.

27 — (17) high Fourth.

28 — (21) diminished Fifth.

29 — low major Seventh, $\frac{50}{27}$.

[*continued on next page*]

A. TONES OF THE MAJOR SCALE.

$$\underline{b} - d \mid f - \underline{a} - c - \underline{e} - g - \underline{b} - d \mid f - a$$
$$\tfrac{6}{5} \quad \tfrac{32}{27} \quad \tfrac{5}{4} \quad \tfrac{6}{5} \quad \tfrac{5}{4} \quad \tfrac{6}{5} \quad \tfrac{5}{4} \quad \tfrac{6}{5} \quad \tfrac{32}{27} \quad \tfrac{5}{4}$$

B. TONES OF THE MINOR SCALE.

$$\underline{b} - d \mid f - \bar{a}\flat - c - \bar{e}\flat - g - \underline{b} - d \mid f - \bar{a}\flat^{1}$$
$$\tfrac{6}{5} \quad \tfrac{32}{27} \quad \tfrac{5}{4} \quad \tfrac{6}{5} \quad \tfrac{5}{4} \quad \tfrac{5}{4} \quad \tfrac{6}{5} \quad \tfrac{32}{27} \quad \tfrac{6}{5}$$

For the minor scale I have assumed the usual form with the major Seventh, because scales with the minor Seventh yield the same intervals as the major scale.

I. THIRDS AND SIXTHS.

The above schemes shew that in the justly-intoned major and minor scales, three kinds of Thirds occur, and their inversions give three kinds of Sixths. These are:

1) The *justly-intoned major Third* $\frac{5}{4}$, [12, roughness 8],[2] and its inversion the *minor Sixth* $\frac{8}{5}$, [28, roughness 20], both consonant.

2) The *justly-intoned minor Third* $\frac{6}{5}$, [11, roughness 20], and its inversion the *major Sixth* $\frac{5}{3}$, [29, roughness 3], also both consonant.

Roughness
30 — (7) diminished Third.
32 — (6) major Tone.
32 — (18) superfluous Fourth.
35 — (22) high diminished Fifth.
38 — (4) minor Tone.
39 — (26) superfluous Fifth.
42 — (37) major Seventh.
44 — (23) low Fifth.
|| 48 — (38) tempered major Seventh, representing (37) just major Seventh, ro. 42.
56 — high Semitone, $\frac{27}{25}$.
70 — (3) just minor Second or natural Semitone.
|| 76 — (2) tempered Semitone, representing (3) just Semitone, ro. 42.—*Translator.*]

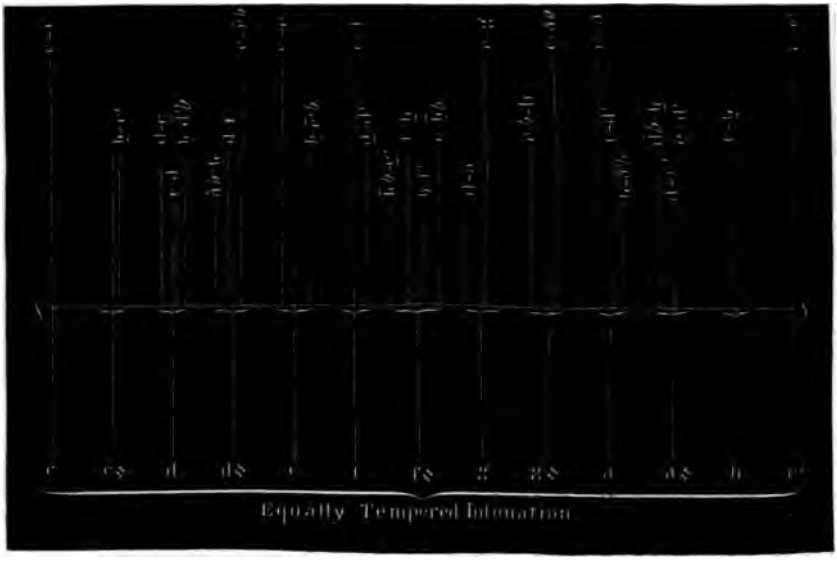
[1] [In the notation of Appendix XIX., using (+) to represent the major Third $\frac{5}{4}$ (−) the minor Third $\frac{6}{5}$, and (|) the Pythagorean minor Third $\frac{32}{27}$, these schemes may be written thus:—

Major $b - d \mid f + a - c + e - g + b - d \mid f + a$
Minor $b - d \mid f - \dagger a\flat + c - \dagger e\flat + g + b - d \mid f - \dagger a\flat$
—*Translator.*]

[2] [For immediate comparison I have, after each interval as it arises, inserted in square brackets, the number of the interval, and its degree of roughness as given in the table on pp. 517–8, note.—*Translator.*]

3) The *Pythagorean minor Third* $\frac{32}{27}$, [9, roughness 26], between the extreme tones of the key, *d* and *f*. If we used *d̤* in place of *d*, this interval would occur between *b̤* and *d̤*. On comparing this dissonant minor Third *d̤* — *f* with the consonant minor Third *d* — *f*, we find first that the former is a comma closer than the latter, since *d* is a comma sharper than *d̤*. The Pythagorean minor Third is somewhat less harmonious than the just minor Third, but the difference between them is not so great as that

FIG. 61.



between the two corresponding major Thirds.[1] The difference of the two cases consists, first, in the major Third being a much more perfect consonance than the minor Third, and consequently much more liable to injury from defects of intonation; and secondly in the nature of the two combinational tones. The just minor Third *d̤‴* — *f‴* has *b♭* for its combinational tone, which completes it into the just major triad of *b♭*. The Pythagorean

[1] [The roughness of the just major Third, *c* + *e* is only 8, while that of the Pythagorean $\frac{81}{64}$ (which is not given because it does not occur in the scale) is necessarily between that of the tempered major Third, 18, and that of the dissonant minor Fourth *c* — ♯*f*♭, which is 25, but scarcely more than 19 or 20, as will be seen by the curve in fig. 61.—*Translator.*]

minor Third $d''' — f'''$ has $\underline{a}$ for its combinational tone, which completes it into the chord $d — f — \underline{a}$, and this is not a perfectly correct minor chord. But as the incorrect Fifth $\underline{a}$ lies among the deep combinational tones and is very weak, the difference is scarcely perceptible. Moreover, it is practically almost impossible to tune the interval so precisely as to insure the combinational tone $\underline{a}$ in place of a. But for the Pythagorean major Third $c'' — e''$ the combinational tone is $c\sharp\sharp$, which is, of course, much more annoying than the rather imperfect Fifth $\underline{a}$ when added to the chord $d — f$.

The Pythagorean major Third does not occur in scales tuned according to the conditions of harmonic music. If we used the minor Seventh $b\flat$ in place of $\bar{b}\flat$ for the minor scale, $b\flat — d$ would be a Pythagorean major Third.[1]

The inversion of the Third $d — f$ is the *Pythagorean major Sixth* $f — d'$, $\frac{27}{16}$, [31, roughness 24], which is a comma wider than the just major Sixth, and is greatly inferior to it in harmoniousness, as is clearly seen in fig. 61 (p. 520).

II. FIFTHS AND FOURTHS.

The Fifth is simply composed of two Thirds, and the different varieties of Fifths depend upon the nature of those Thirds.

4) The *justly-intoned Fifth* $\frac{3}{2}$, [25, roughness 0], consists of a justly-intoned major and a justly-intoned minor Third, or $\frac{3}{2} = \frac{5}{4} \times \frac{6}{5}$. Its inversion is the *justly-intoned Fourth* $\frac{4}{3}$, [15, roughness 2]. Both are consonant. Examples in the major scale, $f — c'$, $\underline{a} — \underline{e}'$, $c — g$, $\underline{e} — \underline{b}$, $g — d$.

5) The *imperfect Fifth* $d — \underline{a}$, $\frac{40}{27}$, [23, roughness 44], a comma less than the just Fifth, consists of a Pythagorean minor and a just major Third, $\frac{40}{27} = \frac{32}{27} \times \frac{5}{4}$. It sounds like a badly-tuned Fifth, and makes clearly sensible beats. In the Octave $c' — c''$, the number of these beats in a second is 11. Its inversion, the *imperfect Fourth*, $\underline{a} — d'$, $\frac{27}{20}$, [17, roughness 27], is also decidedly dissonant. The Fourth $\underline{A} — d$ makes as many beats in a second as the Fifth $d — \underline{a}$, the d being the same in each.

6) The *false Fifth*, $\underline{b} — f'$, $\frac{64}{45}$, [21, roughness 28], consists of a just and Pythagorean minor Third, $\frac{64}{45} = \frac{6}{5} \times \frac{32}{27}$, and is, hence, as the composition shews, about half a Tone closer than the just Fifth.

[1] [That is (in the notation of App. XIX.) if we made the minor chord $g \mid \bar{b}\flat + d$ in place of $g - \dagger\bar{b}\flat + d$. I have found this quite unendurable.—*Translator.*]

It is a tolerably rough dissonance, nearly equal in roughness to a major Second [6, roughness 32]. Its inversion is the *false Fourth* or *Tritone*, f—$\underline{b}$, $\frac{45}{32}$, [19, roughness, 20], consisting of three whole Tones, major f—g, minor g — $\underline{a}$, and major $\underline{a}$ — $\underline{b}$, $\frac{9}{8} \times \frac{10}{9} \times \frac{9}{8} = \frac{45}{32}$; it has very nearly the same degree of roughness as the last, and is about a comma closer. For the false Fifth $\underline{b}$ — f is nearly the same as $c\flat$ — f, and if we diminish this interval by a comma we obtain $c\flat$ — f, which is a false Fourth. Strictly speaking, as $c\flat$ is not precisely the same as $\underline{b}$,[1] the difference between the intervals is not precisely a comma, $\frac{81}{80}$, but about $\frac{89}{88}$, [or $\frac{10}{11}$ of a comma]. On keyed instruments they coincide.

7) The *superfluous* or *extreme sharp Fifth* of the minor scale, $\bar{e}\flat$—$\underline{b}$, $\frac{25}{16}$, [26, roughness 39], consists of two major Thirds, $\bar{e}\flat$—g, and g — $\underline{b}$, $\frac{25}{16} = \frac{5}{4} \times \frac{5}{4}$. It is seen to be nearly two commas closer that the minor Sixth, by putting for $\underline{b}$ the nearly identical $c\flat$, so that $\bar{e}\flat$ — $\underline{b}$ is nearly the same as $\bar{e}\flat$ — $c\flat$, whereas the consonant minor Sixth is $\underline{e}\flat$ — $c\flat$, where $\underline{e}\flat$ is two commas flatter than $\bar{e}\flat$. The superfluous Fifth [26, roughness 39] is markedly rougher than the minor Sixth [28, roughness 20], with which it coincides upon keyed instruments. Its inversion, the *diminished Fourth* $\underline{b}$ —$\bar{e}'\flat$, $\frac{32}{25}$, [14, roughness 25], is about two commas higher than the just major Third, [12, roughness 8], and considerably rougher, although the two intervals coincide on keyed instruments, [13, common roughness 18].

Two just or two Pythagorean minor Thirds cannot occur consecutively in the natural series of Thirds of the just major and minor scales. In the modes of the minor Seventh and of the Fourth, we may find the intervals $\underline{a}$ — $\bar{e}\flat$ and $\underline{e}$ — $\bar{b}\flat$ = $\frac{36}{25}$, [22, roughness 35], composed of two minor Thirds, $\frac{36}{25} = \frac{6}{5} \times \frac{6}{5}$; these are a comma wider than the usual false Fifths $\underline{b}$ — f' (or $\underline{a}$ — $e'\flat$ in the key of $b\flat$ major, and $\underline{e}$ — $b\flat$ in the key of f major), and are decidedly rougher than these, [21, roughness 28].

III. Sevenths and Seconds.

Any three successive Thirds give a Seventh. Beginning with the smallest we obtain the following different magnitudes:

8) The *diminished Seventh* of the minor Scale $\underline{b}$ — $\bar{a}'\flat$ = ($\underline{b}$ — d') + (d' — f') + (f' — $\bar{a}'\flat$), or two just and one Pythagorean

[1] [The tone ♯$c\flat$ is one skhisma flatter than b. See App. XIX., Table V.—*Translator.*]

minor Third. Its numerical ratio is $\frac{128}{75} = \frac{6}{5} \times \frac{32}{27} \times \frac{4}{3}$, [32, roughness 24], about two commas greater than the major Sixth [29, roughness 3], as is seen by putting $\underline{b} - \bar{a}'\flat = c\flat - \bar{a}'\flat$. The interval $c\flat - \underline{a}'\flat$, which is two commas flatter than the last, would be a just major Sixth. Its dissonance is harsh and rough, the same as that of the Pythagorean major Sixth $c - a$, [31], which is about a comma less. But its inversion, the *superfluous Second* $\bar{a}\flat - \underline{b}$ [8, having the same roughness 24], is not much rougher than the just minor Third [11, roughness 20; the tempered minor Third, 10, has exactly the same roughness 24]. Its numerical ratio $\frac{75}{64}$ is very nearly $\frac{7}{6}$ (since $\frac{75}{64} = \frac{7}{6} \times \frac{225}{224}$). If this Second is extended to a Ninth, nearly $\frac{7}{3}$, it becomes tolerably harmonious, as much so as the minor Tenth $\frac{12}{5}$, which, however, is a very imperfect consonance, [see fig. 60 B, p. 292].

9) The *closer minor Seventh* $g - f'$, $\underline{b} - \underline{a}'$, or $d - c$, $\frac{16}{9}$, [34, roughness 23], consists of a just major, a just minor, and a Pythagorean minor Third:

$$g - f' = (g - \underline{b}) + (\underline{b} - d') + (d' - f'), \text{ or } \tfrac{16}{9} = \tfrac{5}{4} \times \tfrac{6}{5} \times \tfrac{32}{27}.$$

It is a comparatively gentle dissonance, gentler than the diminished Seventh [32, roughness 24], and this is of importance for the effect of the chord of the dominant Seventh, in which the Seventh has this form. This closer minor Seventh is the interval of a Seventh in the scale nearest to the natural Seventh or seventh partial tone, $\frac{7}{4}$ ($= \frac{63}{64} \times \frac{16}{9}$), although not so close as the superfluous Sixth [33, roughness 15]. It has been already shewn that the natural Seventh belongs to harmonious combinations (p. 296). The inversion of this Seventh is the *major Second*, $c - d$, $\underline{a} - \underline{b}$, $f - g$, $\frac{9}{8}$, [6, roughness 32], a powerful dissonance.

10) The *wider minor Seventh*, $\underline{e} - d'$, $\underline{a} - g'$, $\frac{9}{5}$, [36, roughness 25], a comma greater than the last, is distinctly harsher than the last [34, roughness 23], because it is nearer the Octave; its roughness [25], is nearly the same as that of the diminished Seventh [24]. It consists of a just major and two just minor Thirds:

$$\underline{e} - d' = (\underline{e} - g) + (g - \underline{b}) + (\underline{b} - d'), \text{ or } \tfrac{9}{5} = \tfrac{5}{4} \times \tfrac{6}{5} \times \tfrac{6}{5}.$$

The last-mentioned *closer minor Seventh* has its root on the dominant side of the scale, and its Seventh on the sub-dominant side, because it contains the Pythagorean minor Third $d - f$. The wider minor Seventh, on the other hand, has its Seventh on the dominant side. Its inversion, the *minor Tone*, $\frac{10}{9}$, $d - \underline{e}$, $g - \underline{a}$,

is somewhat harsher [4, roughness 38] than the major Tone [6, roughness 32].

11) The *major Seventh* $f - \underline{e}'$, $c - \underline{b}$, $\frac{15}{8}$, [37, roughness 42], consists of two just major and one just minor Third:

$$c - \underline{b} = (c - \underline{e}) + (\underline{e} - g) + (g - \underline{b}).$$

It is a harsh dissonance, about the same as the minor Tone [4, roughness 38]. Its inversion, the *minor Second* or *Semitone* $\underline{b} - c'$, $\underline{e} - f$, $\frac{16}{15}$, [3, roughness 70], is the harshest dissonance in the scale.[1]

In the modes of the Fourth or minor Seventh, we find a somewhat closer *major Seventh*, $\bar{b}\flat - \underline{a}'$, which is a comma closer than the usual major Seventh, and hence somewhat softer in effect.[2]

Finally we have to mention an interval peculiar to the Doric mode of the minor Sixth, namely:

12) The *superfluous* or *extreme sharp Sixth* $\bar{d}\flat - \underline{b}$, which arises in this mode from the connecting its peculiar minor Second $\bar{d}\flat$ with the leading note $\underline{b}$.

The numerical ratio is $\frac{225}{128}$, [33, roughness 15], so that it is about a comma less than the closer minor Seventh of the chord of the dominant Seventh, as is seen by putting $\bar{d}\flat - \underline{b} = \bar{d}\flat - c'\flat$; the interval $d\flat - c'\flat$ would be a still closer minor Seventh, and $\bar{d}\flat$ is a comma higher than $d\flat$. The superfluous Sixth may be conceived as composed of two just major Thirds and one just major tone:

$$(\bar{d}\flat - f) + (f - g) + (g - \underline{b}) = \bar{d}\flat - \underline{b}, \text{ or } \tfrac{225}{128} = \tfrac{5}{4} \times \tfrac{9}{8} \times \tfrac{5}{4}.$$

Its harmoniousness is equal to that of the minor Sixth, because it is almost exactly the natural Seventh $\frac{7}{4}$,[3] since $(\frac{225}{128} = \frac{7}{4} \times \frac{225}{224})$. Taken alone it cannot be regarded as a dissonance, but it makes no other consonant combinations, and hence is unfit for use in consonant chords. When it is inverted into the *diminished Third* $\frac{256}{225}$, or nearly $\frac{8}{7}$, it is, as already observed, considerably damaged [*i*, the roughness rises to 30], but it is improved by taking the upper

[1] [That is in the just major scale; the Semitone of the tempered scale, 2, reaches 76 degrees of roughness.—*Translator.*]

[2] [Its numerical ratio is $\frac{50}{27} = \frac{15}{8} \times \frac{80}{81}$, and its log. ·26761, so that it is the interval $c - \ddagger b$, which by fig. 61 (p. 520) should have a roughness of about 29, in place of 42, the roughness of $c - b$.—*Translator.*]

[3] [The diagram, fig. 61, gives the roughness of the superfluous Sixth as 15, and that of the minor Sixth as 20; see p. 518. This would make the former more harmonious than the latter. As I hear these intervals on my justly-intoned concertina, my ear reverses this decision, and makes the minor Sixth $a - f'$ distinctly better than the superfluous Sixth $f - \ddagger d'\sharp$.—*Translator.*]

tone $\underline{b}$ an Octave higher, in which case it is nearly $\frac{1}{4}$. Its near agreement with the natural Seventh and its comparative harmoniousness, seem to have preserved this remarkable interval in certain cadences, although it is quite foreign to our present tonal system. It is characteristic that musicians forbid its inversion into the diminished Third (which lessens its harmoniousness), but allow its extension into the corresponding Thirteenth (which improves its harmoniousness). On keyed instruments this interval coincides with the minor Seventh [35, roughness 24].

Generally, a glance at fig. 61 (p. 520) will shew to what an extraordinary extent different intervals are fused on keyed instruments. On the lower side of the base line X Y are marked the places of the tones of the equally tempered scale, and the small brackets below the base line shew those different tonal degrees which are usually expressed by the corresponding tone of the tempered scale. The interval $\underline{b} - \bar{a}\flat$ is identified on the pianoforte with the major Sixth $c\flat - a\flat$, while the interval $\bar{d}\flat - \underline{b}$ is made a (tempered) Semitone wider, and yet the last is scarcely more different from the first, than the first from a major Sixth.[1] The figure 61 shews also very clearly what an immense difference of harmoniousness ought to exist between the first and either of the two last of these intervals $c - \underline{a}$, $f - d'$, and $\underline{b} - \bar{a}'\flat$, [29, 31, 32, respective roughnesses 3, 24, 24], which are all expressed by the sufficiently harsh sound of the tempered interval $c - a$ [30, roughness 22]. The justly-intoned harmonium with two rows of keys allows all these intervals to be given accurately.[2]

DISSONANT TRIADS.

Dissonant triads with a single dissonant interval are obtained by taking two tones which are consonant to the root, but dissonant to each other. Thus:

1) *Fifth and Fourth:* $c - f - g$.

2) *Third and Fourth:* $c - \underline{e} - f$ or $c - \bar{e}\flat - f$.

[1] [In the notation of App. XIX.,

$\log(d\flat \text{ to } b) = \log(c \text{ to } a\sharp) = \cdot 24497$ difference $\cdot 01282$

$\log(b \text{ to } \dagger a\flat) = \log(c \text{ to } \dagger\dot{b}\flat\flat) = \cdot 23215$ difference $\cdot 01030$

$\log(c \text{ to } a) = \cdot 22185$

The difference of these differences is ·00252, corresponding to half a comma, but as $\dagger\dot{b}\flat\flat$ is identified with a on the piano, this difference appears as a Semitone which is nearly ten times as large.—*Translator.*]

[2] [All the just intervals given by Helmholtz can be studied on the experimental harmonium (Appendix XIX., Sect. G, No. 2), which is generally the most convenient instrument for this purpose.—*Translator.*]

3) *Fifth and Sixth*: $c - g - \underline{a}$ or $c - g - \bar{a}\flat$.

4) *Dissimilar Thirds and Sixths*: $c - \bar{e}\flat - \underline{a}$ or $c - \underline{e} - \bar{a}\flat$.[1]

In all this c is consonant with each of the other two tones. The first chord alone plays a great part in the older polyphonic music as a *chord of suspension*. The others we shall meet with again in the chords of the Seventh.

For modern music *triads with two dissonances*, formed by including the extremes of the key, are more important.

In the series of chords in a major key, major and minor Thirds follow each other alternately, and any two adjacent Thirds produce a consonant triad. But the interval between the extreme tones d and f is a Pythagorean minor Third, and when these are connected as a chord with one of either of the two adjacent tones to make a new triad, it will be dissonant.

$$\text{MAJOR}: \underset{\frac{5}{4}}{c - \underline{e}} \underset{\frac{6}{5}}{- g} \underset{\frac{5}{4}}{- \underline{b}} \underset{\frac{6}{5}}{- d} \underset{\frac{32}{27}}{\mid f} \underset{\frac{5}{4}}{- \underline{a}} \underset{\frac{6}{5}}{- c} \underset{\frac{5}{4}}{- \underline{e}} \underset{\frac{6}{5}}{- g}$$

$$\text{MINOR}: \underset{\frac{6}{5}}{c - \bar{e}\flat} \underset{\frac{5}{4}}{- g} \underset{\frac{5}{4}}{- \underline{b}} \underset{\frac{6}{5}}{- d} \underset{\frac{32}{27}}{\mid f} \underset{\frac{6}{5}}{- \bar{a}\flat} \underset{\frac{5}{4}}{- c} \underset{\frac{6}{5}}{- \bar{e}\flat} \underset{\frac{5}{4}}{- g}$$

The major system gives two such triads[2]:

$$\underset{\frac{6}{5}}{\underline{b} -} \underset{\frac{32}{27}}{d - f} \quad \text{and} \quad \underset{\frac{32}{27}}{d - f} \underset{\frac{6}{5}}{- \underline{a}}$$

The minor scale also gives two[2]:

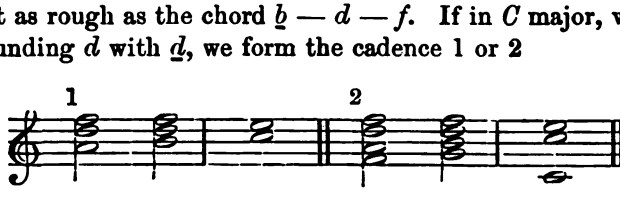
$$\underset{\frac{6}{5}}{\underline{b} -} \underset{\frac{32}{27}}{d - f} \quad \text{and} \quad \underset{\frac{32}{27}}{d - f} \underset{\frac{6}{5}}{- \bar{a}\flat}$$

In the two triads $\underline{b} - d - f$ and $d - f - \bar{a}\flat$, which combine a Pythagorean with a just minor Third, there are also second dissonances, namely the false Fifths $\underline{b} - f$ and $d - \bar{a}\flat$, which make the chord more strongly dissonant than the Pythagorean Third $\frac{32}{27}$ alone could make them. They are hence called *diminished Triads*. The chord $d - f - \underline{a}$, which in the usual musical notation is not distinguished from the minor triad

[1] [These triads I propose to term *con-dissonant*. See Appendix XIX., Section D, *Heptad*. Rearranged in the notation there used, so as to place c' in the centre, those in the text are: 1) $f \times c' \times g'$, 2) $f \times c' + e'$, $f \times c' - \dagger e'\flat$, 3) $a - c' \times g'$, $\dagger a\flat + c' \times g'$, 4) $a - c' - \dagger e'\flat$, $\dagger a\flat + c' + e'$.—*Translator.*]

[2] [In the notation of Appendix XIX., the major dissonant triads are $b - d \mid f$, and $d \mid f + a$; the minor $b - d \mid f$, and $d \mid f - \dagger a\flat$.—*Translator.*]

$\underline{d}$ — f — $\underline{a}$, and may hence be called the *false minor triad*, is, as Hauptmann has correctly shewn, dissonant, and on justly-intoned instruments it is very decidedly dissonant. It sounds almost as rough as the chord $\underline{b}$ — d — f. If in C major, without confounding d with $\underline{d}$, we form the cadence 1 or 2



the chords $\underline{a}'$ — d'' — f'' and f' — $\underline{a}'$ — d'' — f'' are quite as dissonant in their effects as the following $\underline{b}'$ — d'' — f'' and g' — $\underline{b}'$ — d'' — f''. But on account of the incorrect intonation of our musical instruments we cannot produce the same effect without combining an inverted chord of the Seventh with the subdominant in the cadence, as f — $\underline{a}$ — c' — d'. Hauptmann doubts whether in practice the false minor chord of the key of C major can be distinguished from the minor chord of D. I find that this is most distinctly and undoubtedly effected on my justly-intoned harmonium, but allow that we cannot expect the correct intonation from singers. They will involuntarily pass into the minor chord, unless the progression of the parts which execute D, strongly emphasise its connection with the dominant G.[1]

[1] [The chord on the Second of the major scale is in fact the *crux* of the translation of tempered into just intonation, as I have already intimated (p. 504, end of note). It is very easy to play ex. 1 and 2, and a third example as

a'	$\ddagger d''$	f''		f'	a'	$\ddagger d''$	f''		f'	a'	$\ddagger d''$
b'	d''	f''	and	g'	b'	d''	f''	and	g'	b'	d''
c''	c''	e''		c'	c''	c''	e''		f'	b'	d''
									e'	c''	c''

and the effect is not bad. In the first the $\ddagger d''$ might be held on to the second chord, as b' $\ddagger d''$ f'', without materially increasing the harshness of the dissonance, but in the second this would give g' b' $\ddagger d''$ f'', where the low Fifth is very harsh. In the second case, then, there is least harshness in playing d'' in both chords. And in both cases there is most smoothness in playing them as just written. The effect is one on which I have repeatedly experimented, but I find that the small interval $\ddagger d''$ d'' in the highest or lowest part, produces a strange effect, which in singing, and perhaps on the violin, seems to be overcome by a glide, if the other voices are strong enough to pull this voice out of its course, even though the words and parts are written so as to imply that this note is sustained. When the d'' is in the principal part in the melody, as in the third example, I find it best on the whole not to play as written, $\ddagger d''$ d'', but to sustain d''. In some cases an attempt to avoid the dissonance, which is really harsh, would lead to such melodic phrases as d $\ddagger d$ d, which would be simply impossible

These chords, and among them most decisively and distinctly the chord $\underline{b}$ — d — f, have for musical composition the especially important advantage of combining those limiting tones of the key, which separate it from the nearest related keys, and are consequently extremely well suited for marking the key in which the harmony is moving at any given time. If the harmony passed into G major or G minor, f would have to be replaced by $\underline{f}\sharp$. If it passed into F major, d would become $\underline{d}$, and if into F minor d would become $\bar{d}\flat$, and $\underline{b}$ would in the same chords become $b\flat$. Thus—

in G major :	$\underline{b}$	— d	— $\underline{f}\sharp$	d	— $\underline{f}\sharp$	— a
in C major :	$\underline{b}$	— d	— f	d	— f	— $\underline{a}$
in F major :	$b\flat$	— $\underline{d}$	— f	$\underline{d}$	— f	— $\underline{a}$
in G minor :	$\bar{b}\flat$	— d	— $\underline{f}\sharp$	d	— $\underline{f}\sharp$	— a
in C minor :	$\underline{b}$	— d	— f	d	— f	— $\bar{a}\flat$
in F minor :	$b\flat$	— $\bar{d}\flat$	— f	$\bar{d}\flat$	— f	— $\bar{a}\flat$ [1]

for an unaccompanied voice. If in the third example d'' were held throughout, and the accompanying voices sang the minor chord, we should get the succession †f' †a' d'', g' b' d'', f' b' d'', e' c'' c'', which amounts to a modulation into the minor of the dominant, instead of into the subdominant. Whether such is possible, depends on the preceding chords. Played on my just concertina as d' f'‡ b', e' ‡g'‡ b', ‡d' ‡g'‡ b', ‡c'‡ a' a', it produces the best effect of all for this isolated phrase.—*Translator.*]

[1] [It is only in C major that the chords here adduced are chords necessarily between the limiting tones; in the other keys the chords are those nearest related to the first mentioned, the object being to shew the differences. The following table of the six keys in the author's notation will shew this, and the same in the notation of Appendix XIX. will shew how to translate his symbols into mine. In both cases I use my notation of + — | for major, minor, and Pythagorean minor Third as in p. 519, note.

In the Author's Notation.

G major:	a	|	c	+ $\underline{e}$	— g	+ $\underline{b}$	— d	+ $\underline{f}$‡	— a	|	c
C major:	d	|	f	+ $\underline{a}$	— c	+ $\underline{e}$	— g	+ $\underline{b}$	— d	|	f
F major:	g	|	$b\flat$	+ $\underline{d}$	— f	+ $\underline{a}$	— c	+ $\underline{e}$	— g	|	$b\flat$
G minor:	a	|	c	— $\bar{e}\flat$	+ g	— $\bar{b}\flat$	+ d	+ $\underline{f}$‡	— a	|	c
C minor:	d	|	f	— $\bar{a}\flat$	+ c	— $\bar{e}\flat$	+ g	+ $\underline{b}$	— d	|	f
F minor:	g	|	$b\flat$	— $\bar{d}\flat$	+ f	— $\bar{a}\flat$	+ c	+ $\underline{e}$	— g	|	$b\flat$

In the Translator's Notation.

G major:	†a	|	c	+ e	— g	+ b	— d	+ f‡	— †a	|	c
C major:	d	|	f	+ a	— c	+ e	— g	+ b	— d	|	f
F major:	g	|	$b\flat$	+ ‡d	— f	+ a	— c	+ e	— g	|	$b\flat$
G minor:	a	|	c	— †$e\flat$	+ g	— †$b\flat$	+ d	+ f‡	— †a	|	b
C minor:	d	|	f	— †$a\flat$	+ c	— †$e\flat$	+ g	+ b	— d	|	f
F minor:	g	|	$b\flat$	— $d\flat$	+ f	— †$a\flat$	+ c	+ e	— g	|	$b\flat$

Translator.]

This shews that the chords in the nearest related keys are all distinctly different, with the exception of $d - f - \underline{a}$ and $\underline{d} - f - \underline{a}$, which it is practically difficult to distinguish.[1] The rest are much more clearly distinguished from the chords in the nearest adjacent keys. Nevertheless

$$\underbrace{\underline{b} - d}_{\frac{6}{5}}\underbrace{- f}_{\frac{32}{27}} \quad \text{and} \quad \underbrace{d - f}_{\frac{32}{27}}\underbrace{- \bar{a}\flat}_{\frac{6}{5}}$$

are easily confused with

$$\underbrace{\underline{b} - \underline{d}}_{\frac{32}{27}}\underbrace{- f}_{\frac{6}{5}} \quad \text{and} \quad \underbrace{d - \bar{f}}_{\frac{6}{5}}\underbrace{- \bar{a}\flat}_{\frac{32}{27}}$$

of which the former belongs to $\underline{A}$ minor, and the latter to $\bar{E}\flat$ major or minor, where $\underline{A}$ minor is the minor key nearest related to C major, and $\bar{E}\flat$ major is the major key nearest related to C minor.[2]

Finally when we remember that the Pythagorean minor Third $\frac{32}{27}$ is nearer the superfluous second $\frac{75}{64}$ than to the normal minor Third ($\frac{32}{27} = \frac{6}{5} \times \frac{80}{81}$ and $\frac{32}{27} = \frac{75}{64} \times \frac{2048}{2025}$ or nearly $= \frac{75}{64} \times \frac{89}{88}$), it requires comparatively slight changes in intonation to convert the chord $\underline{b} - d - f$ into

$$\underbrace{\underline{b} - d}_{\frac{6}{5}}\underbrace{- \underline{e}\sharp}_{\frac{75}{64}} \quad \text{and} \quad \underbrace{\bar{c}\flat - \underline{d}}_{\frac{75}{64}}\underbrace{- f}_{\frac{6}{5}}$$

which belong to $\underline{F}\sharp$ minor and $E\flat$ minor.[3] Hence the dimi-

[1] [Not in just intonation, where, as was mentioned on p. 527, note, $d \mid f + a$ and $\ddagger d - f + a$ are widely different, but 'practically difficult to distinguish' in musical compositions founded on tempered intonation, the principal object of which was to obliterate the distinction between d and $\ddagger d$.—*Translator.*]

[2] [In the notation of Appendix XIX., these four chords are:

$$b - d \mid f, \quad d \mid f - \dagger a\flat, \text{ and } b \mid \ddagger d - f, \quad d - \dagger f \mid \dagger a\flat$$

respectively. The first is between the limiting tones of C major, the second between those of C minor, the third between those of A minor, and the fourth between those of $\dagger E\flat$ major, the major key nearest related to C minor. There are several misprints in the German text, which the numerical ratios have enabled me to correct in the translation.—*Translator.*]

[3] [In the notation of Appendix XIX., Sections A and B, these chords are $b - d \therefore e\sharp$, and $c\flat \therefore \ddagger d - f$. The following schemes will shew how these chords arise:—

In the Author's Notation.

$$\begin{array}{lllllllll} \underline{F}\sharp \text{ minor}: & \underline{g}\sharp & \mid \underline{b} & - d & - \underline{f}\sharp & - a & - \underline{c}\sharp & - \underline{e}\sharp & - \underline{g}\sharp \mid \underline{b} \\ E\flat \text{ minor}: & f & \mid a\flat & - \bar{c}\flat & - e\flat & - \bar{g}\flat & - b\flat & - \underline{d} & - f \mid a\flat \end{array}$$

In the Translator's Notation.

$$\begin{array}{lllllllll} F\sharp \text{ minor}: & g\sharp & \mid b & - d & + f\sharp & - \dagger a & + c\sharp & + e\sharp & - g\sharp \mid b \\ E\flat \text{ minor}: & f & \mid a\flat & - c\flat & + e\flat & - g\flat & + b\flat & + \ddagger d & - f \mid a\flat \end{array}$$

—*Translator.*]

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[illegible] by slight changes of intonation, never [illegible] can be referred to the keys of [illegible] minor, F♯ minor, and E♭ minor.

Hence although the use of the diminished triad $\underline{b}-d-f$ excludes the keys most nearly related to C, it allows of a confusion with more distant keys, and hence also the characterisation of the key by these triads will not be complete without a fourth note, converting the triad into a tetrad. This leads us to the chords of the Seventh proper.

CHORDS OF THE SEVENTH.

A. *Formed of two Consonant Triads.*

Consonant tetrads, or chords in four parts, as shewn on p. 336, cannot be constructed without using the Octave of one of the tones, but dissonant tetrads are easily constructed. The least dissonant of such chords are those in which only a single interval is dissonant, and the rest are consonant. These are most readily formed by uniting two consonant triads which have two common tones. In this case the tones which are not common to the two chords are dissonant to each other, and the rest are consonant, so that the dissonance is comparatively unobserved amid the mass of consonances. Thus the triads

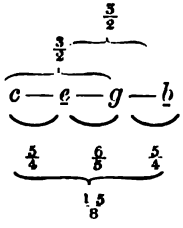
$$c-\underline{e}-g$$
$$\underline{e}-g-\underline{b}$$

on being fused give the tetrad

$$c-\underline{e}-g-\underline{b}$$

in which the major Seventh $c-\underline{b}$ is a dissonant interval and the other intervals are consonant, as the annexed scheme shews:—

$$\overbrace{\overbrace{c-\underline{e}}^{\frac{3}{2}}-g}^{\frac{3}{2}}-\underline{b}$$
$$\underbrace{\underbrace{c-\underline{e}}_{\frac{5}{4}}-\underbrace{g}_{\frac{6}{5}}-\underbrace{\underline{b}}_{\frac{5}{4}}}_{\frac{15}{8}}$$



This position of the chord of the Seventh, deduced from the closest positions of the triads, is regarded as fundamental or primary. The intervals between the individual tones appear as

Thirds, and when chords of the Seventh are formed from the consonant triads of the scale, these Thirds will be alternately major and minor, because consonant triads always unite a major with a minor Third. Hauptmann calls these chords of the Seventh which occur spontaneously in the natural series of Thirds of a key

$$f — \underline{a} — c — \underline{e} — g — \underline{b} — d$$

the *chords of the direct system* or simply *direct chords.*[1] There are two kinds of these chords. In one a minor Third lies between two major Thirds, as in the tetrad $c — \underline{e} — g — \underline{b}$ already cited, and similarly in $f — \underline{a} — c' — \underline{e}'$ in C major, and $\bar{a}\flat — c — \bar{e}\flat — g$ in C minor. In the other a major Third lies between two minor Thirds, as in

$$\overbrace{\overbrace{\underbrace{\underline{a} — c}_{\frac{6}{5}} — \underbrace{c — \underline{e}'}_{\frac{5}{4}} — \underbrace{\underline{e}' — g'}_{\frac{6}{5}}}^{\frac{3}{2}}}^{\frac{3}{2}}$$

$$\frac{9}{5}$$

and similarly in $\underline{e} — g — b — \underline{d}$ in C major and $f — \bar{a}\flat — c — \bar{e}\flat$ in C minor. In this second species the dissonance is a minor Seventh, $\frac{9}{5}$, [roughness 25, see p. 517,] which is much milder than the major Seventh, $\frac{15}{8}$ [roughness 42].

B. *Chords of the Seventh formed of Dissonant Triads.*

Other chords of the Seventh may be formed from the dissonant triads of the key, each united with one consonant triad, and also from the two dissonant triads themselves. By thus uniting the limiting tones of the series of chords in the key,

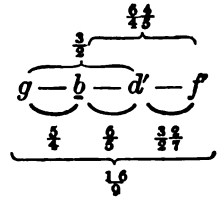
major: $c — \underline{e} — g — \underline{b} — d \mid f — \underline{a} — c$
and minor: $c — \bar{e}\flat — g — b — d \mid f — \bar{a}\flat — c$

we obtain the following *Chords of the Seventh in the reverted system*, or *indirect tetrads*:

[1] [Their construction is plainly shewn by the use of + — in the notation of Appendix XIX.—

$f + a — c + e — g + b — d$.—*Translator.*]

1)



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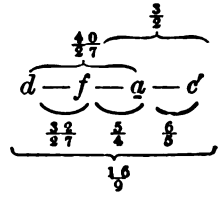

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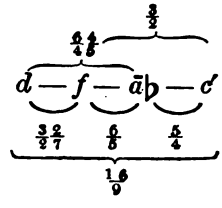

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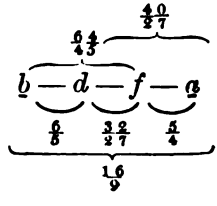

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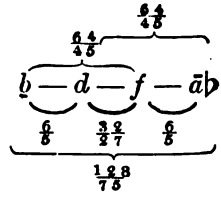


The Sevenths of these chords, all come pretty near to the natural Seventh $\frac{7}{4}$, and are all smaller than the Sevenths in the chords of the Seventh formed from two consonant triads.[1] The

[1] [In the notation of Appendix XIX., these chords may be written: $g + b — d' \mid f'$, $d \mid f + a — c'$, $d \mid f — \dagger ab + c'$, $b — d \mid f + a$, $b — d \mid f — \dagger ab$, respectively, from which forms the intervals may be immediately calculated, on putting $\frac{5}{4}$, $\frac{6}{5}$, $\frac{32}{27}$ for + — | respectively.—*Translator.*]

principal dissonances in these chords are the false and imperfect Fifths $\underline{b} - f$, $d - \underline{a}$, and $d - \bar{a}\flat$, that is, the intervals $\frac{64}{45}$ and $\frac{40}{27}$ [see p. 517]. Hence the first three of these chords of the Seventh, $g - \underline{b} - d - f$, $d - f - \underline{a} - c$, and $d - f - \bar{a}\flat - c$, each of which contains only one of these imperfect Fifths, are less harshly dissonant than the two last, each of which contains both of them. Such of these chords as contain a major triad, namely—

$$\underbrace{g - \underline{b} - d}\,- f \quad \text{and} \quad d - \underbrace{f - \underline{a} - c}$$

are about equal in dissonance to the milder chords of the Seventh in the direct system, which contains the larger and rougher kind of Sevenths, and, at the same time, only perfect Fifths:

$$\overbrace{\underline{a} - c - \underline{e}} - g \quad \text{and} \quad \overbrace{\underline{e} - g - \underline{b}} - d$$

The chord of the dominant Seventh $g - \underline{b} - d' - f'$ can be even rendered much milder by lowering its f' to $\underline{f}'$. The interval $g - \underline{f}'$ corresponds to the ratio $\frac{1280}{729}$, which is very nearly equal to $\frac{7}{4}$, being $= \frac{7}{4} \times \frac{5120}{5103}$ or approximately $\frac{7}{4} \times \frac{301}{300}$. Hence the chord $g - \underline{b} - d - \underline{f}$ is on the verge of consonance.[1]

But the chord of the Seventh which contains a false Fifth and a minor triad, namely—

$$d - \overbrace{f - \bar{a}\flat - c}$$

is about as rough as the tetrads of the direct system containing a major Seventh, namely—

$$\underbrace{f - \overbrace{\underline{a} - c}}\,- \underline{e} \quad \text{and} \quad \underbrace{c - \overbrace{\underline{e} - g}}\,- \underline{b}.$$

It is curious that the first of these three tetrads contains exactly the same intervals as the chord of the dominant Seventh itself, $g - \underline{b} - d - f$, only in inverse order, thus—

$$d \underset{\frac{32}{27}}{\underbrace{- f}} \underset{\frac{6}{5}}{\underbrace{- \bar{a}\flat}} \underset{\frac{5}{4}}{\underbrace{- c'}} \qquad\qquad g \underset{\frac{5}{4}}{\underbrace{- \underline{b}}} \underset{\frac{6}{5}}{\underbrace{- d'}} \underset{\frac{32}{27}}{\underbrace{- f'}}$$

[1] [As $g + b - d$, with $\sharp f$ is composed of the dominant triad in C major, and the subdominant note in $\sharp$ C major, which is a thoroughly unrelated key, it could only occur for the purpose of imitating the natural Seventh, or making an enharmonic change of key, for which it may be useful, as it might be followed by the major Third $\sharp c + \sharp e$, representing the triad $\sharp c + \sharp e - \sharp g$, and thus depressing the tonic from c to $\sharp c$.—*Translator.*]

In the first the consonant portion is a minor triad, and this makes it decidedly harsher than the second where the consonant portion is a major triad.

Here also the difference of harshness depends on the nature of the combinational tones, of which those generated by the closer intervals are most distinctly heard. These are

$$\text{for } \underbrace{g' - \underline{b}'}_{G} \underbrace{- d''}_{G} \underbrace{- f''}_{\underline{A}} \quad \text{and for} \quad \underbrace{d'' - f''}_{\underline{A}} \underbrace{- \bar{a}''\flat}_{\bar{d}\flat} \underbrace{- c'''}_{\bar{a}\flat}$$

Hence one combinational tone in the first chord, and two in the second, are unsuitable to the chord.

The harshest chords of the Seventh are those which each contain two false Fifths, $\underline{b} - d - f' - \underline{a}'$ and $\underline{b} - d' - f' - \bar{a}'\flat$. But the first of these can be made much milder by a slight change in its intonation. Thus $\underline{b} - d - \underline{f}' - a'$ contains tones which all belong to the compound tone of G, and these sound tolerably well together.[1]

[1] [This is only to be taken as an approximative statement, grounded on the assumption that the interval g to f' (in my notation g to $\ddagger f$) is correctly $\frac{7}{4}$, in which case the primes of the tones b, d, $\underline{f}'$, a' are the 5th, 6th, 7th, 9th, partials of G. Mr. Poole (see p. 504, note 1, and App. XIX., section F, where the complete scale of his organ is given) considers it necessary to have a pure natural Seventh to the dominant chord of each key. He thus distinguishes *two* diatonic scales. The first he calls *triple*, because it has *three* roots, as F, C, G, in the usual major scale $C\ D\ E\ F\ G\ A\ B\ c$. The second he calls *double*, apparently because it has only two roots, namely F and C in his scale of $F\ G\ A\ \text{z}B\flat\ c\ d\ e\ f$, the two chords being

F	A	c	and	C	E	G	z$B\flat$	d
4	5	6		4	5	6	7	9

He prefers commencing this on F, because its notes then differ only in one tone z$B\flat$, or the perfectly natural Seventh, from those in the scale of C. In the text it is, in point of fact, proposed to use $\ddagger B\flat$ in the chord $C\ E\ G\ \ddagger B\flat\ d$, as an imitation of Mr. Poole's natural chord, which would be still closer than $C\ E\ G\ A\sharp\ d$, with the superfluous Sixth in place of the natural Seventh. In fact

z$B\flat = \frac{7}{4} \cdot C$,	$A\sharp = \frac{225}{224} \cdot \frac{7}{4} \cdot C$ (p. 524),	and $\ddagger B\flat = \frac{5120}{5103} \cdot \frac{7}{4} \cdot C$, (p. 533).
log. = ·248 0380,	log. = ·244 9725.	log. = ·244 4824.

The logarithms shew that the third is a skhisma flatter than the second, and almost three skhismas sharper than the first, and that the second is almost four skhismas sharper than the first. This not being sufficiently close for Mr. Poole, he provides a natural Seventh for the dominant of each major key and of its relative minor; so that in the key of C major, in addition to the usual seven tones, he has zF, and $\ddagger G\sharp$, $\ddagger D$ and z$\ddagger D$, for the chords $F\ A\ C$, $C\ E\ G$, $G\ B\ D$ zf in the major, and $\ddagger D\ F\ A$, $A\ c\ e$, $E\ \ddagger G\sharp\ B$ z$\ddagger D$ in the relative minor.

To test the effect of septimal intonation I have had an instrument tuned to give the chords —

The chords of the reverted system play an important part in modulations, by serving to mark the key precisely. The most decisive in its action is the chord of the Seventh on the dominant of the key, that is the chord $g - \underline{b} - d - f$ for the tonic C. We saw (p. 530) that the diminished triad $\underline{b} - d - f$ could be adapted by slight changes in its intonation to the keys of

C major, C minor, $\underline{A}$ minor, $\underline{F}\sharp$ minor, and $\underline{E}\flat$ minor.

Of these only the two first contain the tone G, so that the chord $g - \underline{b} - d - f$ can belong to no tonic but C.

The imperfect minor triad $d - f - \underline{a}$, which, when the intonation is correct, belongs only to the key of C major, admits of being confused with $\underline{d} - f - \underline{a}$, which belongs to the keys of $\underline{A}$ minor, F major, and $B\flat$ major. This confusion is not entirely obviated by adding the tone c, and the consequence is the chord of the Seventh $d - f - \underline{a} - c$ is usually employed only in alternation with the chord of the dominant Seventh in the cadence, where it distinguishes C major from C minor. But the addition of the tone $\underline{b}$ to the triad $d - f - \underline{a}$ is characteristic, because this last can at most be confused with $\underline{b} - \underline{d} - f - \underline{a}$, which belongs to $\underline{A}$ minor. The chord $\underline{b} - d - f - \underline{a}$, however, sounds comparatively harsh in every position for which $\underline{a}$ is not the highest note, and hence its application is very limited. It is often united with the chord of the dominant Seventh as a chord of the Ninth, thus $g - \underline{b} - d' - f' - \underline{a}'$, in which g and a' must remain the extreme tones. More upon this hereafter (p. 542).

In the key of C minor, the triad $d - f - \bar{a}\flat$ would, in just

$B\flat\ \ddagger d\ f,\quad F\ A\ C,\quad C\ E\ G\ \zeta B\flat\ d,\quad G\ B\ d,\quad D\ \dagger F\ \dagger A$

perfectly. The effect of the third of these chords far surpasses my expectations, and it is beyond comparison better than the usual chord of the Ninth with $B\flat\ \ddagger d$ in place of $\zeta B\flat\ d$. The chord of the sub-minor Seventh and its inversions

$C\ E\ G\ \zeta B\flat,\quad E\ G\ \zeta B\flat\ c,\quad G\ \zeta B\flat\ c\ e,\quad \zeta B\flat\ c\ e\ g$

are all decidedy superior to the chord of the dominant Seventh, with $B\flat$ in place of $\zeta B\flat$, and its inversions. The septimal minor Triad $G\ \zeta B\flat\ d$ is far superior to the Pythagorean minor triad $D\ F\ \dagger A$, or the false minor triad $D\ F\ A$, and is not far inferior to the true minor triad $\ddagger D\ F\ A$ or $D\ \dagger F\ \dagger A$. The septimal diminished triad $E\ G\ \zeta B\flat$ approaches consonance much more nearly than the usual diminished triad $E\ G\ B\flat$. Though Poole's ascending scale makes too great a gap between $\zeta B\flat$ and c, yet by using $\zeta B\flat\ D$ as alternative tones with $B\flat\ \ddagger D$, ascending with the sharper and descending with the flatter forms, we obtain the perfectly melodious scales of

$F\ G\ A\ B\flat\ c\ d\ e\ f$ and $f\ e\ \ddagger d\ c\ \zeta B\flat\ A\ G\ F.$

These facts shew the acoustic possibility of a septimal theory of harmony, which would include the tertian, as that included the quintal.—*Translator.*]

intonation, be characteristic, but yet it is easily confused with other chords. Thus

$d - f - \bar{a}\flat$ belongs to C minor
$\frac{32}{27}$ $\frac{6}{5}$

$\underline{d} - f - a\flat$ to $E\flat$ major and $E\flat$ minor
$\frac{6}{5}$ $\frac{32}{27}$

$d - \bar{f} - \underline{g}\sharp$ to A minor
$\frac{6}{5}$ $\frac{75}{64}$

$\bar{d} - \underline{c}\sharp - g\sharp$ to $F\sharp$ minor.[1]
$\frac{75}{64}$ $\frac{6}{5}$

The addition of the tone C in the chord of the Seventh $d - f - \bar{a}\flat - c$ would decisively exclude the key $F\sharp$ only, and the addition of the tone $\underline{b}$ (which in imperfect intonation might be confused with b or $c\flat$) would also readily be adapted to all the above keys. This last chord $\underline{b} - d - f - \bar{a}\flat$, called *the chord of the diminished Seventh*, appears on keyed instruments as a series of minor Thirds. In reality a Pythagorean minor Third or superfluous second separates the normal minor Thirds, thus:

$$\underline{b} - d - f - \bar{a}\flat - \underline{b} - d - f - \bar{a}\flat - \underline{b}$$
$$\frac{6}{5} \quad \frac{32}{27} \quad \frac{6}{5} \quad \frac{75}{64} \quad \frac{6}{5} \quad \frac{32}{27} \quad \frac{6}{5} \quad \frac{75}{64}$$

Since the three intervals $\frac{6}{5}$, $\frac{32}{27}$, and $\frac{75}{64}$ differ but very slightly, they are readily confused, and we obtain the following, nearly identical, series of tones:

$\underline{b} - d - f - \bar{a}\flat - \underline{b}$ in C minor
$\frac{6}{5}$ $\frac{32}{27}$ $\frac{6}{5}$ $\frac{75}{64}$

$b - d - \bar{f} - \underline{g}\sharp - b$ in A minor
$\frac{32}{27}$ $\frac{6}{5}$ $\frac{75}{64}$ $\frac{6}{5}$

[1] [In the notation of Appendix XIX. these chords are:—
First, $d \mid f - \dagger ab$ in C minor: $d \mid f - \dagger ab + c - \dagger eb + g + b - d \mid f$
Second, $\ddagger d - f \mid ab$ in $E\flat$ major: $f \mid ab + \ddagger c - eb + \ddagger g - bb + \ddagger d - f \mid ab$
or in $E\flat$ minor: $f \mid ab - cb + eb - gb + bb + \ddagger d - f \mid ab$
Third, $d - \ddagger f \therefore g\natural$ in $\dagger A$ minor: $\dagger b \mid d - \ddagger f + \dagger a - \dagger c + \dagger e + g\natural - \dagger b \mid d$
Fourth, $\dagger d \therefore \dagger e\natural - \dagger g\natural$ in $\dagger F\natural$ minor:
$\dagger g\natural \mid \dagger b - \dagger d + \ddagger f\natural - \dagger\dagger a + \dagger c\natural + \dagger c\natural - \dagger g\natural \mid \dagger b$, —*Translator.*]

$$\underbrace{b - d}_{\frac{6}{5}} \underbrace{- e\sharp}_{\frac{75}{64}} \underbrace{- g\sharp}_{\frac{6}{5}} \underbrace{- b}_{\frac{32}{27}} \text{ in } F\sharp \text{ minor}$$

$$\underbrace{\bar{c}\flat - \underline{d}}_{\frac{75}{74}} \underbrace{- f}_{\frac{6}{5}} \underbrace{- a\flat}_{\frac{32}{27}} \underbrace{- \bar{c}\flat}_{\frac{6}{5}} \text{ in } E\flat \text{ minor.}$$ [1]

These chords of the diminished Seventh do not form so sharp a contrast with the consonances in the minor mode, as the corresponding chord does in the major mode, although if the intonation is just the dissonance is always extremely harsh and cutting. When they are followed by the triad of the tonic, the two chords together contain all the tones of the key, and hence completely characterise it. The chief use of the chord of the diminished Seventh is due to its variability, which readily leads the harmony into new keys. By merely subjoining the minor chords of $F\sharp$, A, C or $E\flat$ the new key will be completely established. It is readily seen that this series of keys itself forms a chord of the diminished Seventh, the tones of which lie a Semitone higher than those of the given chord. This gives a simple means of recollecting them.[2]

The comprehension of the whole of a key by these chords is of

[1] [In the notation of Appendix XIX., as may be seen by the scales in the last note, the chords are:—

$b - d \mid f - \dagger a\flat \therefore b$	in	C minor
$\dagger b \mid d - \dagger f \therefore g\sharp - \dagger b$	in	$\dagger A$ minor
$\dagger b - \dagger d \therefore \dagger e\sharp - \dagger g\sharp \mid \dagger b$	in	$\dagger F\sharp$ minor
$c\flat \therefore \ddagger d - f \mid a\flat - c\flat$	in	$E\flat$ minor.—*Translator.*]

[2] [The last three sentences of the above paragraph must be understood in the sense of equal temperament only, confusing the many tones which this temperament confuses. In just intonation the four keys are $\dagger F\sharp$, $\dagger A$, C and $E\flat$, which have no such connection. Thus on forming the chords of the diminished Seventh $\dagger f\sharp - \dagger\dagger a \mid \dagger c - \dagger\dagger e\flat$, or $f\sharp - \dagger a \mid c - \dagger e\flat$, or $\ddagger f\sharp - a \mid \ddagger c - e\flat$, we see that not one of them contains or can contain the tonics of the four keys in question. In just intonation if any one of the four chords in the last note were followed by the tonic chord of any of the keys except that key on the same line with it in the text, there would be a jump into another key. This jump is easy to execute on keyed instruments, but would it be possible for unaccompanied singers? In making the transitions

	b	d	f	$\dagger a\flat$	or	b	d	f	$\dagger a\flat$
to	$\dagger c$	$\dagger e$	$\dagger e$	$\dagger a$	to	c	e	e	a

what singer could be trusted for making the first rather than the second, when three of the progressions of the first b to $\dagger c$, d to $\dagger e$, f to $\dagger e$ are strange, and three of the second b to c, d to e, and f to e are familiar?—*Translator.*]

special importance in the cadence at the end of a composition or of one of its principal sections. For this purpose we have also to determine what fundamental primary tones can be represented by these chords of the Seventh.

It is clear that a single musical tone can never be more than imperfectly represented by the tones of a dissonant chord. But as a general rule some of these tones can be accepted as the constituents of a musical tone. This gives rise to a practically important difference between the different tones of such a chord. Those tones which can be considered as the elements of a compound tone, form a compact, well-defined mass of tone. Any one or two other tones in the chord, which do not belong to this mass of tone, have the appearance of unconnected tones, accidentally intruding. The latter are called by musicians the *dissonances* or the *dissonant notes* of the chord. Considered independently, of course, either tone in a dissonant interval is equally dissonant in respect to the other, and if there were only two tones it would be absurd to call one of them only *the* dissonant tone. In the Seventh $c - \underline{b}$, c is dissonant in respect to $\underline{b}$, and $\underline{b}$ in respect to c. In the chord $c - \underline{e} - g - \underline{b}$ the notes $c - \underline{e} - g$ form a single mass of tone corresponding to the compound tone of c, and $\underline{b}$ is an unconnected tone sounding at the same time. Hence the three tones $c - \underline{e} - g$ have an independent steadiness and compactness of their own. But the unsupported solitary Seventh $\underline{b}$ has to stand against the preponderance of the other tones, and it could not do so either when executed by a singer, or heard by a listener, unless the melodic progression were kept very simple and readily intelligible. Consequently particular rules have to be observed for the progression of the part which produces this note, whereas the introduction of c, which is sufficiently justified by the chord itself, is perfectly free and unfettered. This practical difference in the laws of progression of parts is indicated by musicians in their terming $\underline{b}$ alone the dissonant note of this chord; and although the expression is not a very happy one, we can have no hesitation in retaining it, after its real meaning has been thus explained.

We now proceed to examine each of the previous chords of the Seventh with a view to determine what compound musical tone they represent, and which are their dissonant tones.

1. The *chord of the dominant Seventh*, $g - \underline{b} - d - f$, contains three tones belonging to the compound tone of G, namely

g, *b*, and *d*, and the Seventh *f* is the dissonant tone. But we must observe that the minor Seventh *g* — *f* (or $\frac{16}{9} = \frac{64}{63} \times \frac{7}{4}$) approaches so near to the ratio $\frac{7}{4}$, which would be almost exactly represented by *g* — *f*, (see p. 534), that *f* may in any case pass as the seventh partial tone of the compound *G*. Singers probably often exchange the *f* of the chord of the dominant Seventh for *f*,[1] partly because it usually passes into *e*, partly because they thus diminish the harshness of the dissonance. This can easily be done when the pitch of *f* is not determined in the preceding chord by some near relationship. Thus if the consonant chord *g* — *b* — *d* had already been struck and then *f* were added, it would readily fall into *f*, because *f* is itself unrelated to *g*, *b*, or *d*. Hence although the chord of the dominant Seventh is dissonant its dissonant tone so nearly corresponds to the corresponding partial tone in the compound tone of the dominant, that the whole chord may be very well regarded as a representative of that compound tone. For this reason, doubtless, the Seventh of this chord has been set free from many obligations in the progression of parts to which dissonant Sevenths are otherwise subjected. Thus it is allowed to be introduced freely without preparation, which is not the case for the other Sevenths.

The chord of the dominant Seventh consequently plays the second most important part in modern music, standing next to the tonic. It exactly defines the key, more exactly than the simple triad *g* — *b* — *d*, or than the diminished triad *b* — *d* — *f*. As a dissonant chord it urgently requires to be resolved on to the tonic chord, which the simple dominant triad does not. And finally its harmoniousness is so extremely little obscured, that it is the softest of all dissonant chords. Hence we could scarcely do without it in modern music. This chord appears to have been discovered in the beginning of the seventeenth century by Monteverde.

2. The *chord of the Seventh upon the Second of a major scale*, *d* — *f* — *a* — *c*, has three tones, *f*, *a*, *c*, which belong to the compound tone of *F*. When the intonation is just, *d* is dissonant with each of the three tones of this chord, and hence must be regarded

[1] [Here ♯*f* must be considered as the representative of ♮*f*. Singers would not naturally take such a strange artificial approximation as ♯*f*, unless led by an instrument. Unaccompanied singers could only choose between *f* and ♮*f*, and singers of unaccompanied melodies are said often to choose ♮*f* when descending to *e*. What is the custom in unaccompanied choirs, which have not been trained to give *f*, has, so far as I know, not been recorded.—*Translator*.]

as the dissonant note. This would make the fundamental position of this chord to be that which Rameau assigned, making *f* the root, thus: *f* — *a* — *c* — *d*, which is a position of the Fifth and Sixth, and the chord is called by Rameau the *chord of the major Sixth.* This is the position in which the chord usually appears in the final cadence of *C* major.[1] Its meaning and its relation to the key is more certain than that of the *false minor chord* *d* — *f* — *a*, mentioned on page 527, which as executed by a singer or heard by a listener is readily apt to be confused with *d* — *f* — *a* in the key of *A* minor. By changing *d* — *f* — *a* into *d* — *f* — *a* we obtain a minor chord, to which there will be a great attraction when the relation of *d* to *g* is not made very distinct. But if we were to change *d* into *d* in the chord *d* — *f* — *a* — *c*, thus producing *d* — *f* — *a* — *c*, although *d* would be consonant with *f* and *a* it would not be so with *c*; on the contrary, the dissonance *d* — *c* is much harsher than *d* — *c*, and after all, it would be only the tone *a* which would enter into the compound tone of *d*, so that, notwithstanding this change, *f*, which contains three tones of the chord in its own compound tone, would predominate over *d*, which has only two. In accordance with this view, I find the chord *f* — *a* — *c* — *d* when used on the justly-intoned harmonium, as subdominant of *C* major, produces a better effect than *f* — *a* — *c* — *d*.

3. The *corresponding chord of the Seventh on the Second of the minor scale*, *d* — *f* — *ā*♭ — *c*, has only one tone, *c*, which can be regarded as a constituent of the compound tone of either *f* or *ā*♭. But since *c* is the third partial of *f* and only the fifth partial of *ā*♭, *f* as a rule predominates, and the chord must be regarded as a subdominant chord *f* — *ā*♭ — *c* with the addition of dissonant *d*. There is still less inducement to change *d* into *d* in this case than in the last.

4. The *chord of the Seventh on the Seventh of the major scale*, *b* — *d* — *f* — *a*, contains two tones, *b* and *d*, belonging to the dominant *g*, and two others, *f*, and *a*, belonging to the subdominant *f*. Hence the chord therefore splits into two equally important halves. But we must observe that the two tones *f* and *a* approach very closely to the two next partial tones of the compound tone of *G*. The partials of this compound tone from the fourth onwards may be written—

[1] [But in its progression to the chord of *G*, *c* and not *d* is treated as dissonant, that is, the chord is assumed to be ♯*d* — *f* + *a* — *c*.—*Translator.*]

$$\begin{array}{cccccc} g & \text{—} & \underline{b} & \text{—} & d & \text{—} & \underline{f} & \text{—} & g & \text{—} & a \\ 4 & & 5 & & 6 & & 7^{1} & & 8 & & 9 \end{array}$$

Hence the chord of the Ninth $g - \underline{b} - d - f - \underline{a}$ may represent the compound tone of the dominant g, provided that the similarity be kept clear by the position of the tones, g being the lowest and $\underline{a}$ the highest; it is also best not to let f fall too low. Since a is the ninth partial tone of the compound g, which is very weak in all usual qualities of tone, and is often inaudible, and since there is the interval of a comma between a and $\underline{a}$ and also between $\underline{f}$ and f, care must be taken to render the resemblance of the chord of the Ninth to the compound tone of g, as strong as possible, by adopting the device of keeping $\underline{a}$ uppermost, and then the use of f, $\underline{a}$, for $\underline{f}$, a, will not be very striking. In this case f and $\underline{a}$ must be considered as the dissonant notes of the chord of the Ninth $g - \underline{b} - d - f - \underline{a}$, because although they are very nearly the same, they are not quite the same, as the partial tones of G. The rules of composition throw no difficulties in the way of introducing $\underline{a}$ into the chord, for the same reasons that f is allowed to be introduced into the chord of the dominant Seventh, $g - \underline{b} - d - f$ without preparation. Lastly, some of the tones of the pentad chord of the Ninth may be omitted, to reduce it to four parts; for example, its Fifth, as in $g - \underline{b} - f - \underline{a}$, or its root, as in $\underline{b} - d - f - \underline{a}$. If only the order of the tones is preserved as much as possible, and especially the $\underline{a}$ kept uppermost, the chord will always be recognised as a representative of the compound tone of G.

This seems to me the simple reason why musicians find it desirable to make $\underline{a}$ the highest tone in the chord $\underline{b} - d - f - \underline{a}$. Hauptmann, indeed, gives this as a rule without exception, and assigns rather an artificial reason for it. The ambiguity of the chord will thus be obviated as far as possible, and it receives a clearly intelligible relation to the dominant of the key of C major, whereas in other positions of the same chord there would be too great a chance of confusing it with the subdominant of $\underline{A}$ minor.[2] When the intonation is just, the chord $g - \underline{b} - d - \underline{f} - a$, which consists (very nearly indeed) of the partial tones of the compound

[1] [Supposing ‡f to be used for ʒf, as already explained, see p. 539, note, so that the above chord represents g b d ʒf g †a.—*Translator.*]

[2] [The rootless chord of the Ninth on the dominant of C major is, in my notation, $b - d \mid f + a$, and the sub-dominant of A minor is $b \mid$ ‡$d - f + a$, which would not be confused in just intonation.—*Translator.*]

tone of g, sounds very soft, and but slightly dissonant; the chord of the Ninth in the key of C major, $g - \underline{b} - d' - f' - \underline{a}'$, and the chord of the Seventh in the position $\underline{b} - d' - f' - \underline{a}'$, sound somewhat rougher, on account of the Pythagorean Third $d' - f'$, and the imperfect Fifth $d' - \underline{a}'$, but they are not very harsh. If, however, $\underline{a}'$ is taken lower, they become very rough indeed.[1]

The chord of the Seventh $\underline{b} - d - f - \underline{a}$ and the following triad $c - \underline{e} - g$, as already observed, contain all the tones in the key of C major, and hence this chordal succession is extremely well adapted for a brief and complete characterisation of the key.

5. The *chord of the diminished Seventh*, $\underline{b} - d - f - \bar{a}\flat$, and the minor chord $c - \bar{e}\flat - g$, have the last mentioned property for the minor key of C, and for this reason as well as for its great variability (p. 537) it is largely, perhaps far too largely (p. 497), employed in modern music, especially for modulations. It contains no note which belongs to the compound tone of any other note in the chord, but the three tones $\underline{b} - d - f$ may be regarded as belonging to the compound tone of g, so that it also presents the appearance of a chord of the Ninth in the form $g - \underline{b} - d - f - \bar{a}\flat$. It therefore imperfectly represents the compound tone of the dominant, with an intruded tone $\bar{a}\flat$, and f and $\bar{a}\flat$ may therefore be regarded as its dissonant tones. But the connection of the three tones $\underline{b} - d - f$ with the compound tone of g is not so distinctly marked as to make it necessary to subordinate the progression of the tones f and $\bar{a}\flat$ to that of $\underline{b}$ and d. At least the chord is allowed to commence without preparation, and it is resolved by the motion of all its tones to those tones of the scale which make the smallest intervals with them, for its elements are not sufficiently well connected with one another to allow of wide steps in its resolution.

6. The *chords of the major Seventh* in the direct system of the key, as $f - \underline{a} - c - \underline{e}$ and $c - \underline{e} - g - \underline{b}$ in C major, and $\bar{a}\flat - c - \bar{e}\flat - g$ in C minor, as already remarked, mainly represent a major chord with the major Seventh as dissonant tone. The major Seventh forms rather a rough dissonance, and is decidedly opposed to the triad below it, into which it will not fit at all.

7. The *chords of the minor Seventh in the direct system of*

[1] [See the observations at the end of note 1, p. 534.—*Translator.*]

the key, as $\underline{a} - c - \underline{e} - g$ and $\underline{e} - g - \underline{b} - d$, give greatest prominence to the compound tone of their Thirds, to which their bass seems to be subjoined. Thus $c - \underline{e} - g - \underline{a}$ is the compound tone of c with an added $\underline{a}$, and $g - \underline{b} - d - \underline{e}$ is the compound tone of g with an added $\underline{e}$. But since $c - \underline{e} - g$ and $g - \underline{b} - d$, being the principal triads of the key, are constantly recurring, this addition of $\underline{a}$ and $\underline{e}$ respectively gives by contrast great prominence to these tones; moreover, the $\underline{a}$ and $\underline{e}$ in these chords of the Seventh are not so isolated as the d in $d - f - \underline{a} - c$, which has no true Fifth in the chord. The $\underline{a}$ in $\underline{a} - c - \underline{e} - g$ has the Fifth $\underline{e}$, and even the Seventh g which belongs to its compound tone; and in the same way the $\underline{b}$ and d of $\underline{e} - g - \underline{b} - d$ may be considered to belong to the compound tone of $\underline{e}$. Hence the tone $\underline{a}$ in the first and $\underline{e}$ in the second are not necessarily subject to the laws of the resolution of dissonant notes.

Writers on harmony are accustomed to consider the normal position of all these chords to be that of the chord of the Seventh, and to call the lowest tone its root. Perhaps it would be more natural to consider $c - \underline{e} - g - \underline{a}$ as the principal position of the chord $\underline{a} - c - \underline{e} - g$ and c as its root. But this latter chord is a compound tone of c with an inclination to $\underline{a}$, and in modulations this intrusion of the tone of $\underline{a}$ is utilised for proceeding to those chords related to $\underline{a}$ which are not related to the chord $c - \underline{e} - g$, for example to $\underline{d} - f - \underline{a}$. In the same way we can proceed from $g - \underline{b} - d - \underline{e}$ to $\underline{a} - c - \underline{e}$, which would be a jump from $g - \underline{b} - d$. For modulation, therefore, the $\underline{a}$ and $\underline{e}$ are essential parts of these chords respectively, and in this practical light they might be called the fundamental tones of their respective chords.

8. The *chord of the Seventh on the tonic of the minor key*, $c - \bar{e}\flat - g - \underline{b}$, is seldom used, because $\underline{b}$ in the minor key belongs essentially to ascending motion, and a resolved Seventh habitually descends. Hence it would be always better to form the chord $c - \bar{e}\flat - g - \bar{b}\flat$, which is similar to the chords considered in No. 7.

CHAPTER XVIII.

LAWS OF THE PROGRESSION OF PARTS.

UP to this point we have considered only the relations of the tones in a piece of music with its tonic tone, and of its chords with its tonic chord. On these relations depends the connection of the parts of a mass of tone into one coherent whole. But besides this the succession of the tones and chords must be regulated by natural relations. The mass of tone thus becomes more intimately bound up together, and, as a general rule, we must aim at producing such a connection, although, exceptionally, peculiar expression may necessitate the selection of a more violent and less obvious plan of progression. In the development of the scale we saw that the connection of all the notes by means of their relation to the tonic, if originally perceived at all, was at most but very dimly seen, and was apparently replaced by the chain of Fifths; at any rate, the latter alone was sufficiently developed to be recognised in the Pythagorean construction of tonal systems. But by the side of our strongly developed feeling for the tonic in modern harmonic music, the necessity for a linked connection of individual tones and chords is still recognised, although the chain of Fifths, which originally connected the tones of the scale, as

$$f — c — g — d — a — e — b,$$

has been interrupted by the introduction of perfect major Thirds, and now appears as

$$f — c — g — d \mid \underline{d} — \underline{a} — \underline{e} — \underline{b}.$$

The musical connection between two consecutive notes may be effected:

1. *By the relation of their compound tones.*

This is either:

a.) *direct*, when the two consecutive tones form a perfectly consonant interval, in which case, as we have previously seen, one

of the clearly perceptible partial tones of the first note is identical with one of the second. The pitch of the following compound tone is then clearly determined for the ear. This is the best and surest kind of connection. The closest relationship of this kind exists when the voice jumps a whole Octave; but this is not usual in melodies, except with the bass, as the alteration of pitch is felt to be too sudden for the upper part. Next to this comes the jump of a Fifth or Fourth, both of which are very definite and clear. After these follow the steps of a major Sixth or major Third, both of which can be readily taken, but some uncertainty begins to arise in the case of minor Sixths and minor Thirds. Esthetically it should be remarked, that of all the melodic steps just mentioned, the major Sixth and major Third have, I might almost say, the highest degree of thorough beauty. This possibly depends upon their position at the limit of clearly intelligible intervals. The steps of a Fifth or Fourth are too clear, and hence are, as it were, drily intelligible; the steps of minor Thirds, and especially minor Sixths, begin to sound indeterminate. The major Thirds and major Sixths seem to hold the right balance between darkness and light. The major Sixth and major Third seem also to stand in the same relation to the other intervals harmonically.

b.) *indirect*, of the second degree only. This occurs in the regular progression of the scale, proceeding by Tones or Semitones. For example:

$$\underbrace{c - d}_{G} \qquad \underbrace{d - \underline{e}}_{G} \qquad \underbrace{\underline{e} - f}_{C}$$

The whole major tone $c - d$ proceeds from the Fourth to the Fifth of the auxiliary tone G, which Rameau supposed to be subjoined as the fundamental bass of the above melodic progression. The minor Tone $d - \underline{e}$ proceeds from the Fifth to the major Sixth of the auxiliary tone G, and the Semitone $\underline{e} - f$ from the major Third to the Fourth of the auxiliary tone C. But in order that these auxiliary tones may readily occur to both singer and hearer, they must be the principal tones of the key. Thus the step $\underline{a} - \underline{b}$ in the major scale of C causes the singers a little trouble, although it is only the interval of a major Tone, and could be easily referred to the auxiliary tone $\underline{E}$. But the sound of $\underline{e}$ is not so firm and ready in the mind, as the sound of C and its Fifths G and F. Hence the hexachord of Guido of Arezzo, which was the normal

scale for singers throughout the middle ages, ended at the Sixth.[1] This hexachord was sung with different pitches of the first note, but always formed the same melody:

	Ut	*Re*	*Mi*	*Fa*	*Sol*	*La*
either	*G*	*A*	*B*	*C*	*D*	*E*
or	*C*	*D*	*E*	*F*	*G*	*A*
or	*F*	*G*	*A*	*B*♭	*C*	*D*

So that the interval *Mi* — *Fa* always marked the Semitone.

For the same reason Rameau preferred, in the minor scale, to refer the steps d — $\bar{a}$♭ and $\bar{e}$♭ — f to *G* and *C* as auxiliary tones, rather than to *B*♭, the Seventh of the descending scale, which had no sufficiently close relationship to the Tonic, and hence was not well enough impressed on the singer's mind for such a purpose. Taking g and c, the Octaves of *G* and *C* as the auxiliary tones, motion in d — $\bar{e}$♭ is from the Fourth below g to the major Third below it, and in $\bar{e}$♭ — f from the major Sixth below c to the Fifth below it. On the other hand, it is impossible to reduce the step $\bar{a}$♭ — $\underline{b}$ in the minor scale to any relationship of the second degree. It is therefore also decidedly unmelodic and had to be entirely avoided in the old homophonic music, just as the steps of the false Fifths and Fourths, as $\underline{b}$ — f', or f' — $\underline{b}'$. Hence the alterations in the ascending and descending minor scales already mentioned.

In modern harmonic music many of these difficulties have disappeared, or become less sensible, because correct harmonisation can exhibit the connections which are absent in the melodic progression of an unaccompanied voice. Hence also it is much easier to take a part at sight in a harmony, written in pianoforte score, which shews its relations, than to sing it from an unconnected part. The former shews how the tone to be sung is connected with the whole harmony, the latter gives only its connection with the adjacent tones.[2]

2. Tones may be connected by *their approximation in pitch.* This relation has been considered previously in relation to the

[1] For the same reason d'Alembert explains the limits of the old Greek heptachord, by means of two connected tetrachords—

$$b - c - d - e - f - g - a$$

in which the step a — b is avoided. But this explanation would only suit a key in which c was the tonic, and this was probably not the case for the ancient Greek scale.

[2] [Hence any means of shewing the relation of each tone to the tonic of the moment, as in the Tonic Solfa system or the use of the *duodenal* (p. 471, note), materially facilitates sight-singing.—*Translator.*]

leading note. The same holds good for the intercalated tones in chromatic passages. For example, if in *C* major, we replace *C* — *D* by *C* — *C*♯ — *D*, this *C*♯ has no relation either of the first or second degree with the tonic *C*, and also no harmonic or modulational significance. It is nothing but a step intercalated between two tones, which has no relation to the scale, and only serves to render its discontinuous progression more like the gliding motion of natural speech, or the continuity of weeping or howling. The Greeks carried this subdivision still further than we do at present, by splitting up a Semitone into two parts in their enharmonic system (p. 408). Notwithstanding the strangeness of the tone to be struck, chromatic progression in Semitones can be executed with sufficient certainty to allow it to be used in modulational transitions for the purpose of suddenly reaching very distant keys.

Italian melodies are especially rich in such intercalated tones. Investigations of the laws under which they occur will be found in two essays of Sig. A. Basevi.[1] The rule is without exception that tones foreign to the scale can be introduced only when they differ by a Semitone from the note of the scale on to which they resolve, while any tones belonging to the scale itself can be freely introduced although out of harmony with the accompaniment, and even requiring the step of a whole Tone for their resolution.

In the same way steps of a whole tone may be made, provided the notes lie in the scale, when they serve merely to connect two other tones which belong to chords. These are the so-called *passing* or *changing notes*. Thus if while the triad of *C* major is sustained, a voice sings the passage *c* — *d* — *e* — *f* — *g*, the two notes *d* and *f* do not suit the chord, and have no relation to the harmony, but are simply justified by the melodic progression of the single voice. It is usual to place these passing notes on the unaccented parts of the bar, and to give them a short duration. Thus in the above example *c*, *e*, *g* would fall on accented parts of the bar. Then *d* is the passing note between *c* and *e*, and *f* between *e* and *g*. It is essential for their intelligibility that they should make steps of Semitones or whole Tones. They thus produce a simple melodic progresssion, which flows on freely, without giving any prominence to the dissonances produced.

[1] 'Introduction à un nouveau Système d'Harmonie,' traduit par L. Delâtre; Florence, 1855. 'Studj sull' Armonia,' Firenze, 1865.

Even in the essentially dissonant chords the rule is, that dissonant tones which intrude on the body of the other tones must proceed in a melodic progression, which can be easily understood and easily performed. And since the feeling for the natural relations of such an isolated tone is almost overpowered by the simultaneous sound of the other tones which force themselves much more strongly on the attention, both singer and hearer are thrown upon the gradual diatonic progression as the only means of clearly fixing the melodic relations of a dissonant note of this description. Hence it is generally necessary that a dissonant note should enter and leave the chord by degrees of the scale.

Chords must be considered essentially dissonant, in which the dissonant notes do not enter as passing notes over a sustained chord, but are either accompanied by an especial chord, differing form the preceding and following chords, or else are rendered so prominent by their duration or accentuation, that they cannot possibly escape the attention of the hearer. It has been already remarked that these chords are not used for their own sakes, but merely as a means of increasing the feeling of onward progression in the composition. Hence it follows for the motion of the dissonant note, that when it enters and leaves the chord, it will either ascend on each occasion or descend on each. If we allow it to reverse its motion in the second half, and thus return to its original position, there would seem to have been no reason for the dissonance. It would in that case have been better to leave the note at rest in its consonant position. A motion which returns to its origin and creates a dissonance by the way, had better be avoided; it has no object.

Secondly it may be laid down as a rule, that the motion of the dissonant note should not be such as to make the chord consonant without any change in the other notes. For a dissonance which disappears of itself provided we wait for the next step, gives no impetus to the progress of the harmony. It sounds poor and unjustified. This is the principal reason why chords of the Seventh which have to be resolved by the motion of the Seventh, can only permit the Seventh to descend. For if the Seventh ascended in the scale, it would pass into the octave of the lowest tone, and the dissonance of the chord would disappear. When Bach, Mozart, and others use such progressions for chords of the dominant Seventh, the Seventh has the effect of a passing note, and must be

so treated. In that case it has no effect on the progression of the harmony.

The pitch of a single dissonant note in a chord of many parts is determined with greatest certainty, when it has been previously heard as a consonance in the preceding chord, and is merely sustained while the new chord is introduced. Thus if we take the following succession of chords:

$$G - d - g - \underline{b}$$
$$c - \underline{e} - g - \underline{b}$$

the $\underline{b}$ in the first chord is determined by its consonance with G. It simply remains while the tones c and $\underline{e}$ are introduced in place of G and d, and thus becomes a dissonance in the chord of the Seventh $c - \underline{e} - g - \underline{b}$. In this case the dissonance is said to be *prepared*. This was the only way in which dissonances might be introduced down to the end of the sixteenth century. Prepared dissonances produce a peculiarly powerful effect: a part of the preceding chord lingers on, and has to be forced from its position by the following chord. In this way, an effort to advance against opposing obstacles which only slowly yield, is very effectively expressed. And for the same reason the newly introduced chord ($c - \underline{e} - g$ in the last example) must enter on a strongly accented part of the bar; as it would otherwise not sufficiently express exertion. The resolution of the prepared dissonance, on the contrary, naturally falls on an unaccented part of the bar. Nothing sounds worse than dissonances played or sung in a dragging or uncertain manner. In that case they appear to be simply mistakes. They are, as a rule, only justified by expressing energy and vigorous progress.

Such prepared dissonances, termed *suspensions*, may occur in many other chords besides those of the Seventh. For example:

Preparation: $G - c - \underline{e}$
Suspension: $G - c - d$
Resolution: $G - \underline{B} - d$

The tone c is the prepared dissonance; in the second chord, which must fall on an accented part of the bar, d the Fifth of G is introduced and generates the dissonance $c - d$, and then c must give way, and according to our second rule, must go further from d, which results in the resolution $G - \underline{B} - d$. The chords might also be played in the inverse order, and then d would be a prepared

dissonance which was forced away by *c*. But this is not so good, because descending motion is better suited than ascending motion to an extruded note. Heightened pitch always gives us involuntarily the impression of greater effort, because we have continually to exert our voice in order to reach high tones. The dissonant note on descending seems to yield suitably to superior force, but on ascending it as it were rises by its own exertion. But circumstances may render the latter course advisable, and its occurrence is not unfrequent.

In the other case, especially frequent for chords of the Seventh, when the dissonant note is not prepared but is struck simultaneously with the chord to which it is dissonant, the signification of the dissonance is different. Since these unprepared Sevenths must usually enter by the descent of the preceding note, they may be always considered as descending from the Octave of the root of their chord, by supposing a consonant major or minor chord having the same root as the chord of the Seventh to be inserted between that and the preceding chord. In this case the entrance of the Seventh merely indicates that this consonant chord begins to break up immediately and that the melodic progression gives a new direction to the harmony. This new direction, leading to the chord of resolution, must be emphasised, and hence the dissonance necessarily falls on the preceding unaccented part of a bar.

The introduction of an isolated dissonant note into a chord of several parts cannot generally be used as the expression of exertion, but this character will attach to the introduction of a chord as against a single note, supposing that this single note is not too powerful. Hence it lies in the nature of things that the first kind of introduction takes place on unaccented and the last on accented parts of a bar.

These rules for the introduction of dissonances may be often neglected for the chords of the Seventh in the reverted system, in which the Fourth and Second of the scale occur, and notes from the subdominant side are mixed up with notes from the dominant side. These chords may also be introduced to enhance the dynamical impression of the advancing harmony, for they have the effect of keeping the extent of the key perpetually before the feeling of the hearer, and this object justifies their existence.

Of several voices which are leaving the chord of the tonic *C*, it is quite easy for some to pass on to notes of the dominant chord *g* — *b* — *d*, and for others to proceed to the notes of the subdomi-

nant chords f — $\underline{a}$ — c or f — $\bar{a}\flat$ — c, as each voice will be able to strike the new note with perfect certainty, on account of the close relationship between the chords. When, however, the dissonant chord has been thus formed and sounded, the dissonant notes will have the feeling for their more distant relations obscured by the strangeness of the other parts of the chord, and must generally proceed according to the rule of resolution of dissonances. Thus the singer who sounds f in the chord g — $\underline{b}$ — d — f, would vainly endeavour to picture to himself the sound of the $\underline{a}$ which is related to f with sufficient clearness to leap up or down to it with certainty; but he is easily able to execute the small step of half a Tone, by which f descends to $\underline{e}$ in the chord c — $\underline{e}$ — g. But the note g itself, on the other hand, having its own compound tone approximatively indicated by the chord of the Seventh, has no difficulty in passing by a leap to its related notes, as c for example, or $\underline{b}$ to g.

In the chords $\underline{b}$ — d | f — $\underline{a}$ and $\underline{b}$ — d | f — $\bar{a}\flat$, in which neither dominant nor subdominant prevails, it would not be advisable to let any note proceed by a leap.

And it would also not be advisable to pass by a leap into the chords of the reverted system from any other chord but the tonic, because that chord alone is related to both dominant and subdominant chords at the same time.

It is not possible to pass from chords of the Seventh, in the direct system, to another chord related to both of its extremities, and hence in this case the dissonance must be introduced in accordance with the strict rules.

Musicians are divided in opinion as to the proper treatment of the subdominant chord with an added Sixth, f — $\underline{a}$ — c — d in C major. The rule of Rameau is probably correct (p. 540), making d the dissonant note, to be resolved by rising to $\underline{e}$. This is also decidedly the most harmonious kind of resolution. Modern theorists regard this chord as a chord of the Seventh on d, and take c as the dissonant note to be resolved by descent; whereas when c remains, d is quite free and may therefore even descend.

Chordal Sequences.

Just as the older homophonic music required the notes of a melody to be linked together, modern music endeavours to link together the series of chords occurring in a web of harmony, and

it thus obtains much greater freedom in the melodic succession of individual notes, because the natural relationship of the notes is much more decisively and emphatically marked in harmonic music than in homophonic melody. This desire for linking the chords together was but slightly developed in the sixteenth century. The great Italian masters of this period allow the chords of the key to succeed each other in leaps which are often surprising, and are always extremely rare at present. But during the seventeenth century the feeling for this peculiarity of harmony was also developed, so that we find Rameau laying down distinct rules on the subject in the beginning of the eighteenth century. In reference to his conception of fundamental bass, Rameau worded his rule thus: '*The fundamental bass may, as a general rule, proceed only in perfect Fourths or Fifths, upwards or downwards.*' According to our view the fundamental bass of a chord is that compound tone which is either exclusively or principally represented by the notes of the chord. In this sense Rameau's rule coincides with that for the melodic progression of a single note to its nearest related notes. The compound tone of a chord, like the voice of a melody, may only proceed to its nearest related notes. It is much more difficult to assign a meaning to progression by relationship in the second degree for chords than for separate notes, and similarly for progression in small diatonic degrees without relationship. Hence Rameau's rule for the progression of the fundamental bass is on the whole stricter than the rules for the melodic progression of a single voice.

Thus if we take the chord c — $\underline{e}$ — g, which belongs to the compound tone of C, we may pass by Fifths to g — $\underline{b}$ — d, the compound tone of G, or to f — $\underline{a}$ — c, the compound tone of F. Both of these chords are directly related to the first c — $\underline{e}$ — g, because each has one note in common with it, g and c respectively.

But we can also allow the compound tone to proceed in Thirds, and then we obtain minor chords, that is, provided we keep to the same scale. The transition from the compound tone of C to that of $\underline{E}$ is expressed by the sequence of chords c — $\underline{e}$ — g and $\underline{e}$ — g — $\underline{b}$, which are related by having two notes, $\underline{e}$, g, in common. The sequence c — $\underline{e}$ — g and $\underline{a}$ — c — $\underline{e}$ from the compound tone of C to that of $\underline{A}$, is of the same kind. The latter is even more natural than the former, because the chord $\underline{a}$ — c — $\underline{e}$ represents imperfectly the compound tone of $\underline{A}$, into which that of C intrudes,

so that the compound tone of C, which was clearly given in the preceding chord, persists with two of its tones, e.g. in the second chord, a relation which did not exist in the former case.

But if we prefer to leave the key of C major, we can pass to perfect compound tones in Thirds, as from c — e — g to e — g♯ — b or a — c♯ — e, as is very usual in modulations.[1]

Rameau will not allow a simple diatonic progression of the fundamental bass of consonant triads, except where major and minor chords alternate, as from c — b — d to a — c — e, that is from the compound tone of G to that of A, but calls this a 'licence.' In reality this progression is readily explicable from our point of view, by considering a — c — e as a compound tone of C with an intrusive a. The transition is then one of the usual close relationship, from the compound tone of G to that of C, and the a appears as a mere dependence of the latter. Every minor chord represents two compound tones in an imperfect manner. Rameau first formulated this ambiguity (*double emploi*) for the minor chord with added Seventh, which in the form d — f — a — c may represent the compound tone of D, and in the form f — a — c — d that of F, or in Rameau's language its fundamental bass might be D or F.[2] In this chord of the Seventh the ambiguity is more marked, because it contains the compound tone of F more completely; but the ambiguity belongs in a less marked degree to the simple minor chord also.

With the false cadence in the major key

$$g - \underline{b} - d \quad \text{to} \quad \underline{a} - c - \underline{e}$$

must be associated the corresponding cadence in the minor key,

$$g - \underline{b} - d \quad \text{to} \quad \bar{a}\flat - c - \bar{e}\flat$$

where the chord $\bar{a}\flat - c - \bar{e}\flat$ replaces the normal resolution $c - \bar{e}\flat - g$. But here there is only a single note of the compound tone of C remaining, and the false cadence therefore becomes much more striking. It will be rendered milder by adding the Seventh f to the G chord, because f is related to $\bar{a}\flat$.

When two chords having only a relationship of the second

[1] [In the notation of Appendix XIX. these major transitions are from c + e — g to e + ‡g♯ — b or a + ‡c♯ — e, the previous minor transitions being from c + e — g to e — g + b or a — c + e.—*Translator.*]

[2] [Of course Rameau, writing in tempered notation, did not distinguish ‡d and d, so that the actual notes in the two chords ‡d — f + a — c and f + a — c ... d were to him identical. See pp. 527, 535, 540.—*Translator.*]

degree, are placed in juxtaposition, we usually feel the transition to be very abrupt. But if the chord which connects them is one of the principal chords of the key, and has consequently been frequently heard, the effect is not so striking. Thus in the final cadence it is not unusual to see the succession $f - \underline{a} - c$ and $g - \underline{b} - d$, the two chords being related through the tonic chord $c - \underline{e} - g$, thus:

$$\overbrace{f - \underline{a} - c} \qquad \overbrace{g - \underline{b} - d}$$
$$\underbrace{c - \underline{e} - g}$$

Generally we must remember that all these rules of progression are subject to many exceptions, partly because expression may require exceptional abruptness of transition, and partly because the hearer's recollection of previous chords may sufficiently strengthen a naturally weak relationship. It is clearly an entirely false position which teachers of harmony have assumed, in declaring this or that to be 'forbidden.' In point of fact nothing musical is absolutely forbidden, and all rules for the progression of parts are actually violated in the most effective pieces of the greatest composers. It would have been much better to proceed from the principle that certain transitions, which are disallowed, produce striking and unusual effects upon the hearer, and consequently are unsuitable except for the expression of what is unusual. Generally speaking, the object of the rules laid down by theorists is to keep up a well-connected flow of melody and harmony, and make its course readily intelligible. If that is what we aim at, we had best observe their restrictions. But it cannot be denied that a too anxious avoidance of what is unusual places us in danger of becoming trivial and dull, while, on the other hand, inconsiderate and frequent infringement of rules makes compositions eccentric and unconnected.

When disconnected triads would come together it is frequently advantageous to transform them into chords of the Seventh, and thus create a bond between them. In place of the preceding sequence of two triads

$$f - a - c \qquad \text{to} \qquad g - \underline{b} - d$$

we can use a sequence of chords of the Seventh which represent the same compound tones

$$f - a - c - d \qquad \text{to} \qquad g - \underline{b} - d - f.$$

In this case two of the four notes remain unchanged; in the chord of *F*, the *d* belongs to the compound tone of the dominant, and in the chord of *G* the *f* to that of the sub-dominant.

Hence chords of the Seventh come to play an important part in modern music, for the purpose of effecting well-connected and yet rapid transitions from chord to chord, and urging them forward by the action of dissonances; and in this way transitions to the compound tone of the subdominant are easily effected.

Thus, for example, beginning with the triad *g* — *b̲* — *d* we can not merely pass to the chord of *C*, or *c* — *e̲* — *g*, but, letting *g* remain as a Seventh, to the chord of the Seventh *a̲* — *c* — *e̲* — *g*, which unites the two chords *c* — *e̲* — *g* and *a̲* — *c* — *e̲*, and then immediately pass to *d̲* — *f* — *a̲*, which is related to the latter chord, so that two steps bring us to the other extremity of the system of *C* major. This transition also gives the best progression for the Seventh (*g* in the example), because it has been prepared in the previous chord, and is resolved by descent (to *f*) in the succeeding chord. If we tried the same transition backwards, we should have to obtain the Seventh *g* by progression from *a̲* in the chord of *d̲* — *f* — *a̲*, and then be compelled to introduce the *c* of the chord of the Seventh abruptly, because we should have a prohibited succession of Fifths (*d̲* — *a̲* and *c* — *g*) if we tried to descend from *d̲*. We must rather obtain *c* by a leap from *f*, because *a̲* in the first triad must furnish both the *a̲* and *g* of the chord of the Seventh. Thus the transition to the dominant is by no means easy, fluent, and natural; it is much more embarrassed than the passage to the subdominant. Consequently the regular and usual progression of the chord of the Seventh is for its Seventh to descend to the triad whose Fifth is the root of the chord of the Seventh. Supposing we denote the root of the chord of the Seventh by I, its Third by III, &c., a falling Seventh will lead us to either of these chords:

```
I — III — V — VII      and      I — III — V — VII
|     \ /   /                   |     |     \ /
I  —  IV — VI                   I — III  —  VI
```

Of these two transitions, the first, which leads to a chord of which IV is the root, is the liveliest, because it introduces a chord with two new tones. The other, which leads to a triad having VI for its root, introduces only one new tone. Hence the first is regarded as

the principal method of resolving chords of the Seventh. For example:

$$\begin{array}{ccccccc} g & — & \underline{b} & — & d & — & f \\ | & & & \diagup\diagdown & & \diagup & \\ g & — & — & c & — & \underline{e} & \end{array} \qquad \begin{array}{ccccccc} \underline{e} & — & g & — & \underline{b} & — & d \\ | & & & \diagup\diagdown & & \diagup & \\ \underline{e} & — & — & \underline{a} & — & c & \end{array}$$

$$\begin{array}{ccccccc} c & — & \underline{e} & — & g & — & \underline{b} \\ | & & & \diagup\diagdown & & \diagup & \\ c & — & & f & — & \underline{a} & \end{array} \qquad \begin{array}{ccccccc} \underline{b} & — & d & — & f & — & \underline{a} \\ | & & & \diagup\diagdown & & \diagup & \\ \underline{b} & — & — & \underline{e} & — & g & \end{array}$$

The descent of the tone VII introduces the tone VI. In the first case this is the Third of the new triad, and in the second its root. But it may be its Fifth:

$$\begin{array}{ccccccc} \text{I} & — & \text{III} & — & \text{V} & — & \text{VII} \\ & \diagdown & & \diagdown\diagup & & \diagup & \\ & \text{II} & — & \text{IV} & — & \text{VI} & \end{array}$$

This, however, could only occur naturally in the two chords:

$$\begin{array}{ccccccc} \underline{b} & — & d & — & f & — & \underline{a} \\ & \diagdown & & \diagdown\diagup & & \diagup & \\ & c & — & \underline{e} & — & g & \end{array} \quad \text{and} \quad \begin{array}{ccccccc} \underline{b} & — & d & — & f & — & \bar{a}\flat \\ & \diagdown & & \diagdown\diagup & & \diagup & \\ & c & — & \bar{e}\flat & — & g & \end{array}$$

because the two chords of the Seventh represent the compound tone of G, and the tonic chord establishes the bond of union between its two sections. In other cases our scheme gives so-called false cadences:

$$\begin{array}{ccccccc} g & — & \underline{b} & — & d & — & f \\ & \diagdown & & \diagdown\diagup & & \diagup & \\ & \underline{a} & — & c & — & \underline{e} & \end{array} \quad \text{and} \quad \begin{array}{ccccccc} g & — & \underline{b} & — & d & — & f \\ & \diagdown & & \diagdown\diagup & & \diagup & \\ & \bar{a}\flat & — & c & — & \bar{e}\flat & \end{array}$$

which are justified (the first as most natural) by the fact that either c — $\underline{e}$ or c — $\bar{e}\flat$ belongs to the chord of the normal resolution. Rameau therefore justly observes that this kind of resolution is only permissible when the IV of the second chord is the normal Fourth of the I in the chord of the Seventh.

This exhausts the resolutions by the descent of the Seventh. Those in which it remains unchanged take place according to the schemes:

$$\begin{array}{ccccccc} \text{I} & — & \text{III} & — & \text{V} & — & \text{VII} \\ & \diagdown & & \diagdown\diagup & & | & \\ & \text{II} & — & \text{IV} & — & \text{VII} & \end{array} \quad \text{and} \quad \begin{array}{ccccccc} \text{I} & — & \text{III} & — & \text{V} & — & \text{VII} \\ & \diagdown\diagup & & & | & & | \\ & \text{II} & — & — & \text{V} & — & \text{VII} \end{array}$$

In the first the Seventh becomes the root, in the second the Third of the new chord. If it were the Fifth, the new chord would coincide with part of the chord of the Seventh:

I — III — V — VII
| | | |
VII — III — V — VII.

In these connections the resolution is towards the dominant side. The transition is most decisive in the first, where the Seventh becomes the root. These resolutions are on the whole less usual, because it has already been found easier to pass from chords on the dominant side into chords of the Seventh of the direct system. In the chords of the reverted system these transitions occur more frequently, because their Sevenths may enter by ascent, and hence we avoid the sequences of Fifths, which greatly embarrass the transitions from a triad to a chord of the Seventh on its dominant side.

As to the transitions from one chord of the Seventh to another, or to a dissonant triad of the direct system which may be regarded as a mutilated chord of the Seventh, all these matters are sufficiently developed in the ordinary manuals of Thorough-Bass, and offer no difficulties that would justify us in dwelling upon them here.

On the other hand, we have to say a few words on certain rules respecting the progression of the individual parts in polyphonic compositions. Originally, as we have already remarked, all these parts were of equal importance, and had usually to repeat the same melodic figures in succession. The harmony was a secondary consideration, the melodic progression of the individual voices was the principal matter. Hence it was necessary to take care that each voice should stand out clear and distinct from all the others. The relation between the importance of harmony and melody has certainly altered essentially in modern music; the former has attained a much higher independent significance. But, after all, perfection of harmony must arise from the simultaneous performance of several voices, each of which has its own beautiful and clear melodic progression, and each of which therefore moves in a direction that the hearer has no difficulty in understanding.

On this rests the prohibition of *consecutive Fifths and Octaves.* The meaning of this prohibition has given rise to much disputation. The meaning of prohibiting consecutive Octaves has been made clear by musical practice. In polyphonic music two voices

which lie one or two Octaves apart, are forbidden to move forward in such a way that after their next step they should be also one or two Octaves apart. But precisely in the same way, two voices in a polyphonic piece are forbidden to go on in unison for several notes, while for complete musical compositions it is not forbidden that two voices, or even all the voices should proceed in Unisons or Octaves, for the purpose of strengthening the melodic progression. It is clear that the reason of this rule must lie in the crippling of harmonic power of progression by, as it were, sacrificing a voice to Unisons and Octaves. This is allowable when it is intentionally introduced for a whole melodic phrase, but it is not suited for a few notes in the course of a piece, where it can only give the impression of reducing the richness of the harmony by an unskilful accident. The accompaniment of a lower part by a voice singing an octave higher, merely strengthens part of the compound tone of the lower voice, and hence where variety in the progression of parts is important, does not essentially differ from a Unison.

Now in this respect the nearest to an Octave are the Twelfth, and its lower octave, the Fifth. Hence then consecutive Twelfths and consecutive Fifths partake of the same imperfection as consecutive Octaves. But the case is somewhat worse. It is possible to accompany a whole melody in Octaves when desirable, without committing any error, but this cannot be done for Fifths and Twelfths without changing the key. It is impossible to proceed by a single diatonic step from the tonic as root with an accompaniment of Fifths, without departing from the key. In *C* major, we ascend from the Fifth $c - g$ to the Fifth $d - a$, but a does not belong to the scale, which requires the deeper $\underset{\cdot}{a}$; we descend to $\underset{\cdot}{b} - \underset{\cdot}{f}\sharp$, and there is no $\underset{\cdot}{f}\sharp$ in the scale at all. The other upward steps from d exclusive to $\underset{\cdot}{a}$, can of course be accompanied by perfect Fifths in the scale, as $\underset{\cdot}{e} - \underset{\cdot}{b}$, $f - c'$, $g - d'$, $\underset{\cdot}{a} - \underset{\cdot}{e}'$. It is therefore impossible to use the Twelfth consistently for increasing the richness of the tone. But again, when the intervals of a Twelfth or Fifth are continued for a few steps in melodic progression, they have simply the effect of strengthening the root. For the Twelfth this arises from its directly corresponding to one of the upper partial tones of the root. For the Fifth $c - g$, the c and g are the two first upper partials of the combinational tone *C*, which necessarily accompanies the Fifth. Hence an accompa-

niment in Fifths above, when it occurs isolated in the midst of a polyphonic piece, is not only open to the charge of monotony, but cannot be consistently carried out. It should therefore be always avoided.

But that consecutive Fifths merely infringe the laws of artistic composition, and are not disagreeable to the natural ear, is evident from the simple fact that all the tones of our voice, and those of most instruments, are accompanied by Fifths, and that our whole tonal system reposes upon that fact. When the Fifths are introduced as merely mechanical constituents of the compound tone, not parts of the harmony, they are fully justified. So in the compound stops of organs. In these stops the pipes which give the prime tones of the compound, are always accompanied by others which give its harmonic upper partials, as the Octaves, Twelfths variously repeated, and even the higher major Thirds. By this means the performer is able to compose a tone of a much more penetrating, piercing quality, than it would be possible to produce by the simple organ pipes with their relatively weak upper partial tones. It is only by such means that an organ is able to regulate the singing of a large congregation. Almost all musicians have blamed an accompaniment of Fifths, or even Thirds, but fortunately have not been able to effect anything against the practice of the organ. In fact the compound stops of an organ merely reproduce the masses of tone which would have been created by bowed instruments, trombones, and trumpets, if they had executed the same music. It would be quite different if we collected independent voices, from each of which we should have to expect an independent melodic progression in the tones of the scale. Such independent voices cannot possibly move with the precision of a machine; they would soon betray their independence by slight mistakes, and we should be led to subject them to the laws of the scale, which, as we have seen, render a consistent accompaniment in Fifths impossible.

The prohibition of Fifths and Octaves extends also, but with less strictness, to the next adjacent consonant intervals, when two of them are so placed as to form a connected group of upper partials in a compound tone. Thus transitions like

$$d - g - \underline{b} \qquad \text{to} \qquad c - f - \underline{a},$$

are ruled by musical theorists to be inferior to transitions like

$$\underline{b} - d' - g' \qquad \text{to} \qquad \underline{a} - c' - f'.$$

For $d - g - \underline{b}$ are the third, fourth, and fifth partial tone of the compound $G_{,}$ but $\underline{b} - d' - g'$ could only be regarded as its fifth, sixth, and eighth. Hence the first position of the chord expresses a single compound tone much more decidedly than the second, which is often allowed to be continued through long passages, when of course the nature of the Thirds and Fourths varies.

The prohibition of consecutive Fifths was perhaps historically a reaction against the first imperfect attempts at polyphonic music, which were confined to an accompaniment in Fourths or Fifths, and then, like all reactions, it was carried too far, in a barren mechanical period, till absolute purity from consecutive Fifths became one of the principal characteristics of good musical composition. Modern harmonists agree in allowing that other beauties in the progression of parts are not to be rejected because they introduce consecutive Fifths, although it is advisable to avoid them, when there is no need to make such a sacrifice.

There is also another point in the prohibition of Fifths to which Hauptmann has drawn attention. We are not tempted to use consecutive Fifths when we pass from one consonant triad to another which is nearly related to it, because other progressions lie nearer at hand. Thus we pass from the triad of C major to the four related triads in the following manner, the fundamental bass proceeding by Thirds or Fifths:

$$\begin{array}{llll} c - \underline{e} - g & c - \underline{e} - g & c - \underline{e} - g & \text{and } c - \underline{e} - g \\ \text{to } c - \underline{e} - \underline{a}, & \text{to } c - f - \underline{a}, & \text{to } \underline{B} - \underline{e} - g, & \text{to } \underline{B} - d - g. \end{array}$$

But when the fundamental bass proceeds in Seconds, and hence does not pass to a directly related chord, the nearest position of the new chord is certainly one which produces consecutive Fifths. For example:

$$\begin{array}{ll} g - \underline{b} - d' & \text{or } g - \underline{b} - d' \\ \text{to } a - c' - \underline{e}', & \text{to } f - \underline{a} - c \end{array}$$

In such cases, then, we must have recourse to other transitions by larger intervals, as:

$$\begin{array}{ll} g - \underline{b} - d' & \text{or } g - \underline{b} - d' \\ \text{to } \underline{e} - \underline{a} - c', & \text{to } \underline{a} - c - f \end{array}$$

which avoid consecutive Fifths.

Hence when the chords are closely connected by near relationship and small distance in the scale, consecutive Fifths do not

present themselves. When they occur, therefore, they are always signs of abrupt chordal transition, and it is then better to assimilate the progression of parts to that which spontaneously arises in the case of related chords.

This consideration respecting consecutive Fifths, which was emphasised by Hauptmann, appears to give the law greater importance. That it is not the only motive for the prohibition of consecutive Fifths appears from the fact, that the forbidden sequence

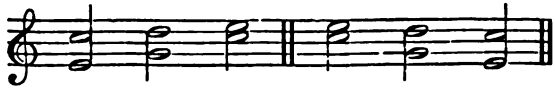
$$g - \underline{b} - d' \quad \text{to} \quad f - \underline{a} - c'$$

is allowed, when the chords are in the position

$$\underline{b} - d' - g \quad \text{to} \quad \underline{a} - c' - f',$$

although the step in the fundamental bass is the same.

The prohibition of so-called *hidden Fifths* and *Octaves* has been added on to the prohibition of consecutive Fifths and Octaves, at least for the two extreme voices of a composition in several parts. This prohibition forbids the lowest and uppermost voice in a piece to proceed by direct motion (that is, both parts ascending or both parts descending) into the consonance of an Octave or Fifth. They should rather come into such a consonance by contrary motion (one descending and the other ascending). In duets this would also hold for the unison. The meaning of this law must certainly be, that whenever the extreme voices unite to form the partial tones of a compound, they ought to have reached a state of relative rest. It must be conceded that the equilibrium will be more perfect when the extreme parts of the whole mass of tone approach their junction from opposite sides, than when the centre of gravity, so to speak, of the sonorous mass is displaced by the parallel motion of the extreme voices, and these voices catch one another up with different velocities. But where the motion proceeds in the same direction, and no relative rest is intended, the hidden Fifths are also not avoided, as in the usual formulae:



in which the $g - d$ is reached by passages involving hidden Fifths.

Another rule in the progression of parts, prohibiting *false*

relations, must have had its origin in the requirements of the singer. What the singer finds a difficulty in hitting, must naturally also appear an unusual and forced skip to the hearer. By *false relations* is meant the case when two tones in consecutive chords, which belong to different voices, form false Octaves or false Fifths. For example, if one voice in the first chord sings *b* and another voice in the next chord sings *b♭*, or the first has *c* and the second *c♯*, there are false Octave relations. False Fifth relations are forbidden for the extreme voices only. Thus in the first chord the bass has *b*, in the second the soprano has *f*, or conversely, where *b* — *f* is a false Fifth. The meaning of this rule is, probably, that the singer would find it difficult to hit the new tone which is not in the scale, if he had just heard the next nearest tone of the scale given by another singer. Similarly, when he has to take the false Fifth of a tone which is prominent in present harmony as lowest or highest. There is therefore a certain sense in the prohibition, but numerous exceptions have arisen as the ear of modern musicians, singers and hearers, has become accustomed to bolder combinations and livelier progressions. All these rules were essentially intended for the old ecclesiastical music, where a quiet, gentle, well-contrived, and well-adjusted stream of sound was aimed at, without any intentional effort or disturbance of the smoothest equilibrium. Where music has to express effort and excitement, these rules become meaningless. Hidden Fifths and Octaves and even false relations of Fifths are found in abundance in the chorales of Sebastian Bach, who is otherwise so strict in his harmonies, but it must be admitted that the motion of his voices is much more powerfully expressed than in the old Italian ecclesiastical music.

CHAPTER XIX.

ESTHETICAL RELATIONS.

LET us review the results of the preceding investigation.

Compound tones of a certain class are preferred for all kinds of music, melodic or harmonic; and are almost exclusively employed for the more delicate and artistic development of music: these are the compound tones which have harmonic upper partial tones, that is compound tones in which the higher partial tones have vibrational numbers which are integral multiples of the vibrational number of the lowest partial tone, or prime. For a good musical effect we require a certain moderate degree of force in the five or six lowest partial tones, and a low degree of force in the higher partial tones.

This class of compound tones with harmonic upper partials is objectively distinguished by including all resonant motions which are generated by a mechanical process that continues to act uniformly, and which consequently produce a uniform and sustained sensation. In the first rank among them stand the compound tones of the human voice, man's first musical instrument in time and value. The compound tones of all wind and bowed instruments belong to this class.

Among the bodies which are made to emit tones by striking, some, as strings, have also harmonic upper partials, and these can be used for artistic music.

The greater number of the rest, as membranes, rods, plates, &c., have inharmonic upper partial tones, and only such of them as have not very strong secondary tones of this kind can be singly and occasionally employed in connection with musical instruments proper.

Although resonant bodies excited by blows may continue to sound for some time, their tones do not continue with uniform force, but diminish more or less slowly and die away. Constant power over the intensity of tone, therefore, which is indispensable for expressive performance, can only be attained on instruments of the

first kind, which can be maintained in a state of excitement, and which produce only harmonic upper partial tones. On the other hand, bodies excited by blows have a peculiar value for clearly defining the rhythm.

A second reason for preferring compound tones with harmonic upper partials is subjective and conditioned by the construction of our ear. In the ear even every simple tone, if sufficiently intense, excites feeble sensations of harmonic upper partials, and each combination of several simple tones generates combinational tones, as I have explained at the end of Chap. VII. (p. 237–9). A single compound tone with irrational partials, when sounded with sufficient force, produces the sensation of dissonance, and simple tones acquire in the ear itself something of the nature of composition out of harmonic upper partial tones.

We are justified in assuming that historically all music was developed from song. Afterwards the power of producing similar melodic effects was attained by means of other instruments, which had a quality of tone compounded in a manner resembling that of the human voice. The reason why, even when constructive art was most advanced, the choice of musical instruments was necessarily limited to those which produced compound tones with harmonic upper partials, is clear from the above conditions.

This invariable and peculiar selection of instruments makes us perfectly certain that harmonic upper partials have from all time played an essential part in musical constructions, not merely for harmony, as the second part of this book shews, but also for melody.

Again, we can at any moment convince ourselves of the essential importance of upper partial tones to melody, by the absence of all expression in melodies executed with objectively simple tones, as, for example, those of wide-stopped organ pipes, for which the harmonic upper partials are formed only subjectively and weakly in the ear.

A necessity was always felt for music of all kinds to proceed by certain definite degrees of pitch; but the choice of these degrees was long unsettled. To distinguish small differences of pitch and intonate them with certainty, requires a greater amount of technical musical power and cultivation of ear, than when the intervals are larger. Hence among almost all uncivilised people we find the Semitones neglected, and only the larger intervals

retained. For some of the more cultivated nations, as the Chinese and Gaels, a scale of this kind has become established.

It might perhaps have seemed most simple to make all such degrees of pitch of equal amount, that is, equally well distinguishable by our sensations. Such a graduation is possible for all our sensations, as Fechner has shewn in his investigations on psychophysical laws. We find such graduations used for the divisions of musical rhythm, and the astronomers use them in reference to the intensity of light in determining stellar magnitudes. Even in the field of musical pitch, the modern equally tempered chromatic scale presents us with a similar graduation. But although in certain of the less usual Greek scales and in modern oriental music, cases occur where some particular small intervals have been divided on the principle of equal graduations, yet there seems at no time or place to have been a system of music in which melodies constantly moved in equal degrees of pitch, but smaller and larger intervals have always been mixed in the musical scales in a way that must appear entirely arbitrary and irregular until the relationship of compound tones is taken into consideration.

On the contrary, in all known musical systems the intervals of Octave and Fifth have been decisively emphasised. Their difference is the Fourth, and the difference between this and the Fifth, is the Pythagorean major Tone 8 : 9, by which (but not by the Fourth or Fifth) the Octave might be approximatively divided.

The sole remnants that I can find in modern music of the endeavour sometimes made in homophonic music to introduce degrees depending on equality of interval and not on relationship of tone, are the chromatic intercalated notes, and the leading note of the key when similarly used. But this is always a Semitone, an interval well known in the series of related tones, which, owing to its smallness, is easily measured by the sensation of its difference, even in places where its tonal relationship is not immediately sensible.

The decisive importance acquired by the Octave and Fifth in all musical scales from the earliest times shews, that the construction of scales must have been originally influenced by another principle, which finally became the sole regulator of every artistic form of a complete scale. This is the principle which we have termed *tonal relationship.*

Relationship in the first degree between two compound tones consists in their each having a partial tone of the same pitch.

In singing, the similarity of two musical tones which stand in the relation of Octave or Fifth to one another, must have been very soon observed. As already remarked, this gives also the Fourth, which has itself a sufficiently perceptible natural relationship to have been remarked independently. To discover the tonal similarity of the major Third and major Sixth, required a finer cultivation of the musical ear, and perhaps also peculiar beauty of voice. Even yet we are easily led by the familiar sharp major Thirds of equal temperament, to endure any major Thirds which are somewhat too sharp, provided they occur melodically and are not sounded together. On the other hand, we must not forget that the rules of Archytas and Abdul Kadir, although both applicable to homophonic music only, gave a preference to the natural major Third, although its introduction obliged both musicians to renounce a musical system so theoretically consistent and invested with such high authority as that of Pythagoras.

Hence the principle of tonal relationship did not at all times exclusively determine the construction of the scale, and does not even yet determine it exclusively among all nations. This principle must, therefore, be regarded to some extent as a *freely selected principle of style*, as I have endeavoured to shew in Chapter XIII. But, on the other hand, the art of music in Europe was historically developed from that principle, and on this fact depends the main proof that it was really as important as we have assumed it to be. The preference first given to the *diatonic scale*, and finally the exclusive use of that scale, introduced the principle of tonal relationship in all its integrity into the musical scale. Within the diatonic scale various methods of execution were possible, and these generated the ancient modes, which had equal claims to attention in homophonic song, and hence stood on a level.

But the principle of tonal relationship penetrated far deeper in its harmonic than it did in its melodic form. In melodic sequence the identity of two partial tones is a matter of memory, but when the notes are sounded together the immediate sensible impression of the beats or of the undisturbed flow of sound forces itself on the hearer's attention. The liveliness of melodic and harmonic impressions differs in the same way as a recollected image differs from the actual impression made by the original. As an immediate

consequence arose that far superior sensibility for the correctness of the intervals which is seen in the harmonic union of tones, and which admitted of being developed into the finest physical methods of measurement.

It must also be remembered that relationship in the second degree can in harmonic music be reduced to audible relationships of the first degree, by a proper selection of the fundamental bass, and that generally more distant relationships can easily be made clearly audible. By this means, notwithstanding the variety of progression, a much clearer connection of all parts with their origin, the tonic, can be maintained and rendered sensible. It cannot be doubted that these are the essential foundations of the great breadth and wealth of expression which modern compositions can attain without losing their artistic unity.

We then saw that the requirements of harmonic music reacted in a peculiar manner on the construction of scales; that properly speaking only one of the old tonal modes (our major mode), could be retained unaltered, and that the rest after undergoing peculiar modifications were fused into our minor mode, which, though most like the ancient mode of the minor Third, can at one time resemble the mode of the minor Sixth, and at another time that of the minor Seventh, but does not perfectly correspond with any one of these.

This process of the development of the elements of our modern musical system lasted down to the middle of the last century. It was not until composers ventured to put a minor chord at the close of compositions written in the minor mode, that the musical feeling of European musicians and hearers can be admitted to have become perfectly and surely habituated to the new system. The minor chord was allowed to be a real, although obscured, chord of its tonic.

Whether this admission of the minor chord expressed a feeling for another mode of unifying its three tones, as A. von Oettingen has assumed,—relying on the fact that the three tones $c - \bar{e}\flat - g$ have a common upper partial g' (p. 478),—must be left to future experience to decide, should it be found practicable to construct long and well-connected musical compositions in *Oettingen's phonic system* (this is the name which he gives to the minor system which he has theoretically developed, and which is essentially different from the historical minor mode). At any rate, the minor mode has historically developed itself as a compromise

between different kinds of claims. Thus it is only major triads which can perfectly indicate the compound tone of the tonic; minor chords contain in their Third an element which, although nearly related to the tonic and its Fifth, does not thoroughly fuse with them, and hence in their final cadence they do not so thoroughly agree with the principle of tonality which had ruled the previous development of music. I have endeavoured to make it probable that the peculiar esthetic expression of the minor mode proceeded partly from this cause and partly from the heterogeneous combinational tones of the minor chord.

In the last part of my book, I have endeavoured to shew that the construction of scales and of harmonic tissue is a product of artistic invention, and by no means furnished by the natural formation or natural function of our ear, as it has been hitherto most generally asserted. Of course the laws of the natural function of our ear play a great and influential part in this result; these laws are, as it were, the building stones with which the edifice of our musical system has been erected, and the necessity of accurately understanding the nature of these materials in order to understand the construction of the edifice itself, has been clearly shewn by the course of our investigations upon this very subject. But just as people with differently directed tastes can erect extremely different kinds of buildings with the same stones, so also the history of music shews us that the same properties of the human ear could serve as the foundation of very different musical systems. Consequently it seems to me that we cannot doubt, that not merely the composition of perfect musical works of art, but even the construction of our system of scales, keys, chords, in short of all that is usually comprehended in a treatise on Thorough Bass, is the work of artistic invention, and hence must be subject to the laws of artistic beauty. In point of fact, mankind has been at work on the diatonic system for more than 2,500 years since the days of Terpander and Pythagoras, and in many cases we are still able to determine that the progressive changes in the tonal system have been due to the most distinguished composers themselves, partly by their own independent inventions, and partly by the sanction which they gave to the inventions of others, by employing them artistically.

The esthetic analysis of complete musical works of art, and the comprehension of the reasons of their beauty, encounter ap-

parently invincible obstacles at almost every point. But in the field of elementary musical art we have now gained so much insight into its internal connection that we are able to bring the results of our investigations to bear on the views which have been formed and in modern times nearly universally accepted respecting the cause and character of artistic beauty in general. It is, in fact, not difficult to discover a close connection and agreement between them; nay, there are probably fewer examples more suitable than the theory of musical scales and harmony, to illustrate the darkest and most difficult points of general esthetics. Hence I feel that I should not be justified in passing over these considerations, more especially as they are closely connected with the theory of sensual perception, and hence with physiology in general.

No doubt is now entertained that beauty is subject to laws and rules dependent on the nature of human intelligence. The difficulty consists in the fact that these laws and rules, on whose fulfilment beauty depends and by which it must be judged, are not consciously present to the mind, either of the artist who creates the work, or the observer who contemplates it. Art works with design, but the work of art ought to have the appearance of being undesigned, and must be judged on that ground. Art creates as imagination pictures, regularly without conscious law, designedly without conscious aim. A work, known and acknowledged as the product of mere intelligence, will never be accepted as a work of art, however perfect be its adaptation to its end. Whenever we see that conscious reflection has acted in the arrangement of a whole work of art, we find it poor.

> Man fühlt die Absicht, und man wird verstimmt.
> (We feel the purpose, and it jars upon us).

And yet we require every work of art to be reasonable, and we shew this by subjecting it to a critical examination, by seeking to enhance our enjoyment and our interest by tracing out the suitability, connection, and equilibrium of all its separate parts. The more we succeed in making the harmony and beauty of all its peculiarities clear and distinct, the richer we find it, and we even regard as the principal characteristic of a great work of art that deeper thought, reiterated observation, and continued reflection shew us more and more clearly the reasonableness of all its individual parts. Our endeavour to

comprehend the beauty of such a work by critical examination, in which we partly succeed, shews that we assume a certain adaptation to reason in works of art, which may possibly rise to a conscious understanding, although such understanding is neither necessary for the invention nor for the enjoyment of the beautiful. For what is esthetically beautiful is recognised by the immediate judgment of a cultivated taste, which declares it pleasing or displeasing, without any comparison whatever with law or conception.

But that we do not accept delight in the beautiful as something individual, but rather hold it to be in regular accordance with the nature of mind in general, appears by our expecting and requiring from every other healthy human intellect the same homage that we ourselves pay to what we call beautiful. At most we allow that national or individual peculiarities of taste incline to this or that artistic ideal, and are most easily moved by it, precisely in the same way that a certain amount of education and practice in the contemplation of fine works of art is undeniably necessary for penetration into their deeper meaning.

The principal difficulty in pursuing this object, is to understand how regularity can be apprehended by contemplation without being consciously felt to exist. And this unconsciousness of regularity is not a mere accident in the effect of the beautiful on our mind, which may indifferently exist or not; it is, on the contrary, most clearly, prominently, and essentially important. For through apprehending everywhere traces of regularity, connection, and order, without being able to grasp the law and plan of the whole, there arises in our mind a feeling that the work of art which we are contemplating is the product of a design which far exceeds anything we can conceive at the moment, and which hence partakes of the character of the illimitable. Remembering the poet's words:

> Du gleichst dem Geist, den du begreifst,
> (Thou'rt like the spirit thou conceivest),

we feel that those intellectual powers which were at work in the artist, are far above our conscious mental action, and that were it even possible at all, infinite time, meditation, and labour would have been necessary to attain by conscious thought that degree of order, connection, and equilibrium of all parts and all internal relations, which

the artist has accomplished under the sole guidance of tact and taste, and which we have in turn to appreciate and comprehend by our own tact and taste, long before we begin a critical analysis of the work.

It is clear that all high appreciation of the artist and his work reposes essentially on this feeling. In the first we honour a genius, a spark of divine creative fire, which far transcends the limits of our intelligent and conscious forecast. And yet the artist is a man as we are, in whom work the same mental powers as in ourselves, only in their own peculiar direction, purer, brighter, steadier; and by the greater or less readiness and completeness with which we grasp the artist's language we measure our own share of those powers which produced the wonder.

Herein is manifestly the cause of that moral elevation and feeling of ecstatic satisfaction which is called forth by thorough absorption in genuine and lofty works of art. We learn from them to feel that even in the obscure depths of a healthy and harmoniously developed human mind, which are at least for the present inaccessible to analysis by conscious thought, there slumbers a germ of order that is capable of rich intellectual cultivation, and we learn to recognise and admire in the work of art, though executed in unimportant material, the picture of a similar arrangement of the universe, governed by law and reason in all its parts. The contemplation of a real work of art awakens our confidence in the originally healthy nature of the human mind, when uncribbed, unharassed, unobscured, and unfalsified.

But for all this it is an essential condition that the whole extent of the regularity and design of a work of art should *not* be apprehended consciously. It is precisely from that part of its regular subjection to reason, which escapes our conscious apprehension, that a work of art exalts and delights us, and that the chief effects of the artistically beautiful proceed, *not* from the part which we are able fully to analyse.

If we now apply these considerations to the system of musical tones and harmony, we see of course that these are objects belonging to an entirely subordinate and elementary domain, but nevertheless they, too, are slowly matured inventions of the artistic taste of musicians, and consequently they, too, must be governed by the general rules of artistic beauty. Precisely because we are here still treading the lower walks of art, and are not dealing with the expression of deep psychological problems, we are able to discover a

comparatively simple and transparent solution of that fundamental enigma of esthetics.

The whole of the last part of this book has explained how musicians gradually discovered the relationships between tones and chords, and how the invention of harmonic music rendered these relationships closer, and clearer, and richer. We have been able to deduce the whole system of rules which constitute Thorough Bass, from an endeavour to introduce a clearly sensible connection into the series of tones which form a piece of music.

A feeling for the melodic relationship of consecutive tones, was first developed, commencing with Octave and Fifth and advancing to the Third. We have taken pains to prove that this feeling of relationship was founded on the perception of identical partial tones in the corresponding compound tones. Now these partial tones are of course present in the sensations excited in our auditory apparatus, and yet they are not generally the subject of conscious perception as independent sensations. The conscious perception of everyday life is limited to the apprehension of the tone compounded of these partials, as a whole, just as we apprehend the taste of a very compound dish as a whole, without clearly feeling how much of it is due to the salt, or the pepper, or other spices and condiments. A critical examination of our auditory sensations as such was required before we could discover the existence of upper partial tones. Hence the real reason of the melodic relationship of two tones (with the exception of a few more or less clearly expressed conjectures, as, for example, by Rameau and d'Alembert) remained so long undiscovered, or at least was not in any respect clearly and definitely formulated. I believe that I have been able to furnish the required explanation, and hence clearly to exhibit the whole connection of the phenomena. The esthetic problem is thus referred to the common property of all sensual perceptions, namely, the apprehension of compound aggregates of sensations as sensible symbols of simple external objects, without analysing them. In our usual observations on external nature our attention is so thoroughly engaged by external objects that we are entirely unpractised in taking as the subjects of conscious observation, any properties of our sensations themselves, which we do not know as the sensible expression of some individual external object or event.

After musicians had long been content with the melodic relationship of tones, they began in the middle ages to make use of

harmonic relationship as shewn in consonance. The effects of various combinations of tones also depend partly on the identity or difference of two of their different partial tones, but they likewise partly depend on their combinational tones. But whereas in melodic relationship the equality of the upper partial tones can only be perceived by *remembering* the preceding compound tone, in harmonic relationship it is determined by *immediate sensation*, by the presence or absence of beats. Hence in harmonic combinations of tone, tonal relationship is felt with that greater liveliness due to a present sensation as compared with the recollection of a past sensation. The wealth of clearly perceptible relations grows with the number of tones combined. Beats are easy to recognise as such when they occur slowly; but those which characterise dissonances are, almost without exception, very rapid, and are partly covered by sustained tones which do not beat, so that a careful comparison of slower and quicker beats is necessary to gain the conviction that the essence of dissonance consists precisely in rapid beats. Slow beats do not create the feeling of dissonance, which does not arise till the rapidity of the beats confuses the ear and makes it unable to distinguish them. In this case also the ear feels the difference between the undisturbed combination of sound in the case of two consonant tones, and the disturbed rough combination resulting from a dissonance. But, as a general rule, the hearer is then perfectly unconscious of the cause to which the disturbance and roughness are due.

The development of harmony gave rise to a much richer development of musical art than was previously possible, because the far clearer characterisation of related combinations of tones by means of chords and chordal sequences, allowed of the use of much more distant relationships than were previously available, by modulating into different keys. In this way the means of expression greatly increased as well as the rapidity of the melodic and harmonic transitions which could now be introduced without losing the connection.

As the independent significance of chords came to be appreciated in the fifteenth and sixteenth centuries, a feeling arose for the relationship of chords to one another and to the tonic chord, in accordance with the same law which had long ago unconsciously regulated the relationship of compound tones. The latter depended on the identity of two or more partial tones, the former on the

identity of two or more notes. For the musician, of course, the law of the relationship of chords and keys is much more intelligible than that of compound tones. He readily hears the identical tones, or sees them in the notes before him. But the unprejudiced and uninstructed hearer is as little conscious of the reason of the connection of a clear and agreeable series of fluent chords, as he is of the reason of a well-connected melody. He is startled by a false cadence and feels its unexpectedness, but is not at all necessarily conscious of the reason of its unexpectedness.

Then, again, we have seen that the reason why a chord in music appears to be the chord of a determinate root, depends once more upon the analysis of a compound tone into its partial tones, that is, once more upon those elements of a sensation which cannot readily become subjects of conscious perception. This relation between chords is of great importance, both in the relation of the tonic chord to the tonic tone, and in the sequence of chords.

The recognition of these resemblances between compound tones and between chords, reminds us of another, an exactly analogous circumstance which we must have often experienced. We recognise the resemblance between the faces of two near relations, without being at all able to say in what the resemblance consists, especially when age and sex are different, and the coarser outlines of the features consequently present striking differences. And yet notwithstanding these differences; and notwithstanding that we are unable to fix upon a single point in the two countenances which is absolutely alike, the resemblance is often so extraordinarily striking and convincing, that we have not a moment's doubt about it. Precisely the same thing occurs in recognising the resemblance between two compound tones.

Again, we are often able to assert with perfect certainty, that a passage not previously heard is due to a particular author or composer whose other works we know. Occasionally, but by no means always, individual mannerisms in verbal or musical phrases determine our judgment, but as a rule we are mostly unable to fix upon the exact points of resemblance between the new piece and the known works of the author or composer.

The analogy of these different cases may be even carried farther. When a father and daughter are strikingly alike in some well-marked feature, as the nose or forehead, we observe it at once, and think no more about it. But if the resemblance is so enigmatically

concealed that we cannot detect it, we are fascinated, and cannot help continuing to compare their countenances. And if a painter drew two such heads having, say, a somewhat different expression of character combined with a predominant and striking, though indefinable, resemblance, we should undoubtedly value it as one of the principal beauties of his painting. Our admiration would certainly not be due merely to his technical skill; we should rather look upon his painting as evidencing an unusually delicate feeling for the significance of the human countenance, and find in this the artistic justification of his work.

Now the case is similar for musical intervals. The resemblance of an Octave to its root is so great and striking that the dullest ear perceives it; the Octave seems to be almost a pure repetition of the root, as it, in fact, merely repeats a part of the compound tone of its root, without adding anything new. Hence the esthetical effect of an Octave is that of a perfectly simple, but little attractive interval. The most attractive of the intervals, melodically and harmonically, are clearly the Thirds and Sixths, —the intervals which lie at the very boundary of those that the ear can grasp. The major Third and the major Sixth cannot be properly appreciated unless the first five partial tones are audible. These are present in good musical qualities of tone. The minor Third and the minor Sixth are for the most part justifiable only as inversions of the former intervals. The more complicated intervals in the scale have no longer any direct or easily intelligible relationship. They have no longer the charm of the Thirds.

Moreover, it is by no means a merely external indifferent regularity which the employment of diatonic scales, founded on the relationship of compound tones, has introduced into the tonal material of music, just as rhythm introduced an external arrangement into the words of poetry. I have shewn, on the contrary, in Chapter XIV., that this construction of the scale furnished a means of measuring the intervals of their tones, so that the equality of two intervals lying in different sections of the scale would be recognised by immediate sensation. Thus the melodic step of a Fifth is always characterised by having the second partial tone of the second note identical with the third of the first. This produces a definiteness and certainty in the measurement of intervals for our sensation, such as might be looked for in vain in the system of colours, otherwise so similar,

or in the estimation of mere differences of intensity in our various sensual perceptions.

Upon this reposes also the characteristic resemblance between the relations of the musical scale and of space, a resemblance which appears to me of vital importance for the peculiar effects of music. It is an esssential character of space that at every position within it like bodies can be placed, and like motions can occur. Everything that is possible to happen in one part of space is equally possible in every other part of space and is perceived by us in precisely the same way. This is the case also with the musical scale. Every melodic phrase, every chord, which can be executed at any pitch, can be also executed at any other pitch in such a way that we immediately perceive the characteristic marks of their similarity. On the other hand, also, different voices, executing the same or different melodic phrases, can move at the same time within the compass of the scale, like two bodies in space, and, provided they are consonant in the accented parts of bars, without creating any musical disturbances. Such a close analogy consequently exists in all essential relations between the musical scale and space, that even alteration of pitch has such a readily recognised and unmistakable resemblance to motion in space, that it is often metaphorically termed the ascending or descending *motion* or *progression* of a part. Hence, again, it becomes possible for motion in music to imitate the peculiar characteristics of motive forces in space, that is, to form an image of the various motives and forces which lie at the root of motion. And on this, as I believe, depends the power of music to picture emotion.

It is not my intention to deny that music in its initial state and simplest forms may have been originally an artistic imitation of the instinctive modulations of the voice that correspond to various conditions of the feelings. But I cannot think that this is opposed to the above explanation; for a great part of the natural means of vocal expression may be reduced to such facts as that its rhythm and accentuation are an immediate expression of the rapidity or force of the corresponding psychical motives, that effort drives the voice up, that desire to make a pleasant impression on another selects a softer, pleasanter quality of tone, and so forth. An endeavour to imitate the involuntary modulations of the voice and make its recitation richer and more expressive, may

therefore very possibly have led our ancestors to the discovery of the first means of musical expression, just as the imitation of weeping, shouting, or sobbing, and other musical delineations may play a part in even cultivated music, (as in operas), although such modifications of the voice are not confined to the action of free mental motives, but embrace really mechanical and even involuntary muscular contractions. But it is quite clear that every completely developed melody goes far beyond an imitation of nature, even if we include the cases of the most varied alteration of voice under the influence of passion. Nay, the very fact that music introduces progression by fixed degrees both in rhythm and in the scale, renders even an approximatively correct representation of nature simply impossible, for most of the passionate affections of the voice are characterised by a gliding transition in pitch. The imitation of nature is thus rendered as imperfect as the imitation of a picture by embroidery on a canvas with separate little squares for each shade of colour. Music, too, departed still further from nature when it introduced the greater compass, the mobility, and the strange qualities of tone belonging to musical instruments, by which the field of attainable musical effects has become so much wider than it was or could be when the human voice alone was employed.

Hence though it is probably correct that mankind, in historical development, first learned the means of musical expression from the human voice, it can hardly be denied that these same means of expressing melodic progression act, in artistically developed music, without the slightest reference to the application made of them in the modulations of the human voice, and have a more general significance than any that can be attributed to innate instinctive cries. That this is the case appears above all in the modern development of instrumental music, which possesses an effective power and artistic justification that need not be gainsaid, although we may not yet be able to explain it in all its details.

Here I close my work. It appears to me that I have carried it as far as the physiological properties of the sensation of hearing exercise a direct influence on the construction of a musical system, that is, as far as the work especially belongs to natural philosophy.

APPENDICES.

APPENDIX I.

On an Electro-Magnetic Driving Machine for the Siren.

(See p. 20.)

I have lately had a small electro-magnetic machine constructed with a constant velocity of rotation, and it has proved of great service in driving the siren. A rotating electro-magnet, in which the direction of current is changed every semi-rotation, moves between two fixed magnetic poles. The current in this electro-magnet is interrupted, as soon as the velocity begins to exceed the desired amount, by the centrifrugal force of a mass of metal fastened to its axis of rotation. Two spiral springs whose elasticity is opposed to the centrifrugal force, may be tightened or loosened, and thus made stronger or weaker at pleasure. By this means the velocity can be maintained at any required rate. A figure and description of this machine were given by Herr S. Exner, in the 'Proceedings' (*Sitzungsberichte*) of the Vienna Academy; Math. Naturw. Cl. vol. LVII. part II., 8 Oct. 1868.

The siren is connected with the machine by a thin driving band, and then it does not require to be blown. Instead of blowing, I placed on the disc a small turbine constructed of stiff paper, which drove the air through the openings whenever they coincided with those in the chest. This arrangement gave me extremely constant tones on the siren, rivalling those on the best constructed organ pipes.

APPENDIX II.

On the Size and Construction of Resonators.

(See p. 69.)

Spherical Resonators with a short funnel-shaped neck for insertion into the ear as shewn in fig. 16 a (p. 68), are most efficient. Their advantage consists partly in their other proper tones being very distant indeed from their prime tones, and hence being very slightly reinforced, and partly in the spherical form giving the most powerful resonance. But the walls of the sphere must be firm and smooth, to oppose the necessary

resistance to the powerful vibrations of air which take place within them, and to impede the motion of the air as little as possible by friction. At first I employed any spherical glass vessels that came to hand, as the receivers of retorts, and inserted into one of their openings a glass tube which had been adapted to my ear. Afterward Herr R. Koenig (maker of acoustical instruments, Paris, 30 rue Hautefeuille [all of whose apparatus can be obtained through Mr. Ladd, Beak Street, Regent Street, London]), constructed a series of these glass spheres properly tuned. The following is a table of their size:

Pitch	Diameter of the Sphere in millimetres [and inches]	Diameter of the Orifice in millimetres [and inches]	Volumes of the Interior in cubic centimetres [and cub. inches]	Remarks
1) *g*	154 [6·06]	35·5 [1·40]	1773 [108·19]	Neck funnel-shaped
2) *b*b	131 [5·16]	28·5 [1·12]	1092 [66·64]	
3) *c'*	130 [5·12]	30·2 [1·19]	1053 [64·24]	
4) *e'*	115 [4·53]	30 [1·18]	546 [33·32]	
5) *g'*	79 [3·11]	18·5 [·73]	235 [14·341]	
6) *b'*b	76 [2·99]	22 [·87]	214 [13·06]	
7) *c''*	70 [2·76]	20·5 [·81]	162 [9·89]	
8) *b'*b	53·5 [2·11]	8 [·31]	74 [4·52]	Neck cylindrical
9) *b''*b	46 [1·81]	15 [·59]	49 [2·99]	Neck cylindrical, mouth at side
10) *d'''*	43 [1·69]	15 [·59]	37 [2·26]	Neck cylindrical

Smaller spheres did not answer well. Latterly metal resonators of the form of fig. 16 a, p. 68, were made for me by Herr Koenig. I broke two glass spheres lying between *c'* and *b''*b, which I endeavoured to replace by cylindrical tubes resembling fig. 16 b (p. 68). Their dimensions were as follows:

No. Pitch	Length in millimetres [and inches]	Width in millimetres [and inches]	Contents in cubic centimetres [and cub. inches]	Remarks
1) *d''*	133 [5·24]	25 [·98]	56 [3·42]	Half covered
2) *f''*	123 [4·84]	21 [·83]	30 [1·83]	ditto
3) *g''*b	114 [4·49]	24 [·94]	50 [3·05]	ditto
4) *a''*b	125 [4·92]	20 [·79]	39 [2·38]	Open

For very low tones I used tubes of pasteboard, of which one end had a circular opening and the other or closed end was fitted with a glass tube adapted to the ear. Those I used had the following dimensions:

No. Pitch	Length of Tube in millimetres [and inches]	Diameter of Tube in millimetres [and inches]	Diameter of Orifice in millimetres [and inches]
5) *B*b	690 [27·16]	96 [3·78]	73 [2·87]
6) *d*b	480 [18·90]	60 [2·36]	23 [·91]

In tubular resonators, however, the second proper tone, which is nearly the Fifth of its prime tone, may also become evident.

Conical tubes of tin, which were made for me by Herr Appun in Hanau, are easy to construct, and applicable for most purposes; but they also reinforce all the harmonic upper partials of their prime tone.

Resonators with a very narrow opening generally produce a much more considerable reinforcement of the tone, but then there must be a much more precise agreement between the pitch of the tone to be heard, and the proper tone of the resonator. It is just as in microscopes; the greater the magnifying power, the smaller the field of view. Reducing the size of the orifice also deepens the pitch of the resonator, and this gives an easy means of tuning it to any required pitch. But, for the above reason, the opening must not be reduced too much.

I should also mention Herr Koenig's plan of transferring vibrations of air to gas flames, and thus making them visible. Flames of this sort act well when connected with resonators, which are then best made of a spherical form, and should have two equal openings. To one of the openings the small gas-chamber is fixed. This chamber is a small flat box, about big enough to contain two shilling pieces laid flat on one another. It is cut out of a plate of wood, and closed on the side next the resonator with a very thin membrane of india-rubber, which, while it completely separates the air of the resonator from the gas in the chamber, allows the vibrations of the air to be freely communicated to the gas. Through the plate of wood two narrow pipes enter the chamber, one introducing the inflammable gas and the other conducting it away. This last ends in a very fine point at which the gas is lighted. The vibrations of the air in the resonator being communicated to the gas cause the flame to leap up and down. These oscillations of the flame are so rapid and regular that, when viewed directly, the flame appears to be quite steady. Its altered condition, however, betrays itself by an altered form and colour. Thus to recognise the beats of two tones reinforced by the resonator, it is enough to look at the flame and observe how it alternates between its forms of rest and of oscillation. But to see the separate oscillations the flames should be viewed in a rotating glass, in which the flame at rest appears to be drawn out into long uniform ribbon, while the oscillating flame appears as a series of separate images of flames. It is thus possible to allow a large number of persons at once to determine whether or not a given tone is reinforced by the resonator.[1]

[1] [All these instruments and appliances can be obtained of Herr Koenig, by whom they were exhibited in London at the International Exhibition of 1872. Large drawings of the appearances of the flames just described as viewed in the rotating mirror while two octaves were sung to the French vowels, were also exhibited. See Koenig's paper on the subject, with plates, in Philosophical Magazine, 1873, vol. 45, pp. 1–18, 105–114.—On the principles of the use of revolving mirrors, first experimentally used by Sir C. Wheatstone, see Donkin's 'Acoustics,' 1870, p. 142.—*Translator.*]

APPENDIX III.

ON THE MOTION OF PLUCKED STRINGS.

(See p. 83.)

LET x be the distance of a point in the string from one of its extremities, and l the length of the string, so that for one extremity $x=0$, and for the other $x=l$. It is sufficient to investigate the case for which the motion takes place in one plane passing through the position of rest. Let y be the distance of the point x from its position of rest at the time t. And let μ be the weight of the unit of length, and T the tension of the string. The differential equation of motion is then

$$\mu.\frac{d^2y}{dt^2} = T.\frac{d^2y}{dx^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Then, since the extremities of the string are assumed to be at rest, we must have

$$y=0 \text{ when } x=0, \text{ and also when } x=l \quad \ldots\ldots\ldots \quad (1a)$$

The most general integral of the equation (1) which fulfils the conditions (1a) and corresponds to a periodic motion of the string, is

$$\left.\begin{aligned} y=A_1.\sin\frac{\pi x}{l}.\cos 2\pi nt + A_2.\sin\frac{2\pi x}{l}.\cos 4\pi nt \\ + A_3.\sin\frac{3\pi x}{l}.\cos 6\,\pi nt + \&c. \\ + B_1.\sin\frac{\pi x}{l}.\sin 2\pi nt + B_2.\sin\frac{2\pi x}{l}.\sin 4\pi nt \\ + B_3.\sin\frac{3\pi x}{l}.\sin 6\pi nt + \&c. \end{aligned}\right\} \ldots \quad (1b)$$

where $n^2 = \frac{T}{4\mu l^2}$ and A_1, A_2, A_3, &c., and B_1, B_2, B_3, &c., are any constant co-efficients, which can be determined when the form and velocity of the string are known for any determinate time t.

For $t=0$, the form of the string will be

$$y=A_1.\sin\frac{\pi x}{l} + A_2.\frac{\sin 2\pi x}{l} + A_3.\sin\frac{3\pi x}{l} + \quad \ldots \quad (1c)$$

and the velocity of the string will be

$$\frac{dy}{dt} = 2\pi n\left(B_1.\sin\frac{\pi x}{l} + 2\,B_2.\sin\frac{2\pi x}{l} + 3\,B_3.\sin\frac{3\pi x}{l}\right) \ldots \quad (1d)$$

Now suppose the string to have been drawn aside by a sharp point, and

that the point was withdrawn at the time $t=0$, so that the vibration commenced at that moment, then for $t=0$ there was no velocity, that is $\frac{dy}{dt}$ was $=0$ for all values of x. This can only be the case when in equation (1d), $0=B_1=B_2=B_3=$&c.

The coefficients A_1, A_2, A_3, &c., depend on the shape of the string at the time $t=0$. At the moment the sharp point quitted it the string must have assumed the position of fig. 18 A (p. 86), that is, it must have formed two straight lines proceeding from the sharp point to the fixed extremities of the string. Supposing the position of the sharp point at that moment to be determined by $x=a$ and $y=b$, then for the time $t=0$, the value of y,

$$\text{if } a>x>0, \quad \text{was } y=\frac{bx}{a} \quad \ldots\ldots\ldots\ldots \quad (2)$$

$$\text{and if } l>x>a, \quad \text{was } y=b.\frac{l-x}{l-a} \quad \ldots\ldots\ldots\ldots \quad (2a)$$

and the values of y in (1c) and (2), or else (2a) respectively, must be identical.

To find the co-efficient A_m, the well-known method is to multiply both sides of the equation (1c) by $\sin\frac{m\pi x}{l}.dx$, and to integrate between the limits $x=0$ and $x=l$. In this case equation (1c) reduces to

$$\int_0^l y.\sin\frac{m\pi x}{l}.dx = A_m.\int_0^l \sin^2\frac{m\pi x}{l}.dx \quad \ldots\ldots\ldots\ldots \quad (2b)$$

in which y must be replaced by its values in (2) and (2a). Performing the integrations indicated in (2b) we find

$$A_m = \frac{2bl^2}{m^2\pi^2 a(l-a)}.\sin\frac{m\pi a}{l} \quad \ldots\ldots\ldots\ldots \quad (3)$$

Hence A_m will $=0$, and consequently the mth tone of the string will disappear when $\sin\frac{m\pi a}{l}=0$, that is, when $a=\frac{l}{m}$ or $=\frac{2l}{m}$, or $=\frac{3}{m}$ &c. Hence if we suppose the string to be divided into m equal parts, and to be plucked in one of these divisions, the mth tone disappears, and this is the tone whose nodes fall upon these points.

Every node for an mth tone is also a node for the $2m$th, $3m$th, $4m$th, &c., tone, and hence all these tones also disappear.

The integral of equation (1) may also, as is well known, be exhibited in the following form :—

$$y=\phi(x-at) + \psi(x+at) \quad \ldots\ldots\ldots\ldots \quad (4)$$

where $a^2 = \frac{T}{\mu}$, and ϕ, ψ are arbitrary functions. The function $\phi(x-at)$ denotes any form of the string which advances in the direction of positive x with the velocity a, but without any other change; and the function $\psi(x+at)$ denotes a similar form proceeding with the same velocity in the direction of negative x. For any given value of the time t we must suppose both functions to be given from $x=-\infty$ to $x=+\infty$, and then the motion of the string is determined.

The determination of the motion of a plucked string will result in this second form of solution, from determining the functions ϕ and ψ, so that

1) for the values $x=0$ and $x=l$, the value of y for any value of t will be constantly $=0$. This will be the case, if for any value of t,

$$\phi(-at) = -\psi(+at) \quad \text{.....} \quad (4a)$$

and

$$\phi(l-at) = -\psi(l+at) \quad \text{.....} \quad (4b)$$

If in the first equation we put $at=-v$, and in the second $l+at=-v$, we obtain

$$\phi(v) = -\psi(-v)$$

and

$$\phi(2l+v) = -\psi(-v)$$

so that

$$\phi(2l+v) = \phi(v) \quad \text{.....} \quad (5)$$

Hence the function ϕ is periodic, for its value becomes the same when its argument is increased by $2l$. The same results for ψ.

2) For $t=0$, we must have $\frac{dy}{dt}=0$ between the limits $x=0$ and $x=l$. Hence writing $\psi'(v)$ for $\frac{d\psi(v)}{dv}$, and putting $\frac{dy}{dt}=0$ in equation (4), we obtain

$$\psi'(x) = \psi'(x)$$

And integrating this with respect to x, we have

$$\phi(x) = \psi(x) + C$$

Now since neither y nor $\frac{dy}{dt}$ are altered by adding the same constant to ϕ and substracting it from ψ, the constant C is perfectly arbitrary, and we may consequently assume it to be $=0$, and hence write

$$\phi(x) = \psi(x) \quad \text{.....} \quad (5a)$$

3) Since finally at the time $t=0$, and within the limits $x=0$, $x=l$, the magnitude

$$y, \text{ which is } = \phi(x) + \psi(x) = 2\phi(x),$$

must have the value shewn in fig. 18 A (p. 86), the ordinates of this figure immediately give the value of $2\phi(x)$ and of $2\psi(x)$, by means of equation (5):—

$$\begin{array}{lll} \text{between } x=0 & \text{and } x=l \\ \quad\text{,,}\quad x=2l & \text{,,}\quad x=3l \\ \quad\text{,,}\quad x=4l & \text{,,}\quad x=5l \end{array}$$

and so forth.

But since from (4a, 4b, 5) it follows that $\phi(-v)=-\phi(v)$, and $\phi(l-v)=-\phi(l+v)$, the value of $2\phi(x)$ is given by the triangle in fig. 18 G (p. 86),

$$\begin{array}{lll} \text{between } x=-l & \text{and } x=0 \\ \quad\text{,,}\quad x=-3l & \text{,,}\quad x=-2l \\ \text{and also between } x=l & \text{,,}\quad x=2l \\ \quad\text{,,}\quad x=3l & \text{,,}\quad x=4l \end{array}$$

and so forth.

By this means the functions ϕ and ψ are completely determined, and on supposing that the two wave-lines proceed in opposite directions with the velocity a, we obtain the forms of the string given in fig. 18, p. 86, which represent the changes of the string for every twelfth part of the periodic time of its vibration.

[See Donkin's 'Acoustics,' Chaps. V. and VI.]

APPENDIX IV.

ON THE PRODUCTION OF SIMPLE TONES BY RESONANCE.

(See pp. 87 and 113.)

I HAVE given the theory of tubes and hollow spaces filled with air, so far as it can be at present mathematically expressed, in my paper, entitled: 'The Theory of Aerial Vibrations in Tubes with open Ends' (*Theorie der Luftschwingungen in Röhren mit offenen Enden*), in Crelle's 'Journal für Mathematik,' vol. lvii. A comparison of the upper partial tones of tuning-forks and the corresponding resonance tubes, will be found in my paper: 'On Combinational Tones' (*Ueber Combinationstöne*), in Poggendorff's 'Annalen,' vol. xcix. pp. 509 and 510.

I add here the dimensions of the resonance tubes mentioned on p. 87, which were made for me by Herr Fessel, in Cologne, in connection with the tuning-forks kept in motion by electricity as described in Appendix VIII. These were cylindrical tubes of pasteboard, with terminal surfaces of tin plate, one entirely closed, the other provided with a circular opening. These tubes therefore had only *one* opening, not two like the resonators

which were intended for insertion in the ear. A resonance tube of this kind can have its tone flattened by diminishing its opening. To sharpen the tone, when necessary, I threw in a little wax, and placed the closed end of the tube on a warm stove or hob, until the wax was melted, and uniformly distributed over the surface. It was then allowed to cool in the same position. To try whether the tone of a tube is a little sharper or flatter than that of the fork, cover its opening slightly while the excited fork is held before it. If the covering strengthens the resonance the tube was too sharp. But if the resonance begins to decrease decidedly as soon as any part of the opening is covered, the tube was too flat. The dimensions in millimetres [and inches] are as follows:—

No.	Pitch	Length of Tube		Diameter of Tube		Diameter of Opening.	
1	*B*b	425	[16·73]	138	[5·43]	31·5	[1·24]
2	*b*b	210	[8·27]	82	[3·23]	23·5	[·93]
3	*f'*	117	[4·61]	65	[2·56]	16	[·63]
4	*b'*b	88	[3·46]	55	[2·17]	14·3	[·56]
5	*d''*	58	[2·28]	55	[2·17]	14	[·55]
6	*f''*	53	[2·09]	44	[1·73]	12·5	[·49]
7	*a''*b	50	[1·97]	39	[1·54]	11·2	[·44]
8	*b''*b	40	[1·57]	39	[1·54]	11·5	[·45]
9	*d'''*	35	[1·38]	30·5	[1·20]	10·3	[·41]
10	*f'''*	26	[1·02]	26	[1·02]	8·5	[·34]

The theory of the sympathetic resonance of strings is best developed by means of the experiments mentioned on page 88. Retaining the notations of Appendix III. assume that the end of the string for which $x=0$, is connected with the stem of the tuning-fork, and must move in the same way, and that its motion is given by the equation

$$y=A.\ \sin mt, \quad \text{for } x=0 \qquad (6)$$

Suppose the other end of the string to rest on the bridge which stands on the sounding board. The following forces act upon the bridge:—

1) The pressure of the string, which will increase or diminish according to the angle under which the extremity of the string is directed against the bridge. The tangent of this angle between the variable direction of the string and its position of rest is $\frac{dy}{dx}$, and hence we can put the variable pressure of the string $= -T.\frac{dy}{dx}$, for $x=l$, supposing the bridge to lie on the side of negative y.

2) The elastic force of the sounding board, which acts to bring the bridge back into its position of rest, may be put $= -f^2y$.

3) The sounding board, which moves with the bridge, is resisted by the air, to which it imparts some of its motion. The resistance of the air

may be considered to be approximatively proportional to the velocity of its motion, and hence be put $= -g^2 \frac{dy}{dt}$.

Then putting M for the mass of the bridge, we obtain the following equation for the motion of the bridge, and hence for the extremity of the string which rests upon it:—

$$M.\frac{d^2y}{dt^2} = -T.\frac{dy}{dx} - f^2y - g^2.\frac{dy}{dt}, \text{ for } x=l \quad \text{.....} \quad (6a)$$

For the motion of the other points in the string, we have, as in Appendix III., the condition

$$\mu.\frac{d^2y}{dt^2} = T.\frac{d^2y}{dx^2} \quad \text{.....} \quad (1)$$

Since part of every motion of the string must be constantly given off to the air in the resonance chamber, the motion would gradually die away if it were not kept up by some continuous cause. Hence we may neglect the variable initial conditions of the motion, and proceed at once to determine the periodic motion, which finally remains constant under the influence of the periodic agitation of the tuning-fork. It is manifest that the period of the motion of the string must be the same as the period of the motion of the fork. Hence the required integral of (1) must be of the form

$$\left.\begin{aligned} y = D.\cos px.\sin mt + E.\cos px.\cos mt \\ + F.\sin px.\sin mt + G.\sin px.\cos mt \end{aligned}\right\} \quad \text{.....} \quad (7)$$

And to satisfy equation (1) we must then have

$$\mu m^2 = Tp^2 \quad \text{.....} \quad (7a)$$

From the equation (7) we have, when $x=0$,

$$y = D.\sin mt + E.\cos mt,$$

and on comparing this with equation (6) we find

$$D = A, \text{ and } E = 0 \quad \text{.....} \quad (8)$$

The two other coefficients of the equation (7), namely F and G, must be determined by means of equation (6a). On substituting in (6a) the values of y from (7), the equation (6a) splits into two, as we must put the sum of the terms multiplied by $\sin mt$ separately $=0$, and also the sum of those multiplied by $\cos mt$ separately $=0$. These two equations are:

$$\left.\begin{aligned} F.[(f^2 - Mm^2).\sin pl + pT.\cos pl] - Gmg^2.\sin pl \\ = -A.[(f^2 - Mm^2).\cos pl - pT.\sin pl] \\ Fmg^2.\sin pl + G.[(f^2 - Mm^2).\sin pl + pT.\cos pl] \\ = -Ag^2m.\cos pl \end{aligned}\right\} \quad \text{.....} \quad (8a)$$

Assume for abbreviation

$$\left.\begin{aligned} \frac{pT}{f^2-Mm^2} &= \tan k \\ (f^2-Mm^2)^2+p^2T^2 &= C^2 \end{aligned}\right\} \quad \text{.....} \quad (8b)$$

Then the values of F and G will be as follows:

$$\left.\begin{aligned} F &= -\frac{A}{2} \cdot \frac{C^2.\ \sin 2(pl+k)+g^4m^2.\ \sin 2pl}{C^2.\ \sin^2 (pl+k)+g^4m^2.\ \sin^2 pl} \\ G &= -A \cdot \frac{Cmg^2.\ \sin k}{C^2.\ \sin^2 (pl+k)+g^4m^2,\ \sin^2 pl} \end{aligned}\right\} \quad \text{.....} \quad (8c)$$

Putting the amplitude of the vibration of the extremity of the string which rests upon the bridge $= V$, equation (7) becomes

$$V^2=[F.\ \sin pl+A.\ \cos pl]^2+G^2.\ \sin^2 pl,$$

and on putting in this equation the values of F and G from (8c) we find

$$V=\frac{AC.\ \sin k}{\sqrt{[C^2.\ \sin^2 (pl+k)+g^4m^2.\ \sin^2 pl]}} \quad \text{.....} \quad (9)$$

The numerator in this expression is independent of the length of the string. Any alteration of its length therefore affects the denominator only. Under the radical sign is the sum of two squares, which can never $=0$, because m, g, p, T, and hence k, can never $=0$. The coefficient of the resistance of air, g, must certainly be considered as infinitesimal. Hence the denominator is a maximum, and V is a minimum, when $\sin (pl+k)=0$, or when

$$pl=\nu\pi-k \quad \text{.....} \quad (9a)$$

where ν is any whole number. The maximum value of V is

$$V_M=\frac{AC}{g^2m}.$$

Hence, other circumstances being the same, this maximum value increases as g, the co-efficient of the resistance of the air, decreases, and as C increases. To ascertain the circumstances on which the magnitude of C depends, in (8b), which defines the meaning of C, put for p^2 its value from (7a), and also put $n^2=\frac{f^2}{M}$; this gives

$$C^2=M^2.(n^2-m^2)^2+T\mu m^2.$$

Now n is the number of vibrations which the bridge would perform in 2π seconds, under the influence of the elastic sounding board alone, with-

out the string and the resistance of the air; and m is the same number of vibrations for the tuning-fork. Hence the maximum value of V can now be written

$$V_M = \frac{A}{g^2}\sqrt{\left[M^2 \cdot \left(1-\frac{n^2}{m^2}\right)^2 + T\mu\right]}$$

in which everything is reduced to the weights M, T, μ and the magnitude of the interval $1-\frac{n}{m}$. If $m > n$, which is usually the case, it is advantageous to make the weight of the bridge M, rather large. Hence I have had it constructed of a plate of copper. If M is very large, k will be very small in consequence of (8b), and then the equation (9a) shows that the various tones of greatest resonance approach very nearly to those which correspond with the series of simple whole numbers. The heavier the bridge, the sharper the boundaries of the tones of the string.

The rules here given for the influence of the bridge hold only for the assumption that the string is excited by a tuning-fork, and not, so far as this investigation extends, for other cases.

APPENDIX V.

ON THE VIBRATIONAL FORMS OF PIANOFORTE STRINGS.

(See pp. 121–128.)

WHEN a stretched string is struck by a perfectly hard and narrow-edged metal wedge, which is immediately withdrawn, the blow conveys a certain velocity to the point struck, while the rest of the string receives no motion. Let the moment of impact correspond to $t=0$; the motion of the string can then be determined on the condition that at that moment the string as a whole was in its position of equilibrium, and that it was only the point struck that had any velocity. Hence in equations (1c) and (1d) of Appendix III. (p. 582) put $y=0$ and $\frac{dy}{dt}=0$ for $t=0$, at all points except that which is struck, for which suppose the co-ordinate to be a.

Hence it follows that in those equations

$$0 = A_1 = A_2 = A_3 = \&c.,$$

and the values of B are determined by an integration similar to that in (2b), p. 583, giving

$$2\pi nmB_m \cdot \int_0^l \sin^2 \frac{m\pi x}{l} \cdot dx = \int_0^l \frac{dy}{dt} \cdot \sin \frac{m\pi x}{l} \cdot dx,$$

and
$$\pi nmlB_m = c. \sin \frac{m\pi a}{l},$$

where c is the product of the velocity imparted to the struck portion of the string and of its infinitesimal length. Consequently

$$y = \frac{c}{\pi nl} \cdot \left(\sin \frac{\pi a}{l} \cdot \sin \frac{\pi x}{l} \cdot \sin 2\pi nt\right.$$
$$\left. + \frac{1}{2} \cdot \sin \frac{2\pi a}{l} \cdot \sin \frac{\pi x}{l} \cdot \sin 4\pi nt + \frac{1}{3} \cdot \sin \frac{3\pi a}{l} \cdot \sin \frac{3\pi x}{l} \cdot \sin 6\pi nt + \&c.\right)$$

and
$$B_m = \frac{c}{\pi nlm} \cdot \sin \frac{m\pi a}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

The mth partial tone of the string, therefore, disappears in this case also when it is struck in a node of this string. Also the upper partial tones are stronger in comparison with the prime tone, than when the string is plucked, because the value of A_m in equation (3), p. 583, has m^2, as a divisor, whereas the value of B_m in (10) has only m as a divisor. This is immediately confirmed by experiment, on striking the strings with the sharp edge of a metal ruler.

For a pianoforte, the discontinuity in the motion of the string is diminished by covering the hammer with an elastic pad. This sensibly diminishes the force of the higher upper partials, because the motion is no longer conveyed to a single point, but is imparted to a sensible length of string, and this too, not in an indivisible moment of time, as would be the case for a blow with a perfectly hard body. On the contrary, the elastic pad yields to the blow at first, and then recovers itself, so that while the hammer is in contact with the string, the motion is capable of extending over a considerable length. An exact analysis of the motion of a string excited by the hammer of a pianoforte would be rather complicated. But observing that the string moves but very slightly from its position of rest, and that the elastic pad of the hammer is very yielding and admits of much compression, we may simplify the mathematical theory, by assuming the pressure exerted by the hammer during the blow which it gives to the string to be as great as it would be if the string were a perfectly fixed and perfectly unyielding body. We are then able to assume the pressure of the hammer to be

$$P = A \sin mt,$$

for such a time that $0 < t < \frac{\pi}{m}$. This last magnitude $\frac{\pi}{m}$ is the length of

time during which the hammer is in contact with the string. The magnitude of m increases therefore as the elastic power of the hammer increases and its weight decreases.

We have first to determine the motion of the string during the interval of time that the hammer is in contact with it, that is, from $t=0$ to $t=\frac{\pi}{m}$. During this time, the hammer divides the string into two sections, and the motion of each section has to be separately determined. At the place of impact let x be written x_0. When $x<x_0$, distinguish the values of y by writing them y_1, and when $x>x_0$, by writing them y^1. At the point struck the pressure of the string against the hammer must be equal to the pressure P, which the hammer exerts against the string. The pressure of the string is to be calculated as in Appendix IV., equation (6a) (p. 587), and we consequently obtain the equation

$$P=A.\sin mt=T.\left(\frac{dy_1}{dx}-\frac{dy^1}{dx}\right) \quad \text{.....} \quad (11)$$

Waves proceed towards both ends of the string from the place struck. Hence y_1 must have the form

$$y_1=\phi\,(x-x_0+at)$$

for values of t, for which $0<t<\frac{\pi}{m}$, and $x_0>x>x_0-at$, and y^1 must have the form

$$y^1=\phi\,(x_0-x+at)$$

for the same values of t, and for values of x for which $x_0<x<x_0+at$. Using ϕ' for the differential co-efficient of the function ϕ, equation (11) gives

$$P=A.\sin mt=2T.\,\phi'\,(at) \quad \text{.....} \quad (11_a)$$

Integrating with respect to t we find

$$C-\frac{A}{m}\cdot\cos mt=\frac{2T}{a}\cdot\phi\,(at),$$

and then, determining the constant C, so that $y_1=0$ when $x=x_0+at$, and $y^1=0$ when $x=x_0-at$, we have

$$y_1=\frac{aA}{2mT}\cdot\left\{1-\cos\left[\frac{m}{a}(x-x_0)+mt\right]\right\},$$

$$y^1=\frac{aA}{2mT}\cdot\left\{1-\cos\left[\frac{m}{a}(x_0-x)+mt\right]\right\}.$$

This determines the motion of the string for the time t, when

$0 < t < \frac{\pi}{m}$, and on the supposition that the two waves proceeding from the place of impact have not reached the ends of the string. If the latter had been the case, they would have been reflected there.

When at has become greater than $\frac{\pi}{m}$, the pressure P will be $=0$, and hence it follows from equation (11a) that from thenceforward

$$\phi'(at)=0, \text{ and hence } \phi=\text{constant, when } at > \frac{\pi}{m}.$$

Hence both y_1 and y^1 remain $=\frac{aA}{mT}$ for all those parts of the string over which the waves have already advanced, until portions of the waves reflected from the extremities reach those parts of the string on their return.

To introduce the influence of the extremities of the string properly into calculation, suppose another string to be infinitely long, and that in all points distant from x_0 by multiples of $2l$, similar blows are given to it, so that from all these places waves proceed similar to those which proceed from x_0. Moreover suppose that in all those places for which $x=-x_0 \pm 2\nu l$, a blow be applied equal to that given to x_0 and at the same time, but in the opposite direction, so that from all these latter points waves will proceed of an identical form, but with a negative height. Those points of the infinite string which correspond with the extremities of the finite string will then be agitated by positive and negative waves of equal magnitude at the same time, and will hence be completely at rest, and consequently all the conditions of the real finite string will be fulfilled by the state of this section of the infinite string.

From the moment that the hammer quits the string, the motion of the string may be regarded as two systems of waves, one advancing (or in the direction of positive x), and the other retreating (or in the direction of negative x). Of these systems of waves we have as yet found only certain isolated portions, namely, those which correspond to the sections of the string lying nearest the point struck. We have now to complete these waves properly and obtain a connected advancing and retreating system.

Advancing in the direction of the positive x on the string, we have $y=0$ until we come to a positive retreating wave, and then it rises to $\frac{aA}{mT}$, which is its value in the points of positive impact. If we proceed beyond the point of impact, and over the wave thence proceeding, we again find values of y which $=0$, and sink to $-\frac{aA}{mT}$ as soon as the first negative retreating wave has been passed over. This is the value of y in the first point of negative impact. To connect the positive and negative retreating waves

properly with each other, we must suppose the values of y_1 to be increased between every point of positive impact and the next following point of negative impact, by the magnitude $+\frac{aA}{mT}$, so that the height of the wave retains this value, which it had at x_0, until the corresponding negative wave begins. Here then the height of the wave becomes $\frac{aA}{2mT}-y_1$ and sinks to zero. Similarly, suppose that $-\frac{aA}{mT}$ is added to the height of the wave in advancing waves between any point of negative impact and the next following point of positive impact. The retreating waves will thus be everywhere positive, and the advancing waves everywhere negative, and the waves at the same time are so constituted that their continued motion will generate that kind of motion which we have found to exist in the string after the hammer quits it.

We have now to express this system of waves as the sum of simple waves. The length of the wave is $2l$, because the points of simultaneous impact lie at intervals of $2l$. Let us take the positive retreating waves at the time $t=\frac{\pi}{m}$, then

1) $y_1=0$, from $x=0$ to $x=x_0-\frac{a\pi}{m}$

2) $y_1=\frac{aA}{2mT}\cdot\left\{1+\cos\left[\frac{m}{a}(x-x_0)\right]\right\}$

from $x=x_0-\frac{a\pi}{m}$ to $x=x_0$

3) $y_1=\frac{aA}{mT}$, from $x=0$ to $x=2l-x_0-\frac{a\pi}{m}$

4) $y_1=\frac{aA}{2mT}\cdot\left\{1-\cos\left[\frac{m}{a}(2l-x_0-x)\right]\right\}$

from $x=2l-x_0-\frac{a\pi}{m}$ to $x=2l-x_0$

5) $y_1=0$, from $x=2l-x_0$ to $x=2l$.

Hence if we assume

$$y_1=A_0+A_1\cdot\cos\frac{\pi}{l}(x+c)+A_2\cdot\cos\frac{2\pi}{l}(x+c)+A_3\cdot\cos\frac{3\pi}{l}(x+c)+\text{ \&c.}$$

$$+B_1\cdot\sin\frac{\pi}{l}(x+c)+B_2\cdot\sin\frac{2\pi}{l}(x+c)+B_3\cdot\sin\frac{3\pi}{l}(x+c)+\text{ \&c.}\ldots\ (12)$$

we shall have

$$\int_0^{2l} y_1 \cdot \cos\frac{n\pi}{l}(x+c) \cdot dx = A_n l,$$

$$\int_0^{2l} y_1 \cdot \sin\frac{n\pi}{l}(x+c) \cdot dx = B_n l.$$

If we put $c=\frac{a\pi}{2m}$, all B's become $=0$, because y_1 has the same values for $\frac{a\pi}{2m}+\xi$ and $\frac{a\pi}{2m}-\xi$, and the limits of the integration may be selected at pleasure provided only that their difference is $2l$. But on the other hand

$$A_n = -\frac{2aAml^2}{Tn\pi \cdot (n^2\pi^2a^2-m^2l^2)} \cdot \sin\left(\frac{n\pi}{l} \cdot x_0\right) \cdot \cos\left(\frac{n\pi}{l} \cdot \frac{a\pi}{2m}\right) \ldots (12a)$$

This equation gives the amplitude A_n of the several partial tones of the compound tone of the string which has been struck. When the point of impact is a node of the nth partial, the factor $\sin\left(\frac{n\pi}{l} \cdot x_0\right)$ will $=0$, and hence all those partial tones disappear which have a node at the point of impact. The table on p. 126 was calculated from this equation.

To determine the complete motion of the string we must further substitute $x+at$ for x in the equation (12) for y_1. The corresponding expression for y^1 then becomes

$$y^1 = -A_0 - A_1 \cdot \cos\frac{\pi}{l}(x+at-c) - A_2 \cdot \cos\frac{2\pi}{l}(x-at-c) - \text{ \&c.}$$

and finally

$$y = y_1 + y^1 = 2A_1 \cdot \cos\frac{\pi}{l}x \cdot \cos\frac{\pi}{l}(at+c)$$
$$+ 2A_2 \cdot \cos\frac{2\pi}{l}x \cdot \cos\frac{2\pi}{l}(atc+) + \text{ \&c.}$$

which completely solves the problem.

If m be infinite, that is, if the hammer be perfectly hard, the expression for A_n in (12a) becomes identical with that of B_m in equation (10). It must be remembered that m in (10) is identical with n in (12a), [and that a in (10) is then identical with x_0 in (12a), while a in (12a) has a different meaning].

If m is not infinite, as n increases the coefficients of A_n decrease as $\frac{1}{n^3}$, but if m be infinite they decrease as $\frac{1}{n}$; for plucked strings they decrease

as $\frac{1}{n^2}$. This corresponds to theorems developed by Stokes ('Cambridge Transactions,' vol. viii. pp. 533 to 584) concerning the effect of the discontinuity of a function, when developed in Fourier's series, upon the magnitude of the terms with high ordinal numbers. Thus, if y is the function to be developed in a series

$$y = A_0 + A_1 \,.\, \sin(mx + c_1) + A_2 \,.\, \sin(2mx + c_2) + \&c.$$

the coefficient of A_n, when n is very great,

1) is of the order $\frac{1}{n}$ when y itself suddenly alters;

2) is of the order $\frac{1}{n^2}$ when the first differential coefficient $\frac{dy}{dx}$ suddenly alters;

3) is of the order $\frac{1}{n^3}$, when the second differential coefficient $\frac{d^2y}{dx^2}$ suddenly alters;

4) is at most of the order e^{-n} when the function itself and all its differential coefficients suddenly alter. [See p. 54, note.]

Hence follow the laws of musical tones so often mentioned in the text, that their upper partial tones generally increase in power, with the greater discontinuity of the corresponding motion of the resonant body.

[See Donkin's 'Acoustics,' pp. 119–126, where, on p. 124, equation (14) corresponds to equation (12a) above.]

APPENDIX VI.

ANALYSIS OF THE MOTION OF VIOLIN STRINGS.

(See p. 133.)

ASSUME the lens of the vibration microscope to make horizontal vibrations, then vibrational curves will be observed like those represented in fig. 23, p. 131. Call the vertical ordinate y and the horizontal x; then y is directly proportional to the elongation of the vibrating point, and x to that of the vibrating lens. The latter performs a simple pendular vibration. If the number of its vibrations be n and the time t, we have generally

$$x = A \,.\, \sin(2\pi nt + c)$$

where A and c are constant.

Now if y also makes n vibrations, x and y are both periodic and have the same periodic time. Hence at the end of each period x and y have the same values as before, and the observed point is at exactly the same place as at the beginning of the period. This holds for every point in the curve and for every fresh repetition of the vibratory motion, so that the curve appears stationary.

Suppose a vibrational form of the kind depicted in fig. 5, p. 30, fig. 6, p. 30, fig. 8, p. 32, fig. 9, p. 32, or fig. 10, p. 35, in which the horizontal abscissa is directly proportional to the time, to be wrapped round a cylinder, of which the circumference is equal to a single period of those curves, so that the time t is now to be measured along the circumference of the cylinder. Call x the distance of a point from a plane drawn through the axis of the cylinder. Then in this case also

$$x = A \,.\, \sin\,(2\pi nt + c),$$

where $A \,.\, \sin c$ is the value of x for $t=0$, and A is the radius of the cylinder. Hence, if the curve drawn upon the cylinder be viewed by an eye at an infinite distance in the line $x=0$, $y=0$, the curve has exactly the same appearance as in the vibration microscope.

If x and y have not exactly the same period; if, for example, y makes n vibrations while x makes $n+\Delta n$, where Δn is a very small number, the expression for x may be written

$$x = A \,.\, \sin\,[2\pi nt + (c + 2\pi t\Delta n)].$$

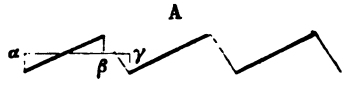
In this case, then, c which was formerly constant, increases slowly. But c represents the angle between the plane $x=0$ and the point in the drawing for which $t=0$. In this case, then, the imaginary cylinder round which the drawing is supposed to be wrapped, revolves slowly.

Since a magnitude which is periodic after the period π, may be also considered as periodic after the periods 2π, or 3π, or $\nu\pi$, where ν is any whole number, these remarks apply also for the case when the period of y is an aliquot part of the period of x, or conversely, or both are aliquot parts of the same third period, that is, for the case when the tones of the tuning-fork and of the observed body stand in some consonant ratio; the only limitation is that the common period of vibration must not exceed the time required for a luminous impression to become extinct in the eye. [See Donkin's 'Acoustics,' pp. 36-44.]

From the observed curves, fig. 23 B, C, p. 131, and fig. 24 A, B, p. 132, it follows that all points of the string ascend and descend alternately, that the ascent is made with a constant velocity, and also the descent with a constant velocity, which is however different from the velocity of ascent. When the bow is drawn across a node of one of the upper partials of the string, the motion takes place in all nodes of the same tone precisely in

the manner described. For other points of the string, little crumples are perceptible in the vibrational figure, but they do not prevent us from clearly recognising the principal motion.

FIG. 62.



If in fig. 62 we reckon the time from the abscissa of the point α, so that for α, $t=0$, and further for the point β put $t=\tau$, and for the point γ put $t=T$, so that the last represents the length of a whole period; then

$$\left.\begin{array}{l} y=ft+h, \text{ from } t=0 \text{ to } t=\tau; \\ y=g\,(T-t)+h, \text{ from } t=\tau \text{ to } t=T, \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

whence for $t=\tau$, it results that

$$f\tau=g\,(T-\tau).$$

Now suppose y to be developed in one of Fourier's series:

$$y=A_1 \,.\, \sin\frac{2\pi t}{T}+A_2 \,.\, \sin\frac{4\pi t}{T}+A_3 \,.\, \sin\frac{6\pi t}{T} \text{ \&c.}$$

$$+B_1 \,.\, \cos\frac{2\pi t}{T}+B_2 \,.\, \cos\frac{4\pi t}{T}+B_3 \,.\, \cos\frac{6\pi t}{T} \text{ \&c.}$$

then it will follow by integration that

$$A_n \,.\, \int_0^T \sin^2\frac{2n\pi t}{T} \,.\, dt=\int_0^T y \,.\, \sin\frac{2n\pi t}{T} \,.\, dt$$

$$B_n \,.\, \int_0^T \cos^2\frac{2n\pi t}{T} \,.\, dt=\int_0^T y \,.\, \cos\frac{2n\pi t}{T} \,.\, dt,$$

and this gives the following values for A_n and B_n:

$$A_n=\frac{(g+f)\,.\,T}{2n^2\pi^2} \,.\, \sin\frac{2n\pi\tau}{T}$$

$$B_n=-\frac{(g+f)\,.\,T}{2n^2\pi^2} \,.\, \left\{1-\cos\frac{2n\pi\tau}{T}\right\}$$

and y may then be written in the form

$$y=\frac{(g+f)\,.\,T}{\pi^2} \,.\, \sum_{n=1}^{n=\infty} \left\{\frac{1}{n^2} \,.\, \sin\frac{\pi n\tau}{T} \,.\, \sin\frac{2\pi n}{T}\left(t-\tau\right)\right\} \quad \ldots\ldots (2)$$

In equation (2), y denotes merely the distance of any determinate point of the string from its position of rest. If x denotes the distance of this point from the beginning of the string, and L the length of the string, then the general form of y, as in equation (1b) of App. III., p. 582, is

$$y=\sum_{n=1}^{n=\infty}\left\{C_n\,.\sin\frac{n\pi x}{L}\,.\sin\frac{2\pi n}{T}\left(t-\frac{\tau}{2}\right)\right\}$$
$$+\sum_{n=1}^{n=\infty}\left\{D_n\,.\sin\frac{2\pi x}{L}\,.\cos\frac{2\pi n}{T}\left(t-\frac{\tau}{2}\right)\right\}\ldots\ldots\ldots\ldots\quad(3)$$

By comparing equations (2) and (3) we find immediately that all D's vanish, or

$$D_n=0,\qquad\text{and}$$

$$C_n\,.\sin\frac{n\pi x}{L}=\frac{g+f}{\pi^2}\,.\,\frac{T}{n^2}\,.\sin\frac{n\pi\tau}{T}\ldots\ldots\ldots\ldots\quad(3a)$$

Here $g+f$ and τ are independent of x, but not of n. On taking the equations for $n=1$ and $n=2$, and dividing one by the other, there results

$$\frac{C_2}{C_1}.\cos\frac{\pi x}{L}=\frac{1}{4}\,.\cos\frac{\pi\tau}{T}$$

From which it follows that for $x=\frac{L}{2}$, τ is also $=\frac{T}{2}$, as observation shews. But if $x=0$, then according to observations τ is also $=0$. Hence

$$C_2=\frac{1}{4}\,.\,C_1,\quad\text{and}\ \frac{x}{L}=\frac{\tau}{T}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\quad(3b)$$

so that $g+f$ is independent of x. Let v be the amplitude of the vibration of the point x in the string, then

$$f\tau=g\,(T-\tau)=2v,$$

$$g+f=\frac{2v}{\tau}+\frac{2v}{T-\tau}=\frac{2vT}{\tau\,(T-\tau)}=\frac{2vL^2}{Tx\,(L-x)}.$$

And since $g+f$ is independent of x, we must have

$$v=4\,V\,.\,\frac{x\,(L-x)}{L^2}$$

where V is the amplitude in the middle of the string. From equation (3b) it follows that the sections $\alpha\beta$ and $\beta\gamma$ of the vibrational figure, fig. 62,

p. 597, must be proportional to the corresponding parts of the string on both sides of the observed point. Hence we have finally

$$y=\frac{8V}{\pi^2}\cdot\sum_{n=1}^{n=\infty}\left\{\frac{1}{n^2}\cdot\sin\frac{n\pi x}{L}\cdot\sin\frac{2\pi n}{T}\left(t-\frac{\tau}{2}\right)\right\}\quad\ldots\quad(3c)$$

for the complete expression of the motion of the string.

If we put $t-\frac{\tau}{2}=0$, y will $=0$ for all values of x, and hence all parts of the string pass through their position of rest simultaneously. From that time the velocity f of the point x is

$$f=\frac{2v}{\tau}=\frac{8V(L-x)}{LT}.$$

But this velocity lasts only from the time τ, where $\tau=\frac{Tx}{L}$. Hence after the time t, and

$$\text{as long as } t<\frac{Tx}{L},\ \text{we have } y=ft=\frac{8V}{LT}\cdot(L-x)\,t\quad\ldots\quad(4)$$

$$\text{and hence } y<\frac{8V}{L^2}\cdot x(L-x).$$

Then the point x returns with the velocity

$$g=\frac{2v}{T-\tau}=\frac{8Vx}{LT}.$$

And hence after the time $t=\tau+t_1$,

$$y=\frac{8V}{L^2}\cdot x(L-x)-\frac{8Vx}{LT}\cdot t_1.$$

And since

$$L-x=\frac{T-\tau}{T}\cdot L$$

we find

$$y=\frac{8Vx}{LT}\cdot\left\{T-(\tau+t_1)\right\}$$

$$=\frac{8Vx}{LT}\cdot(T-t)\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\quad(4a).$$

The deflection on one part of the string is therefore given by the equation (4), and on the other part by the equation (4a). Both equations shew that the form of the string is a straight line, which in (4) passes through $x=L$, and in (4a) through $x=0$. These are the two extremities

APPENDIX VII.

INFLUENCE OF RESONANCE IN REED PIPES.

(See p. 152.)

THE laws of resonance for cylindrical tubes have been developed mathematically in my paper on the 'Theory of Aerial Vibrations in Tubes with open Ends' (*Theorie der Luftschwingungen in Röhren mit offenen Enden*, 'Journal für reine und angewandte Mathematik,' vol. lvii.) The example treated in § 7 of that paper is applicable to reed pipes, where the motion at the bottom of the pipe is assumed to be given. Let Vdt be the volume of air which enters the reed pipe in the infinitesimal time dt, then as this magnitude is periodical we can express V in one of Fourier's series, thus:—

$$V = C_0 + C_1 . \cos(2\pi nt + T_1) + C_2 . \cos(4\pi nt + T_2) + \text{\&c.} \quad \ldots \quad (1)$$

The resonance must be determined separately for each term, because the vibrations corresponding to each partial tone are superposed without modification. If we assume l to be the length of the tube, S its section, $l+a$ the reduced length of the tube (where in cylindrical tubes the difference a is equal to the radius multiplied by $\frac{8r}{4}$), k the magnitude $\frac{2\pi}{\lambda}$ (where λ is the length of the wave), and put the potential of the wave in free space, for the tone having the vibrational number νn,

$$= \frac{M_\nu}{r} . \cos(\nu kr - 2\pi\nu nt + c)$$

where r is the distance from the middle of the mouth; then the equations (15) and (12b) of the paper referred to, give

$$M_\nu = \frac{C_\nu}{\sqrt{(4\pi^2 \cos^2 \nu k (l+a) + \nu^4 k^4 S^2 \sin^2 \nu kl)}}.$$

Since the magnitude $k^2 S$ must always be considered as very small to make our theory applicable, this equation, for cases in which $l+a$ is not an uneven multiple of the length of a quarter of a wave, becomes approximatively,

$$M_\nu = \frac{C_\nu}{2\pi . \cos \nu k(l+a)}$$

Hence the resonance is weakest when the reduced length of the tube is an even multiple of the length of a quarter of a wave, and becomes

stronger as it approaches an uneven multiple of that length. When it absolutely reaches such a multiple the complete formula gives

$$M_\nu = \frac{C_\nu}{\nu^2 k^2 S}.$$

Hence the maximum of resonance increases as the length of the wave of the tone in question increases and the transverse section decreases. The smaller the tranverse section, the sharper is the limit of the pitch of the tone which is strongly reinforced by resonance; while when the transverse section is large, the reinforcement of resonance extends over a much greater length of the scale.

For hollow bodies of other shapes, with narrow mouths, similar equations may be obtained by means of the propositions given in § 10 of the paper cited.

Since the condition of powerful resonance is that $\cos \nu k\,(l+a)=0$, cylindrical tubes (clarinet) reinforce only the prime and other uneven partial tones.

In the interior of conical tubes we may assume the potential of the motion of the air for the tube n to be

$$V = \frac{A}{r} \,.\, \sin\,(kr+c) \cos 2\pi nt,$$

where r is the distance from the vertex of the cone. If the tone is introduced at a distance a from the vertex, and l be the length of the tube, so that for the open end $r=l+a$, we may assume as an approximatively correct limiting condition for the free end, that the pressure there vanishes. This is the case when

$$\frac{dV}{dt} = -2\pi n \,.\, \frac{A}{l+a} \,.\, \sin\,[k(l+a)+c] \,.\, \sin 2\pi nt = 0, \quad \text{and hence}$$

$$\sin\,[k(l+a)+c] = 0.$$

Hence we may assume

$$c = -k(l+a)$$

and

$$V = \frac{A}{r} \,.\, \sin k(r-l-a) \,.\, \cos 2\pi nt.$$

The most powerful resonance, then, in this case, as well as in cylindrical tubes, belongs to those tones which have a minimum velocity at the place where the vibrator is placed. For as during the development of velocity in the mouthpiece in equation (1) the coefficients C_ν have a determinate value which depends only on the motion of the vibrator and the pulses of

air which it occasions, the coefficient A of the last equation must increase, as the velocity produced by the corresponding train of waves in the mouthpiece of the tube decreases. The velocity in the other parts of the tube will then increase. Now the velocity of a particle of air is

$$\frac{dV}{dr}=\frac{A}{r^2}.\cos 2\pi nt.[kr.\cos k(r-l-a)-\sin k(r-l-a)].$$

Hence for maximum resonance we have the condition that

for $$r=a, \text{ either } kr=\tan k(r-l-a)$$

or $$ka=-\tan kl.$$

If, then, the magnitude a, that is, the distance of the tongue from the vertex of the cone, be very small, ka and also $\tan kl$ will be very small, and $kl-\nu\pi$ must also be very small, if ν is any whole number. Hence we may develop the tangents according to powers of their arc, and retaining only the first term of this development we have

$$ka=\nu\pi-kl, \quad \text{and} \quad k(a+l)=\nu\pi$$

or, putting $k=\frac{2\pi}{\lambda}$, we have

$$a+l=\nu.\frac{\lambda}{2}.$$

This shews that conical tubes reinforce all those tones for which the whole length of the cone, reckoned up to its imaginary vertex, is a multiple of half the length of the wave, on the assumption that the distance of the vibrator from this imaginary vertex is infinitesimal in comparison with the length of the wave. Hence if the prime of the compound tone is reinforced by the tube, all the partial tones, both odd and even, will be reinforced up to a pitch where the wave lengths of the higher partial tones cease to be very large in comparison with the distance a.

APPENDIX VIII.

PRACTICAL DIRECTIONS FOR PERFORMING THE EXPERIMENTS ON THE COMPOSITION OF VOWELS.

(See p. 179.)

To make the forks vibrate powerfully, it is necessary that the ratios of their vibrational numbers should agree with the simple arithmetical ratios to the utmost nicety. After the forks have been tuned by the maker by ear and to the piano as accurately as is possible in this way, the necessary greater exactness is obtained by the electrical current itself. First the interrupting fork, fig. 33, p. 178, is connected with the fork that gives the prime tone, and the movable clamp on the former is arranged so as to make the unison perfect. This gives a maximum intensity to the prime tone, easily recognised by both eye and ear. The vibrations of this lowest fork are so powerful that the excursions of the extremities of the prongs under favourable circumstances amount to 2 or 3 millimètres (from ·08 to ·12 inch). It should also be observed that if the unison has not been perfectly attained, a few beats of the fork are heard when the electric current is first brought to bear upon it, although these disappear when the apparatus is in full action. [This is accounted for in Appendix IX.]

After perfect unison has been accomplished between the interrupting fork and that of the prime tone, the other forks are successively brought into electrical connection, with their resonance chambers wide open, and they are tuned until they reach a maximum intensity when excited by the current. The tuning is first performed with the file. The forks are sharpened, as is well known, by taking off some of their extremities, and flattened by reducing the thickness of the root of the prongs. Both must be done with great uniformity to each prong. To discover whether the fork is too sharp or too flat, stick a little piece of wax at the ends of its prongs (which flattens the fork) and observe whether the tone becomes louder or weaker. If louder, the fork was too sharp; if weaker, it was too flat. Since alterations of temperature, and, perhaps, other causes exert a slight influence on the pitch of the forks, I have preferred to make the higher forks a little too sharp by filing, and to bring them into exact tune by attaching small quantities of wax to the extremities of the prongs. The quantity of wax is easily altered at pleasure, and by this means slight accidental variations of pitch can be readily corrected.

It is not necessary to tune the resonance tubes so accurately; if, when blown across, they give the same pitch as the forks, they are sufficiently

well tuned. If they are too flat, some melted wax may be poured in to sharpen them. If they are too sharp, the opening must be reduced.

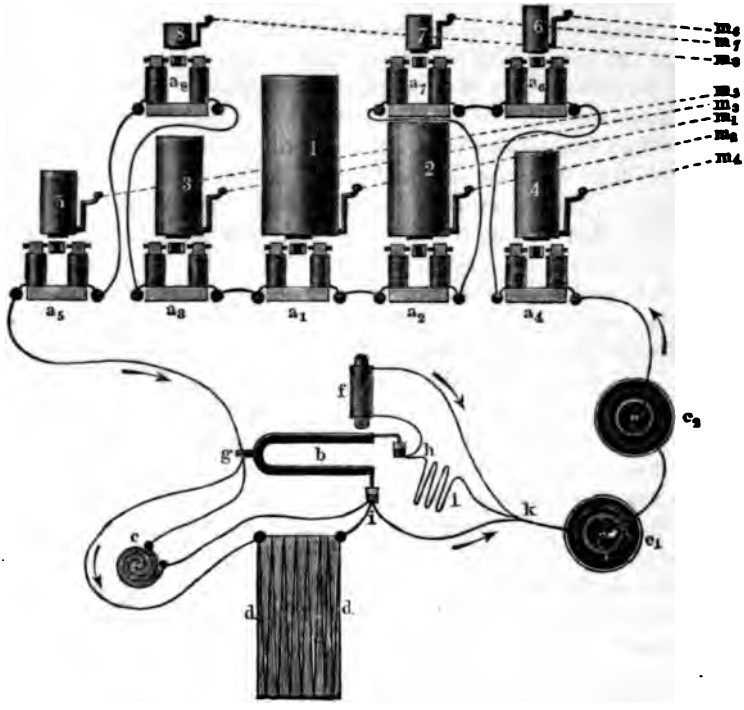
It cost me some trouble to get rid of the noise of the spark at the point of interruption. At first I inserted a large condenser of tin foil such as is used in induction machines. But this merely reduced the spark to a certain size. No good effect followed from increasing the size of the condenser. The layers of the condenser are separated by thin varnished paper; one is connected with the interrupting fork, and the other with the cup of quicksilver into which its end dips. After many vain attempts, I at last found that, by inserting a very long and thin wire between the two extremities of the conduction at the point of interruption, the noise of the spark was almost entirely destroyed, without injuring the action of the current on the forks. The wire thus inserted must have an amount of resistance far greater than that occasioned by the coils in all the electro-magnets taken together. When this is the case no sensible portion of the current will go through this wire. It is not till the conduction is broken and the thin wire forms the only connection for the extra current of the electro-magnets, that the current discharges itself through the wire. But to prevent the thin wire itself from generating any secondary current, it must not be coiled round a cylinder, but must be stretched up and down on a board in such a way that two adjacent pieces of the wire should be traversed by two currents proceeding in opposite directions. For this purpose I screwed two hard india-rubber combs at the two ends of a board (one foot long), and passed a thin plated copper wire, such as is used for spinning over silken threads, backwards and forwards (90 times) between the teeth of these combs. By this means a great length (90 feet) of this wire was brought well insulated into a comparatively small space, in such a way as not to produce any sensible secondary current. For when on breaking the primary current a secondary current would be formed in the wire, this would have a direction in the circuit formed by the electro-magnets and the thin wire, opposite to the secondary current in the electro-magnets, and the latter would be entirely or partly prevented from discharging itself through the thin wire.

For moving the forks I used two or three cells of a Grove's battery. The electro-magnets were placed in two rows beside one another. The whole arrangement is shown in a diagram in fig. 64, p. 606. The figures 1 to 8 show the resonant chambers of the tuning-forks; the dotted lines which lead to m_1 and m_8 are the threads which remove the cover from the opening of the resonance chambers; a_1 to a_2 are the electro-magnets which set in motion the tuning-forks between their legs; b is the interrupting fork, and f its own electro-magnet. The relative position of the two last has been somewhat changed in order to make the connection of the direction of the currents more intelligible. The cells of the galvanic battery

are marked e_1 and e_2; the great resistance-wire dd; the condenser c, but its plates which are rolled in a spiral are seen only in section.

The electric current passes from e_2, through all the electro-magnets in order, up to the handle of the interrupting fork g. It is sometimes more advantageous to arrange this part of the conduction, so that it should be separated into two parallel branches, and that the three highest forks, which are the most difficult to move should be inserted into one branch, and the five lower forks into the other, thus allowing a stronger stream to pass through the former than the latter.

FIG. 64.



The remainder of the conduction from g to the second pole of the battery e_1 contains the interrupting apparatus, which is here so arranged that each vibration of the fork makes the current twice; once when the upper prong dips into the cup of mercury h, and once when the lower prong dips into the cup i. When the conduction is closed at h, it passes from g through the fork to h, and then through the electro-magnet f to k and e_1. Between h and k it is generally necessary to insert a lateral branch h l k, having moderate resisting power, to weaken the current in

the electro-magnet f, sufficiently to prevent the fork b from making violent vibrations. The zig-zag at l shews this branch.

When the prongs of the fork move apart, the conduction will be broken at h, and after a short interval again completed at i, so that the current now passes from g through the lower prong of the fork to i, and thence by k to the battery at e_1. But at the moment the conduction is broken either at h or at i, powerful secondary currents are formed by induction in the 8 electro-magnets of the tuning-forks, which would emit luminous and noisy sparks at the points of interruption, if the rush of electricity were not partly stored for the moment in the condenser c, and partly discharged through the very great resistance dd.

This resistance dd, as is seen by the figure, forms a continual connection between g and the battery, but it conducts so badly that no sensible part of the current can pass through it, except at the moment when, on breaking the conduction, the great electro-motive force of the secondary currents is generated.

The arrangement just described is preferable when the fork 1 is the Octave above the fork b. But if the fork 1 makes the same number of vibrations as the fork b, the wire i k must be removed, and both the other wires ending in i must be connected with h.

To exclude particular forks from the circuit, short secondary connections of the coils of wire of their electro-magnets are introduced. The arrangement is shewn in fig. 32, p. 176. The metal knobs h h are connected conductively with the clamping screw g in which the wire of the electro-magnet terminates. If the lever i is moved down, it presses with some friction on the nearer knob h, and forms so good a secondary conducting connection for the wire of the electro-magnets, that the greater part of the electric current passes by h h, and only an infinitesimally small part travels round by the much longer path of the electro-magnets.

As regards the theory of the motion of the forks, it is immediately seen that the force of the current in the electro-magnets must be a function of the time. The length of the period is equal to the period of a vibration of the interrupting fork b. Let the number of interruptions in a second be n. Then the strength of the current in the electro-magnets, and consequently the magnitude of the force exerted by the electro-magnets on the forks, will be of the form:

$$A_0 + A_1 \,.\, \cos(2\pi nt + c_1) + A_2 \,.\, \cos(4\pi nt + c_2) + A_3 \,.\, \cos(6\pi nt + c_3) + \text{\&c.}$$

The general term of this series $A_m \,.\, \cos.\,(2\pi mnt + c_m)$ is adapted for setting in motion a fork making $m\,n$ vibrations in a second, but would have little effect on forks otherwise tuned.

APPENDIX IX.

ON THE PHASES OF WAVES CAUSED BY RESONANCE.

(See p. 182.)

LET a tuning-fork be brought near the opening of a resonance chamber, and suppose the ear of the listener to be at a great distance off in comparison with the dimensions of the opening. In the *Journal für reine und angewandte Mathematik*, vol. lvii., pp. 1–72, in my paper on the theory of aerial vibrations in tubes with open ends, I have shewn that if a sonorous point exists at B in a space partly bounded by firm walls, and partly unbounded, the motion of sound at another point A, in the same space, will be identical in intensity and phase with that which would have existed at B if A had been the sonorous point. Let B be the position of the tuning-fork (or more properly of the end of one of its prongs), and A that of the ear. The motion of the air which begins when the tuning-fork is placed near the opening of the resonance chamber, is not easily determined, but I have been able (in pp. 47 and 48 of the paper quoted above) to determine the motion when the tuning-fork is at a great distance. Let us suppose, then, that the fork is removed to the position of the ear A, and we shall then have to determine the motion of the sound at the point B near the opening. This motion is composed of two parts; the first, having its potential denoted by Φ in the paper cited, corresponds with the motion which would also exist if the opening to the resonance chamber were closed, and in the above case is too small to be sensible; the second, there marked Ψ, has, in open space and at some distance from the opening, the following value, using the notations explained in the above paper (p. 28, equation 12 h),

$$\Psi = -\frac{AQ}{2\pi\rho} \cdot \cos(k\rho - 2\pi nt) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where Q is the sectional area of the resonance tube, ρ the distance from the middle point of the opening, n the vibrational number, $\frac{2\pi}{k}$ the length of the wave. The motion at an infinitesimal distance r from the sonorous point A is given by the equation,

$$\Phi = H. \frac{\cos(2\pi nt - c)}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

and if r_1 be the distance of the imaginary sonorous point A from the middle of the opening of the resonance tube, we find from equations (16c) and (13a) of the paper cited:

$$-\tan(kr_1+c)=\tan.r_2=-\frac{k^2.Q.\sin kl.\cos ka}{2\pi.\cos k(l+a)}\quad\ldots\quad(2a)$$

(l length of tube, and a a constant depending on the form of its opening), and finally by the same equations, the magnitude there called I is:

$$I=K.\frac{2k.\sin kl}{r_1}=\pm AQ.\frac{k^2.\sin kl}{2\pi.\sin r_2},$$

whence
$$A=\pm H.\frac{4\pi.\sin r_2}{kQr_1}\quad\ldots\ldots\ldots\ldots\ldots\ldots\quad(3)$$

The sign $\pm$ is to be so determined that the constants A and H have the same sign, and in that case r_2 must lie between 0 and π.

In this case the strength of the resonance A is expressed in terms of the intensity of the sonorous point H, the section of the resonance tube Q, the distance r_1 of the sonorous point from the opening of the tube, and the magnitude r_2. The difference of phase between the points A and B is shewn by equations (1), (2), and (2a) to be

$$\pi-kp+c=\pi-kp-kr_1-r_2.$$

But the magnitude kp at all such distances of the point B from the middle of the opening as we can use, may be regarded as infinitesimally small, so that when we weaken the tone by withdrawing the fork further from the opening of the tube, we do not sensibly change the phase of the aerial vibration. But if we change the pitch of the tube, the expression for the phase will be altered only through a change in r_2, which by equation (2a) depends on kl, and to this change there always corresponds a change in the strength of the resonance, since $\sin r_2$ appears as a factor in the expression for that resonance in equation (3). The resonance is strongest when $\sin r_2=1$, or $r_2=\frac{1}{2}\pi$. Calling this maximum resonance A_1, we have

$$A_1=\frac{4\pi H}{kQr_1},$$

and for other pitches of the tube, supposing its sectional area Q to remain unchanged

$$\sin r_2=\frac{A}{A_1}.$$

Whether r_2 is to be taken smaller or greater than a right angle, depends upon whether the value of $\tan r_2$ from equation (2a) is positive or negative. But since k, Q and $\cos ka$ are always positive, the value of $\tan r_2$ depends on the factor $\frac{\sin kl}{\cos k(l+a)}$. The maximum resonance cor-

responds to $\cos k(l+a)=0$; the minimum to $\sin kl=0$. Hence $r_2 < \frac{\pi}{2}$ when by elongation of the tube the resonance is brought towards its minimum; but $r_2 > \frac{\pi}{2}$ when the resonance is brought towards its maximum. In actual application the tube is always near its position of maximum resonance, and hence $r_2 < \frac{\pi}{2}$ when the tube is too flat in pitch, and $r_2 > \frac{\pi}{2}$ when the tube is too sharp in pitch.

If we put the tube out of tune to such an extent that $A^2 = \frac{1}{2} . A_1^2$, the phase of vibration alters by $\frac{1}{4}\pi$. Hence we are always able to estimate the amount of alteration of phase by the alteration of strength of resonance.

A similar law holds for the phases of the vibrating forks as compared with those of the exciting current. To simplify the treatment, I will consider the case of a single vibrating heavy point, which is constantly restored to its position of rest by an elastic force. When the heavy point is moved to the distance x from its position of rest, let $-a^2x$ be the elastic force. Suppose moreover that there act, first a periodic force, similar to that generated by the electrical currents in our experiments, which may be represented by $A . \sin nt$, and secondly a force which damps the vibrations and is proportional to the velocity, so that we may write it $-b^2 \frac{dx}{dt}$. A force of the latter kind arises in our experiments partly from friction and resistance of the air, and also partly from the currents induced by the tuning-forks set in motion, and this latter part has most effect in damping the vibrations. If m is the mass of the heavy vibrating point, we have, therefore,

$$m . \frac{d^2x}{dt^2} = -a^2x - b^2 \frac{dx}{dt} + A . \sin nt \qquad (4)$$

The complete integral of this equation is

$$x = \frac{A . \sin \varepsilon}{b^2 n} . \sin (nt - \varepsilon) + Be^{-\frac{b^2 t}{2m}} . \sin \left\{ \frac{t}{m} . \sqrt{(a^2 m - \tfrac{1}{4} b^4)} + c \right\} \qquad (4a)$$

where

$$\tan \varepsilon = + \frac{b^2 n}{a^2 - m n^2} \qquad (4b)$$

The term having the coefficient B in (4a) is sensible only at the beginning of the motion; on account of the factor $e^{-\frac{b^2 t}{2m}}$ it decreases with the increase of the time t, and ultimately vanishes. But its existence at

the beginning of the motion occasions those transient beats mentioned in App. VIII., p. 604, when n is slightly different from

$$\frac{1}{m}\sqrt{(a^2 m - \tfrac{1}{4}b^2)}.$$

The term with the coefficient A in equation (4a), on the other hand, corresponds to the sustained vibration of the heavy point. The vis viva i^2 of this motion is equal to the maximum value of $\frac{1}{2}\, m.\left(\frac{dx}{dt}\right)^2$, or to

$$i^2 = \frac{mA^2.\ \sin^2 \epsilon}{2b^4} \quad \text{.....} \quad (5)$$

When the pitch of the exciting tone, that is, n, can be altered, i^2 will reach its maximum (which we will call I^2), when

$$\sin^2 \epsilon = 1, \quad \text{or} \quad \tan \epsilon = \pm\infty,$$

giving

$$I^2 = \frac{mA^2}{2b^4}.$$

Hence we may also write

$$i^2 = I^2.\ \sin^2 \epsilon \quad \text{.....} \quad (5a)$$

The same magnitude ϵ therefore determines in equation (4a) the difference of the phases between the periodically changing elongations x of the heavy point and the changing values of the force, and in equation (5a) the strength of the resonance.

The condition $\tan \epsilon = \pm\infty$ is by (4b) fulfilled when $a^2 = m\, n^2$.

Hence if N be the value of n which answers to the maximum of the sympathetic vibration, we shall have

$$N^2 = \frac{a^2}{m} \quad \text{.....} \quad (5b)$$

This tone of *strongest resonance* is the same as the tone which the heavy point would occasion, if it were set in vibration solely by the influence of the elastic force, without friction and without external excitement. Somewhat different from this is the *proper tone of the body*, which it produces under the influence of friction and resistance of the air. The pitch of this proper tone ν is given in the second term of the equation (4a)

$$\nu = \frac{1}{m}.\ \sqrt{(a^2 m - \tfrac{1}{4}b^2)}.$$

Not until $b=0$, that is, not until the friction and resistance of the air vanish, will

$$\nu^2 = \frac{a^2}{m} = N^2.$$

In the table on p. 213, it is assumed that $L=10\,l$, and the value of T is calculated on this assumption, as follows, after finding the value of β. In equation (6), $\sin^2\epsilon$ is put $=\frac{1}{10}$, corresponding with the condition that the strength of the tone of the sympathetically resonant body should be $\frac{1}{10}$ of the maximum strength it can attain; and the ratio $N:n$ is calculated from the numerical ratios corresponding to the intervals mentioned in the first column of the table on p. 213.

Equation (4b) in App. IX., p. 610, may be written

$$\tan\epsilon=\frac{b^2}{mN.\left(\frac{N}{n}-\frac{n}{N}\right)}=\frac{\beta}{\pi.\left(\frac{N}{n}-\frac{n}{N}\right)}.$$

In this equation N, giving the pitch of strongest resonance; b^2, the strength of the friction; and m, the mass for various fibres of Corti, may be different. Hence in applications to the ear, we must consider b^2 and m to be functions of N. Now since the degree of roughness of the closer dissonant intervals remains tolerably constant for constant intervals throughout the scale [see the precise relations in App. XIX., Section H., art. 1b], the magnitude represented by $\tan\epsilon$ must assume approximatively the same values for equal values of $\frac{N}{n}$, and hence the magnitude $\frac{b^2}{mN}=\frac{\beta}{\pi}$ must be tolerably independent of the values of N. No very exact result can be obtained. Hence in the following calculations β is assumed to be independent of N.

APPENDIX XI.

VIBRATION OF THE MEMBRANA BASILARIS IN THE COCHLEA.

(See p. 219.)

THE mechanical problem here attempted is to examine whether a connected membrane with properties similar to those of the membrana basilāris in the cochlĕa, could vibrate as Herr Hensen has supposed this particular membrane to do; that is, in such a way that every bundle of nerves in the membrane could vibrate sympathetically with a tone corresponding to its length and tension, without being sensibly set in motion by the adjacent fibres. For this investigation we may disregard the spiral expansion of the basilar membrane, and assume it to be stretched between the legs of an angle, of the magnitude 2η. Let the axis of x bisect this angle, and the

axis of y be drawn at right angles to it through the vertex of the angle. Let the tension of the membrane parallel to the axis of x be $=P$, and that parallel to the axis of y be $=Q$, both measured by the forces which when exerted on the sides of a unit square, parallel to x and y respectively, would balance the tension of the membrane. Let μ be the mass of this unit square, t the time, and z the displacement of a point in the membrane from its position of equilibrium. Moreover let Z be an external force, acting on the membrane in the direction of positive z, and setting it in vibration. The equation of the motion of the membrane, deduced without material difficulty from Hamilton's principle by Kirchhoff's process, is then

$$Z + P \cdot \frac{d^2z}{dx^2} + Q \cdot \frac{d^2z}{dy^2} = \mu \cdot \frac{d^2z}{dt^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

The limiting conditions are

1) that $z = 0$ along the legs of the angle, that is, $z = 0$, when $y = \pm x . \tan \eta$.
2) that $z = 0$, when $x = y = 0$, that is, at the vertex of the angle, and finally
3) that z is finite, when x is infinite.

The further development of the problem will shew how these two last limiting equations, which suffice for our purpose, may be replaced by certain determinate curves acting as fixed boundaries between the legs of the angle (p. 620).

By putting $x = \xi . \sqrt{P}$ and $y = v . \sqrt{Q}$, the equation (1) may be reduced to the better known form

$$Z + \frac{d^2z}{d\xi^2} + \frac{d^2z}{dv^2} = \mu \cdot \frac{d^2z}{dt^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1a)$$

which is the equation of motion for a membrane stretched uniformly in all directions, ξ and v being the rectangular co-ordinates on its surface.

For this notation the limiting conditions become

1) $z = 0$ for $v = \pm \xi . \sqrt{\frac{P}{Q}} . \tan \eta$,
2) $z = 0$ for $\xi = v = 0$,
3) z finite, for $\xi = \infty$.

The transformed problem consequently differs from the original merely in having a uniformly stretched membrane, and a different amount of angle, which we will represent by 2ϵ.

Since in the applications which we have to make of the result, P will be very small in comparison with Q, the angle ϵ for the transformed mem-

brane will also be very small, and upon this circumstance mainly depend the analytical difficulties of the problem.

After these preliminary remarks, we proceed to the analytical treatment of the equations (1) and (1a) by introducing polar co-ordinates, assuming

$$\left.\begin{aligned} x=\xi\,.\,\sqrt{P}=r\,.\,\sqrt{P}\,.\cos\omega \\ y=\upsilon\,.\,\sqrt{Q}=r\,.\,\sqrt{Q}\,.\sin\omega \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(1b)}$$

The equations (1) and (1a) then take the form

$$\frac{d^2z}{dr^2}+\frac{1}{r}\cdot\frac{dz}{dr}+\frac{1}{r^2}\cdot\frac{d^2z}{d\omega^2}=\mu\cdot\frac{d^2z}{dt^2}-Z \quad \ldots\ldots\ldots\ldots \text{(1c)}$$

The limiting conditions are now, that

1) $z=0$, when $\omega=+\varepsilon$, and hence $\tan\varepsilon=\sqrt{\frac{P}{Q}}\,.\tan\eta$,

2) $z=0$ for $r=0$,

3) z is finite, when r is infinite.

As regards the nature of the force Z, we shall assume that it consists of two parts; the *first* depending on the friction, which we may put $=-\nu\,.\,\frac{dz}{dt}$, where ν is a positive real constant; the *second*, depending on a periodically variable pressure exerted by the surrounding medium on the membrane, uniformly over its whole surface. Consequently we put

$$Z=-\nu\,.\frac{dz}{dt}+A\,.\cos nt,$$

and obtain as the equation of motion

$$\frac{d^2z}{dr^2}+\frac{1}{r}\cdot\frac{dz}{dr}+\frac{1}{r^2}\cdot\frac{d^2z}{d\omega^2}=\mu\cdot\frac{d^2z}{dt^2}+\nu\cdot\frac{dz}{dt}-A\,.\cos nt \ldots \text{(2)}$$

Of the various motions which the membrane could execute under these circumstances, we are interested solely in those which are maintained by the continuous periodical action of the force, and which must themselves have the same period. Let us consequently assume

$$z=\zeta\,.\,e^{int},\ \text{where } i=\sqrt{(-1)} \quad \ldots\ldots\ldots\ldots\ldots \text{(2a)}$$

and determine ζ by the equation

$$\frac{d^2\zeta}{dr^2}+\frac{1}{r}\cdot\frac{d\zeta}{dr}+\frac{1}{r^2}\cdot\frac{d^2\zeta}{d\omega^2}+(\mu n^2-i n\nu)\,.\zeta=-A \quad \ldots\ldots \text{(2b)}$$

In this case the real part of the value of z will satisfy equation (2) and correspond to a uniformly sustained oscillation of the membrane.

Having thus eliminated the variable t from the differential equation, we proceed to do the same for ω by means of the first limiting equation, after transforming both ζ and the constant A into a series of cosines of uneven multiples of the angle $\frac{\pi\,\omega}{2\epsilon} = h\omega$. It is well known that between the limits $h\omega = +\frac{\pi}{2}$ and $-\frac{\pi}{2}$,

$$A = \frac{4A}{\pi} \cdot \left(\cos h\omega - \frac{1}{3} \cdot \cos 3h\omega + \frac{1}{5} \cdot \cos 5h\omega + \ldots\right) \quad \ldots (3)$$

If in the same way we put

$$\zeta = s_1 \cdot \left(\cos h\omega - \frac{1}{3} \cdot s_3 \cdot \cos 3h\omega + \frac{1}{5} \cdot s_5 \cdot \cos 5h\omega + \ldots\right) \quad \ldots (3a)$$

then for each coefficient s_m we must have

$$\left.\frac{d^2 s_m}{dr^2} + \frac{1}{r} \cdot \frac{ds_m}{dr} + \left(\mu n^2 - i n \nu - \frac{m^2 h^2}{r^2}\right) \cdot s_m = -\frac{4A}{\pi}\right\} \ldots\ldots (3b)$$

And since the first of our limiting conditions is satisfied by the equation (3a), whenever the series converges, there remain only the conditions that

1) $s_m = 0$ for $r = 0$,
2) s_m finite, for $r = \infty$.

It is easily seen that every s_m is perfectly determined by these conditions. For if there were two different functions which satisfied the equation (3b) and the two limiting conditions, then their difference, which we will call δ, would satisfy the conditions

$$\frac{d^2\delta}{dr^2} + \frac{1}{r} \cdot \frac{d\delta}{dr} + \left(\mu n^2 - i n \nu - \frac{m^2 h^2}{r^2}\right) \cdot \delta = 0 \ldots\ldots\ldots (3c)$$

and hence be a Bessel's function, and at the same time we should have

1) $\delta = 0$ for $r = 0$,
2) δ finite for $r = \infty$.

But these two conditions cannot be satisfied at the same time by a Bessel's function, when ν has a value differing ever so little from 0. It is only when $\nu = 0$, that is, when there is no friction, that the determination is insufficient. In that case oscillations once induced may continue for ever, even when there is no force to give fresh impulses.

Particular integrals of the equation (3b) may be easily developed in the form of series, resembling the series for the related Bessel's functions which satisfy equation (3c). One of these series proceeds according to powers of r and is always convergent. But when the angle ϵ is very small, the number of terms in this series which are necessary to determine s is very large, and hence the series cannot be used for determining the progress of the function. A second series which proceeds according to negative powers of r and gives a second particular integral is semi-convergent, and will not become an algebraical function, unless h is an uneven number. But in the latter case the first mentioned series will be infinite in its separate terms.

It is therefore preferable for our present purpose to obtain the expression for s in the form of definite integrals.

Let ϕ and ψ denote the following pair of integrals:—

$$\left.\begin{aligned} \psi &= \int^{\frac{1}{2}\pi} e^{-i\kappa r.\sin t}.\sin mht\,.\,dt \\ \phi &= \int_1^{\infty} u^{-mh-1}.\,e^{-\frac{i\kappa r}{2}.\left(u+\frac{1}{u}\right)}.\,du \end{aligned}\right\} \quad \text{.....} \quad (4)$$

where $$\kappa = \sqrt{(\mu n^2 - i n r)} \quad \text{.....} \quad (4a)$$

and the sign of the root is so chosen that the real part of $i\kappa$ is positive.

Then

$$s_m = \frac{4A}{\pi\kappa^2}\,.\left(mh\,.\,\psi + mh\,.\,\phi\,.\cos\tfrac{1}{2}mh\pi - 1\right) \quad \text{.....} \quad (4b)$$

which is the required expression for s_m.

To shew that the expression in (4b) really satisfies the equation (3b), substitute this value for s_m in that equation, and in differentiating under the integral signs of ψ and ϕ, use partial integration to eliminate the factors $\cos t$ and $\left(u - \frac{1}{u}\right)$ which appear under the integral signs.

For $r=0$ we find

$$\psi = \int_0^{\frac{1}{2}\pi} \sin mht\,.\,dt = \frac{1}{mh}\,.\left(1 - \cos\frac{mh\pi}{2}\right)$$

$$\phi = \int_1^{\infty} \frac{du}{u^{mh+1}} = \frac{1}{mh}$$

and hence $s_m = 0$.

For $r=\infty$, we have $\phi = \psi = 0$, and hence

$$s_m = -\frac{4A}{\pi\kappa^2}.$$

Hence the function s_m satisfies the two limiting conditions, which have been already shewn to be sufficient to determine it.

The equation (4b) may be used to determine the value of s_m when P, the tension of the membrane in the direction x, is infinitesimal. In this case, as (1b) shews, r must be the infinite; as also h, of which the value is

$$h=\frac{\pi \cdot \sqrt{Q}}{2 \cdot \sqrt{P} \cdot \tan \eta}.$$

Hence putting

$$r=h\rho$$

ρ will be the finite, namely

$$\rho=\frac{2x \cdot \tan \eta}{\pi \cdot \sqrt{Q}}$$

It is easily seen that under these circumstances $mh\phi$ will $=0$. For we may write

$$mh\phi=\int_1^\infty mh \cdot e^{-mh \cdot \log u-(l-i\lambda) \cdot \frac{h\rho}{2} \cdot \left(u+\frac{1}{u}\right)} \cdot \frac{du}{u} \ldots\ldots (5)$$

where I have put

$$i\kappa=l-i\lambda,$$

and l according to the above supposition will be positive. Since within the whole extent of the integration $u > 1$ and hence $\log u > 0$, the real part of the exponent will be negative throughout the same extent, and will contain the infinite factor h. Consequently every part of the integral vanishes, and hence also the whole value $h\phi$.

On the other hand the integral ψ, or

$$\psi=\int_0^{\frac{1}{2}\pi} e^{-(l-i\lambda) \cdot h\rho \cdot \sin t} \cdot \sin mht \cdot dt$$

will have the real part of the exponent negative and infinite for all those parts of the integral for which t is not infinitesimal, so that these will all $=0$; but this is not the case for those parts of the integral for which t vanishes.

Hence for an infinite h we may replace the above equations for ψ by the following:

$$\psi=\int_0^\infty e^{-(l-i\lambda) \cdot h\rho t} \cdot \sin mht \cdot dt$$

In this last form the integration may be effected and gives

$$\left.\psi=\frac{m}{h \cdot [(l-i\lambda)^2 \cdot \rho^2+m^2]}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5a)$$

and $$s_m = \frac{4A\rho^2}{\pi.\ (m^2-\rho^2 k^2)}$$

or, by (4a),

$$\left.\begin{aligned} s_m = \frac{4A\rho^2}{\pi.\ (m^2-\rho^2\mu n^2+i\rho^2 n\nu)} \end{aligned}\right\} \quad \text{(5b)}$$

Or if, in order to get rid of the auxiliary magnitude ρ, we represent by $\frac{1}{2}\beta$ the value of y on the limits of membrane, we have

$$\tfrac{1}{2}\beta = x \tan \eta,$$

and hence $$\rho = \frac{\beta}{\pi\ .\ \sqrt{Q}},$$

so that, using S_m for the modulus of s_m,

$$S_m = \frac{4A}{\pi.\ \sqrt{\left[\left(\frac{m^2\pi^2 Q}{\beta^2} - \mu n^2\right)^2 + n^2\nu^2\right]}} \quad \text{(5d)}$$

This value is therefore independent of the magnitude of the angle through which the membrane is stretched. In place of the distance ρ or x from the vertex, we have only β the breadth of the membrane at the point in question. Hence this expression will still hold when the angle is $=0$, and the membrane vibrates like a string between two parallel lines, thus forming m vibrating segments which are separated by lines of nodes parallel to the edges.

The same expression also results for a string, if from the first z is regarded in equation (1) as only a function of y in a line, and supposed to be independent of x, but the limiting condition is retained that when $y = \pm\beta$, then $z=0$. Hence the motion of the membrane is the same as that of a series of juxtaposed but unconnected strings.

The value of $\frac{1}{m}\ .\ S_m$ in (5d) gives us the amplitude of the corresponding form of vibration having the vibrational number $\frac{n}{2\pi}$, and having m vibrating transverse divisions of the membrane. The maximum of S_m will occur when

$$m^2\pi^2 Q - \beta^2\mu n^2 = 0 \quad \text{(6)}$$

The value of this maximum, which we call Σ_m is

$$\Sigma_m = \frac{4A}{\pi n \nu}.$$

The smaller the coefficient of friction ν, the larger will be this maximum at the point in question.

If we call b the value of β which satisfies the equation (6), we may write the equation (5d) thus

$$S_m = \frac{\Sigma_m}{\sqrt{\left[1 + \frac{m^4 \pi^4 Q^2}{n^2 \nu^2} \cdot \left(\frac{1}{\beta^2} - \frac{1}{b^2}\right)^2\right]}}$$

When ν is infinitesimal, and the condition of the maximum is not fulfilled in equation (6), the denominator of this expression becomes infinite, and hence S_m infinitesimal. The amplitude of the vibrations $\frac{1}{m} \cdot S_m$, will become finite, for those values of β only which are so nearly $= b$, that $b - \beta$ is of the same order as ν. Under these circumstances, therefore, each simple tone sets in vibration only some narrow strips of the membrane in the direction of x, of which the first has one, the second three, the third four, &c., vibrating segments, and in which $\frac{\beta}{m}$, that is the length of the vibrating segments, has always the same value.

The greater the coefficient of friction ν, the greater in general will be the extent of the vibrations of every tone over the membrane.

The present mathematical analysis shews that every superinduced tone must also excite all those transverse fibres of the membrane on which it can exist as a proper tone with the formation of nodes. Hence it would follow, that if the membrane of the labyrinth were of completely uniform structure, as the membrane here assumed, every excitement of a bundle of transverse fibres by the respective fundamental tone must be accompanied by weaker excitements of the uneven harmonic subtones, the intensity of which would, however, be multiplied by the factors $\frac{1}{9}$, $\frac{1}{25}$, and generally $\frac{1}{m^2}$. There is nothing of the kind observable in the ear. But I think that this cannot necessarily be urged as an objection against the present theory, because the appendages of the basilar membrane greatly impede the formation of tones with nodes.

The solution can also be extended without difficulty to the case where the membrane in the field of ξ, υ is bounded by two circular arcs, with their centre at the vertex of the angle. To this case correspond as boundaries in reality, that is, in the field of x, y, two elliptic circular arcs, which when P vanishes become straight lines. It is only necessary to add to the value of s_m in (4b), a complete integral of the equation (3c) which can be expressed by Bessel's functions with two arbitrary constants,

and to determine these constants in such a manner as to make $s_m=0$ on the limiting curves selected. When ν is small this change in the limits has no essential effect on the motion of the membrane, except when the maximum of vibration itself falls in the neighbourhood of the limiting curves.

APPENDIX XII.

THEORY OF COMBINATIONAL TONES.

(See pp. 229 and 237.)

IT is well known that the principle of the undisturbed superposition of oscillatory motions, holds only on the supposition that the motions are small,—so small, indeed, that the moving forces excited by the mutual displacements of the particles of the oscillating medium should be sensibly proportional to these displacements. Now it may be shewn that *combinational tones must arise whenever the vibrations are so large that the square of the displacements has a sensible influence on the motive forces.* It will suffice for the present to select as the simplest example, the motion of a single heavy point under the influence of a system of waves, and develop the corresponding result. The motions of the air and other elastic media may be treated in a perfectly similar manner.

Suppose that a heavy point having the mass m is able to oscillate in the direction of the axis of x. And let the force which restores it to its position of equilibrium be

$$k=ax+bx^2.$$

Suppose two systems of sonorous waves to act upon it, with the respective forces

$$f.\sin pt, \quad \text{and} \quad g.\sin(qt+c)$$

then its equation of motion is

$$-m\,.\,\frac{d^2x}{dt^2}=ax+bx^2+f.\sin pt+g.\sin(qt+c).$$

This equation may be integrated by a series, putting

$$x=\epsilon x_1+\epsilon^2 x_2+\epsilon^3 x_3+\ldots.$$
$$f=\epsilon f_1$$
$$g=\epsilon g_1,$$

and then equating the terms multiplied by like powers of ϵ, separately to zero. This gives

$$1)\ ax_1+m.\frac{d^2x_1}{dt^2}=-f_1.\sin pt-g_1.\sin qt+c,$$

$$2)\ ax_2+m.\frac{d^2x_2}{dt^2}=-bx_1^2,$$

$$3)\ ax_3+m.\frac{d^2x_3}{dt^2}=-2bx_1x_2, \text{ and so on.}$$

From the first equation we obtain

$$x_1=A.\sin\left(t.\sqrt{\frac{a}{m}}+b\right)+u.\sin pt+v.\sin(qt+c)$$

where $$u=\frac{f_1}{mp^2-a} \text{ and } v=\frac{g_1}{mq^2-a}.$$

This is the well known result for infinitesimal vibrations, shewing that the body which vibrates sympathetically produces only its proper tone $\sqrt{\frac{a}{m}}$, together with those communicated to it, p and q. Since the proper tone in this case rapidly disappears, we may put $A=0$. And then equation (2) gives

$$\begin{aligned}x_2=&-\frac{b}{2a}.(u^2+v^2)\\&-\frac{u^2}{2(4mp^2-a)}.\cos 2pt-\frac{u^2}{2(4mq^2-a)}.\cos 2(qt+c)\\&+\frac{uv}{m(p-q)^2-a}.\cos[(p-q)t+c]-\frac{uv}{m(p+q)^2-a}.\cos[(p+q)t+c).\end{aligned}$$

The second term of the series for x_1 [involving x_2], contains, then, a constant, and also the tones $2p$, $2q$, $(p-q)$, and $(p+q)$. If the proper tone $\sqrt{\frac{a}{m}}$ of the sympathetic vibrator is deeper than $(p-q)$, as may be certainly assumed in most cases for the drumskin of the ear in connection with the auditory ossicles, and if the intensities u and v are nearly the same, the tone $(p-q)$ will have the greatest intensity among the terms of x_2; it corresponds with the well-known deep combinational tone. The tone $(p+q)$ will be much weaker, and the tones $2p$ and $2q$ will be heard with difficulty as weak harmonic upper partial tones of the generating tones.

The third term x_3 of the series for x contains the tones $3p$, $3q$, $2p+q$, $2p-q$, $p+2q$, $p-2q$, p and q. Of these $2p-q$ or $2q-p$ is a combinational tone of the third order according to Hallstroem's nomenclature

(p. 233). Similarly the fourth term x_4 of the series for x gives combinational tones of the third order; and so on.

If then we assume that in the vibrations of the tympanic membrane and its appendages, the square of the displacements has an effect on the vibrations, the preceding mechanical developments give a complete explanation of the origin of combinational tones. Thus the present new theory explains the origin of the tones $(p+q)$, as well as of the tones $(p-q)$, and shews us, why when the intensities u and v of the generating tones increase, the intensity of the combinational tones, which is proportional to $u\,v$, increases in a more rapid ratio.

The previous assumption respecting the magnitude of the force called into action, namely

$$k = ax + bx^2$$

mplies that when x changes its sign, k changes not merely its sign, but also its absolute value. Hence this assumption can hold only for an elastic body which is unsymmetrically related to positive and negative displacements. It is only in such that the square of the displacement can affect the motion, and combinational tones of the first order arise. Now among the vibrating parts of the human ear, the drumskin is especially distinguished by its want of symmetry, because it is forcibly bent inwards to a considerable extent by the handle of the hammer, and I venture therefore to conjecture that this peculiar form of the tympanic membrane conditions the generation of combinational tones.

APPENDIX XIII.

DESCRIPTION OF THE MECHANISM EMPLOYED FOR OPENING THE SEVERAL SERIES OF HOLES IN THE POLYPHONIC SIREN.

(See p. 243.)

FIG. 65 (p. 624) shews the vertical section of the upper box of the double siren, in order to display its internal construction. E is the wind pipe which is prolonged into the interior of the box, and firmly fixed in the cross beam AA of the support of the apparatus. The prolongation of the wind pipe into the box B has conical surfaces at its upper and lower ends, on which slide corresponding hollow surfaces in the bottom and top surfaces of the box, so that this box can revolve freely about the wind pipe as an axis. At α may be seen a section of the toothed wheel fastened to the cover of the box. At β is the driving wheel which is turned by the

APPENDIX XIV.

VARIATION IN THE PITCH OF SIMPLE TONES THAT BEAT.

(See p. 248.)

LET v be the velocity of a particle, which vibrates under the influence of two tones, so that

$$v = A \,.\, \sin mt + B \,.\, \sin (nt + c)$$

where m differs very slightly from n, and $A > B$. We may then put

$$nt + c = mt - [(m-n)\, t - c], \text{ and}$$

$$v = \{A + B \,.\, \cos [(m-n)\, t - c]\} \,.\, \sin mt - B \,.\, \sin [(m-n)\, t - c] \,.\, \cos mt.$$

Assume

$$A + B \,.\, \cos [(m-n)\, t - c] = C \,.\, \cos \epsilon,$$

and

$$B \,.\, \sin [(m-n)\, t - c] = C \,.\, \sin \epsilon,$$

so that

$$v = C \,.\, \sin (mt - \epsilon).$$

in which C and ϵ are functions of the time t, which will alter slowly, if, as we have assumed, $m - n$ is small in comparison with m.

The intensity of C^2 of this oscillation is determined by

$$C^2 = A^2 + 2\, AB \,.\, \cos [(m-n)\, t - c] + B^2,$$

and it will be a maximum,

$$C^2 = (A + B)^2, \quad \text{when } \cos [(m-n)\, t - c] = +1,$$

and a minimum,

$$C^2 = (A - B)^2, \quad \text{when } \cos [(m-n)\, t - c] = -1.$$

The variable phase ϵ of the motion is determined by

$$\tan \epsilon = \frac{B \,.\, \sin [(m-n)\, t - c]}{A + B \,.\, \cos [(m-n)\, t - c]}$$

As $A > B$, this tangent never becomes infinite, and hence ϵ remains included between the limits $+\frac{1}{2}\pi$ and $-\frac{1}{2}\pi$, to which it alternately approaches. As long as ϵ increases, $mt - \epsilon$ increases more slowly than mt; as long as ϵ diminishes, $mt - \epsilon$ increases faster than mt; hence in the first case the tone flattens and in the second it sharpens.

The vibrational number of the variable tone, multiplied by 2π, is under these circumstances equal to

$$m - \frac{d\epsilon}{dt} = \frac{m\, A^2 + (m+n) \,.\, AB \,.\, \cos [(m-n)\, t - c] + n\, B^2}{A^2 + 2 \,.\, AB \,.\, \cos [(m-n)\, t - c] + B^2}.$$

The limits for the vibrational number therefore correspond

to $\cos[(m-n)t-c]$ becoming $+1$ or -1,

and hence also to a maximum or minimum strength of tone.

1) When the strength of tone is a maximum, the vibrational number varies as

$$\frac{mA+nB}{A\cdot B}=m-\frac{(m-n)\,B}{A+B}=n+\frac{(m-n)\,A}{A+B}.$$

2) When the strength of tone is a minimum, the vibrational number varies as

$$\frac{mA-nB}{A-B}=m+\frac{(m-n)\,B}{A-B}=n+\frac{(m-n)\,A}{A-B}.$$

Hence in the first case, or during the maximum strength, the pitch of the variable tone lies between the pitches of the two separate tones. But during the minimum strength, if the stronger tone is also the sharper, the pitch of the variable tone is sharper than that of either of the single tones; and if the stronger tone is the flatter, the pitch of the variable tone is flatter than that of either of the single tones.

These differences are well heard with two stopped pipes; and also with two tuning-forks when first the higher and then the lower is placed nearer to the resonance chamber.

[See Mr. Sedley Taylor's paper on this subject, 'Philosophical Magazine,' July 1872, pp. 56-64, where he gives several figures illustrating the variability of the pitch, and deduces the above results (1) and (2) from the figures only.]

APPENDIX XV.

Calculation of the Intensity of the Beats of Different Intervals.

(See pp. 283 and 292.)

We shall again employ the formulae for sympathetic vibration developed in Appendix IX., equations (4a) and (4b), p. 610, and (5) and (5a), p. 611. For the tone of strongest resonance in one of Corti's elementary organs, let n be its number of vibrations in 2π seconds, n_1 and n_2 the corresponding vibrational numbers for two tones heard, and B', B'' the greatest velocities of the vibrations which they superinduce in those Corti's organs which have the same pitch, and B_1, B_2 the greatest velocities which both attain in their representation of the vibrational number n. Then by equation (5a) of Appendix IX., p. 611, we have

$$B_1=B'\sin\epsilon_1,\ \text{and}\ B_2=B''.\sin\epsilon_2$$

where $\pi . \tan \epsilon_1 = \dfrac{\beta}{\dfrac{n}{n_1} - \dfrac{n_1}{n}}$, and $\pi . \tan \epsilon_2 = \dfrac{\beta}{\dfrac{n}{n_2} - \dfrac{n_2}{n}}$,

and β is a magnitude which may he regarded as independent of n. Hence the intensity of the vibrations of the organ for the vibrational number n, when both tones n_1 and n_2 affect it simultaneously, fluctuates between the values

$$(B_1 + B_2)^2 \text{ and } (B_1 - B_2)^2.$$

The difference of these two magnitudes, which measures the strength of the beats, is

$$4\, B_1\, B_2 = 4\, B'\, B''. \sin \epsilon_1 . \sin \epsilon_2 \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

Hence for equal differences in the amount of pitch, the strength of the beats is dependent on the product $B'\, B''$. For the mth partial tone of the compound tone of a violin, we may, by Appendix VI., p. 597, put $B'^2 = \dfrac{A'^2}{m^2}$, and hence if the m_1th and m_2th partial tone of two compound tones of a violin, beat, we may put the intensity of their beats for equal differences of interval $= \dfrac{A'^2}{m_1 . m_2}$.

This is the expression from which the numbers in the last column of the table on p. 284 have been calculated. [They are therefore 100 times the reciprocals of the products of the two numbers, giving the ratio of the vibrational numbers in the corresponding line of its third column.]

For the calculation of the degree of roughness of the various intervals, mentioned in pp. 292, 293, and 516-520, the following abbreviations of notation are introduced :

$$\begin{aligned} n_1 + n_2 &= 2\ N. \\ n_1 &= N\,(1+\delta). \\ n_2 &= N\,(1-\delta). \\ n &= N\,(1+\nu). \end{aligned}$$

So that

$$\pi . \tan \epsilon_1 = \frac{\beta}{\dfrac{1+\nu}{1+\delta} - \dfrac{1+\delta}{1+\nu}} \text{ and } \pi . \tan \epsilon_2 = \frac{\beta}{\dfrac{1+\nu}{1-\delta} - \dfrac{1-\delta}{1+\nu}}$$

Since powerful sympathetic resonance ensues only when ν and δ are very small, we may assume that, approximatively,

$$\tan \epsilon_1 = \frac{\beta}{2\pi\,(\nu+\delta)}, \text{ and } \tan \epsilon_2 = \frac{\beta}{2\pi\,(\nu+\delta)}.$$

Putting these values for ϵ_1 and ϵ_2, in equation (7) we have

$$4\,B_1\,B_2 = 4\,B'\,B''.\frac{\beta^2}{\sqrt{[\beta^2+4\pi^2.(\nu-\delta)^2]}\,.\,\sqrt{[\beta^2+4\pi^2.(\nu+\delta)^2]}}\;\ldots(7a)$$

If then we consider ν, that is, the pitch of the Corti's organ which vibrates sympathetically, to be variable, $4\,B_1\,B_2$ will reach its maximum when $\nu=0$, and hence $n=N=\frac{1}{2}\,(n_1+n_2)$, and if we call the value of this maximum s we have

$$s = 4\,B'\,B''.\frac{\beta^2}{\beta^2+4\pi^2\,\delta^2}\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\;(7b)$$

In calculating the degree of roughness arising from sounding two tones together which differ from each other by the interval 2δ, I have thought it sufficient to consider this maximum value, which exists in those Corti's organs which are most favourably situated. Undoubtedly other beats of a weaker kind will be excited in the neighbouring fibres, but their intensity rapidly diminishes. It might therefore appear to be a more exact process to integrate the value of $4\,B_1\,B_2$ in equation (7a) with respect to ν, in order to obtain the sum of the beats in all Corti's organs. This would require an at least approximate knowledge of the density of Corti's organs for different values of ν, that is, for different parts of the scale, and of that we know nothing. In sensation the highest degree of roughness is certainly more important than the distribution of a less degree of roughness over many sensitive organs. Hence I have preferred to take only the maximum of the vibrations from (7b) into account.

Finally we have to consider that very slow beats cause no roughness, and that when the intensity of the beats remains unaltered, and their number increases, the roughness reaches a maximum and then decreases. To express this, the value of s must be also multiplied by a factor, which vanishes when the number of beats is small, attains a maximum for about 30 beats in a second, and then diminishes, and again vanishes when the number of such beats is infinite. Suppose then that the roughness r_p, due to the pth partial tone, is expressed by

$$r_p = \frac{4\,\theta^2\;\delta^2.\,p^2}{(\theta^2+p^2\,.\,\delta^2)^2}\,.\,s_p.$$

The factor of s_p reaches its maximum value $=1$, when $p\delta=\theta$; and becomes $=0$, when δ, that is, half the interval between the two tones in the scale, is $=0$ or ∞. Since δ may be indifferently positive or negative, the expression can only involve even powers of δ. The above is the simplest expression which satisfies the conditions, but it is of course to a certain extent arbitrary.

For θ we must put half the extent of the interval which at the pitch of the lower beating tone causes 30 beats to be made in a second.

Since we have taken c' with 264 vibrations in a second, as the lower tone, θ has been put $= \frac{15}{264}$. Hence we have finally

$$r_p = 16\, B'B''.\ \frac{\beta^2 \theta^2 \delta^2 p^2}{(\beta^2 + 4\pi^2 c^2)(\theta^2 + p^2 \delta^2)}.$$

And from this formula I have calculated the roughness of the intervals, shewn graphically in the diagrams, fig. 60, *A* and *B*, p. 292, and fig. 61, p. 520. The roughnesses due to the separate partial tones have been drawn separately and superimposed on one another in the drawing.

Although the theory leaves much to be desired in the matter of exactness, it at least serves to shew that the theoretical view we have proposed is really capable of explaining such a distribution of dissonances and consonances as actually occurs in nature.

[See Prof. Mayer's Researches, App. XIX., Section H, art. (1b)].

APPENDIX XVI.

ON BEATS OF COMBINATIONAL TONES, AND ON COMBINATIONAL TONES IN THE SIREN AND HARMONIUM.

(See pp. 302 and 234 to 237.)

LET a, b, c, d, e, f, g, h be whole numbers. Let an and $bn+\delta$ be the vibrational numbers of the primes of two compound tones sounded simultaneously, where δ is supposed to be very small in comparison with n, and a and b are the smallest whole numbers by which the ratio $a:b$ can be expressed. The vibrational numbers of any pair of partial tones of these two compound tones will be

$$acn \text{ and } bdn + d\,\delta,$$

These will beat with each other $d\delta$ times in a second, if

$$ac = bd \quad \text{or} \quad \frac{a}{b} = \frac{d}{c}.$$

And since the ratio $\frac{a}{b}$ is expressed in its lowest terms, the smallest values of d and c are

$$d = a \quad \text{and} \quad c = b$$

and their other values are

$$d = ha \quad \text{and} \quad c = hb.$$

Now c and d represent the ordinal numbers of the partial tones which beat together. Hence the lowest partials of this kind will be the bth partial of the compound an, and the ath partial of the compound $bn+\delta$. The resulting number of beats is $a\delta$.

In the same way the $2b$th partial of the first compound, and the $2a$th of the second give $2a\delta$ beats, and so on.

The first differential combinational tone of the two partials acn and $bdn+d\delta$ are

$$\pm[(bd-ac)\,n+d\delta]$$

where the + or − sign has to be taken so that the whole expression is positive.

Two other partials fan and $gbn+g\delta$ give the differential combinational tone

$$\pm[(gb-af)n+g\delta].$$

When both sound together they produce $(g\mp d)\delta$ beats, if

$$bd-ac=\pm(gb-af)$$

or

$$\frac{a}{b}=\frac{g\mp d}{f\mp c}.$$

As before, it follows that the least value of $g\mp d$ is $=a$, and the other, greater, values are $=ha$, so that the smallest number of beats is $a\delta$.

To find the lowest values of the partials which must be present in order to beat with the first differential tones, we will take the lower signs for c and d, and we thus obtain:

$$g=d=\tfrac{1}{2}a,\text{ or } g=\tfrac{1}{2}(a+1),\text{ and } d=\tfrac{1}{2}(a-1)$$
$$f=c=\tfrac{1}{2}b,\text{ or } f=\tfrac{1}{2}(b+1),\text{ and } c=\tfrac{1}{2}(b-1),$$

according as a and b are even or odd. If b is the larger number, $\frac{1}{2}b$ or $\frac{1}{2}(b+1)$ is the number of partials which any compound must have in order to produce beats when the two tones composing the interval are sounded. If the combinational tones are neglected, about double the number, that is b, are required.

When simple tones are sounded together, the beats arise from the combinational tones of higher orders. The general expression for a differential tone of a higher order arising from two tones with the vibrational numbers n and m is $\pm(an-bm)$, and this tone is then of the $(a+b-1)$th order. Let the vibrational number of a combinational tone of the $(c+d-1)$ order arising from the tones an and $(bn+\delta)$ be

$$\pm[(bd-ca)\,.\,n+d\delta],$$

and of another of the $(f+g-1)$th order be

$$+[(gb-fa)\,.\,n+g\delta],$$

then both produce $(g \mp d) . \delta$ beats, when

$$bd - ac = \pm(bg - af)$$

or

$$\frac{a}{b} = \frac{\mp d}{f \mp c}.$$

The lowest number of beats is therefore again $a\delta$, and the lowest values of c, d, f, g are found as in the former case, so that the ordinal numbers of combinational tones need not exceed $\frac{1}{2}(a+b-2)$, if a and b are both odd, or $\frac{1}{2}(a+b-1)$, if only one of them is odd, the other being even.

To what has been said in Chap. VII., pp. 234-237, I will add the following remarks on the origin of combinational tones.

Combinational tones must always arise when the displacement of the vibrating particles from their position of rest is so large that the force of restitution is no longer simply proportional to the displacement. The mathematical theory of this case for a heavy vibrating point is given in App. XII., pp. 621 to 623. The same holds for aerial vibrations of finite magnitude. The principles of the theory are given in my essay on the 'Theory of Aerial Vibrations in Tubes with Open Ends,' in Crelle's 'Journal für Mathematik,' vol. lvii., p. 14. I will here draw attention to a third case, where combinational tones may also arise from infinitely small vibrations. This has already been mentioned in pp. 234-7. It occurs with sirens and harmoniums. We have here two openings, periodically altering in size, and with a greater pressure of air on one side than on the other. Since we are dealing only with very small differences of pressure, we may assume, that the mass of the escaping air is jointly proportional to the size of the opening ω, and to the difference of pressure p, so that

$$q = c\omega p$$

where c is some constant. If we now assume for ω the simplest periodic function which expresses an alternate shutting and opening, namely

$$\omega = A . (1 - \sin 2\pi nt),$$

and consider p to be constant, that is, suppose ω to be so small and the influx of air so copious, that the periodical loss through the opening does not essentially alter the pressure, q will be of the form

$$q = B . (1 - \sin 2\pi nt)$$

where

$$B = cAp.$$

In this case the rapidity of the motion of sound at any place of the space filled with air, must have a similar form, so that only a tone with the vibrational number n can arise. But if there is a second greater opening of

variable size, through which there is sufficient escape of air to render the pressure p periodically variable, instead of being constant, as the air passes out through the other opening, that is, if p is of the form

$$p = P \,.\, (1 - \sin 2\pi m t),$$

then q will have the form

$$\begin{aligned} q &= cAP \,.\, (1 - \sin 2\pi n t) \,.\, (1 - \sin 2\pi m t) \\ &= cAP \,.\, [1 - \sin 2\pi n t - \sin 2\pi m t - \tfrac{1}{2} \cos 2\pi (m+n) t \\ &\qquad\qquad + \tfrac{1}{2} \cos 2\pi (m-n) t]. \end{aligned}$$

Hence, in addition to the two primary tones n and m, there will be also the tones $m+n$ and $m-n$, that is, the two combinational tones of the first order.

In reality the equations will always be much more complicated than those here selected for shewing the process in its simplest form. The tone n will influence the pressure p, as well as the tone m; even the combinatinal tones will alter p; and finally the magnitude of the opening may not be expressible by such a simple periodic function as we have selected for ω. This will occasion not merely the tones m, n, and $m+n$, $m-n$, to be produced, but also their upper partials, and the combinational tones of those upper partials, as may really be observed in experiments. The complete theory of such a case becomes extraordinarily complicated, and hence the above account of a very simple case may suffice to shew the nature of the process.

I will mention another experiment which may be similarly explained. The lower box of my double siren vibrates strongly in sympathy with the fork a' when held before its lower opening, when the holes are all covered, but not when the holes are open. On putting the disc of the siren in rotation so that the holes are alternately open and covered, the resonance of the tuning-fork varies periodically. If n is the vibrational number of the fork, and m the number of times that a single hole in the box is opened, the strength of the resonance will be a periodic function of the time, and consequently in its simplest case equal to $1 - \sin 2\pi m t$.

Hence the vibrational motion of the air will be of the form

$$\begin{aligned} (1 - \sin 2\pi m t) \,.\, \sin 2\pi n t = \sin 2\pi n t &+ \tfrac{1}{2} \cos 2\pi (m+n) t \\ &- \tfrac{1}{2} \cos 2\pi (m-n) t, \end{aligned}$$

and consequently we hear the tones $m+n$, and $m-n$ or $n-m$. If the siren is rotated slowly, m will be very small, and these tones being all nearly the same, will beat. On rotating the disc rapidly, the ear distinguishes them.

APPENDIX XVII.

PLAN FOR JUSTLY INTONED INSTRUMENTS WITH A SINGLE MANUAL.

(See p. 497.)

To arrange an organ or harmonium with twenty-four tones to the octave in such a way as to play in just intonation in all keys, the tones of the instrument must be separated into four pairs of groups, thus

1 a)	*f*	*g*	*c*♯	1 b)	*f*	*a*	*c*
2 a)	*c*	*e*	*g*♭	2 b)	*c*	*e*	*a*♭
3 a)	*g*	*b*	*e*♭	3 b)	*g*	*b*	*e*♭
4 a)	*d*	*f*♯	*b*♭	4 b)	*d*	*f*♯	*b*♭

Each of these groups must have a separate wind-conduit from the bellows, and valves must be introduced in such a way that the wind may be driven at pleasure either to the right or left group of any horizontal series. This would not be difficult on the organ. On the harmonium the digitals would have to be placed in a different order from the tongues, and hence it would be necessary to have a more complicated arrangement for conducting to the valve the effect of pressing down a digital, which would be similar to that on the organ.

Hence four valves are to be arranged by stops or pedals in a different way for every key. The following is a table of the arrangement of the stops for the four horizontal series of the tones named:—

Major Keys	Series				Minor Keys
	1	2	3	4	
C♭*	b	a	a	a	(*E*♭)
G♭*	b	b	a	a	(*B*♭)
D♭*	b	b	b	a	(*F*)
A♭*	b	b	b	b	(*C*)
E♭*	a	b	b	b	(*G*)
B♭*	a	a	b	b	(*D*)
F	a	a	a	b	*A*
C	a	a	a	a	*E*
G	b	a	a	a	*B** or *C*♭
D	b	b	a	a	*F*♯* or *G*♭
A	b	b	b	a	*C*♯* or *D*♭
E	b	b	b	b	*G*♯* or *A*♭
B	a	b	b	b	*D*♯* or *E*♭
	a	a	b	b	*A*♯* or *B*♭

The minor keys which have their names in parentheses, namely *E*♭, *B*♭, *F*, *C*, *G*, *D*, have a true minor Seventh, but too high a leading tone.

For the six keys marked with *, the arrangement of the stops is the same both for major and minor.[1]

In order to have a complete series of tonics, each with a perfect major and minor case, it would be necessary to cut off $\underline{a}$♭, $\underline{e}$♭, $\underline{b}$♭, $\underline{f}$, $\underline{c}$, $\underline{g}$, from the other notes, and to allow them to be replaced when needed by g♯, d♯, a♯, e♯, b♯, and f♯♯, by means of a fifth stop. We should thus have 30 tones to the octave. By drawing out this stop we should have the following system of keys:—

Major Keys	Series marked with accented letters to shew that they are affected by the fifth stop				Minor Keys
	1	2	3	4	
F	a′	a′	a′	b′	*F*
C	a′	a′	a′	a′	*C*
G	b′	a′	a′	a′	*G*
D	b′	b′	a′	a′	*D*
A	b′	b′	b′	a′	*A*
E	b′	b′	b′	b′	*E*[2]
B	a′	b′	b′	b′	*D*♯
F♯	a′	a′	b′	b′	*A*♯
C♯	a′	a′	a′	b′	*E*♯
G♯	a′	a′	a′	a′	*B*♯
D♯	b′	a′	a′	a′	*F*♯♯
A♯	b′	b′	a′	a′	*C*♯♯
E♯	b′	b′	b′	a′	*G*♯♯

To have a complete series of minor keys, 28 instead of 30 tones to the octave would be enough. They would suffice for the 12 minor keys of $\underline{A}$, $\underline{E}$, $\underline{B}$, $\underline{F}$♯ or G♭, $\underline{C}$♯ or D♭, $\underline{G}$♯ or A♭, $\underline{D}$♯ or E♭, B, F, C, G and D, and for 17 major keys from C♭ major to G♯ major.[3]

[1] [It will be observed that the series in the first six lines is the same as in the six last.—*Translator.*]

[2] [The *E* minor has the leading note, but not the minor Seventh. The other minor keys have both.—*Translator.*]

[3] [The question of the number of tones necessary for all usual modulations will be discussed in App. XIX., Sections E and F, and various schemes for playing in just or practically just intonation will be explained in Section G, where, under No. 3 and No. 8, the present arrangements will be reconsidered.—*Translator.*]

APPENDIX XVIII.

JUST INTONATION IN SINGING.

(See p. 509.)

SINCE the publication of the first edition of this book, I have had an opportunity of seeing the *Enharmonic Organ*, constructed by General *Perronet Thompson*,[1] which allows of performance in 21 different tonics harmonically connected. This instrument is much more complicated than my harmonium. It contains 40 pipes to the octave, and has three distinct manuals, with, on the whole, 65 digitals to the octave, as the same note has to be sometimes struck on two or all of the manuals. This instrument allows of the performance of much more extensive harmonies than my harmonium, without requiring any enharmonic interchange. It is even possible to execute tolerably quick passages and ornamentations upon it, notwithstanding its apparently involved fingering. The organ was erected in the Sunday School Chapel, 10 Jewin Street, Aldersgate, London,[2] and was built by Messrs. Robson, 101 St. Martin's Lane, London. It contains only one stop of the usual *principal* work, has Venetian shutters to the swell, and is provided with a peculiar mechanism for correcting the influence of temperature on the intonation.

Mr. H. W. Poole has lately transformed his organ[3] so as to get rid of

[1] 'On the Principles and Practice of *Just Intonation*, with a view to the Abolition of Temperament, and embodying the results of the Tonic *Sol-fa* Associations, as illustrated on the *Enharmonic Organ* . . . presenting the power of performing correctly in 21 keys (with the minors to the extent of involving not more than 5 flats), and a correction for changes of temperature. . . . Calculated for taking the place of the choir organ in a cathedral, and learned by the blind in six lessons. With an Appendix tracing the identity of design with the Enharmonic of the Ancients.' By T. Perronet Thompson, F.R.S. Ninth Edition, 1866. [The exact compass of this organ will be explained in Appendix XIX., Section F. General Thompson was born at Hull, in 1783, and died at Blackheath, 6 September 1869. He had been four years in the navy before joining the army, and was prominent during the Corn Law Abolition agitation. He was many years editor of the 'Westminster Review,' and was first returned to Parliament for Hull in 1835.—*Translator*.]

[2] [Shortly before his death General Perronet Thompson presented this organ to Mr. John Curwen, mentioned in the note to p. 637. The General's executors had it reconstructed in a schoolroom at Plaistow, Essex, but as that room is now (1874) wanted for School Board purposes, the organ has been again taken down, and its present destination is not yet fixed.—*Translator*.]

[3] 'Silliman's American Journal of Science and Arts.' Vol. xliv., July 1867. [See above, p. 504, note 1. It does not appear from this article in 'Silliman' that this new form of organ has been actually constructed; on the contrary, Mr. Poole seems to appeal for means to construct it.—*Translator*.]

stops for changing the intonation, and has constructed a peculiar arrangement of the digitals, which enables him to play in all keys with the same fingering. His scale contains not merely the just Fifths and Thirds in the series of major chords, but also the natural or sub-minor Sevenths for the tones of both series. There are 78 pipes to the octave, and *F*♭ has been identified with *E*, &c, as upon my harmonium.[1]

Successions of chords on General Thompson's instrument are extraordinarily harmonious, and, perhaps, on account of their softer quality of tone, even more surprising in their agreeable character than on my harmonium.[2] I had an opportunity, at the same time, of hearing a female singer, who had often sung to it, perform a piece to the accompaniment of the enharmonic organ, and her singing gave me a peculiarly satisfactory feeling of perfect certainty in intonation, which is usually absent when a pianoforte accompanies. There was also a violinist[3] present who had not been much accustomed to play with the organ, and accompanied well-known airs by ear. He hit off the intonation exactly as long as the key remained unchanged, and it was only in some rapid modulations that he was not able to follow it perfectly.

In London I had also an opportunity of comparing the intonation of this instrument with the natural intonation of singers who had learned to sing without any instrumental accompaniment at all, and are accustomed to follow their ear alone. This was the Society of Tonic Sol-faists, who

[1] [The precise compass of this organ will be also given in Appendix XIX., Section F, with a short account of the principles of its scale and fingering. There are 100, not 78, pipes to the octave.—*Translator.*]

[2] ['On organs of many stops, one or more ought certainly to be tuned with mathematically correct intonation, on account of their wonderful effect, to be employed (of course without using any others at the same time) as the music of the spheres (*als Gesang der Sphären*). It is impossible to form any notion of the effect of a chord in mathematically just intonation, without having heard it. I have such a one to compare with the others. Everyone who hears it expresses his delight and surprise at a correctness of intonation that it does one good to hear (*Jeder, der ihn hört, spricht sein frohes Erstaunen über diese wohlthuende Reinheit aus*).'—Scheibler, 'Ueber mathematische Stimmung, Temperaturen und Orgelstimmung nach Vibrations-Differenzen oder Stössen, 1838.' I have given the original words of the last German sentence, as it was impossible to do justice to its homely force in any translation. Everyone who has heard just intonation will understand it.—*Translator.*]

[3] [A blind man, who had therefore no notes to guide him. I had the pleasure of taking Professor Helmholtz to hear the organ on this occasion, and can corroborate his statements. Unfortunately the proper blind organist was not present. It is to this lady that General Thompson dedicates his little book, already recited, in these words: 'To Miss E. S. Northcote, Organist of St. Anne and St. Agnes, St. Martin's-le-Grand. In commemoration of the talent by which, after six lessons, she was able to perform in public on the enharmonic organ with 40 sounds to the octave; thereby settling the question of the practicability of just intonation on keyed instruments, and realizing the visions of Guido and Mersenne, and the harmonists of classical antiquity.'—*Translator.*]

are spread in great numbers (there were 150,000 in 1862[1]) over the large cities of England, and whose progress is of great importance for the theory

[1] [It is difficult in any way to estimate the numbers in 1874. The Tonic Sol-fa College has since been formed, to examine in harmony and grant certificates of competency to teach, and holds public session once or twice a year. The School Board of London has approved of the use of the Tonic Sol-fa system, requiring results to be produced equal to the several grades represented by its certificates, and as a consequence all the teachers under its control, about 300 in number, have voluntarily adopted the Tonic Sol-fa method. The Glasgow, Paisley, Aberdeen, and other School Boards have adopted similar arrangements with respect to this method. The printing and publication of works in this notation, which are sold at an extremely moderate price, gives employment to a large office and staff at Plaistow, in Essex, near London, with three steam presses (a fourth will soon be needed), conducted by the firm of John Curwen and Sons. There is an especial depôt for their sale at the Tonic Sol-fa Agency, 8 Warwick Lane, Paternoster Row, E.C., and Mr. Curwen calculates that at the present time twice as much Tonic Sol-fa music is printed by other firms besides his own. In lieu of the foot-note respecting the Tonic Sol-fa publications, which Professor Helmholtz added to this Appendix (3rd German edition, p. 632), and which is now much out of date, I refer, first, to Mr. Curwen's new edition (re-written in 1872) of his 'Standard Course of Lessons and Exercises in the Tonic Sol-fa Method of Teaching Music' (founded on Miss Glover's 'Scheme for Rendering Psalmody Congregational, A.D. 1835'); secondly, to his 'How to Observe—Harmony,' a little work of great practical value, which uses the scientific construction of the Tonic Sol-fa system to impart elementary but strictly exact notions on harmony: thirdly, to his elaborate 'Tract on Musical Statics; an Attempt to shew the Bearing of the most Recent Discoveries in Acoustics on Chords, Discords, Transitions, Modulations, and Tuning, as used by Modern Musicians, being Part A of the Commonplaces of Music, a Student's Handbook originally prepared for the first course of Euing Lectures in Anderson's University, Glasgow, Session 1866-7,' a copy of which reached me as this was passing through the press, and which is full of interesting information, in a form suitable for general use; the other parts, B to M, contain constructive exercises and historical specimens: and fourthly, to the long list of instruction books, apparatus, and pieces of music given in the Standard Course, containing already 17 Cantatas by Macfarren, Baker, Romberg, Lahee, de Rille, Root, Bradbury, and Thomas, the oratorios or other sacred pieces of Handel, Haydn, Bach, Mozart, Beethoven, Mendelssohn, &c., all printed in the Tonic Sol-fa notation. The organ of the system is 'The Tonic Sol-fa Reporter,' published fortnightly for one penny, half consisting of intelligence, and the other half of vocal music. The number for October 1, 1874, which came to hand as I was revising this note, contains an account of a concert of 3,000 Sol-faists at the Crystal Palace on September 12, beginning with a chorale by Bach, and passing through choruses by Handel, to modern part songs; between the pieces was a 'sight test,' a piece in B, with 'two extended transitions to the dominant key, and at the close a sort of coda with separate entry of the parts.' This had not been previously seen by the singers, and 'was taken at once to the words, that is without any sol-faing; it was read with perfect ease, and the only defect we noticed was a little uncertainty in the sopranos.' Also referring to the 'result of inquiries as to the systems used by the 26 schools honourably mentioned for their singing by Her Majesty's Inspectors,' this 'Reporter' states that six sent no reply, 1 answered 'by the Hullah system,' 1 'not by Tonic Sol-fa,' 2 'by ear,' and 16 'use the Tonic Sol-fa Method,' adding that as 'several inspectors avoid mentioning cases of excellence, the list is only of value as far as it goes.'—*Translator.*]

of music. The Tonic Sol-faists represent the tones of the major scale by the syllables *Do*, *Re*, *Mi*, *Fa*, *So*, *La*, *Ti*, *Do*, where *Do* is always the tonic [vowels as in Italian]. Their vocal music is not written in ordinary musical notation, but is printed with common types, the initial letters of the above words representing the pitch.[1]

When the tonic is changed in modulations, the notation is also changed. The *new* tonic is now called *Do*, and the change is pointed out in the notation by giving two different marks to the note on which it occurs, one belonging to the old, and one to the new key. This notation, therefore, gives the very first place to representing the relation of every note to the tonic, while the absolute pitch in which the piece has to be performed is marked at the commencement only. Since the intervals of the natural major scale are transferred to each new tonic as it arises in the course of modulation, all keys are performed without tempering the intervals. That in the modulation from *C* major to *G* major, the *Mi* (or *b̲*) of the second scale answers precisely to the *Ti* of the first is not indicated in the notation, and is only taught in the further course of instruction. Hence the pupil has no inducement to confuse *a* with *a̲*.[2]

It is impossible not to acknowledge that this method of notation has the great advantage to the singer of giving prominence to what is of the greatest importance to him, namely, the relation of each tone to the tonic. It is only persons unusually gifted, who are able to fix in their mind, and re-discover absolute pitches, when other tones are sounded at the same time. But the ordinary notation[3] gives directly nothing but absolute pitch, and that too only for tempered intonation. Anyone who has frequently sung at sight is aware how much easier it is to do so from a pianoforte vocal score, in which the harmony is shewn, than from the separate voice part. In the first case it is easy to see whether the note to be sung is the root, Third, Fifth, or dissonance of the chord which occurs, and it is then comparatively easy to find one's way;[4] in the second case

[1] [Great care has also been bestowed on the representation of rhythm, and exercises in rhythm form an important part of the 'Standard Course' and the practice of Tonic Sol-fa teachers.—*Translator.*]

[2] [In a footnote to this passage Prof. Helmholtz gives a list of the Tonic Sol-fa works, which is superseded by the note I have inserted on the last page, and at the end of it he says:] In France singing is taught by the *Galin-Paris-Chevé* system, on similar principles and with a similar notation. ['Méthode Élémentaire de Musique Vocale, par Mme. Émile Chevé (Nanine Paris),' 1864 and 1866, pp. 400, large 8vo. The notation (Rousseau's figures 1, 2, 3, &c.) and method are totally distinct from those of the Tonic Sol-fa, and the two systems grew up without knowledge of each other. Miss Glover began 1812, published 1835. M. Galin published 1818, died 1822. Mr. Curwen published 1842, and in 1867 adopted from Mme. Chevé's book M. Aimé Paris's 'langue des durées.' This remains the only real point of similarity.—*Translator.*]

[3] [Usually called 'the Staff Notation' by the Tonic Sol-faists, by way of distinction.—*Translator.*]

[4] [After a pupil has thoroughly acquired music on the Sol-fa notation, it becomes

the only resource of the singer is to go up and down by intervals as well as he can, and trust to the accompanying instruments and the other voices to force his own to the right pitch.

Now the instruction conveyed to a singer who is acquainted with musical theory by an examination of the pianoforte vocal score, is conveyed by the notation itself to the Tonic Sol-faist as an immediate consequence of the method of teaching. I have convinced myself that by using this notation, it is much easier to sing from the separate part than in ordinary musical notation, and I had an opportunity when in one of the primary schools in London, of hearing more than forty children of between eight and twelve years of age, that performed singing exercises in a manner that astonished me ('mich in Erstaunen setzten') by the certainty with which they read the notes, and by the accuracy of their intonation.[1] Every year the London schools of Sol-faists are accustomed to give a concert of two to three thousand children's voices in the Crystal Palace at Sydenham, which, I have been assured by persons who understand music, makes the best impression on the audience by the harmoniousness and exactness of its execution.[2]

The Tonic Sol-faists, then, sing by natural, and not by tempered

part of his duty to learn the other, and a course of instruction has been prepared for this purpose by Mr. Curwen, called '*The Staff Notation*, a practical introduction on the Tonic Sol-fa method of teaching music,' p. 38. This when properly mastered (a comparatively easy task) puts the pupil in a condition to sing at sight from the old notation as readily as from the new.—*Translator.*]

[1] [On April 20, 1864, I had the pleasure of taking Prof. Helmholtz to hear the singing of the children in the British and Foreign School here alluded to, which was situate behind the chapel in Tottenham Court Road. The master of the school, Mr. Gardiner, was a very good Tonic Sol-fa teacher, but the children were those who ordinarily attended (about 40 were then present) and had received only ordinary instruction. After hearing them sing a few tunes in parts, from the Tonic Sol-fa notation, Prof. Helmholtz himself 'pointed' out an air on the 'modulator' or scale drawn out large on a chart, from which the pupils learn to sing, (that is, by means of a pointer shewed the Tonic Sol-fa names of the tones the children were to sing) and the class followed in unison at sight. Then, on the suggestion of Mr. Gardiner, the class was divided into two sections, and Prof. Helmholtz pointed a piece in two parts, one with each hand, while the class took them at sight. Of course the piece was simple, but the dissonance of a Semitone was purposely introduced in one place between the parts, and Prof. Helmholtz was delighted at the firmness and correctness with which the children took it. I recollect his saying to me afterwards, 'We could not do that in Germany!' meaning, as he subsequently explained, that there was no German system of teaching to sing which could produce such results on such materials.—*Translator.*]

[2] [Latterly two concerts yearly have been given, one by young children who have gained the 'elementary certificate' of proficiency, and one by older children, girls and youths, who have gained the 'intermediate certificate,' with older voices for tenors and basses, and the number of voices has increased to four and five thousand. At these concerts it has frequently happened that a 'sight test' has been given, one of which is described at the end of the note on p. 637.—*Translator.*]

intervals. When their choirs are accompanied by a tempered organ, there are marked differences and disturbances, whereas they are in perfect unison with General Thompson's Enharmonic Organ. Many expressions used are very characteristic. A young girl had to sing a solo in *F* minor, and took it home to study it at her pianoforte. When she returned she said that the *A*♭ and *D*♭ on her piano were all wrong. These are the Third and Sixth of the key in which the deviation of tempered from just intonation is most marked. Another girl was so charmed with the Enharmonic Organ that she remained practising for three hours in succession, declaring that it was pleasant to be able to play *real notes.* Generally in a large number of cases, young people who have learned to sing by the Sol-fa method, find out by themselves, without any instruction, how to use the complicated manuals of the Enharmonic Organ, and always select the proper intervals.

Singers find that it is easier to sing to the accompaniment of this organ, and also that they do not hear the instrument while they are singing, because it is in perfect harmony with their voice and makes no beats.

I have myself observed, that singers accustomed to a pianoforte accompaniment, when they sang a simple melody to my justly intoned harmonium, sang natural Thirds and Sixths, not tempered, nor yet Pythagorean. I accompanied the commencement of the melody, and then paused while the singer took the Third or Sixth of the key. After he had struck it, I touched on the instrument the natural, or the Pythagorean, or the tempered interval. The first was always in unison with the singer, the others gave shrill beats.

After this experience, I think that no doubt can remain, if ever any doubt existed, that *the intervals which have been theoretically determined in the preceding pages, and there called natural, are really natural for uncorrupted ears; that moreover the deviations of tempered intonation are really observed and disliked by uncorrupted ears; and lastly that, notwithstanding the delicate distinctions in particular intervals, correct singing by natural intervals is much easier than singing in tempered intonation.* The complicated calculation of intervals which the natural scale necessitates, and which undoubtedly much increases the manual difficulty of performance on instruments with fixed tones, does not exist for either singer or violinist, if the latter only lets himself be guided by his ear. For in the natural progression of correctly modulated music they have always and only to proceed by the intervals of the natural diatonic scale. It is only the theoretician who finds the calculation complicated, when at the end of numerous such progressions he sums up the result, and compares it with the starting point.

That the natural system can be carried out by singers, is proved by the English Tonic Sol-faists. That it can also be carried out on bowed instruments, and is really carried out by distinguished players, I have no

doubt at all after the experiments of Delezenne already mentioned (p. 506, note 2), and what I myself heard when I was listening to the violinist who accompanied the enharmonic organ. Among the other orchestral instruments, the brass instruments naturally play in just intonation, and can only be forced to the tempered system by being blown out of tune. The wooden instruments could have their tones slightly changed so as to bring them into tune with the rest. Hence I do not think that the difficulties of the natural system are invincible. On the contrary, I think that many of our best musical performances owe their beauty to an unconscious introduction of the natural system, and that we should oftener enjoy their charms if that system were taught pedagogically, and made the foundation of all instruction in music, in place of the tempered intonation which endeavours to prevent the human voice and bowed instruments from developing their full harmoniousness, for the sake of not interfering with the convenience of performers on the pianoforte and the organ.

[The remainder of this Appendix, which concludes the work in the third edition of the original German, was occupied with a description of my musical notation (see p. 425, note), and is therefore omitted in this translation. See Appendix XIX.—*Translator.*]

APPENDIX XIX.

ADDITIONS BY THE TRANSLATOR.

NOTICE.—The greater part of this Appendix consists of a re-arrangement of four papers read by me before the Royal Society, in 1864 and 1874, namely, 'On the Conditions, Extent, and Realisation of a Perfect Musical Scale on Instruments with Fixed Tones,' read 21 Jan. 1864, printed at length in 'Proceedings,' vol. xiii. p. 93; 'On the Physical Constitution and Relations of Musical Chords;' and, 'On the Temperament of Musical Instruments with Fixed Tones,' both read on 16 June 1864, and printed at length in the 'Proceedings,' vol. xiii. p. 392 and p. 404, and lastly, 'On Musical Duodenes, or the Theory of Constructing Instruments with Fixed Tones in Just or Practically Just Intonation,' read on 19 Nov. 1874, and printed at length in the 'Proceedings,' vol. xxiii. p. 3. These papers were suggested by the text of the preceding work, and are here especially adapted to develop and systematise some of its views, especially those presented in Part III. For this purpose much of the originals has been omitted, and other parts have been extended, and new Tables have been added. A condensed account of the whole subject of Just and Tempered Intonation, including necessarily the formation of all varieties of harmonisable scales, and their relations during modulation, is thus conveyed, and principles are furnished on which musical instruments can be constructed so as to satisfy the demands of Just Intonation, or the better systems of Temperament. At the close of section G I have been able to describe Mr. Bosanquet's Generalised Keyboard, from information with which he has supplied me, and in section H to give an account of some recent important researches. It will be understood that Prof. Helmholtz is in no way responsible for any statements made in this Appendix XIX.—*Translator.*

SECTION A.

ON THE NOTATION AND CALCULATION OF PITCH.

(See p. 425.)

1. *Just Intonation.*—The notation is of two kinds, one adapted to ordinary, and one to musical typography.

Ordinary Typography.—The Roman letters C, D, E, F, G, A, B, whether capital or small, represent compound musical tones, and *also* the vibrational numbers of their primes for the major scale of C in just intonation, independently of Octave. They have, therefore, the values in Table I. The *Italic* letters, capital and small, and accented, have the relative values assigned on p. 25.

The signs ♯, ♭, †, ‡, ȝ, ʒ, ¶, ♭̸, σ, ρ, express certain fractional multipliers, given in Table I., A, such that the intervals B to F♯, B♭ to F, D to †A, ‡D to A, should be perfect Fifths; that the intervals ȝG to F, or G to ʒF should be natural or sub-minor sevenths; that ¶E♭ should be identical with D♯, and ♭̸D♯ with E♭ (see p. 486, note 2), and σC to G, or C to ρ G should be that skhistic Fifth, or Fifth diminished by $\frac{1}{8}$ part of a comma, mentioned on p. 492, line 1. The values and names of these fractions are given in Table I., A.

By the use of these fractions the exact vibrational number of the prime of any compound musical tone in just intonation obtained by successions of Octaves, Fifths, Major Thirds, or sub-minor (natural) Sevenths is given by the form of the symbol itself in terms of that of C. Thus †*B*♯ read 'high *B* sharp,' $= \frac{81}{80} \cdot \frac{15}{8} \cdot \frac{135}{128} \cdot C =$ ¶*c*.

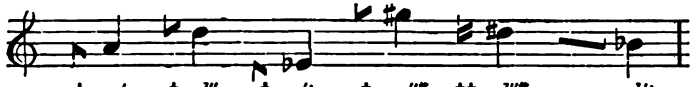
When other pitches are required in just intonation the appropriate fractions should be prefixed. Thus the first 32 partial tones of *C* are to be written thus:

N°.	Symbol	Vib. N°.	N°.	Symbol	Vib. N°.	N°.	Symbol	Vib. N°.	N°.	Symbol	Vib. N°.
1	$C_{,}$	33	9	d'	297	17	$\frac{255}{256}\,d''$♭	561	25	‡g''♯	825
2	C'	66	10	e'	330	18	d''	594	26	$\frac{39}{40}\,a''$	858
3	G	99	11	$\frac{33}{32}\,f'$	363	19	$\frac{513}{512}\,e''$♭	627	27	†a''	891
4	c	132	12	g'	396	20	e''	660	28	ʒb''♭	924
5	e	165	13	$\frac{39}{40}\,a'$	429	21	ʒf''	693	29	$\frac{261}{256}\,b''$♭	957
6	g	198	14	ʒb'♭	462	22	$\frac{33}{32}\,f''$	726	30	b''	990
7	ʒb♭	231	15	b'	495	23	$\frac{40}{45}\,f''$♯	759	31	$\frac{31}{30}\,b''$	1023
8	c'	264	16	c''	528	24	g''	792	32	c'''	1056

The correspondence of this notation with Herr v. Oettingen's, used in the text, is explained on p. 425, note, and in Table V. below. The typographical inconvenience of that notation is so great that it delayed the printing of this Translation by many weeks, and occasioned great difficulties in correcting the press. Also for ordinary writing it is often difficult

in consecutive lines to tell whether a bar is above the lower, or below the upper letter, as may be seen in fig. 61, p. 520, although in re-cutting that figure, I endeavoured, by means of two columns of letters, to obviate this inconvenience as much as possible. An alternative notation possessing all the advantages of Herr v. Oettingen's without its typographical inconvenience, consists in using inverted acute accents for bars over, as ,e, ,,e, for ē, ē̈, and inverted grave accents for bars under, as ‸e, ‸‸e for e̤, e̤̤, and is shewn also in Table V. It is a typographically convenient representative of Mr. Bosanquet's notation; see Section G, No. 15.

Musical Typography.—Assume ♯, ♭, and the marks corresponding to *C*, *D*, &c. to have the values above assigned to those marks and letters. In theoretical writing use ⚑, ⚑, being tails of quavers and semiquavers with the angle pointing upwards, for †, ††; and ⚐, ⚐, being the same with the angle pointing downwards, for ‡, ‡‡; and ⌣, being the bar for joined quavers, for ʒ. These signs exist typographically for all positions on the scale, thus:—



† *a'* ‡ *d''* † *e'*♭ ‡ *g''*♯ ‡‡ *d''*♯ ʒ *b'*♭

Mr. Bosanquet's ,e ‸e may also be represented in musical notation by similar sloping lines placed before the head of the note.

Practically no such notation will be required. The *duodenal* (p. 471, note, and p. 504, note 2,) of which the theory is given below (in Section E, p. 671,) entirely supersedes the necessity of making any changes in the ordinary staff notation, to indicate just intonation as founded on Octaves, Fifths, and Thirds.

Absolute Pitch.—The above contrivances refer the pitch of all tones to that of C. By prefixing to any piece of music the vibrational number of c' or any other tone, thus $c' = 264$, or $c'' = 512$, or $a' = 440$, the vibrational numbers, and hence the absolute pitch of all other tones is determined. See data in Table VI., C, below.

2. *Tempered Intonations.*—Prefix to the commencement of the piece of music the name of the temperament in which it has to be played, as given in Section C, below. This is not necessary for just intonation, as the *duodenal* suffices for marking it; nor for equal temperament, as the absence of any mark or duodenal suffices. The pitch of one tone should also be prefixed. For the *commatic* temperaments of Section C (in which an ascent by four Fifths is identical with an ascent by a major Third and two Octaves) the letters and symbols ♯, ♭ will have a modified meaning, given in Table VI., A, and the marks †, ‡ will not be required. But for the *skhismatic* temperaments of Section C (in which, though the last-named identity is not admitted, yet an ascent of eight Fifths and a major Third is taken to be identical with an ascent of five Octaves,) the

letters and symbols ♯, ♭, and †, ‡ will have to be used with the modified meaning given in Table VI., A. The vibrational numbers cannot be conveniently calculated for any tempered intonation except by means of logarithms as shewn in Table VI., A and C.

3. *Calculation of Pitch.*—The ratios of the just tones are given in Table I., A, together with the logarithms of those ratios to twenty places of decimals, and in Table I., B, to facilitate calculations of temperaments and on very high upper partials, the logarithms of the numbers 1 to 32, and the primes 37 to 101 are given to the same extent, from W. Gardiner's Tables of Logarithms (London, 4to, 1752, very rare). In Table I., C, will be found tables and rules for converting the expression of pitch as given by equal Semitones (with which equal temperament has rendered musicians familiar), into the expression by logarithms (which is far the most convenient for calculations) and conversely. Logarithms will be generally used in this Appendix, but their equivalents in equal Semitones will be added in the Tables.

SECTION B.

ON THE NOTATION OF INTERVALS.

(See p. 453, note.)

In writing notes in succession to form the fundamental position of a chord, or of a scale, it is very convenient to connect and yet separate them by signs which shew the intervals between them, two and two. In the foot-notes to the text (from p. 453 onwards, and on p. 400, note) I have used part of the following notations, which was necessary for the construction of Tables II. and III.

For Chords			For Scales		
Sign	Interval	Ratio	Sign	Interval	Ratio
+	Major Third . .	5÷4	(')	Low Semitone . .	256÷243
–	Minor Third . .	6÷5	(.)	Semitone. . . .	16÷15
×	Fifth	3÷2	(:)	High Semitone. .	27÷25
+	Pythagorean major Third	81÷64	(..)	Minor Tone. . .	10÷9
			(...)	Major Tone. . .	9÷8
|	Pythagorean minor Third	32÷27	(∴)	Augmented Tone .	75÷64
♯+	Super major Third	9÷7			
♭|	Sub minor Third .	7÷6	(,,) or (to) unnamed interval.		

The starting tone is written first, and the ratio is that of the greater to the smaller vibrational number. Thus $c \times g$, $c + e$, $e - g$, $d \mid f$.

When the pitch of the two notes is not precisely fixed by italicising the letters, the difference of an Octave is not noticed, but it is supposed that the first-named note has the lower pitch. But when the precise Octave is fixed by italic letters, after proceeding from the first by the intervals pointed out, we must ascend or descend the requisite number of Octaves to reach the second. Thus $c \times g$ is a proper Fifth, but for $c \times G$, after ascending the Fifth $c \times g$, we must descend an Octave, so that $c \times G$ is really a descending Fourth. Similarly $c + E$ is a descending minor Sixth, and $e - G$ a descending major Sixth. But $c \times g'$ is an ascending Twelfth; and so on.

To shew that an interval is repeated, put an exponent above the symbol of the interval, thus $c \times^2 d'$ is made up of $c \times g \times d'$, omitting the auxiliary g, and using $\times^2$ for $\times\times$. But $c \times^2 d$ shows that after ascending to d' we have descended an Octave, and have hence reached a major Second d.

To shew that an interval is taken downwards, write the exponent below. Even in the case of a single descent the exponent must then be written. Thus $c +_1 \dagger Ab$ shows that we descend a major Third from c to $\dagger Ab$; but $c +_1 \dagger ab$ shows that we afterwards ascend an Octave, and hence that the interval is a minor Sixth.

In Tables IV. and V. the tones in every horizontal line proceed regularly in major Thirds up (or minor Sixths down) from left to right, and those in every vertical column proceed in Fifths up (or Fourths down) from bottom to top. Hence we have for the horizontal line $5m$ in Table V.

$$\dagger Fb + \dagger Ab + C + E + \ddagger G\sharp + \ddagger B\sharp$$

and

$$\ddagger B\sharp +_1 \ddagger G\sharp +_1 E +_1 C +_1 \dagger Ab +_1 \dagger Fb$$

and for the middle section of the vertical column III.

$$Eb \times Bb \times F \times C \times G \times D \times \dagger A \times \dagger E$$

and

$$\dagger E \times_1 \dagger A \times_1 D \times_1 G \times_1 C \times_1 F \times_1 Bb \times_1 Eb.$$

We can evidently proceed from any tone in this table to any other by going up and down by Fifths (or Fourths), and to the left or right by major Thirds (or minor Sixths). Hence any interval can be expressed and noted by means of Fifths and major Thirds. Thus C to $f\sharp$ is $C \times G \times d + f\sharp$, or omitting the auxiliary tones $C \times\times + f\sharp$, that is $C \times^2 + f\sharp$.

The ratio of the interval, supposing both tones to lie within the same Octave, is then calculated by putting $\times = \frac{3}{2}$, $+ = \frac{5}{4}$, multiplying them together, and subsequently multiplying or dividing the result by 2 as often as is necessary to make the result greater than 1 (which would indicate a Unison), and less than 2 (which would indicate an Octave). Thus for C to $F\sharp$ we have $\times^2 + = (\frac{3}{2})^2 \times \frac{5}{4} = \frac{45}{16}$, and as this is greater

than 2, divide by 2, and the result is $\frac{45}{32}$, known as the Tritone, No. 26 of Table II. But for C to $f\sharp$ we are in the Octave above, and hence $\frac{45}{16}$ is the correct result. The logarithms of the ratios to twenty places of decimals may then be found by Table I. A.

In Table II. fifty of the special intervals which have received names are thus expressed, and their ratios thus calculated, and the logarithms of these ratios are added together with the number of equal Semitones which they contain. One advantage of this method of expressing an interval by Fifths and major Thirds is that by means of Tables IV. and V. it becomes immediately possible to take the same interval from any required tone, and when an interval is given between any two tones to discover what the second tone will be when the first is C. The logarithm and number of equal Semitones, will also immediately point this out by Table VI., B, which at the same time shews the great confusion of intervals caused even by the best temperaments. Compare the Table on pp. 516-7, and fig. 61, p. 520.

In commatic and skhismatic systems of tempered intonation (see Section C), these intervals all reduce to successions of Fifths.

For the commatic series (Section C) we omit †, ‡, and put $+ = \times^4$, where $\times$ now represents the tempered Fifth. Then the first twenty-one intervals on Table II. become: 1) $c \times^{12} B\sharp$; 2) $c \times_{12} Dbb$; 3) $c = e$; 4) same as 1; 5) same as 2; 6) $c \times^7 c\sharp$, the 'seven-Fifth Semitone'; 7) $c \times_5 db$, the 'five-Fifth Semitone'; 8) same as 6; 9) same as 7; 10) same as 6; 11) same as 7; 12) $c \times^3 d$; 13) same as 12; 14) $c \times^9 d\sharp$; 15) same as 14; 16) $c \times_{10} ebb$; 17) same as 16; 18) $c \times_3 eb$; 19) same as 18; 20) $c \times e$; 21) same as 20.

For the skhismatic series, we put $+ = \times^8$, using $\times$ for the tempered Fifth, so that $\times^8 + = 1$, or a unison when reduced to the same Octave. Also $\dagger = \times^{12}$, $\ddagger = \times_{12}$. Hence the first twenty-one intervals of Table II. reduce to: 1) $c = \dagger B\sharp$, a unison, or synonym; 2) $c \times^{12} dbb$, thus $dbb = \dagger c$, as seen by Table V.; 3) same as 2; 4) same as 2; 5) $c \times^{24} \dagger dbb$, or two commas; 6) $c \times_{17} \ddagger c\sharp$; 7) $c \times_5 \ddagger db$, 'the five-Fifth Semitone' again, but different from the above commatic N°. 7; 8) same as 7, for $c\sharp = \ddagger db$, see Table V.; 9) $c \times^7 db$, 'the seven-Fifth Semitone' again, but different from the above commatic N°. 6; 10) same as 9, for $\dagger c\sharp = db$, see Table V.; 11) $c \times^{19} \dagger db$; 12) $c \times_{10} \ddagger d$; 13) $c \times^3 d$; 14) $c \times_{27} \ddagger\ddagger d\sharp$; 15) $c \times_{15} \ddagger d\sharp$; 16) $c \times^{14} \dagger ebb$; 17) $c \times^{26} \dagger\dagger ebb$; 18) $c \times_3 eb$; 19) $c \times^9 \dagger eb$; 20) $c \times_8 e$; 21) $c \times^4 \dagger e$. All these may be readily verified by counting up and down on the middle section only of Table V., or lines p to x of Table IV.

It is, however, still convenient to use the symbols + and − in the commatic series for $\times^4$ and $\times_3$, and in the skhismatic series for $\times_8$ and $\times^9$ respectively, with the different tempered meanings of $\times$, as in the notation of dyads in Table VI., D.

SECTION C.

ON TEMPERAMENTS.

(See Chap. XVI.)

It is impossible to form Octaves by just Fifths or just Thirds, or both combined, or to form just Thirds by just Fifths, because it is impossible by multiplying any one of the numbers $\frac{3}{2}$ or $\frac{5}{4}$ or 2, each by itself, or one by the other, any number of times, to produce the same result as by multiplying any other one of these numbers by itself any number of times. Temperament, however, consists essentially in replacing Fifths and Thirds by other intervals of nearly the same amount which in the new intonation will give a relation between Thirds and Fifths, or Thirds and Octaves, or Fifths and Octaves, and hence, as all intervals can be expressed in terms of Fifths, Thirds, and Octaves (Table II.), will allow all intervals to be expressed by Fifths and Octaves, as shewn at the end of Section B. Any such alteration necessarily injures or even destroys the harmoniousness of chords. Nevertheless, considerations of instrumental convenience have led musicians to submit to the noises of false intervals, and the object of temperament should be to make that inharmoniousness a minimum. In Table VI. the best temperaments have been carefully compared with just intonation, tone for tone, the effect of the alterations on the harmoniousness of consonant triads shewn by calculating the resulting beats and differential tones, and the various systems thought worthy of consideration, placed in order of harmoniousness. The following is a new, simple, and general mathematical theory by which the pitch of the various tones there given, and those of all possible uniform temperaments, may be calculated.

The following relations are easily found by repeating Fifths and major Thirds, that is by actual multiplication:

$$\left(\frac{3}{2}\right)^{12} \times \left(\frac{1}{7}\right)^{7} = \frac{531441}{524288} = \dagger\P, \text{ the comma of Pythagoras...} \quad (1)$$

$$\left(\frac{4}{5}\right)^{3} \times 2 = \frac{128}{125} = \dagger\dagger\flat, \text{ the diesis} \quad (2)$$

Multiplying these equations together, and extracting the cube root,

$$\left(\frac{3}{2}\right)^{4} \times \frac{4}{5} \times \left(\frac{1}{2}\right)^{2} = \frac{81}{80} = \dagger, \text{ the comma (of Didymus)...} \quad (3)$$

Dividing (1) by (3),

$$\left(\frac{3}{2}\right)^{8} \times \frac{5}{4} \times \left(\frac{1}{2}\right)^{4} = \frac{32805}{32768} = \P, \text{ the skhisma} \quad (4)$$

This last is in fact the relation employed in the text on pp. 431, 486, and 491.

Temperament is of two kinds, *Uniform* and *Unequal*. In uniform temperament the intervals are the same for all keys. In unequal temperament they are varied for different keys to suit pleasure or convenience (p. 499).

I. *Uniform Temperaments.*

Let V, T, k, s be any four fractions having relations similar to $\frac{3}{2}$, $\frac{5}{4}$, † and ¶ respectively in (1) and (2), so that they will be the *tempered* Fifth, major Third, Comma, and Skhisma of the tempered system. Then from (1) and (2)

$$V^{12} \div 2^7 = ks, \text{ and } 2 \div T^3 = k^3 \div s \qquad (5, 6)$$

For calculation it is most convenient to use the logarithms of equations (1, 2, 5, 6), thus:—

$$\log \frac{3}{2} = \frac{7}{12} \cdot \log 2 + \frac{1}{12} (\log † + \log ¶) \qquad (1a)$$

$$\log \frac{5}{4} = \frac{1}{3} \cdot \log 2 - \frac{2}{3} \log † + \frac{1}{3} \log ¶ \qquad (2a)$$

$$\log V = \frac{7}{12} \cdot \log 2 + \frac{1}{12} (\log k + \log s) \qquad (5a)$$

$$\log T = \frac{1}{3} \cdot \log 2 - \frac{2}{3} \log k + \frac{1}{3} \log s \qquad (6a)$$

Next, subtracting (1a) from (5a), and (2a) from (6a) we have:—

$$\log V = \log \frac{3}{2} - \frac{1}{12} \times (\log † - \log k + \log ¶ - \log s) \qquad (7)$$

$$\log T = \log \frac{5}{4} + \frac{2}{3} \times (\log † - \log k) - \frac{1}{3} (\log ¶ - \log s) \qquad (8)$$

These are the fundamental equations of uniform temperament. Two of the four unknown expressions which they contain can be assumed at pleasure, and the others determined from them, giving rise to an endless variety of temperaments. But no temperament will be more convenient to the performer than just intonation itself, unless either $k = 1$ or $s = 1$. This, therefore, is always one assumption for practical temperaments. But the other may depend upon various conditions, of which I have considered 50 for $k = 1$, in my paper on *Temperament* (see p. 641).

i. *Commatic System* of Uniform Temperament.

Characteristic; $k = 1$, but s is not $= 1$ in equations (7, 8).

Then $\log k = 0$

$$\log s = 12 \log V - 7 \log 2, \quad \text{from (5a)} \ldots\ldots\ldots\ldots \quad (9)$$

$$= 3 \log T - \log 2, \quad \text{from (6a)} \ldots\ldots\ldots\ldots \quad (10)$$

$$\log V = \frac{1}{12} (\log s + 7 \log 2), \quad \text{from (5a)}$$

$$= \frac{1}{4} \log T + \frac{1}{2} \log 2, \quad \text{from (10)}$$

$$\log T = \frac{1}{3} (\log s + \log 2), \quad \text{from (6a)}$$

$$= 4 \log V - 2 \log 2, \quad \text{from (9)}$$

That is, there is a series of uniform tempered Fifths; the tempered major Third is formed by taking four tempered Fifths up, and two just Octaves down, and the tempered skhisma is the difference (sharper or flatter) between three tempered major Thirds and a just Octave, and also between twelve tempered Fifths and seven just Octaves. These properties are common to all these temperaments.

1. *Pythagorean* or Quintal Temperament.

Assume $k = 1$, and $V \frac{3}{2}$, then by (7)

$$\log \dagger + \log \P = \log s = \cdot 00588\ 51390\ 20306\ 53858$$

whence by (8),

$$\log T = \log \frac{5}{4} + \log \dagger = \cdot 10230\ 50448\ 94762\ 57790$$

That is, each major Third is a whole comma sharper than a just major Third. This is the Gaelic, Chinese, and old Greek system. See the 'five-Fifths' and 'seven-Fifths' scales, Table III., A and B. Extended to seventeen tones it is the Arabic system. See the complete series of tones in Table VI., B, col. Pythagoras; and the proof in Table VI., D, that it is totally unfit for harmony, which incontrovertibly establishes that the Greeks were unacquainted with anything approaching to the modern harmony of Thirds.

2. *Mean*, Mean-tone, Mesotonic or Tertian Temperament.

Assume $k = 1$ and $T = \frac{5}{4}$, then by 8,

$$\log \P - 2 \log \dagger = \log s = -\cdot 01029\ 99566\ 39811\ 95204 = -\log \text{diesis}.$$

whence by (7),

$$\log V = \log \frac{3}{2} - \frac{1}{4} \log \dagger = \cdot 17474\ 25010\ 84004\ 70121.$$

That is, each mean Fifth is $\frac{1}{4}$ comma less than a just Fifth. This was the oldest temperament admitting of harmonisation. In my paper on Temperament (see p. 641) I have shewn this to be the most harmonious of this system; see also Table VI., D. Ordinary musical notation is especially adapted to this system and the last. It requires twenty-seven tones at least. Reduced to twelve, described on p. 499, note 3, it formed the old organ tuning. In Table VI., B, col. Mean, the pitch of thirty-five tones is given.

ii. *Commato-Skhismatic System* of uniform temperament, for which both $k = 1$ and $s = 1$ in equations (7, 8).

Characteristic; $k = 1$ and $s = 1$ in equations (7, 8).

Then
$$\log k = 0 \text{ and } \log s = 0$$
$$\log V = \frac{7}{12} \log 2, \text{ from (5a)}$$
$$\log T = \frac{1}{3} \log 2, \text{ from (6a)}$$

and there is, therefore, only one such temperament.

3. *Equal*, or Hemitonic Temperament.

Assume $k = 1$, and also $s = 1$, then by (7),

$$\log V = \log \frac{3}{2} - \frac{1}{12} \times (\log \dagger + \log \P) = \cdot 17560\ 08308\ 03989\ 03054$$

and by (8),

$$\log T = \log \frac{5}{4} - \frac{1}{3} \times (2 \log \dagger - \log \P) = \cdot 10034\ 33318\ 87993\ 73174$$

by the approximations in Table I., A, it is seen that $\frac{1}{12}(\log \dagger + \log \P)$ agrees with $\frac{1}{11} \log \dagger$ to seven places of decimals; so that we may, for all acoustical purposes,[1] assume $\log \P = \frac{1}{11} \log \dagger$, and hence that

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[1] Not for mathematical, that is exact theoretical purposes. On examining the temperaments for the condition

$\log k = 11 \log s$, we find by (7, 8, 1a, 2a), that

if $V = \frac{3}{2}$, then $12 \log s = \log \dagger + \log \P$

and $\log T = \frac{1}{3} \log 2 - 7 \log s$

$= \frac{1}{3} \log 2 - \frac{7}{12} \times (\log \dagger + \log \P)$

$= \cdot 096\ 9103\ 345$

whereas $\log \frac{5}{4} = \cdot 096\ 9100\ 130$

and if $T = \frac{5}{4}$, then $21 \log s = 2 \log \dagger - \log \P$

and $\log V = \frac{7}{12} \log 2 + 12 \log s$

$= \cdot 176\ 0912\ 225$

whereas $\log \frac{3}{2} = \cdot 176\ 0912\ 590$

so that in both cases we have real temperaments, mathematically different, although audibly indistinguishable from just intonation.

$$\log V = \log \frac{3}{2} - \frac{1}{11} \times \log \dagger = \log \frac{3}{2} - \log \P,$$

$$\log T = \log \frac{5}{4} + \frac{7}{11} \times \log \dagger = \log \frac{5}{4} + 7 \log \P.$$

That is each equal Fifth is a skhisma or $\frac{1}{11}$ comma flatter than the just Fifth, and each equal major Third is 7 skhismas or $\frac{7}{11}$ comma sharper than the just major Third, and consequently each minor Third is 8 skhismas or $\frac{8}{11}$ of a comma flatter than a just minor Third. The harmony is therefore very bad, as appears by Table VI., D, except under the circumstances mentioned on pp. 502-3. But it is the sole uniform temperament which requires only 12 tones, and hence it is universally assumed, although musical notation is not adapted for it. See the pitch of its tones, and their synonyms in Table VI., col. Equal, and the logarithm of every tone to 20 places of decimals in Table I., C.

iii. *Skhismatic System* of Uniform Temperament.

Characteristics, $s = 1$, and k not $= 1$ in equations (7, 8).

Then $\log s = 0$

$$\log k = 12 \log V - 7 \log 2, \quad \text{from (5a)} \ldots\ldots\ldots\ldots\ldots \quad (11)$$

$$= \frac{1}{2}(\log 2 - 3 \log T), \quad \text{from (6a)} \ldots\ldots\ldots\ldots\ldots \quad (12)$$

$$\log V = \frac{1}{12}(7 \log 2 + \log k), \quad \text{from (5a)}$$

$$= \frac{1}{8}(5 \log 2 - \log T), \quad \text{from (12)}$$

$$\log T = \frac{1}{3}(\log 2 - 2 \log k), \quad \text{from (6a)}$$

$$= 5 \log 2 - 8 \log V, \quad \text{from (11)}.$$

We have therefore a series of uniform tempered Fifths, and the tempered major Third is found by taking 8 tempered Fifths down and 5 just Octaves up, so that to ascend 8 tempered Fifths and a tempered major Third is the same as to ascend 5 Octaves; and the comma is now the difference (sharper or flatter) between 12 tempered Fifths and 7 just Octaves, and half the difference between 2 tempered major Thirds and a just Octave.

4. *Skhismic* or Arabic, according to Prof. Helmholtz's indication (p. 431).

Assume $s = 1$ and $V = \frac{3}{2}$, then by (7),

$$\log k = \log \dagger + \log \P = \cdot 00588\ 51390\ 20306\ 53858$$

and by (8),

$$\log T = \log \frac{5}{4} - \log \P = \cdot 09641\ 99058\ 74456\ 03932.$$

The absolute Pitch of the 17 Arabic tones is given on p. 432, note 3. The Skhismic tones for a complete series are given in Table VI., B, col. Skhismic, and the harmonic effect is shewn to be much superior to that of any commatic temperament in Table VI., D. The Arabs, however, do not seem to have used any tertian harmony.

5. *Skhistic*, or Helmholtzian, as it may be called, because suggested by Prof. Helmholtz (p. 491).

Assume $s = 1$ and $T = \frac{5}{4}$, then by (8),

$$\log k = \log \dagger - \frac{1}{2} \log \P = \cdot 00514\ 99783\ 19905\ 97602$$

and by (7),

$$\log V = \log \frac{3}{2} - \frac{1}{8} \log \P = \cdot 17602\ 99956\ 63981\ 19521.$$

The name Skhistic is derived from skhist, or by abbreviation σ (see Table I., A), which I use for the small interval that is one-eighth of the Greek skhisma. It appears by Table I, A, that $88 \log \sigma = \log \dagger$ almost exactly. This allows of the following comparison of the consonant intervals in Mean, Equal, Skhismic, and Skhistic temperaments, using '22 flat' to mean, 'flatter than the corresponding justly intoned interval by 22 skhists,' that is, by a quarter of a comma.

The Interval of a	is in Skhistic Temperament	is in Skhismic Temperament	is in Mean Temperament	is in Equal Temperament
Minor Third	1 flat	8 sharp	22 flat	64 flat
Major Third	exact	8 flat	exact	56 sharp
Fourth	1 sharp	exact	22 sharp	8 sharp
Fifth	1 flat	exact	22 flat	8 flat
Minor Sixth	exact	8 flat	exact	56 flat
Major Sixth	1 sharp	8 sharp	22 sharp	64 sharp

The complete series of skhistic tones will be found, with their synonyms, in Table VI., col. Skhistic. By the examination made in Table VI., D, the skhistic appears to be the most harmonious of this series of temperaments, and hence of all existing temperaments, and probably no ear would distinguish it from just intonation, but it is very difficult to realise in practice, since the skhistic Fifths are individually indistinguishable from just Fifths, although the accumulated difference would, if neglected, entirely destroy the nature of the temperament. Hence the contrivance described on p. 656, No. 13.

iv. *Cyclic Systems* of Uniform Temperament.

Characteristic. The interval of an Octave is divided into an integral number of aliquot parts, for each of which a definite tone is tuned. Thus as many intervals and tones are created as there are aliquot parts selected, and integral numbers of these intervals being assigned to the tempered Fifth and Third, this original limited number of tones serves for every possible case, and the tones recur cyclically, under different names. By Section F., p. 671, it appears that no cyclic system has any but theoretical value, when the number of its tones exceeds 117.

These temperaments do not necessarily belong to either the commatic or skhismatic system, that is, they may have neither $k = 1$, nor $s = 1$. The following is a general method of deducing them from the preceding equations:

Take $m \,.\, \log \mathrm{V} = v \,.\, \log 2, \qquad m \,.\, \log k = q \,.\, \log 2 \;\ldots\; (9, 10)$

$m \,.\, \log \mathrm{T} = t \,.\, \log 2, \qquad m \,.\, \log s = z \,.\, \log 2 \;\ldots\; (11, 12)$

and, after substituting these values for log V, log T, log k, log s in the equations (1a, 2a), divide out by log 2, and multiply up by m. Then

$$12v - 7m = q + z, \qquad m - 3t = 2q - z \;\ldots\ldots\ldots\ldots\; (13, 14)$$

Take any integral values for q and z, and find the integral values which satisfy one of these indeterminate equations for v, m, or t, m, and substitute in the other, taking the resulting integral values of t or v respectively. The five integral values determine a *cycle* in which the Octave is divided into m aliquot parts, which may be termed *octs*, v of which make a Fifth, t a major Third, q a comma, and z a skhisma of 'the cycle of m.' Most of the results are valueless, because they do not make T and V sufficiently near to $\frac{5}{4}$ and $\frac{3}{2}$, but the following present either theoretical convenience or historical interest. Those for which $q = 0$ belong to the commatic system, and those for which $z = 0$ to the skhismatic system.

N°.	Cycle of m.	Fifth, v.	Major Third, t.	Comma, q.	Skhisma, z.
6	30103	17609	9691	539	48
7	3010	1761	969	55	7
8	301	176	97	5	0
9	53	31	17	1	0
10	53	31	18	0	1
11	31	18	10	0	−1
12	12	7	4	0	0

As these numbers do not sufficiently convey the relation of these temperaments to just intonation, the value of log 2 ÷ m, and corresponding values of log V, log T, log k, log s, up to 7 places of decimals, are added for each, and the values of the latter in just intonation subjoined.

N°.	m	log 2+m	log V	log T	log k	log s
6	30103	·000 0100	·176 0900	·096 9100	·005 3900	·000 4800
7	3010	·000 1000	·176 1000	·096 9000	·005 5000	·000 7000
8	301	·001 0001	·176 0176	·097 0097	·005 0005	0
9	53	·005 6798	·176 0741	·096 5568	·005 6798	0
10	53	·005 6798	·176 0741	·102 2366	0	·005 6798
11	31	·009 7106	·174 7916	·097 1065	0	—·009 7106
12	12	·025 0858	·175 6008	·100 3433	0	0
Just			·176 0913	·096 9100	·005 3950	·000 4901

6. *Cycle of* 30103. This is the closest possible cyclic approximation to just intonation. Call each *oct* in this case a *jot* (as used in Curwen's 'Musical Statics,' p. 100, on my suggestion), or an *atom* (as proposed by De Morgan, 'Trans. of Cam. Phil. Soc.,' vol. 10, p. 130, but merely as an abbreviation of 5 figure logarithms, not assuming an atom to be a true oct), and the number of jots will form the best means of discussing questions of intonation for the great majority of musicians, who, as a rule, are not familiar with logarithms. The true cyclic character of this temperament was, I believe, first shown in my paper on 'Duodenes' (see above, p. 641). By calculating out the numbers of jots and comparing them with the numbers in Table VI., B, col. Logarithm, it will be found that they agree with those of just intonation up to five places, within a unit of the last fifth place.

7. *Cycle of* 3010. This may be considered to be a mere abbreviation of the former. By calculating out the numbers of octs, and comparing with the numbers in Table VI., B, col. Logarithm, it will be seen that they agree with those of just intonation up to four places, within one or at most two units in the last place.

8. *Cycle of* 301. On calculating out the numbers of octs as follows, and comparing them with the numbers in Table VI., B, col. Logarithm, it will be seen that they agree with those of just intonation up to three places, within one or at most two units. But they agree to three places much more closely with those of skhistic intonation (in the same table, col. Skhistic), for which it is often extremely convenient to use them, for purposes of illustration, as in the following table, in which the value in octs of the cycle of 301 is given for 73 tones, being those in cols. I. to VI. of Table IV., less those in col. I., lines *l*, *m*, *n*, and col. VI., lines *y*, *z*. The 48 of those tones contained in the middle section of Table V. are in Roman capitals, the other of the 73 tones, which are some of their skhistic synonyms, are in Roman small letters, and other synonyms from Table V. are added in *Italics*; the whole are divided into groups of 4, the constituents of which differ from one another by 5 octs, or one skhistic comma. The possibility of such a division in skhistic temperament is an important

practical result. The names in the central column of Tones are those used in Table VIII.

Table of the Cycle of 301.

Tones.	Octs.	Tones.	Octs.	Tones.	Octs.
‡C♯ ‡‡*d*b b♯♯	18	‡E♯ ‡‡*f* ‡‡‡*g*bb	115	‡G♯♯ ‡a ‡‡*b*bb	217
C♯ ‡db †*b*♯♯	23	E♯ ‡f ‡‡*g*bb	120	g♯♯ A ‡*b*bb	222
†c♯ Db ††*b*♯♯	28	†e♯ F ‡*g*bb	125	†g♯♯ †A bbb	227
††c♯ †Db †††*b*♯♯	33	††e♯ †F gbb	130	††g♯♯ ††a †Bbb	232
‡C♯♯ ‡‡*d* ‡‡*e*bb	41	‡F♯ ‡‡*g*b e♯♯	143	‡A♯ ‡‡*b*b	240
c♯♯ ‡D ‡*e*bb	46	F♯ ‡*g*b †e♯♯	148	A♯ ‡*b*b	245
†c♯♯ D *e*bb	51	†f♯ Gb ††e♯♯	153	†a♯ Bb	250
††c♯♯ †D †*e*bb	56	††*f*♯ †Gb †††e♯♯	158	††a♯ †Bb	255
‡‡D♯ ‡‡*e*b	64	‡F♯♯ ‡‡*g* ‡‡*a*bb	166	a♯♯ ‡B ‡‡*c*b	268
‡D♯ ‡*e*b	69	f♯♯ ‡G ‡*a*bb	171	†a♯♯ B ‡*c*b	273
d♯ Eb	74	†*f*♯♯ G *a*bb	176	††a♯♯ †b Cb	278
†d♯ †Eb	79	††*f*♯♯ †g †Abb	181	†††a♯♯ ††*b* †Cb	283
‡D♯♯ ‡e ‡‡*f*b	92	‡G♯ ‡*a*b	194	‡B♯ ‡‡*c* ‡‡‡*d*bb	291
d♯♯ E ‡*f*b	97	G♯ ab	199	B♯ ‡c ‡‡*d*bb	296
†*d*♯♯ †E fb	102	†*g*♯ †Ab	204	†*b*♯ C ‡*d*bb	0
††*d*♯♯ ††*e* †Fb	107	††*g*♯ ††Ab	209	††*b*♯ †C dbb	5

9. *Bosanquet's Cycle of* 53. This is the cycle proposed by Nicholas Mercator, as a theoretical representation of just intonation, for which purpose it was much employed by Gen. T. Perronet Thompson, but as it was first practically used in music on an instrument constructed by Mr. Bosanquet (see below, section G, No. 15), who has invented a special notation for it (explained in Table VI., B, Introduction), it is proper to speak of it as Bosanquet's cycle. The logarithms of every division are given in Table VI., B, col. Bosanquet, by which it will be seen to be a very accurate representative of skhismic intonation, but (by the same Table, D) to be superior to it in harmoniousness. It is no doubt the best cyclic temperament that can be formed without exceeding 117 tones to the Octave.

10. *Commatic Cycle of* 53. The value of the Octs and of the Fifths remains the same, but the major Third is taken sharper. This makes the cycle a very accurate representative of Pythagorean intonation, as may be readily verified by Table VI., B. But as that intonation is unfit for harmony (Table VI., D), the present cycle may be disregarded.

11. *Cycle of* 31. This is Huyghen's Cyclus Harmonicus. The value of all its octs is given in Table VI., B, col. Huyghens, by which it will be

small indeed. If we tuned all the other columns in the middle section of Table V. in the same way, and the tuning were absolutely correct, we should have an ascending series of perfectly just Fifths from ‡‡D♯ to ‡D♯♯, from ‡B to B♯, from ‡G to G♯, from E♭ to †E, from C♭ to †C, and from †A♭♭ to ††A♭, each series being exactly a just major Third lower than the following. But we should have the *five* flat Fifths ‡D♯♯ to ‡B (for A♯♯), B♯ to ‡G (for F♯♯), G♯ to E♭ (for D♯), †E to C♭ (for †B), †C to †A♭♭ (for †G), each such Fifth being a skhisma too flat. For the same reason each of the four minor Thirds ‡D♯♯ — ‡G (for ‡E — ‡G or ‡D♯♯ — F♯♯), B♯ — E♭ (for ‡C—E♭ or B♯ —D♯), G♯ — C♭ (for A♭ — C♭ or G♯ — †B), †E— †A♭♭ (for F♭ — †A♭♭ or †E — †G) will be a skhisma too flat. The resulting eight imperfect triads are specified, and their harmoniousness is investigated in Table VI., D, and shewn to be of a high character. For all other triads the harmoniousness would be that of just intonation. In future, skhistic intonation is supposed to be practically replaced by this *unequally just* intonation, and both are regarded as audibly identical with uniformly just intonation.

The practical direction for tuning the forty-eight tones of the middle section of Table V. in this practically just intonation is: *Tune* C *to fork, tune three just major Thirds up* C + E, E + ‡G♯, ‡G♯ + ‡B♯, *and two down* C $+_1$ †A♭, †A♭ $+_1$ †F♭. *From each of the six notes thus obtained, tune four just Fifths up, as* C × G, G × D, D × †A, †A × †E, *and three down, as* C $\times_1$ F, F $\times_1$ B♭, B♭ $\times_1$ E♭, *and verify each set of Fifths by the two adjacent sets, as the individual tones are tuned, by seeing that the tones written on the same horizontal line in Tables IV. and V. make just major Thirds with each other.* As, however, it is practically impossible to tune the major Thirds by mere absence of beats with sufficient accuracy to make them starting-points (or 'bearings') for the series of Fifths, the result will seldom be satisfactory to the ear when carried out to the full extent, unless tuning-forks be obtained for the first series of major Thirds. From the vibrational numbers for $c' = 264$, in Table VI., B, either unison forks, or (still better) Scheibler's beating forks (p. 253, note 2) can be constructed for these bearings.

14. *Practical Intonation for Bosanquet's Cycle.*—Two methods (14a and 14b) are proposed, depending on the following equations,

$$53 \log \frac{3}{2} - 31 \log 2 = \cdot 00090\ 68643\ 67689 = \log r$$

$$= 6 \times \cdot 00013\ 68851\ 87576 + \cdot 00008\ 55532\ 42235 = 6 \log m + \log n$$

$$= \cdot 00098\ 02142\ 67201 - \cdot 00007\ 33498\ 99512 = 2 \log \P - \log p$$

so that, very nearly, $\log \P = 3\frac{1}{2} \log m = 6 \log n = 7 \log p$, and hence the intervals m, n, p are practically inaudible, but $\log r$ is seen to be very nearly $2 \log \P$, and hence r is quite sensible. The tempered major Thirds, which now constitute the 'bearings,' are tuned as follows:—

Table of Bearings.

Intonation of	in (14a)	in (14b)	Beats in 1 sec. of	in (14a)	in (14b)
†f'♭	338·47	338·30	†f'♭ + †a'♭	1·37	1·91
†a'♭	422·74	422·4	†a'♭ + c'	1·72	0
c'	264	264	c' + e'	1·07	0
e'	329·73	330	e' + ‡g'♯	1·34	1·86
‡g'♯	411·83	412·04	‡g'♯ + ‡b'♯	1·68	2·32
‡b'♯	514·37	514·46	‡b'♯ + ‡‡d''♯♯	2·09	2·91
‡‡d'♯♯	321·22	321·18			

(14a) *Practical Cyclic Intonation.*—Tune the seven notes in col. 1, by Scheibler's or other forks, or from c', tuned to 264, derive the other six tones by beats, as in cols. (14a). Then from †f'♭ tune 4, and from each of the other six notes tune 7 just Fifths up. Verification, the 5th Fifth from †f'♭ will be flatter than ‡‡d'♯♯ by n, and may be assumed as identical with it; and the 8th Fifth from each of the other bearings will be flatter than the preceding bearing in the table by m, and may be assumed as identical with it. Assume then that †f'♭♭ = ‡‡d'♯♯, and that ‡‡g'♯♯ = ††a'♭ and repeat the series of 53 Fifths up from †f'♭♭ and down from ‡‡d'♯♯. This unequally recurrent system will be audibly indistinguishable from the true cycle.

(14b) *Skhismo-Cyclic Intonation*, invented and kindly communicated by Captain J. Herschel, Royal (late Bengal) Engineers, and named by me from its predominantly skhismic and recurrent character. Tune the bearings by forks or beats, as before, taking cols. (14b). From c' and e' tune 4 just Fifths up, and from †a'♭ and c' tune 3 just Fifths down. From e', ‡g'♯, ‡b'♯, tune 7 just Fifths down; from †a'♭ tune 7 just Fifths up, and from †f'♭ tune 4 just Fifths up. Verification, the major Thirds in columns 4, 5, 6 of Table IV. will be just, and hence each of the two Fifths g♯ × e♭, and †E × c♭ will be too flat by one skhisma; the 8th just Fifth down from e', ‡g'♯, ‡b'♯ will be respectively the ‡g♯, ‡b♯, ‡‡d'♯♯ already tuned by Octaves from the bearings; the 8th just Fifth up from †a'♭ will coincide with f''♭, and the 5th just Fifth up from †f'♭ will be sharper than ‡d''♯♯ by p, which is indistinguishable. Assume the identities and repeat as in (14a). The major Thirds between columns 1 and 2, 3 and 4, 6 and 7, 7 and 8, and 8 and 9 as far as line t in the last, will be too flat by a skhisma as in skhismic intonation, the remaining major Thirds between columns 8 and 9, and between columns 9 and 10 to line t, and the major Thirds between columns 2 and 3 will be too flat by about $\frac{8}{7}$ skhisma, which is audibly indistinguishable from a skhisma, and so on for further columns for ever.

Equal Temperament is realised on an ordinary manual.

Commatic Uniform Temperament, so far as the tones E♭, B♭, F, C, G, D, A, E, B, F♯, C♯, G♯, and no others, are concerned, can also be played on the same manual, which was specially constructed for that purpose.

Saunders's 'Tilting Action,' as I shall call it in future references, allows the number of tones on harmoniums to be doubled by twelve stops. Thus the harmonium vibrators are arranged in two series, the large letters representing the tones excited by long or white digitals, and the small letters those excited by short or black digitals, on a manual of the usual form, the *two* tones written over one another being excited by the *same* digital:

Back.	B♯	d♭	C♯♯	e♭	F♭	E♯	g♭	F♯♯	a♭	G♯♯	b♭	C♭
Front.	C	c♯	D	d♯	E	F	f♯	G	g♯	A	a♯	B

When no stops are drawn out the arrangement is that of the *front* series, where the long white digitals are natural and the short black digitals are the sharps. On pulling out a stop, the vibrators of its name in the front series throughout the octaves of the instrument are damped, and the corresponding vibrators in the back series come into action. Thus a♯ is changed to b♭, and d♯ to e♭, and then the series of notes speaking to the pressure of the digitals is

C c♯ D e♭ E F f♯ G g♯ A b♭ B

which are the twelve tones of the old-fashioned manual. On pushing the stop in the previous intonation is restored. The arrangement, which I have personally tested, works well, and, as will be seen in Section G, is valuable for many purposes.[1]

The Skhismatic System of Uniform Temperament requires altogether new contrivances, described in Section G, of which the principal is that of Mr. Bosanquet, Table IX.

It must be always understood, in future investigations, that all theory refers to just intonation *only*, and that any tempered intonation, even when so good as skhistic, or practically just, or Bosanquet's, is a *mere practical substitute*. To found any harmonic theories on the synonymity of tones in any temperament, when there is known to be no synonymity in nature, and when the artificial synonymity thus engendered varies from tempera-

[1] It consists essentially of applying twelve rollers, below the vibrators, one for each note, carrying arms which on the rollers being made to revolve slightly by the stops, damp a note in each Octave of the front and back vibrators alternately. For details reference should be made to Mr. Saunders, at E. Lachenal's Concertina Manufactory, 4 Little James Street, Bedford Row, W.C. His harmonium was shewn when my paper on Musical Duodenes (p. 641) was read before the Royal Society, and also before the Musical Association.

ment to temperament, is only comparable to deducing geometrical conclusions from the mere practical construction of figures. Both tempered intonation and practical drawing are of great value for giving conceptions of the results with more or less correctness, which are difficult to obtain otherwise, but no further. Even equal temperament, like the hypothesis of circular planetary orbits all in one plane, is extremely useful for rough results. But musical theory founded on equal temperament would be just as rickety a contrivance as astronomy founded on circular co-planar orbicity—that is, simply ridiculous as a part of science.

SECTION D.

SCALES OF QUINTAL AND TERTIAN HARMONY.

Harmony, which is due to consonance, is of four kinds:

1) *Octaval Harmony*, taking account of the consonance of an Octave 2 only, as in equally tempered intonation, which has only this one consonance, and a variety of rougher or smoother dissonances.

2) *Quintal Harmony*, taking account of both Octave and Fifth, and hence possessing such consonances as the Octave 2, double Octave 4, Fifth $\frac{3}{2}$, Twelfth 3 (or the extension of a Fifth by an Octave), Fourth $\frac{4}{3}$ (or the defect of a Fifth from an Octave), and Eleventh $\frac{8}{3}$ (or the extension of a Fourth by an Octave), but no others.[1] On this were founded the quintal scales mentioned below, but whether the nations using those scales ever *sang* any consonances but the Octave remains doubtful.

3) *Tertian Harmony*, taking account of the consonances of the Octave 2, Fifth $\frac{3}{2}$, and major Third $\frac{5}{4}$, and those derived from them; this is the modern theoretical harmony, considered below.

4) *Septimal Harmony*, taking an account of the consonances of the Octave 2, Fifth $\frac{3}{2}$, major Third $\frac{5}{4}$, subminor Seventh $\frac{7}{4}$, and those derived from them.

[1] This is thus expressly laid down by Euclid, 'Introd. Harm.,' p. 12, ed. Meibom: *σύμφωνα δέ ἐστιν ἐν τῷ ἀμεταβόλῳ συστήματι ἕξ: ἐλάχιστον μὲν τὸ διὰ τεσσάρων, τόνων δύο ἡμίσεως, ... δεύτερον δὲ τὸ διὰ πέντε, τόνων τριῶν ἡμίσεως ... τρίτον δὲ τὸ διὰ πασῶν, τόνων ἕξ ... τέταρτον δὲ τὸ διὰ πασῶν καὶ διὰ τεσσάρων, τόνων δικτὼ ἡμίσεως ... πέμπτον δὲ τὸ διὰ πασῶν καὶ διὰ πέντε, τόνων ἐννέα ἡμίσεως ... ἕκτον δὲ τὸ δὶς διὰ πασῶν, τόνων δώδεκα ... ὁ δὲ τῆς φωνῆς τόπος αὔξεται μέχρι τοῦ ὀγδόου συμφώνου, ὅπερ ἐστὶ δὶς διὰ πασῶν καὶ διὰ τεσσάρων* ($\frac{16}{3}$, an eighteenth), *καὶ δὶς διὰ πασῶν καὶ διὰ πέντε* ($\frac{20}{3}$, a nineteenth). *διάφωνα δέ ἐστι τά τε ἐλάττω τῶν διὰ τεσσάρων* (especially enumerated on p. 8 as *δίεσις, ἡμιτόνιον, τόνος, τριημιτόνιον, δίτονον*, the two last being the Pythagorean minor and major Thirds $\frac{32}{27}$ and $\frac{81}{64}$), *καὶ μεταξὺ τῶν εἰρημένων συμφώνων πάντα.* Compare Gaudentius, ed. Meibom. p. 12.

This is the harmony mentioned on p. 534, note, and provided for by Mr. Poole (below, Section F). It is yet far too undeveloped to be further considered here, and the observations on p. 296 (text and note 2) and p. 330 tend to render it doubtful whether any substantive theory of harmony founded upon the subminor Seventh should be attempted. Even the advantage of using this interval to diminish the roughness of certain dissonances in tertian harmony, is doubtful, because of the trouble that it would occasion the ear in following the progression of parts, and grasping the tonality of the whole piece of music.

A. Scales of Quintal Harmony.

Five-Fifths Scales consist of five tones which can be arranged so as to form four Fifths. The pitch of all of these is known when that of one is known. By 5 . C . 2 . represent such a scale, of which C is the second tone in order of Fifths, as F × C × G × D × †A, implying also that the piece of music in which it occurs *ends* on this C. See the complete series of these scales in Table III., A.

Seven-Fifths Scales consist of seven tones, which can be arranged so as to form six Fifths. The pitch of all of these is also known when that of one is known. By 7 . C . 2 . represent such a scale, of which C is the second tone in order of Fifths, as F × C × G × D × †A × †E × †B, implying also that the piece of music in which it occurs *ends* on this C. These comprise the Greek modes. See the complete series in Table III., B.

Seventeen-Fifths Scales. These are scales formed by selecting tones from a series of seventeen, forming sixteen perfect fifths, and include the Arabic and Persian scales considered on pp. 431–436. After re-examining Kiesewetter's work (p. 431, note 2) I feel that we cannot perfectly comprehend the meaning of the Arabic and Persian theoreticians without referring to the original texts, and deducing conclusions on a scientific basis, independent of that equally tempered notion of the nature of music which seems to have prevented Kiesewetter from thoroughly understanding what he had to explain.

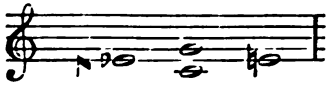
B. Scales of Tertian Harmony.

Such scales will be called simply *harmonic.* They are based on the principle that *every tone in the scale will form a major or minor consonant triad with two other tones in the same scale.* This excludes quintal and septimal scales, and all tempered scales, and reduces the latter to merely mechanical attempts at conveying a more or less accurate notion of the nature of tertian harmonies, which imply just intonation. The principle which renders the construction of such scales possible is shewn by the table on p. 480. The following development of this principle was first given in my paper on Duodenes (p. 641) for the purpose of determining the number of tones required by modern modulational music.

The Harmonic Elements are the Fifths, the major and minor Thirds, with their extensions, inversions, and extensions of their inversions, which will include all the forms on p. 289, note 1. Referred to the same First these may be considered to be included in the forms C × G, C + E, C — †E♭, where the Roman letter, as it does not define any Octave, allows either constituent or both of the constituents of an element, to be taken any number of Octaves higher or lower.

The Harmonic Cell or Unit of Concord.

Staff Notation. Letter Notation. Solfeggio Notation.

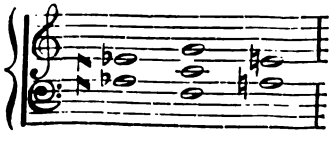


†E♭ G *Mo So*
C E *Do Mi*

This consists of the three harmonic elements C × G, C + E, C—†E♭, referred to the same First, C. The staff notation is symmetrical, the major Third being placed on the right, and the minor Third on the left. The Letter and Solfeggio notations are unsymmetrical. The major Thirds C + E, †E♭ + G are horizontal to the right; the Fifth C × G is vertical upwards, and the minor Thirds C — †E♭, E — G oblique from below to the left upwards. This relative position replaces the symbols +, ×, —. In all three diagrams the form is rhombic. The Fifth C × G forms the *vertical axis*, the major Third E the *right pole*, and the minor Third †E♭ the *left pole*. The low sharp †E♭ $\times_1$ $+^2$ E forms the *polar axis*. This *cell*, whence all tertian harmony is *developed*, contains the major triad C + E — G on the right, and the minor triad C — †E♭ + G on the left, having the same First C, and therefore the same Fifth G. By the convention of pitch in the elements, and a further convention by which Octaves may be used simultaneously with the primary tones, these triads represent all concords in all positions, as in Chap. XII. The example is a C-cell. Cells to any First may be formed by taking any tone in Tables IV. and V. as the First, with the tone on its right, the tone above it, and the tone to the left of this last. The solfeggio names throughout have Italian vowels, and are modelled (with some necessary changes) on those used in the Tonic Sol-fa system, for the use of singers. The *Do* may have any pitch whatever.

The Harmonic Heptad or Unit of Chord-Relationship.

Staff Notation. Letter Notation. Solfeggio Notation.

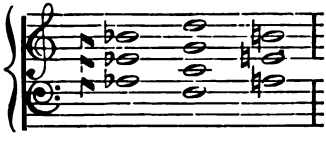


†E♭ G *Mo So*
†A♭ C E *Lo Do Mi*
F A *Fa La*

This consists of two cells, the First of the upper cell being the Fifth of the lower cell. The staff notation forms a regular hexagon, with the First in its centre. These seven notes give rise to the name. The vertical *axis* is F × C × G, the line of Fifths; the major Thirds are on the right, the minor on the left. The horizontal axis †A♭ + C + E contains two major Thirds. The poles are A the major Sixth, and †E♭ the minor Third. This form contains all the three major and three minor triads, into which the first enters; namely, the four *cell triads* F + A − C, C + E − G, F − †A♭ + C, C − †E♭ + G; and the two *union triads* (which connect the cells and arise from their union), the minor A − C + E on the right, and the major †A♭ + C − †E♭ on the left. It also contains all the *con-dissonant triads* (p. 526, note 1) which have two tones consonant with C and dissonant with each other. Of these the horizontal axis †A♭ + C + E, which I term a *trine*, is an important element in future constructions. Hence the heptad contains all consonant and con-dissonant triads related by having one common tone, as C. The example is the heptad of C, and the central tone is its First. By Tables IV. and V. heptads may be constructed to any First. The reader will feel the nature of the connection of these chords better if, on a justly-intoned instrument (as the just harmonium, Section G, No. 2), he holds down the C, and then plays the six triads, in every position given on pp. 333–8.

The Harmonic Decad or Unit of Harmony.

Staff Notation.



Letter Notation.			Solfeggio Notation.		
†B♭	D		*To*	*Re*	
†E♭	G	B	*Mo*	*So*	*Ti*
†A♭	C	E	*Lo*	*Do*	*Mi*
	F	A		*Fa*	*La*

On a given harmonic cell with C as its First construct two others, having the First and Fifth of the original cell for their Fifth and First respectively (in inverse order therefore). The decad (so called from its having *ten* tones) is the complete *vertical* development of the original cell, upwards and downwards, and the First, C, of that cell is called the *tonic* of the decad. The three cells from lowest to highest are called the *subdominant*, *tonic*, and *dominant cells*.

The decad, therefore, contains two heptads having a common cell, forming its *nucleus*. Its vertical axis is the line of Fifths. Its horizontal axis is doubled, and has two trines. Its poles are the major Sixth A and high minor Seventh †B♭. It contains ten consonant triads; three cell major triads, and three cell minor triads; two union minor triads on the right or major side, and two union major triads on the left or minor side. It has all the discords possible without modulating. By help of Tables IV. and V. decads may be constructed to any tonic.

Harmonic Trichordals and their Scales. A trichordal consists of one triad from each of the three cells of a decad. Eight such trichordals may be formed. Name them by abbreviating the words 'major triad' and 'minor triad' into *ma*, *mi* (with Italian vowel sounds), repeated, with equal stress, in the order of the chords from the lowest or subdominant, through the middle or tonic, up to the highest or dominant, thus immediately recalling their construction by the words: *mamama*, *mimama*, *mamima*, *mimima*; *mamami*, *mimami*, *mamimi*, *mimimi*. The seven tones contained in any trichordal, when reduced to a single Octave, necessarily form an *harmonic scale*, as above defined, because each *tone* in them is *harmonic* or referable to one or more consonant triads belonging to the related triads of a decad.

It is possible to commence and end the scale by any one of these harmonic tones. Thus are formed the seven *modes* or *scales* of each of the eight trichordals, or fifty-six in all. These modes answer to the Greek modes, but they are not the same, because the Greek scales did not consist of harmonic tones. To indicate whether the scale begins with the First (*p*rima), the Third (*t*ertia), or the Fifth (*qu*inta) of any triad in the trichordal, change the initial letter *m* of the name of that chord in the trichordal to *p*, *t*, *qu* respectively, and in reading accent that syllable. Since the Fifth of the first or second chord is the First of the next following, the initial *qu* is required only for the third chord. Thus the six cases of the table on p. 480 respectively, and hence the whole table, will be represented by the six words: *mipími*, *mapími*, *mapámi*, *mapáma*, *mipáma*, *mipíma*. The name of the tonic of the decad should be prefixed to the name of the scale, as C *mapáma*, which is equivalent to C major. Many scales in different trichordals of the same decad are included under the name of C minor, but this expression may be generally held to mean the three scales, C *mipíma*, or C *mapíma* ascending, and C *mipími* descending. For this reason it is better to speak of a piece of music as being in a certain *decad*, than as of being in a certain *key*.

In Table III. C, a complete list is given of all the trichordals and their scales. Each trichordal has its chordal system exhibited as on p. 519, but with solfeggio names, which apply equally to all decads, as the *do* is variable. The scales are taken from different decads in order to make every scale commence with C, and the whole may be immediately verified by forming these decads by Tables IV. and V. In writing out the scales the exact interval between each pair of tones is marked, as on p. 644, shewing their different nature. Whenever any of those scales has been referred to, and named in the text, the name proposed is added with a reference to the page. Whenever in Table III. A and B, one of the Five-Fifths or Seven-Fifths scales can be reduced by altering some of its tones by a comma to one of these harmonic scales, cross references are given to facilitate comparison. It is possible on the experimental harmonium de-

scribed in Section G, No. 2, to examine the effect of all these trichordals and scales in the decad of C, and try the effect of all their peculiar and distinctive harmonies and cadences. Such examinations are indispensable for their proper appreciation, as an equally tempered instrument obliterates their delicate distinctions.

SECTION E.

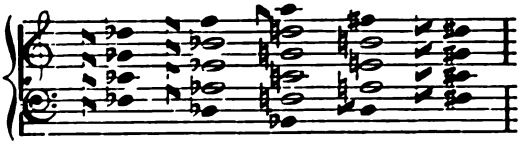
ON MODULATION, DECADATION, AND DUODENATION.

Modulation is properly a change of *mode*, that is, of scale within the same trichordal. But such changes of mode are no longer made consciously, each trichordal being generally represented by a single scale beginning with the tonic. Modulation should then be applied to the change of trichordal within the same decad. But as the modern minor comprises three trichordals, *mimima*, *mamima*, *mimimi*, and is yet looked upon as a single mode, these changes are not regarded as modulations, although a change from *mamama* to any other trichordal in the same decad, or conversely, is considered as a modulation from the major into the *synonymous* minor, or conversely, as from C *mapáma* to C *mipíma*. The more usual changes require an alteration of decad as well as of trichordal, and although by a great extension of language they may still be called modulations, I prefer calling them *decadations*.

They arise thus: On examining the decad of C in the scheme on p. 666 it is easily seen that the C and G cells might be the sub-dominant and dominant cells of a new tonic G; and that the F and C cells might be the tonic and dominant cells of a new tonic F. Similarly the three tones A, E, B, to the right only require completion by a Fifth above, as F♯, or a Fifth below, as ‡D, to form the vertical axis of a new decad with tonic E or tonic A as the case may be, of which the two cell triads, A — C + E, E — G + B, already exist as union triads in the original decad. And in the same way the three tones, †A♭, †E♭, †B♭, on being completed by a Fifth †F above, or a Fifth D♭ below, would be the vertical axis of a new decad with tonic †E♭ or †A♭, as the case may be, of which the two cell triads, †A♭ + C — †E♭, †E♭ + G — †B♭ appear as union triads in the original decad. Every chord of a decad is therefore *ambiguous*, or may belong to other decads. Each new decad is liable to similar alterations. This change of one decad to another is called *decadation*. To determine the number of tones necessary to provide on instruments with fixed tones in order to execute all such music as now exists (which abounds in such decadations) it is necessary to determine the extent of tonic decadation (that is, such decadation as is consistent with a conscious retention of relationship to the original tonic) and also the extent of variability in the original tonic itself.

The Harmonic Hepta-Decad or Unit of Tonic Decadation.

Staff Notation.



The white notes and capital letters shew the original decad, the black notes and small letters the additions necessary to form new decads.

Letter Notation.						Solfeggio Notation.				
†db	†f	†a				*ru*	*fo*	*le*		
†gb	†Bb	D	f♯			*su*	*To*	*Re*	*fi*	
†cb	†Eb	G	B	‡d♯		*du*	*Mo*	*So*	*Ti*	*ri*
†fb	†Ab	C	E	‡g♯		*fu*	*Lo*	*Do*	*Mi*	*se*
	db	F	A	‡c♯			*ro*	*Fa*	*La*	*de*
		bb	‡d	‡f♯				*ta*	*ra*	*fe*

On every tone of a heptad (as the C or *Do* heptad, in the example) taken as a tonic, construct a decad, and thus form seven related decads, indicated by the name *hepta-decad.*

There are two vertical decads, which, as is seen by the capital letters, each retain seven tones of the original decad, and consequently introduce only three new tones. Extracted from the above figure these decads are as follows:

Dominant Decad, Or *So* decad of C, being that of G.			Subdominant Decad, Or *Fa* decad of C, being that of F.		
†f	†a		†Eb	G	
†Bb	D	f♯	†Ab	C	E
†Eb	G	B	db	F	A
	C	E		bb	‡d

Of the tones in the dominant or *So* decad, two, (†f, †a), differ from two (F, A) in the original decad, by a comma (†) only, and one (f♯) by a sharp (♯). But in the subdominant or *Fa* decad, two, (db, bb), differ from two original tones by a sharp, (♯ or ♭), and one (‡d) by a comma, († or ‡). Hence the dominant decad is the more nearly related to the tonic decad, and, as a matter of fact, dominant is more frequent than subdominant decadation.

There are four lateral decads which when similarly treated appear as:

Left relative Decad, Or *Mo* decad of C, being that of †Eb.			Right relative Decad, Or *La* decad of C, being that of A.		
†db	†f		G	B	
†gb	†Bb	D	C	E	‡g♯
†cb	†Eb	G	F	A	‡c♯
	†Ab	C		‡d	‡f♯

Left correlative Decad, Or *Lo* decad of C, being that of †A♭.			Right correlative Decad, Or *Mi* decad of C, being that of E.		
†g♭	†B♭		D	f♯	
†c♭	†E♭	G	G	B	‡d♯
†f♭	†A♭	C	C	E	‡g♯
	d♭	F		A	‡c♯

It is thus seen that the relative decads have each six tones in common with the tonic decad, and hence introduce four new tones, of which three differ by a low sharp (or high flat) and one by a comma from the original tones. The correlative decads have likewise six tones in common with the tonic decad, and of the four new tones, three likewise differ by a low sharp (or high flat), but the fourth differs by a whole sharp (or flat). Hence the correlative are less nearly related to the tonic than the relative. As the right relative or *La* decad is most nearly related to the major or *mamama* trichordal of the tonic decad, and as this trichordal is by far the most commonly used, decadations into the *La* decad are most frequent. And as in commatic temperaments, on which so much of musical practice (and theory, too, unfortunately) is founded, ‡d was not distinguished from D, the *La* decad was supposed to have seven tones in common with the tonic, sufficient to form the *mimimi* trichordal. This view was, of course, suggested by the false identification of Tertian with Seven-Fifths scales, so that the major was supposed to be C D †E F G †A †B c, that is to say, 7 . C . 2, and the minor to be †A †B c d †e f g †a, that is to say, 7 . †A . 5, a difference merely of mode. Hence the modulations from the major of the tonic to the minor of the *La* decad is most common, and hence the minor is generally called the relative minor of the tonic simply; and upon this relationship the theory of the minor scale has sometimes been founded, though such a view appears to be imperfect and unsystematic.

The heptadecad as first figured contains all these six decads in addition to the tonic, and forms them, as will be seen by the capitals and small letters, by using fourteen new tones, or twenty-four tones in all. On account of the frequency of these *first* decadations or modulations the singer should become familiar with the 'mental effects' of all the tones in a heptadecad (involving many in addition to those cited on p. 429, note), and for this the solfeggio names will be found peculiarly serviceable.

Tetrachordal Scales produce the ambiguities referred to on p. 460, note. A tetrachordal consists of one triad from each of four cells, of which the three first and three last separately form a decad. A tetrachordal therefore contains 9 tones. Arranging them in order of Thirds, if we reject the two first or the two last tones we have two ordinary trichordals, but if we reject only the first and last tone we have a real tetrachordal scale. Sufficient details will be found in Table III., D. It is observable that even when these seven tetrachordal tones are used in the melody, the harmony generally fills up the complete tetrachordal, and

The Harmonic Hepta-Decad or Unit of ecadation, greatly tempered treatment or scale as a real minor y the received theory of ave frequent recourse to one Table III., D, I., Ex. 1 and 2), such changes as decadations.

Staff Notation.

Letter Notation.

†db	†f	†a
†gb	†Bb	D
†eb	†Eb	
†fb	†Ab	

f Duodenation, is a series of twelve ordinary pianoforte manual, having the an oblong on the diagram of the hepta- e duodene of C and *Do*. It consists of are *trines* of major Thirds; the lowest or *sub-* + A; the next or *tonic* trine †Ab + C + E; e †Eb + G + B; and the highest or *superdominant* + F♯. It also consists of three vertical columns or Fifths; the minor Third, *minor*, or left-hand quaternion †Eb × †Bb; the Fifth, *radical*, or central quaternion G × D; and the major Third, *major*, or right-hand quaternion B × F♯. From these determinate conditions a duodene can be constructed when there are given one of its tones, and the position of that tone in the duodene, which is best assigned by its solfeggio name, of the duodene for which A = *La*, or A = *Mo*. By means of Tables IV. and V. such duodenes can be formed at sight. The *Do* of the duodene is called its *root*, and the name of its tone is that of the duodene; thus if A = *La*, the duodene is C; if A = *Lo*, the duodene is ‡C♯; if A = *Mi*, the duodene is F; and if A = *Mo*, the duodene is ‡F♯. A duodene has three *tonics*, forming the tonic trine; which, in the duodene of C, are †Eb, the tonic of †Eb *mapima*; C, the tonic of C decad, and all its trichordals and scales; and E, the tonic of E *mipimi*.

On taken ind

The smallest intervals between two tones in a duodene are the low sharp ‡♯ (as †Bb to B, †Eb to E, †Ab to A), the sharp ♯ (as F to F♯, Db to D), and the semitone †♯ ♭ = $\frac{16}{15}$ (as F♯ to G, B to C, E to F, D to †Eb, G to †Ab, C to Db). Hence any smaller intervals imply a change of duodene. Thus on referring to Table IV., where the duodene of C fills the central oblong, it will be seen that if we take the trine †F + †A + C♯ above the superdominant trine, and compare it with the subdominant trine Db + F + A, two tones, †F, †A, in the former are a comma higher than two, F, A, in the latter; and one tone C♯, is flatter than the remaining tone of the latter Db, by a diaskhisma (see Table II., No. 2), or nearly $\frac{19}{11}$ comma, †♭. Comparing the trine below the subdominant, Gb + Bb + ‡D, with the superdominant trine †Bb + D + F♯, we find two tones Bb, ‡D lower by a comma, and one tone Gb higher by a diaskhisma, or nearly $\frac{19}{11}$ comma, in the first than in the last. But if we take the quaternion to the right of the major quaternion,

G♯ × ‡D♯ × A♯, and compare it with the original minor
D♭ × †A♭ × †E♭ × †B♭, we find that each tone of the
r than the corresponding tone of the latter by a diesis ††‖
) or nearly $\frac{21}{11}$ comma. These small intervals then arise
dene or *duodenation*; and their absence serves strictly
a single duodene.[1]

ay be *vertical* or *lateral*. Vertical duodenation proceeds
trines, one being cut off at bottom and another added at top,
y. Hence the same trine forms part of *four* different duodenes
by vertical duodenations. Thus †B♭ + D + F♯ is a super-
nant trine in the C, a dominant trine in the G, a tonic trine in the
, and a subdominant trine in the †A duodene. But after 4 vertical duodenations there is no trace of the original or *radical* duodene. Hence vertical *radical duodenation*, or vertical change into a duodene which has at least one trine in common with the duodene of the original root, cannot exceed *three* duodenations up and as many down, and hence all the tones of the vertical radical duodenation from C must be contained in the 10 trines in Table IV., columns 4, 5, 6, lines *n* to *y*, which are limited above and below by a thick rule.

Lateral duodenation, as from the duodene of C to that of E, changes a quaternion at each move, so that each quaternion forms part of three

[1] It is worth while observing that in equal temperament, where both † and ‖ are replaced by 1, all these intervals †, †‖, ††‖ become replaced by unisons, so that there is no change of pitch, but merely alteration of position when the duodene is changed. In other words there is but one duodene, in equal temperament, which is that of the ordinary piano. This will be still better shewn by using

C cd D de E F fg G ga A ab B

for the tones played by the 12 digitals on a piano, thus avoiding the ambiguities of notation, so that the central duodene and its adjacent trines and quaternions reduce to

cd	F	A	cd	F
fg	ab	D	fg	ab
B	de	G	B	de
E	ga	C	E	ga
A	cd	F	A	cd
D	fg	ab	D	fg

where it is seen that every line is repeated by the fourth below it, and every column by the third to its right. The extreme imperfections of such a system lie on the surface. But there is a less apparent and more dangerous error. Since all duodenes contain the same tones as one and no others, all duodenes are identical. Hence after the most extravagant written duodenations, implying the most distant modulations from the original trichordal, we are always striking tones which are a Third or a Fifth (in tempered intonation) distant from some tone in the original key, and thus closely related to that key, to which therefore the music can immediately return. This is threatening to destroy the feeling of tonality and to make music little more than a game of permutations and combinations of twelve tones.

duodenes, and after 3 lateral duodenations there is no trace of the original or *radical* duodene. Hence lateral radical duodenation extends to two quaternions to the right and two to the left. For the duodene of C it embraces the tones in Table IV., cols. 2 to 8, lines *r* to *u*, limited right and left by thick rules.

Each vertical duodenation may be combined with a lateral duodenation, and if we do not exceed the radical limits each way, we shall always have a duodene which contains at least *one tone* that exists in the radical duodene, from which therefore the transition to the original tonic decad or trichordal is ready and direct without in any respect impairing the feeling of tonality. The tones which lie within the limits of general radical duodenation for the root C are therefore those $7 \times 10 = 70$ tones in Table IV. within the large interior oblong, forming $(7 - 2) \times (10 - 3) = 35$ duodenes.

Wider limits for related tones are obtained by using duodenation in place of decadation. The harmony still works through decads, one of which occupies the whole of each duodene except the two tones at its upper right (or superdominant major) and lower left (or subdominant minor) extremities. These tones, which are F♯ and D♭ in the duodene of C, I propose to call *mutators*, as they are essentially those which are concerned in decadation, because each is a constituent of two cells belonging to vertical and lateral decadation which are themselves left incomplete in the duodene. Thus, using the small and capital letters as in the diagram of the hepta-decad, f♯ and d♭ enter as follows into the

Vertical Cells			and	Lateral Cells.		
†f	†a			D	f♯	
	D	f♯			B	‡d♯
d♭	F			†f♭	†A♭	
	b♭	‡d			d♭	F

Hence *the duodene represents a decad in the act of decadation*, and it thus first presented itself to my mind, as the appropriate unit for appreciating modulation in its most extensive sense.

The limits of variation of the pitch of the root of the original duodene are fixed, by the practice of musicians, as those of the tones in the one duodene on ordinary manuals, independently of temperament and of the pitch of C. The variations of that pitch are given by means of the pitch of *c′* and *a′* in Table VI., C. This is a question rather of the mechanical construction of instruments and the compass of individual voices, than of general musical science. Hence passing it over, and also disregarding the differences between any tempered and just intonation for the duodene of C, which so far as regulating initial pitch is concerned are insignificant, we may take as the twelve original roots of duodenes, the twelve tones which

form the duodene of C. Any other roots may be regarded as only occurring in the process of duodenation, and therefore already provided for. Forming then the limits of general radical duodenation of each of these twelve tones as an original root, it is immediately seen that we obtain the $9 \times 13 = 117$ tones of Table IV., extending two lines higher, one line lower, and one column further both to the right and left than those for C, and forming only $(9 - 2) \times (13 - 3) = 70$ duodenes. It seems to me that any further duodenation than is thus shewn may be attributed simply to the confusions of temperament, and that we are therefore justified in disregarding them from a scientific point of view. These 117 tones and 70 duodenes are therefore the just tones and just duodenes for which we have to provide. They are given in order of pitch, within the Octave of c' to c'', in Table VI., B, col. Just.

The Duodenal is the letter-name of the root of a duodene. The symbols in the ordinary staff notation have been for so long referred to equal temperament, that they may be considered as merely indicating, within limits of a few commas, the pitch of the tones in any of the seventy duodenes of Table IV. Hence by laying down as a rule that *when the duodenal is prefixed to any piece in staff notation the twelve tones thus indicated are to receive precisely the just values which they must have to form the duodene thus pointed out, until such time as a new duodenal is employed*, we shall have a means of reducing ordinary to just notation without introducing any new sign on the staff itself. Thus on pp. 471, 472, 475, and 476, duodenals (followed in these examples by the numbers of the duodenes in Tables VII. and VIII.; but this is merely for practical convenience, and is not only quite unnecessary, but inexpedient for general use,) point out the exact duodenes to which the tones are to be accommodated, as shewn precisely by the analysis given in the foot-note to p. 471 for the first two cases. Observe in this note that we have first the radical duodene of B, and then for two chords the subdominant duodene of E, showing a vertical descent in order to give the subdominantal chord A + ‡C♯ — E, which alters two tones. If, however, we had preferred the dominantal form †A + C♯ — †E, which alters only one tone, and had thus entered the dominant duodene of F♯, we should have used the duodenal F♯, and then have had †*a* *c*♯ †*e*′ *c*′′♯, †*a* ‡*d*′♯ *f*′♯ *c*′♯, as the third and fourth chords. In either case we have to return to the radical duodeno of B for the next chord, and remain in it till the last bar, where we must ascend to the dominant. It is thus seen how the ambiguity of the modulation in tempered notation may be resolved one way or the other by merely prefixing the duodenal.

Occasionally the duodenal will force equally tempered G♭, which is equivalent to equally tempered F♯, to represent just F♯; as, for example, by prefixing the duodenal C, which excludes G♭. Similar cases often arise for passing notes and appoggiature, where the ordinary notation,

founded on old Pythagorean conceptions, is incompatible with just intonation. Again the chord of the dominant Seventh f+a−c | e♭ is in the duodene B♭ or ‡D; but if we marked the duodene as A or E, we must mean f + a − c × +² ‡d♯, or the chord of the German Sixth, which in tempered intonation would be the same, but as much more harmonious might occasionally be substituted for it in just intonation. Again the condissonance ‡g♯ −b−d, can only occur in duodenes B or E, but if we wrote the duodenal C we should mean the chord †a♭ × +² b − d, which is not uncommon as a mutilated form of the chord of the diminished Seventh b − d | f − †a♭ with the f omitted.

As the duodene has three tonics and ten trichordals (eight belonging to the tonic of its decad, and one to each of the others) it may often happen that it is more convenient to the performer (with such a manual as the *duodenary*, compare Table VIII.) to have the duodenal marked irrespective of the tonic in which the tonality moves, but in such a way as will best guide the fingers to the right tones. Thus on p. 471 the duodenal F♯ is continued throughout the last bar, although the tonic is evidently B at the close. Similarly when the piece is in C major modulating into A minor, or with a tendency to a right lateral modulation, it is best perhaps to mark it, as in the duodene of E (which contains C major), descending to that of A, because this being a simple vertical descent is more direct and easier than a combined vertical and lateral duodenation from C to A. But when the scale is minor it employs each of the three quaternions of the duodene, which must therefore be always shewn by the duodenal.

If, however, the root of the duodene be always taken as the tonic of the scale in use at the time being, the duodenal will mark the progress of modulation whenever that requires a change of decad. By combining these duodenals with a study of Table IV., the meaning, extent, and connection of the most complicated modulations can be readily traced, and the mere confusions of temperament sometimes resulting in impossible harmonies, can be detected and explained. By means of the duodenal and this table also, it is possible to shew how commatic changes can be made with the least sacrifice of the connections pointed out by just intonation, so as to restore the pitch and revert to the original duodene instead of to one a comma lower or higher to which just intonation would lead, contrary to the intentions of the composer, who thought in tempered intonation. This may often be effected by taking the chord of the dominant g+b−d | f, which is in the duodene of C or E, as g+b−d−†f, which is in the duodene of D or †A; or again by taking the 'false minor' chord d | f + a (p. 527), which is in the duodene of C or E, as either d−†f+†a (in the duodenes of G, D, †A, †E♭, †B♭, or †F) or as ‡d−f+a (in the duodenes of F, B♭, E♭, A, ‡D, or ‡G). Details belong to practical works.

SECTION F.

ON PRACTICAL TONAL SYSTEMS.

The theoretical limitation of the requirements of just intonation to the 117 tones and 70 duodenes of Table IV., in place of the infinity usually assumed, is, however, not a sufficient limitation for the purposes of execution on instruments with fixed tones, and would even occasion great difficulties with instruments of variable tones, as the human voice and violin. The first object is therefore to reduce the number by practical considerations, retaining just intonation, and, secondly, to reduce this result again by the synonymity of some tempered intonation. Equal temperament reduces the whole to 12 tones, which cannot be accepted. Commatic temperament reduces all to 27 tones, or cyclically to 31, which, though a great improvement, is not sufficiently harmonious, and, by the reduction of commas to unisons, is too destructive of theory. Bosanquet's cycle of 53 retains the comma, and is, I believe, sufficiently harmonious; but its cyclic character, which reduces distant to close relations, is opposed to the nature of just intonation, where continued duodenation in one direction leads to ever more distant relations. As Bosanquet's cycle is better than skhismic temperament, there remain only the skhistic and practically just intonations, where the synonymity is merely practical, and does not lead to cyclical duodenation. Added to this, practically just intonation is absolutely just, except in 8 triads, where it is scarcely distinguishable from just, while the cycle of 53 is sensibly, although very slightly, distinct from just for every major and minor Third (Table VI., D).

Now, first, it is not the practice in one piece of music, nor does it seem consistent with a steady preservation of tonality, to modulate freely in all directions. Hence it is sufficient if we can, at different times, allow great power to modulate from the same, or nearly the same, tonic in different directions. Next, the modulations to the left are much less frequent than those to the right, or than vertical modulations. Thirdly, in most music hitherto composed we must allow for the composer's commatic tempered confusion of lateral with vertical modulation, A being confused with †A, and †E♭ with E♭. When a composer uses, as an original root, a tone which he writes B♭, it is impossible to say whether he means to begin with †B♭ (here theoretically assigned because belonging to the duodene of C), or with B♭, or even A♯ or ‡A♯. Hence we may reject columns 1 and 2 of Table IV. Similarly, column 9 may be rejected, but not column 8, because modulations to the right are more frequent and extensive than modulations to the left. We have, then, the six columns I. and VI., with $6 \times 13 = 78$ tones, and $(6-2) \times (13-9) = 40$ duodenes. Of these tones we may, perhaps, reject the three in column 3, lines *l*, *m*, *n*, as not being required for the duodene of †B♭, and the two in column 8, lines

y and *z*, as being wanted only for the duodenes of ‡B and ‡E, derived modulationally from A as an original tonic. If the music required us to modulate beyond the duodene of †B♭ to the left or upwards, and beyond the duodene of ‡F♯ to the right or downwards, both of which cases are not very likely to happen in one piece (although the passage from *Don Giovanni*, cited on p. 477, goes quite to the limits here assigned), we can generally overcome the difficulty by altering the original root in the manner just suggested (taking B♭ for †B♭, or †A for A), or, if necessary, by commatic changes suggested on p. 672 for preserving the pitch of the original duodene. These rejections reduce the practical system of just tones to 78 − 5 = 73, forming 35 duodenes. But it must be borne in mind in practical constructions of instruments, that some unusual music may require the 78 or even the whole 117 tones, and provision should be made for this contingency.

By introducing skhistic or practically just intonation (which for brevity will be assumed as that really used for skhistic intonation without further indication, see p. 657), we see that after rejecting the 5 tones in Table IV., col. 1, lines *l*, *m*, *n*, and col. 9, lines *y*, *z*, the remaining 112 tones would be represented by the 9 × 8 = 72 tones in lines *p* to *x*. This is shewn clearly by Table VI., B, col. Just, where all the 77 tones in order of pitch are printed in capitals (either Roman or Italic), and the remaining 40 are in small letters (Roman only), and every small letter represents a note, which differs by one skhisma (log. = ·000 4901) exactly from some note represented by a capital, while in col. Skhistic these large and small letters come into the same line, as skhistic unisons. These 72 tones may be regarded as forming an uninterrupted series of 71 skhistic Fifths proceeding *downwards* from col. 1, line *p* to *x*, then to col. 2, line *p* to *x*, and so on, and hence forming (9 − 2) × 8 = 56 trines, and 56 − 3 = 53 duodenes, from that of ††E♭ down by skhistic Fifths to that of ‡A♯, corresponding to one duodene for each tone in the cycle of 53, from No. 15 in that cycle in Table VI., B, which corresponds to both ††E♭♭ and ‡‡D♯, down by its own Fifths to No. 46 in the same cycle, which corresponds to ‡A♯. But all of these 72 tones form only 6 *independent* duodenes, that is, duodenes which have no tones in common, of which the roots are ††E♭♭ and †C♭♭ in col. 2, D and B♭ in col. 5, and ‡C♯♯ and ‡A♯ in col. 8, a circumstance of some practical value in constructions. See Section G, No. 7, at bottom of p. 685.

If we confine ourselves to the 78 − 5 = 73 tones in Table IV., cols. I. to VI., already mentioned, we find that by skhistic intonation they reduce to the 48 in lines *p* to *x*, in the same columns, within the oblong contained by three dotted lines and one strong line. These 48 tones form the middle sections of Table V., where in the upper and lower sections their synonyms are printed in small letters, and are continued much further than is practically necessary for playing music, in order to shew the

law of skhistic synonymity, and for other theoretical purposes. If in any section of Table V. we proceed from any tone to 8 Fifths up and then to a major Third, we reach a tone which is sharper by one skhisma in just, but identical with it in skhistic intonation (Section C, iii. p. 651). These synonyms are printed in small letters, and the lines in each section have corresponding numbers, to assist in making the comparison. In Table VI., B, col. Just, these 48 tones in just intonation, forming the series of practically just tones, are printed in *Italic* capitals, and all the synonyms are given in small letters (Roman or Italic), while in col. Skhistic such of these synonyms as appear in Table V. are printed also in Italics, and are now found to be unisons with the corresponding tones in Italic capitals. These 48 tones form 4 independent duodenes, being those of †B♭ and G♭ in col. II., and A♯ and ‡F♯ in col. V., as shewn by the dotted cross-lines in the middle section. This circumstance becomes of considerable importance for future constructions. See Section G, Nos. 7 and 14.

These 48 tones form a series of skhistic Fifths,[1] and consequently also form $(6 - 2) \times 8 = 32$ trines, and $32 - 3 = 29$ duodenes. These trines are arranged in three principal columns in Table VII., where trines in the same line are skhistic synonyms, capital letters corresponding to the middle section of Table V. and small letters to the other sections. In just intonation, however, trines 1 to 8 in col. 1 are flatter by one skhisma than those in col. 2; trines 9 to 16 in col. 1 are also flatter by one skhisma than those in col. 2, and these are again flatter by one skhisma than those in col. 3; trines 17 to 24, in col. 2, are flatter by one skhisma than those in col. 3, which are again flatter by one skhisma than those in col. 1 (which, had space allowed, would have formed a fourth column as for the names of the roots of duodenes in Table VIII.); and, lastly, trines 25 to 32 in col. 3 are flatter by one skhisma than those in col. 1 (which, again, would have been in col. 4 had space allowed.)

The 29 duodenes formed from these trines have, therefore, roots which have two or three names, and in just intonation differ in pitch by one or two skhismas, though all become identical in skhistic intonation; and it is this identification without cyclic periodicity, and without any appreciable confusion, and without appreciable injury to the Fifths and Minor Thirds,

[1] To obtain these we began by omitting 5 tones from the 78 in columns 3 to 8 of Table IV., namely col. 3, lines *l*, *m*, *n*, and col. 8, lines *y*, *z*. If we restored these and played them by skhistic intonation, they would appear as col. 2, lines *u*, *w*, *x*, and col. 9, lines *p* and *q*, respectively. We should then have 53 tones from †G♭♭ (col. 2, line *u*) to ‡B♯♯♯ (col. 9, line *q*), which correspond to the 53 tones of Bosanquet's cycle, namely (in Table VI., B), No. 28 to No. 6, which last corresponds both to ‡B♯♯♯ and †D♭♭, the Fifth above †G♭♭. The first and second reduction of the number of tones, therefore, both lead us to 53, and hence shew the theoretical importance of that number as an approximation to a correct representation of just intonation, but a glance at Table VI., B, shews it to be only an approximation.

while the major Thirds are absolutely just, which gives skhistic intonation its great practical value. In Table VII. the name of the root of each duodene is written against its *first* trine. The duodenes in the same vertical column are formed by vertical duodenation, but it is not possible to continue a descent of 29 duodenes without synonymity. Trines 1 to 8 give 5 duodenes, and similarly 9 to 16, 17 to 24, and 25 to 32, or 20 duodenes in all; and between each set of 5 there are 3 (that is, 9 in all) formed by synonyms, namely, duodenes 6, 7, 8; 14, 15, 16; and 22, 23, 24. But skhistic intonation has made all duodenation vertical. Lateral duodenation from root C, No. 11, to the correlative root E, No. 19, is a vertical descent of 8 duodenes, as seen in the table; while the relative root A, No. 20, is 9 duodenes below. This is of much practical importance.

In Table VIII. these 29 duodenes are arranged linearly, and such synonyms are chosen as will make clear to the eye that each tone persists through 4 vertical duodenes (a matter of practical importance), and that each vertical duodenation changes 3 tones and no more (pp. 669–70). The roots of these duodenes are written in the margin.

The number of tones to be provided is a point of great importance in the construction of instruments with fixed tones, and I therefore subjoin an account of those selected in the three principal attempts which have been made to furnish complete instruments in just intonation.

1. *Mr. Liston's Organ.*—The pitch of the notes has been calculated from the data furnished by Mr. Farey, 'Phil. Mag.' vol. 39, p. 418. The arrangement of columns, capitals, small letters, and sections is not that of Mr. Liston, but that of Table V. The intonation is just.

	0	I	II	III	IV	V	VI	
6*t*			—	†c♯	†e♯	—		6*t*
7*t*			†d	†f♯	†a♯	c♯♯		7*t*
8*t*			†g	†b	d♯	f♯♯		8*t*
1*m*	††Fb	††Ab	†C	†E	G♯	B♯	—	1*m*
2*m*	—	†Db	†F	†A	C♯	E♯	—	2*m*
3*m*	††Ebb	†Gb	†Bb	D	F♯	A♯	‡C♯♯	3*m*
4*m*		†Cb	†Eb	G	B	‡D♯	‡F♯♯	4*m*
5*m*		†Fb	†Ab	C	E	‡G♯	‡B♯	5*m*
6*m*		†Bbb	Db	F	A	‡C♯	‡E♯	6*m*
7*m*		—	Gb	Bb	‡D	‡F♯	‡A♯	7*m*
8*m*		—	Cb	Eb	‡G	—	—	8*m*
1*b*			fb	ab	‡c			1*b*
2*b*			bbb	—	—			2*b*

This is a total of 58 just tones, but if the skhistic identification is carried out the tones †d, †g in II. will appear as †Eb♭, †Ab♭ in I., and the other tones in the top and bottom sections are already expressed by their

synonyms. In this case 44 of my 48 skhistic tones are used, omitting ‡B, ‡‡D♯, and ‡D♯♯, ‡G♯♯; but two new ones are added, namely ††F♭, ††E♭♭, which were I believe due to the system of tuning. Mr. Liston's system of tones is then practically the same as mine, but its curious gaps shew that it did not arise from a systematic examination of the extent of modulation.

2. *Mr. Poole's Organ.*—The pitch of the notes is derived from the ratios furnished by Mr. Poole in his paper cited on p. 635, note 3. Mr. Poole (as was mentioned on p. 534, note) considers it necessary that the chord of the dominant Seventh in every major and minor scale should be made with the subminor or natural Seventh, and hence has throughout provided for them. In the annexed account of his tones a ƶ subjoined to a note, thus: B♭ ƶ, means that both B♭ and ƶB♭ occur; but ƶ‡‡c♭ shews that only that septimal tone occurs. It is thus possible to arrange his 100 just tones according to Table V., as in the margin. Rejecting the 39 septimal tones for the reasons on pp. 296, 330, and 660, there remain 61 ordinary notes in tertian harmony. If skhistic intonation is introduced the 7 tones in Table V., col. III., lines 2*t* to 8*t*, become those in col. II., lines 2*m* to 8*m*, and the five tones in col. V., lines 1*b* to 5*b*, become those in col. VI., lines 1*m* to 5*m*, so that there remain only 36 tones from †F, col. II., line 2*m* to ‡B♯, col. VI., line 5*m*. By arranging these 36 tones in columns of 8 thus:

	III	IV	V
1*t*	—	—	—
2*t*	††e♯	—	—
3*t*	††a♯	†c♯	—
4*t*	†d♯	†f♯	†a♯♯
5*t*	†g♯ ƶ	†b♯	d♯♯
6*t*	†c♯ ƶ	†e♯ ƶ	g♯♯
7*t*	†f♯ ƶ	†a♯ ƶ	c♯♯
8*t*	†b ƶ	d♯ ƶ	f♯♯
1*m*	†E ƶ	G♯ ƶ	B♯
2*m*	†A ƶ	C♯ ƶ	E♯
3*m*	D ƶ	F♯ ƶ	A♯
4*m*	G ƶ	B ƶ	‡D♯
5*m*	C ƶ	E ƶ	‡G♯
6*m*	F ƶ	A ƶ	‡C♯
7*m*	B♭ ƶ	‡D ƶ	‡F♯
8*m*	E♭ ƶ	‡G ƶ	‡B
1*b*	a♭ ƶ	‡c ƶ	‡e
2*b*	‡d♭ ƶ	‡f ƶ	‡a
3*b*	‡g♭ ƶ	‡b♭ ƶ	‡‡d
4*b*	‡c♭ ƶ	‡e♭ ƶ	‡‡g
5*b*	‡f♭ ƶ	‡a♭ ƶ	‡‡c
6*b*	‡b♭♭ ƶ	‡‡d♭ ƶ	
7*b*	‡e♭♭ ƶ	‡‡g♭ ƶ	
8*b*	ƶ‡a♭♭	ƶ‡‡c♭	

†F	†A	C♯	E♯	‡G♯♯
†B♭	D	F♯	A♯	‡C♯♯
†E♭	G	B	‡D♯	‡F♯♯
†A♭	C	E	‡G♯	‡B♯
D♭	F	A	‡C♯	
G♭	B♭	‡D	‡F♯	
C♭	E♭	‡G	‡B	
F♭	A♭	‡C	‡E	

it will be immediately seen that they consist of the two independent d denes of G and E♭, and of 12 tones forming consecutive skhistic Fift but not a duodene, and that the whole will give the 17 duodenes, No. 10 28 of Table VII., forming a smaller arrangement which may often pr useful, but necessitating rather frequent commatic changes (as from minor to ‡D minor).[1]

3. *General T. Perronet Thompson's Organ.*—The pitch has be determined from the lengths of the monochord given in his book (see 635, note 1). The arrangement as before follows Table V. The inton tion is just.

	II	III	IV	V	VI	
1*m*	†C	†E	—	—	—	1*m*
2*m*	†F	†A	C♯	—	—	2*m*
3*m*	†B♭	D	F♯	A♯	—	3*m*
4*m*	†E♭	G	B	‡D♯	‡F♯♯	4*m*
5*m*	†A♭	C	E	‡G♯	‡B♯	5*m*
6*m*	D♭	F	A	‡C♯	‡E♯	6*m*
7*m*	G♭	B♭	‡D	‡F♯	‡A♯	7*m*
8*m*	—	E♭	‡G	‡B	‡‡D♯	8*m*
1*b*		a♭	‡c	‡e	‡‡g♯	1*b*
2*b*		—	‡f	‡a	‡‡c♯	2*b*

Total, 40 just tones. If the skhistic synonyms are introduced t tones a♭, ‡c ‡f, and ‡e ‡a in the bottom section will appear in t

[1] Mr. Poole proposes *two* diatonic scales, explained on p. 534, note. His extreme ingenious keyboard, which cannot well be described without a figure, and does appear to have advanced beyond a model, gives 19 series of connected manuals, whi though quite different from those proposed by me in Section G, Nos. 13 and 14, fi gave me the conception of that arrangement. Each manual plays 10 (and not tones, which, translating Mr. Poole's notation into the solfeggio notation of the h tadecad (p. 666), and using an added *s* to indicate the Septimals, are thus arrang for *Do* = *g*, *c*, or *f* by Mr. Poole. The underwritten numbers are given by Mr. Po to determine the ratios of the vibrational numbers; and the order of the tones, wh as respects *re*, *ras*, *mi* is not that of pitch, is also that adopted by Mr. Poole.

	Do	*Re*	*ras*	*Mi*	*Fa*	*So*	*se*	*La*	*tas*	*Ti*
	48	54	52·5	60	64	72	75	80	84	90
	g	†*a*	♎*a*	*b*	*c*	*d*	‡*d*♯	†*e*	♎*f*	*f*♯
or	*c*	*d*	♎‡*d*	*e*	*f*	*g*	‡*g*♯	*a*	♎*b*♭	*b*
	f	*g*	♎‡*g*	*a*	*b*♭	*c*	‡*c*♯	‡*d*	♎*e*♭	*f*

Hence each manual contains the *triple scale* of *Do* and the *double scale* of *Fa* (p. note), and the chords *la* − *do* + *mi*, *mi* − *so* + *ti*, and *mi* + *se* − *ti*, with *ras* out of relative minor scale of *la*; the subdominant chord *ra* − *fa* + *la* of the relative m has to be taken from the subdominant key when the false minor *re* | *fa* + *la* is not in its place. Each manual is so arranged that the player beginning with any *Do* f all the other nine tones given above, in the same line, and in the same order, so the fingering is the same in all major and in all minor keys whatever be the tonic

middle section of cols. IV., V. and VI., as G♯, B♯, E♯, and ‡D♯♯, ‡G♯♯, and then we shall have 38 of my tones (the 40 in cols. II., III., IV., V. and VI., less the tones col. II. line 8*m* and col. VI. line 3*m*, or C♭, and ‡C♯♯), together with two additional tones, ‡‡g♯, ‡‡c♯ (Table IV., col. 8, lines *y* and *z*), which were rejected as scarcely necessary on p. 674, line 1.

Hence, with the exception of Mr. Bosanquet, (who uses all the 53 tones of his cycle, and employs them cyclically to represent a perfectly infinite succession of Fifths,) those who have made the most practical experiments in constructing instruments seem content with tones which skhistic intonation reduces to about the 48 tones of p. 675 as the practical number. But in view of the great extension of modulations introduced by equal temperament, and founded partly on its misleading synonymities, it may be occasionally necessary to increase that number to 60 (from col. 2, line *p*, to col. 9, line *s*, in Table IV.), or even 72 (lines *p* to *x* in Table IV.), and for this occasional necessity provision should be made. The practical problem is therefore how to place the corresponding 29, or 41, or 53 duodenes within convenient reach of a performer on an organ or harmonium. The pianoforte may be left untouched as the especial instrument of equal temperament (pp. 502-3).

SECTION G.

ON THE CONSTRUCTION OF EXPERIMENTAL, EDUCATIONAL, AND PRACTICAL INSTRUMENTS WITH FIXED TONES IN JUST, SKHISMATIC, AND PRACTICALLY JUST INTONATION.

Three purposes for the construction of instruments must be recognised—experimental, educational, and practical. The first are intended purely for the purpose of illustration in explaining the theory of music, and shewing the meaning of just intonation, and its difference from tempered intonation. It is of importance, therefore, either that they should involve no new mechanism, so that existing instruments should be available on being merely re-tuned (as in Nos. 1, 2, 3, 4, 5, 6, 7), or that the new mechanism should be very simple (as in Nos. 8, 9, 10). As also such instruments would be only occasionally employed it is of great importance that the fingerboard and mode of fingering should not differ at all from that of the old tempered instruments (as in Nos. 2, 5, 6, 7, 10 last), or, at least, but slightly (as in Nos. 1, 10 first). The difficulty of fingering on Prof. Helmholtz's double-manual instrument, which may be studied from the diagrams of the manuals on p. 493, note, has doubtless deterred many experimentalists from using it. The educational class of instruments are for the use of singers, to carry out the suggestions on pages

509-10. It is not necessary that these instruments should play in a great variety of keys, but it is necessary that they should be able to shew every description of modulation, and distinguish commatic changes (as in No. 10). Finally, practical instruments are such that a single performer may have at his command a range of tones sufficient to play any modern music of any degree of complexity in just or skhistic intonation. Although it may be advisable at first to make the fingering identical with that now in use, it is not too much to demand of a performer on such an instrument that he should also undertake to learn a new system of fingering as on Gen. Perronet Thompson's or Mr. Poole's organ. I have endeavoured (in Nos. 13 and 14) to suggest a fingerboard which will preserve the usual fingering, but the new *generalised fingerboard* of Mr. Bosanquet, of which a description is given in No. 15, is every way so much superior in philosophical and mechanical construction, while it has had the advantage of having been sufficiently tested in practice, that it is certainly the only keyboard which ought to be finally adopted. The one and only objection to it, that it involves a slightly new system of fingering, ought to weigh as nothing against its other merits, especially as it solves the difficulty of retaining the same fingering in all keys or duodenes.

It will be seen then that there are two plans of construction—*simplex*, in which old instruments are re-tuned; and *complex*, in which new instruments have to be constructed.

I. SIMPLEX INSTRUMENTS, being existing instruments re-tuned.

1. *Just English Concertina.*

Intonation.—As the English concertina possesses 14 tones to the octave, is a portable instrument playing any combinations of chords with sustained tones on harmonium vibrators, and makes all deficiencies of intonation strongly apparent, giving especial prominence to the combinational tones, it is peculiarly well suited for private experiments. The only objection is the absence of bass, the tones reaching from violin *g* to *c''''*. The intonation selected gives part of a heptadecad, having the whole decad of E, the duodene of E less the tone A♯; the decad and duodene of A less the tone ‡F♯; the *mapáma* or major scales of F, C, G and E (with, however, only the false minor chord on the second in F and E), and A *mipími* and *mipíma* (but not *mapíma*). Having 6 tones, and also 5 tones forming Fifths, it can play the Five-Fifths scales 5.B♭.1, 5.F.1, 5.‡D.1, and their derivatives (Table III., A.), and hence shew the effect of Quintal scales. The compass is sufficient to allow of the performance of many harmonised airs, and of studying the effects of modulation and commatic change. The fingering is slightly different from that ordinarily used, but ten years' experience has shewn me that it

†A		
D	F♯	
G	B	‡D♯
C	E	‡G♯
F	A	‡C♯
B♭	‡D	

offers no real difficulty. Using capitals for white, and small letters for black studs, its 14 notes are tuned thus:—

C ‡c♯, ‡D d, E ‡d♯, F f♯, G ‡g♯, A †a, B b♭.

The directions to the tuner are: Tune the following eight major chords without beats, putting low D on the usual *D* stud, D on the usual *D♯* stud, D♯ on the usual E♭ stud, and high A on the usual *A♭* stud: B♭, low D, F; F, A, C; C, E, G; G, B, D; D, F♯, high A; A, C♯, E; E, G♯, B; B, D♯, F♯. The concertina is ordinarily tuned in the mean tone temperament, with the studs as

C c♯, D d♯, E e♭, F f♯, G g♯, A a♭, B b♭,

and sometimes in equal temperament, in which case d♯, e♭, and g♯, a♭, are tuned as unisons, and the double studs merely facilitate fingering. I keep two concertinas thus tuned, and one tuned on the Pythagorean system, for constant comparison with just intonation.

2. *Just Experimental Harmonium.*

Intonation.—For ordinary lecture and illustrative purposes, this is the cheapest and best instrument, and it is almost indispensable for those who wish to study the musical effects mentioned in Part III., above. It contains the portion of a heptadecad shewn in the margin, and hence contains the duodene of C with the exception of F♯, the whole decad of C, and the major scales of †E♭ and †A♭. The two last shew vertical modulation. Again, †E♭ major or †A♭ major to C decad shews right lateral, and inversely, C decad to †E♭ major or †A♭ major shews left lateral modulation. The F and †F shew the influence of a comma, and the difference between the just triad, D♭ + F − †A♭, and the Pythagorean triad, D♭ + †F | †A♭. Also in the key of †E♭ major the difference can be shewn between the true minor chord, F − †A♭ + C, and the false minor chord, †F | †A♭ + C. This instrument also contains the German Sixth D♭ + F − †A♭ $\times +^2$ B, which is a close imitation of the chord of the natural Seventh; the Italian Sixth, D♭ + F $\times^2$ + B, and the French Sixth, D♭ + F . . . G + B. The five tones, D♭, †A♭, †E♭, †B♭, F, allow of playing all Scotch airs in Five-Fifths scales on the *black* digitals (p. 400, note 1). Using capitals and small letters for the black and white keys, they are arranged thus:—

†F		
†B♭	D	
†E♭	G	B
†A♭	C	E
D♭	F	A

C d♭ D †e♭ E F †f G †a♭ A †b♭ B.

The †F is placed on the F♯ digital, and the other fingering is normal.

The following directions were found sufficient for the tuner of a common four-octave harmonium, which he tuned in my presence in two hours,

VIII. as the regular distribution. The fingering is abnormal; thus each of the chords, F + A − C, C + E − G, and G + B − D is distributed over two manuals. The tuning may be effected by the general rule, p. 657, as in No. 3. For a single manual arrangement see No 9.

5. *Duóni.*—The effects of either of these instruments could be produced on two instruments with ordinary single manuals, played by two performers, on simply tuning the two instruments (by major Thirds and Fifths) according to the two duodenes selected, and rewriting the piece of music, distributing the notes properly between the instruments in the manner shewn on p. 684. It is, however, hardly worth while to do so, as these instruments have not sufficient compass for practical purposes and the arrangements of Nos. 8 and 9 below make it perfectly easy to play their tones on one ordinary manual for experimental purposes.

6. *Trióni.*—Tune three instruments thus:—

(I.)	C	‡db	D	†eb	E	†F	‡gb	G	†ab	†A	†bb	B
(II.)	‡C	db	‡D	eb	†E	F	gb	‡G	ab	A	bb	†B
(III.)	‡B♯	‡c♯	‡C♯♯	‡d♯	‡E	E♯	‡f♯	‡F♯♯	‡g♯	‡G♯♯	a♯	†B

Then (I.) is in the duodene of G, No. 10, and (II.) in that of E♭, No. 14, of Tables VII. and VIII.; and this readily gives the method of tuning them. (III.) consists of 12 tones forming consecutive Fifths from E♯ to ‡E, col. 7, lines *q* to *y*, and from ‡G♯♯ to ‡B♯, col. 8, lines *q* to *t*, of Table IV. For (III.) begin by tuning ‡G♯ a major Third without beats to E in (I.), and then work up to E♯ and down to ‡E by Fifths, verifying with the corresponding major Thirds below in (I.) and (II.). Then tune ‡B♯ a major Third above ‡G♯, and tune up by Fifths to ‡G♯♯, verifying by the major Thirds below, which lie all in (III.). The three instruments are thus completely in tune, and give the 17 duodenes, Nos. 10 to 26, Table VII., or Mr. Poole's scale of 36 tones (p. 677).

Music for *Trióni* is arranged thus. Lines are drawn in each column of Table VII., similar to those in the extract from the third column in the margin, the figures (I.), (II.), (III.) shewing to which instrument they belong. All the three columns in Table VII. have to be thus marked in order to

(I.)	†F♯♯ †B♯ †E♯ †A♯	10 11 12 13	††e♯ ††a♯ †d♯ †g♯	†g♯♯ †c♯♯ †f♯♯ †b♯	†b♯♯ †e♯♯ †a♯♯ d♯♯	(I.)
(II.)	D♯ G♯ C♯ F♯	14 15 16 17	†c♯ †f♯ †b †E	†e♯ †a♯ d♯ G♯	g♯♯ c♯♯ f♯♯ B♯	(II.)
(I.)	B E A ‡D	18 19 20 21	†A D G C	C♯ F♯ B E	E♯ A♯ ‡D♯ ‡G♯	(III.)
(II.)	‡G ‡C ‡F ‡B♭	22 23 24 25	F B♭ E♭ a♭	A ‡D ‡G ‡c	‡C♯ ‡F♯ ‡B ‡e	(III.)
(I.)	‡E♭ ‡A♭ ‡‡D♭ ‡‡G♭	26 27 28 29	‡d♭ ‡g♭ ‡c♭ ‡f♭	‡f ‡b♭ ‡e♭ ‡a♭	‡a ‡‡d ‡‡g ‡‡c	(III.)

hod of distributing the tones among several
ıe practice of a Russian horn-band, which
ı, each performer playing a single tone. It
of handbell-ringers. In the present case
ɔmmand. As keyed wind instruments can
commanding a duodene, three of them
le, and thus orchestral performance in
ithout altering the fingering, and
uments. Compare p. 641.
r three pianofortes in the
le day, so that we may
y of playing harmoniums
rmers would merely require
the case of harmoniums with
be produced. It is evident, how-
nly for experimental purposes.
estóni.—The effect of all the 48 skhistic
harmoniums tuned to the 4 independent
d lines in the middle section of Table V.,

(III.) to A♯, No. 25,
(IV.) to ‡F♯, No. 29,

e arrangement of the music would be so
rióni in No. 6, that it need not be further
ırmonium plays a complete duodene, and all
VII. and VIII. are at the command of the
nary modulations a fifth harmonium (V.)
3 tones col. 2, lines *p* to *x*, and the 4 tones
., arranged thus:—

Gbb ‡e♯ ♯ ††Abb ‡f♯ ♯ ♯ ††Bbb †cbb ‡A♯ ♯

††F ‡‡f♯ ††G ‡‡g♯ †††A ††bb ‡‡B

Table IV. The tuning would be as follows—
. below †Cb on (I.), and then work up by
Fbb, verifying by the major Thirds above
ɜ ‡A♯ ♯ as a major Third above ‡F♯ ♯ in
to ‡F♯ ♯ ♯, verifying by the major Thirds
be a mere auxiliary instrument, rarely if
he five instruments would form *quintóni.*
which six harmoniums are tuned to the 6
by the 72 tones in Table IV. (see p. 674),

giving 53 duodenes, and playing by skhistic or unequally just intonation, 112 out of the 117 tones in that Table. The only tones omitted would be the first 3 and last 2. By this contrivance it is as easy to get up a piece in just intonation as to get up any quartet, quintet, or sestet with the sole exception of the necessity of retuning the instruments and transcribing the parts.

II. COMPLEX INSTRUMENTS requiring new mechanical contrivances.

8. *Helmholtz's Harmonium with one Manual.*—Instead of the arrangement in Appendix XVII., arrange and tune two sets of vibrators, as in lines 14 and 18 in Table VIII., for back and front respectively. Introduce Mr. Saunders's 'tilting action,' already explained for the mean tone harmonium, on p. 659, and draw the stops when necessary to change the note. Assume that the front row of vibrators, played by the manual when no stops are drawn, represents the duodene of B, lines 18 to 21 of Table VII. in columns 2 and 3. Arrange the stops, and mark them in the following order, where the capitals and small letters are used as in Table VII., and the letters below the transverse line give the tones of the duodene of B, or ‡c♭, arranged in the order of trines, but with the order of the tones in the trines corresponding to those marked on the stops, above which are the four successive trines of the duodene of E♭ or d♯.

(1)			(2)			(3)			(4)		
D♭	F	A	G♭	B♭	‡D	C♭	E♭	‡G	f♭	a♭	‡c
†c♯	†e♯	g♯♯	†f♯	†a♯	c♯♯	†b	d♯	f♯♯	†E	G♯	B♯
C♯	E♯	†A	F♯	A♯	D	B	‡D♯	G	E	‡G♯	C
‡d♭	‡f	b♭♭	‡g♭	‡b♭	e♭♭	‡c♭	‡e♭	a♭♭	‡f♭	‡a♭	‡d♭♭

For scientific experiments each stop should have an independent action, but the stops should have links which can be attached to them, so that they could be pulled out and pushed in by threes, as shewn by the braces above, and for practical purposes they might be consolidated, so that four stops only need be used, and this would materially simplify the mechanism. The names on the separate stops shew the tones which would be substituted for those in the duodene of B. The practical stops need only be numbered, as shewn above and tabulated below. Of course the two sets of vibrators would be arranged in the usual way, as lines 14 and 18 in Table VIII., the above arrangement being introduced solely to shew the action of the stops. We see, then, that if all four compound stops be drawn, we have the duodene of E♭ or D♯. Pushing in the stop (1) the tones D♭, F, A are exchanged for C♯, E♯, †A, or ‡d♭, f, b♭♭, and hence the duodene becomes that of A♭ or G♯. Pushing in both (1) and (2) we have the duodene of ‡G♭ or F♯. And pushing in all four (1), (2), (3), and (4), we have the duodene of ‡c♭ or B. These movements conse-

quently place at the command of the player the five duodenes of E♭, a♭, ‡d♭, ‡g♭, ‡c♭, or d♯, G♯, C♯, F♯, B, and consequently their five major and minor scales in all forms, and of course also the major scales of C♭, f♭, b♭♭, e♭♭, a♭♭, or †b, †E, †A, D, G. Again, by pulling out (1) only we have the major scale of C or †b♯; by pulling out (1) and (2), the major scale of F or †e♯; by pulling out (1), (2), and (3), the major scale of B♭ or †a♯. The complete series of major scales from †b to B, and the minor scales (marked *) from E♭ to ‡c♭ or d♯ to B, are given as in the margin, where only the compound stops which have to be pulled out are marked.

For Scales					Pull out
†b	or	C♭			(1, 2, 3, 4)
†E	or	f♭			(2, 3, 4)
†A	or	b♭♭			(3, 4)
D	or	e♭♭			(4)
G	or	a♭♭			(none)
C	or	‡d♭♭	or	†b♯	(1)
F	or	‡g♭♭	or	†e♯	(1, 2)
B♭	or	‡c♭♭	or	†a♯	(1, 2, 3)
* E♭	or	‡f♭♭	or	d♯	(1, 2, 3, 4)
* a♭			or	G♯	(2, 3, 4)
*‡d♭			or	C♯	(3, 4)
*‡g♭			or	F♯	(4)
*‡c♭			or	B	(none)

Compare the arrangements in Appendix XVII., p. 633.

The minor scales are very deficient. But the additional eight required, with other major and minor scales also, may be given by a second harmonium having its vibrators tuned as follows (see Table V.) :—

‡B♯ ‡c♯ ‡C♯♯ †e♭ ‡D♯♯ †F ‡f♯ ‡F♯♯ †a♭ ‡G♯♯ †b♭ ‡B

which would be played by a second performer, thus giving the trines 10 to 29 and the duodenes 10 to 26 of Table VII., as for *Triôni*, p. 683.

9. *Guéroult's harmonium with one manual*, if we tuned B♯ in the usual way, and not as B♯, No. 4, p. 682, would be treated in a perfectly similar manner. The front row of vibrators would be tuned for the duodene of A, Table VIII., No. 20, and the back row for that of C♯ or ‡d♭, Table VIII., No 16. The four compound stops would be—

1			2			3			4		
C♭	E♭	‡G	f♭	a♭	‡c	b♭♭	‡d♭	‡f	e♭♭	‡g♭	‡b♭♭
†b	d♯	f♯♯	†E	G♯	B♯	†A	C♯	E♯	D	F♯	A♯

and the auxiliary harmonium would have for its vibrators—

‡B♯ d♭ ‡C♯♯ †e♭ ‡D♯♯ E♯ g♭ ‡F♯♯ †a♭ ‡G♯♯ †b♭ ‡B

giving the trines 11 to 30 and the duodenes 11 to 27 of Table VII.

10. *Hephtharmonium and Hexharmonium.*—These are both educational instruments, especially adapted for singers. Both of them use Mr. Saunders's 'tilting action,' p. 659, and have both two rows of vibrators.

For the *hephtharmonium* the vibrators are disposed thus:—

Back.....	†D♭	‡c♯	‡D	‡d♯	†F♭	†F	‡f♯	†G♭	‡g♯	‡A	b♭	†C♭
Front.....	C	d♭	D	†e♭	E	F	f♯	G	†a♭	A	†b	B.

This harmonium contains all the tones in a heptadecad (whence its name), and consequently illustrates every kind of modulation in the first degree from C. The front vibrators, to which correspond the digitals when the stops are all pushed in, contain the whole duodene of C, and the stops enable the player to exchange their tones for those in the six other decads. All lie on the usual digitals except †D♭ and †G♭, which are placed on the long white digitals of C and G, in place of the short black digitals next on their right, as these were wanted for ‡c♯ and ‡g♯. This makes a slight difference in the fingering of †E♭ minor and similar scales. But the chief deficiency consists in the absence of tones for shewing the difference between the false minor Third on the second, and its just substitutes in several keys. To obviate this I found it would be necessary to sacrifice one of the decads, and thus to form the

Hexharmonium. The vibrators are disposed as follows:—

Back.....	†C	‡c♯	‡D	‡d♯	‡e	†F	‡f♯	‡G	‡g♯	†A	b♭	‡b.
Front.....	C	d♭	D	†e♭	E	F	f♯	G	†a♭	A	†b♭	B

†C			
†F	†A		
†B♭	D	F♯	
†E♭	G	B	‡D♯
†A♭	C	E	‡G♯
D♭	F	A	‡C♯
	B♭	‡D	‡F♯
		‡G	‡B
			‡E

Intonation.—Comparing the diagram in the margin with Table IV., cols. 4, 5, 6, 7, it will be seen that 6 tones from each column are selected, giving the *six* decads of F, C, G; ‡D, A, E, and the major scales of †A♭, †E♭, †B♭. In all the nine major scales both or one of the just substitutes for the false minor chord on the second exist, thus:—

Tonic.		Dominant Form.			Subdominant Form.
in †A♭				only	B♭ — D♭ + F
in †E♭				only	F — †A♭ + C
in †B♭				only	C — †E♭ + G
in F	both	G — †B♭ + D		and	‡G — B♭ + ‡D
in C	both	D — †F + †A		and	‡D — F + A
in G	only	†A — †C		and	A — C + E
in ‡D	both	E — G + B		and	‡E — ‡G + ‡B
in A	both	B — D + F♯		and	‡B — ‡D + ‡F♯
in E	only	F♯ — †A		and	‡F♯ — A + ‡C♯

This is very valuable as a guide for singing in one of the most difficult points of just intonation (p. 527, note 1). This instrument also possesses the complete duodene of C and its right relative A. It has major scales on all the tones of the heptad of C, and decads upon five of them, and also an additional decad ‡D. The succession of scales is broken, however, by commatic gaps, thus—

Major.....†A♭, †E♭, †B♭, — F, C, G, — ‡D, A, E
Minor..... F, C, G, — ‡D, A, E

This gives an opportunity of studying the effect of commatic changes in modulation, as compared with just modulation.

11. *Triolet.*—Helmholtz's harmonium being arranged as in No. 8, the third set of vibrators there proposed to be placed on a second harmonium are placed upon the same instrument as the two first, and are brought into action by a set of valves moved by *studs* which pass up through about the centre of the black digitals and a little higher up through the white digitals. One performer would thus have the 17 duodenes, 10 to 26 of Table VII., under command. He would have to remember that the *minor* Thirds of D, G, C, F, and the *major* Thirds of A, ‡D, ‡G, B♯, E♯, A♯, ‡D♯, ‡G♯ lie on *studs*. This instrument containing 36 tones to the octave might easily be arranged for Guéroult's original compass with the third series as in No. 8, or for Poole's, with the vibrators tuned as for Trióni No. 6.

12. *Double Triolet.*—This is a double-manual instrument, with three rows of vibrators to each manual, opened by Saunders's 'tilting action' (p. 659), and by studs as for the Triolet No. 11.

Vibrators for Upper Manual.

Back; for the duodene of †B♭, Table VIII., No. 1.

Front; for the duodene of G♭, Table VIII., No. 5.

Extra, the 12 tones in Table V., col. IV., line 1*m* to 8*m*, and col. V., lines 1*m* to 4*m*, arranged thus, and played by studs:—

B♯ c♯ ‡D ‡d♯ E E♯ f♯ ‡G g♯ A a♯ B

When the stops are pushed in, the manual is arranged for the duodene of G♭. The four compound stops when drawn out give the trines Nos. 1 to 4 of Table VII. The upper manual therefore plays the duodenes, Table VII., Nos. 1 to 17, trines 1 to 20.

Vibrators for Lower Manual.

Back; for the duodene of F♯, Table VIII., No. 17.

Front; for the duodene ‡D, Table VIII., No. 21.

Extra; the 12 tones in Table IV., col. 8, line *p* to *x*, and col. 9, lines *p* to *s*, arranged thus, and played by studs:—

‡B♯ ‡b♯♯ ‡C♯♯ ‡‡d♯ ‡D♯♯ ‡E♯ ‡e♯♯ ‡F♯♯ ‡f♯♯♯ ‡G♯♯ ‡a♯ ‡A♯♯

When the stops are pushed in the manual is arranged for the duodene of ‡D. The 4 compound stops when drawn out give the 4 trines, Nos. 17 to 20 of Table VII. The lower manual therefore plays in the duodenes of Table VII., Nos. 17 to 29, and also in the duodenes of ‡B, ‡D♯♯ = ‡E, ‡G♯♯ = ‡A, and ‡C♯♯ = ‡‡D. The whole instrument, therefore, plays in 33 duodenes, using 52 tones to the octave, and is very complete.

All the mechanism is already invented and constructed. The existence of the duodene of F♯ on each manual renders transition easy from one manual to the other. The use of the studs could be avoided for all major keys from †G♭ to ‡D inclusive, so that studs would be used exclusively in minor keys.

But this instrument requires six rows of vibrators instead of four, and two manuals with awkward fingerings in the minor keys, and would necessarily be very expensive.

13 and 14. *Small and Great Duodenaries.*—Stops being always inconvenient, I suggest a manual founded on the following principles. The *Small* Duodenary will play the duodenes 10 to 26 in Table VIII., using three rows of vibrators as in Nos. 6 or 11, and the *Great* Duodenary will play all the 29 duodenes, using the four rows of vibrators in Nos. 7 or 11. From a model which I have had constructed for two octaves the arrangement seems practicable, but would doubtless require much mechanical treatment from harmonium-builders before it would act properly. It will be enough to indicate the form of the great duodenary.

Conceive the manual as a set of 29 'steps,' with $\frac{3}{4}$-inch 'tread' and $\frac{1}{4}$-inch 'rise,' the lowest step next the performer. Each step for the length of an Octave is divided into 12 digitals corresponding to the columns in Table VIII. The width of the digitals to be as follows for No. 11 of Table VIII., *in eighths of an inch*:—

C	d♭	D	†e♭	E	F	‡g♭	G	†a♭	A	†b♭	B
5	3	5	3	5	5	3	5	3	5	3	5

The digitals corresponding to the small letters are to rise $\frac{1}{4}$ inch above the others and to be bevelled, so that they are $\frac{3}{8}$ inch wide at bottom, and $\frac{1}{4}$ inch wide at top. Each step is then a miniature finger-board in the ordinary arrangement. Whenever any note occurs in 4 consecutive steps, as shewn by the cross lines in Table VIII., its 4 digitals are to be consolidated into one, so that, except in 'steps' 1 to 3, and 26 to 29, the digitals will be practically 3 inches long. To shew which digitals are consolidated, colour the low white digitals alternately white and light red, and the high narrow digitals alternately light blue and light brown, distinctions of colour easily seen. By colouring line 1 in Table VIII. (continued for a second octave), in the following manner, and observing to change the colour for each crossbar in descending, the effect is

pleasant to the eye. The numbers give the columns carried on to two octaves :—

White;	5, 6, 8,—	13, 15, 22, 24—
Red;	1, 3, 10, 12—	17, 18, 20—
Blue;	2, 4, 9, 11—	19—
Brown;	7,—	14, 16, 21, 23—

To mark the duodene, draw a black line, $\frac{1}{4}$ inch broad, across the digital bearing the name of the duodene, and put a black circle of $\frac{1}{4}$ inch in diameter on the tonic of the major scale which it contains. The lines thus marked, together with the alternation of colour, will clearly distinguish each duodene.

The depth of this manual from front to back would be $21\frac{3}{4}$ inches, and the rise $7\frac{1}{4}$ inches; the width of an octave from *C* to *B* is $6\frac{1}{4}$ inches, and from *C* to *c* is $6\frac{7}{8}$ inches. The last width is $7\frac{3}{8}$ inches on the piano; but as the hand would on the duodenary always have to dip between high digitals to strike octaves of low digitals, it must be held more upright, and hence its span will be less. A manual of five octaves and one note, *C* to *c''''*, will be $31\frac{7}{8}$ inches long. The number of movable digitals in each column of Table VIII. is 8, which open only 4 valves. This will necessitate coupling, the details resulting from Table VIII., which may be considered as the ground-plan of this manual.

This manual possesses the advantages of preserving the ordinary fingering, except in so far as it depends upon the low flat digitals coalescing in front of the short high digitals. And it also clearly indicates the duodenes in which a piece of music necessarily moves, as shewn by the duodenal. In fact, if the duodenal were always to bear one of the 12 forms B, E, A, D, G, C, F, B♭, E♭, A♭, D♭, G♭, with a mark to shew whether it was to be taken high up, or low down, or in the middle of the manual, a performer might play in skhistic intonation on this manual from ordinary notation, in quite as mechanical a manner as in equal temperament, without giving a thought to the reason of the changes. Although this mechanism is highly undesirable, yet the fact that playing in skhistic or practically just intonation can be so reduced without altering our present fingering or our present staff notation, is a result of the present theories to which no others have hitherto approached. This board also serves to shew very clearly the whole nature of duodenation as in Tables IV., V., and VII. But there are several objections to ordinary fingering, especially that it differs from key to key, so that at least 24 different fingerings have to be acquired for the major and minor scales. Also the fact that, though only four different tones are used in each column, they are played by eight different movable digitals, is a clear waste of mechanism. These and other difficulties are overcome by Mr. Bosanquet's arrangement, which is in every way far superior, and has stood the test of actual practical use.

I have thought it expedient, however, for those who prefer the old arrangement of the digitals to record the duodenary proposal contained in my paper on 'Duodenes,' p. 641.

15. *Bosanquet's 'Generalised Keyboard.'*—Mr. R. H. M. Bosanquet, Fellow of St. John's College, Oxford, has invented, and has actually had in use since the beginning of 1873, a most ingenious manual, adapted for playing in any commatic or skhismatic temperament. This keyboard seems to leave nothing to be desired in theory, compactness, and convenience, for those who will take the trouble to learn a new system of fingering, which, however, has the unexampled advantage of being the same for all scales of the same kind whatever be their tonic. Mr. Bosanquet, whose experience as a player on the organ gives great weight to an opinion confirmed by nearly two years of practice, states that his manual is well adapted for all the combinations of harmonies which occur in skhismatic systems; that it admits of considerable rapidity of execution, and that it may possibly be adapted for use as a pedal. It was invented for playing in his cycle of 53 (p. 655); but in order to facilitate fingering for 'working round,' it was constructed to play 84 tones to the octave, and this feat it accomplishes in about the same space as is required for 48 tones on my duodenary manual. Consequently the 24 tones of Helmholtz, the 36 of Poole, the 48 that I propose, supplemented to 60 or even 72 (the whole of lines p to x in Table IV.), could be played upon it with ease. It may therefore be said to solve the constructional problem of just intonation, as represented by skhistic or practically just intonation. Mr. Bosanquet read papers on his keyboard and its theory before the Royal Society on January 30, 1873, and the Musical Association on November 2, 1874, but as no description of his keyboard has yet been printed, he has kindly favoured me with the data from which I have drawn up the following description. For brevity and convenience I have here endeavoured to give a theory of its construction in accordance with the preceding sections. Mr. Bosanquet himself founds its theory upon the relations of temperaments, chiefly cyclic, to equal temperament as a point of departure.[1] Hence Mr. Bosanquet is in no respect responsible for the theory here given. Represent, both in sound and pitch, 12 tones which are arranged in order of any uniform Fifths, by the figures (1) to (12), inclosed in a parenthesis. Then the 13th tone, forming the 12th Fifth above (1), will be (1) ks by the fundamental relation, section C, equation (5), p. 648, where $k = \dagger$, and $s = \P$ for just Fifths, and vary according

[1] An abstract of his theory as read to the Musical Association, is printed in the 'Philosophical Magazine' for January 1875, vol. 48, No. 321, p. 507. His paper 'On the Theory of the Division of the Octave, and the Practical Treatment of the Musical Systems thus obtained,' with a wood-cut of part of his keyboard, is contained in the 'Proceedings of the Royal Society,' vol. 23 (in type, but not issued, when this page was printed).

to the temperament used. Similarly, the 13th tone below (12), that is the first below (1), will be $(12) \div ks$. We may therefore separate any series of tones forming uniform Fifths into sets of twelves, where each ascending set is formed from the one below by multiplying by ks. Also by Section C, equation (3), p. 647, the major Third to any tone, as (6), is found by taking the fourth Fifth above it, as (10), and dividing by k, or is $(10) \div k$, and the major Third below (6) will be found by taking the fourth Fifth below it, as (2), and multiplying by k, or is $(2)\,k$. Hence the tones $(2)\,k$, (6), $(10) \div k$ form major Thirds, and from these we can complete the whole series of Fifths, as in the margin, which we may suppose to represent columns II., III., IV. of Table V., indefinitely extended. Now, in *just intonation* no tone in any such series of columns, which may be extended indefinitely each way, will coincide with any other tone. But for any *skhismatic temperaments* $s = 1$; hence we may suppose s omitted throughout in these columns, the value of k varying according to the temperament, and then each succeeding column repeats the preceding, beginning 8 Fifths lower down. Hence there is in fact only one column so far as sound is concerned, and the major Third above any tone is the eighth Fifth below it. All other intervals are also given in numbers of Fifths up and down, as shewn in Table II., see the end of Section B, (p. 646.) As was shewn in Section C, skhismatic temperaments embrace skhistic, skhismic, Bosanquet's, and other, and may be conceived to be represented by unequally or practically just intonation. But for *commatic* temperament $k = 1$, and s varies from one to another. In this case, again, each succeeding column repeats the preceding, beginning 4 Fifths higher up, so that there is still but one column so far as sounds go, but the major Third above any tone is now the fourth Fifth *above* (instead of the eighth below). The only valuable temperaments of this class are the mean tone temperament and Huyghens's cycle of 31. If both $k = 1$ and $s = 1$, then all the columns reduce to a single set of Fifths. This is the case of equal temperament provided for by the usual fingerboard.

II	III	IV
$(10)k$	$(2)ks$	$(6)s$
$(9)k$	$(1)ks$	$(5)s$
$(8)k$	(12)	$(4)s$
$(7)k$	(11)	$(3)s$
$(6)k$	(10)	$(2)s$
$(5)k$	(9)	$(1)s$
$(4)k$	(8)	$(12)+k$
$(3)k$	(7)	$(11)\div k$
$(2)k$	(6)	$(10)\div k$
$(1)k$	(5)	$(9)+k$
$(12)+s$	(4)	$(8)\div k$
$(11)+s$	(3)	$(7)+k$
$(10)+s$	(2)	$(6)\div k$
$(9)\div s$	(1)	$(5)\div k$
$(8)\div s$	$(12)+ks$	$(4)\div k$
$(7)\div s$	$(11)\div ks$	$(3)+k$
$(6)+s$	$(10)+ks$	$(2)+k$
$(5)+s$	$(9)+ks$	$(1)\div k$
$(4)+s$	$(8)+ks$	$(12)+kks$
$(3)\div s$	$(7)+ks$	$(11)+kks$
$(2)\div s$	$(6)+ks$	$(10)+kks$
$(1)\div s$	$(5)+ks$	$(9)+kks$
$(12)\div kss$	$(4)\div ks$	$(8)+kkss$
$(11)+kss$	$(3)\div ks$	$(7)+kkss$

For skhismatic and commatic temperaments, then, the problem reduces

to placing under the hand of a player, successive sets of twelve tones forming tempered Fifths, each set differing from the preceding by a fixed interval, or *degree*, which answers to a tempered comma for skhismatic, and a tempered skhisma for commatic temperaments. For skhistic, skhismic, and Bosanquet's intonations, each successive set of twelve Fifths is *sharper* than the preceding, by an interval whose logarithm is ·005 1500, ·005 8851, and ·005 6798 (or ·205, ·235, and ·226 equal Semitones) respectively; and for mean and Huyghens's intonation each successive set of twelve Fifths is *flatter* than the preceding by an interval whose logarithm is ·010 3000 (a diesis) and ·009 7107 (or ·411 and ·387 equal Semitones) respectively.[1] The *amount* of change is of no consequence to the theory of the board, and Mr. Bosanquet has found it practically convenient for actual fingering not to take account of the sharpening or flattening, but merely of the *direction* of change, reckoning by *ascending* Fifths in each case.

If the set of twelve tones forming tempered Fifths be reduced to one Octave commencing on the second of them, they will stand in the following order of pitch. They are here associated above with the names of the tones in equal temperament, and below, first, with the names of the 48 tones in the middle section of Table V., below (each tone being *sharper* by a skhistic comma than the one here below it), and, secondly, with the 27 mean tones (each tone being *flatter* by a just diesis than the one here below it). Each series begins with an F tone and ends with a B♭ tone.

Equal	C	c♯	D	e♭	E	F	f♯	G	g♯	A	b♭	B
Fifths	2	9	4	11	6	1	8	3	10	5	12	7
Skhistic ′	†C	†d♭	†E♭♭		†F♭	†F	†g♭	†A♭♭	††a♭	†B♭♭		†C♭
Skhistic	C	d♭	D	†e♭	†E	F	g♭	G	†a♭	†A	†b♭♭	C♭
Skhistic ′	B♯	c♯	‡D	e♭	E	E♯	f♯	‡G	g♯	A	b♭♭	B
Skhistic ″	‡B♯	‡c♯	‡C♯♯	‡d♯ ‡‡d♯	‡D♯♯	‡E♯	‡f♯	‡F♯♯	‡g♯	‡G♯♯	a♯ ‡a♯	‡B
Mean ′	B♯		C♯♯					F♯♯		G♯♯		
Mean	C	c♯	D	d♯	E	E♯	f♯	G	g♯	A	a♯	B
Mean ′		d♭	E♭♭	e♭	F♭	F	g♭	A♭♭	a♭	B♭♭	b♭♭	C♭

To avoid the increasing complexities of notation, Mr. Bosanquet uses the equal temperament symbols with all their synonyms (given in Table VI., B, col. Equal) to represent the twelve tones forming Fifths in one set arbitrarily selected, for example the second skhistic line above, and prefixes a sloping line drawn upwards from bottom to top (here represented by a turned *acute* accent) to indicate the line above, and a sloping line drawn

[1] The sharpening and flattening, indeed, is not necessarily confined to skhismatic and commatic temperaments as in the examples just given. For example, in the Pythagorean temperament, which is commatic, 12 Fifths are *sharper* than 7 Octaves.

downwards from top to bottom (here represented by a second *grave* accent) to indicate a line below, as shewn above. The application of this notation to the cycle of 53 is shewn in Table VI., B, col. Bosanquet. In just intonation *two* increments have to be considered, for sets of twelve Fifths and for columns. The series of Fifths in each column might then be represented in the usual way (without † and ‡, as in Table VI., B, col. Pythagoras), and Mr. Bosanquet's degree-marks might be employed to indicate the *column* to which any tone belongs, in place of Herr von Oettingen's bars above and below, as suggested in Table IV., because these sloping lines (,,, ,,,) are much more convenient typographically, and also because, unlike the other, they can be applied to the staff notation, on which Mr. Bosanquet uses them in all music for his keyboard.

Now, referring to Table IX., it will be seen that it is divided by columns and cross lines into equal oblongs, 4 in each of its 13 columns, the last being merely a repetition of the first. Each column is headed by the numbers of Fifths as above, and contains, in capital letters (one in each oblong), the four skhistic tones given above; in small letters, the two or three mean tones also given above, and four numbers of the tones in the cycle of 53 (as in Table VI., B, col. Bosanquet). At each side of the table the lines are numbered in sets of 12, and it will be found that the cross lines dividing any column are on the lines bearing the same number as that at the head of the column. That is, each oblong advances one line upwards for every Fifth that its tone advances upwards, and as the length of every oblong is 12 times the interval between two lines, at the end of twelve removes the bottom line of the oblong is on a level with its original upper line, as may be seen for the oblong C line 2 in the column headed 2, which moved up to line 3 would be in a position parallel to G in the column headed 3, and moved up to line 4 would be in a position parallel to D in the column headed 4, and so on. It will be found that the numbers which head the columns, or 2, 9, 4, 11, 6, 1, 8, 3, 10, 5, 12, 7, increase by 7 and diminish by 5 alternately (adding or subtracting 12 where necessary), and that the alternate numbers increase by 2, as 2, 4, 6, 8, 10, 12 and 9, 11, 13 (= 1), 3, 5, 7. In the same way, taking the lower lines of the oblongs, it will be seen that we ascend 7 lines and descend 5 lines alternately in passing from column to column, but if we pass to alternate columns we ascend 2 lines every time. This gives a ready means of constructing the figure. The table might be continued indefinitely up or down, and in point of fact, Mr. Bosanquet has 7 oblongs in each column, bearing the numbers 1 to 7, 6 to 12, 10 to 16, &c., as shewn at the bottom of the table; so that, as there would be 84 oblongs to the octave and only 53 numbers, because, as already implied, the convenience of playing requires several numbers to occur on two oblongs, one at the top and the other at the bottom of the table.

The following are practical points of construction. Each interval

between two lines is $\frac{1}{4}$ inch; each oblong is therefore 3 inches long. Each column is $\frac{3}{8}$ inch wide; the thickness of the line dividing the columns is $\frac{1}{8}$ inch; hence the distance from the centre of one oblong to the next is $\frac{1}{2}$ inch. Used as a digital each oblong rises $\frac{1}{12}$ inch when its base is displaced $\frac{1}{4}$ of an inch; hence the difference of the numbers at the head of the column shews the number of *quarters* of an inch in longitudinal, and the number of *twelfths* of an inch in vertical displacement. Thus the digital for C is nearer to the performer by $\frac{7}{4}$ inch, and vertically $\frac{7}{12}$ inch lower than that for D♭; but further from the performer by $\frac{5}{4}$ inch, and vertically $\frac{5}{12}$ inch higher than that for C♯. After proceeding through a complete set of Fifths, the digital is further from the performer by 3 inches, and vertically 1 inch higher than before. The end of each digital next the performer is provided with a *lip*, ending in a semicircle, and extending half-an-inch beyond the 3 inches of the digital proper; this lip forms a thin projecting ledge, and from the commencement of the digital proper the lower wall is bevelled away at about an angle of 45° to allow of the finger coming readily below the lip and reaching to the end of the digital immediately under it. As a guide to the eye, Mr. Bosanquet blackens the 'chromatic' columns of digitals, as on an ordinary piano, and shews the sets of twelve Fifths, by placing his marks of degrees (, ,, ,,,) on the vertical thickness of the lip of each digital. In Table IX. I have suggested marking the 'natural' columns alternately white and red, and the 'chromatic' columns alternately blue and black, because these colours are readily seen, and mark the sets of Fifths distinctly. The colour suggested for the commencing and concluding digital in each column is placed over and under it in Table IX. Mr. Bosanquet, however, prefers keeping to the usual white and black.[1]

[1] Mr. Bosanquet's harmonium of $4\frac{1}{2}$ Octaves with 84 vibrators to the Octave, was built by Mr. T. A. Jennings (address 127, Pentonville Road, London, N.), who had already built him an organ. Being an experimental instrument, the expense was increased by time occupied in designing, experimental actions, alterations, and additional windchests and the construction of a tuning machine, but the whole expense from first to last did not exceed £150. The keyboard itself was made direct from Mr. Bosanquet's drawings by Messrs. Henry Brooks & Co., pianoforte action makers, 31 to 35, Cumberland Market, Regent's Park, London, W. As only 12 patterns were required (for a single set of twelve Fifths repeated 7 times), the cost was only £29 (included in the above £150), as against the £50 paid for Gen. T. Perronet Thompson's keyboard. There is no patent in the way, and the description in the text (which has been made very full for this purpose) might suffice for any English or foreign builder to complete an instrument. But the experience of Mr. Jennings would certainly facilitate the work and lighten the expense. He roughly estimates that he could make an harmonium on Mr. Bosanquet's plan, with the generalised keyboard, in an ordinary mahogany case, with $4\frac{1}{2}$ Octaves, and 36 digitals to the Octave, for £52, with 48 for £70, with 60 for £90, with 72 for £108, and with 84 for £126, all expenses included. No musical lecture room should be considered complete without an in-

It is clear that Prof. Helmholtz's or M. Guéroult's compass could most readily be played with this keyboard. It is only necessary to select the tones on Table IX. from the descriptions already given, Nos. 3 and 8, and Nos. 4 and 9. The same also holds for the hephtharmonium and hexharmonium of No. 10. In all instruments it is of great importance to avoid stops for regulating intonations, retaining them merely for the regulation of quality of tone. But considering the ease with which 36 tones to the Octave can be mastered on this board as for Mr. Poole's compass (No. 6), and the importance of these additional 12 tones, even for experimental purposes—no practical instrument can dispense with them—it would scarcely be worth while to make an instrument with a less compass; Table IX. shews the positions occupied by the 36 tones named in No. 6, or No. 8, or No. 9. In Table IX., for convenience, 48 tones alone have been considered, and the fingerboard is represented as complete for that number in order to shew the terminations at top and bottom, but any number of tones could be used consistently with the performer's being easily able to reach across the whole manual. The dimensions given shew that for these 48 digitals, the extreme depth (from the bottom of the E♭ or 11 column, to the top of the A♭ or 10 column) is 15 inches, and the width for an Octave is 6 inches, and for 5 Octaves and a note 30½ inches, which is much smaller than for my great duodenary with the same compass For 60 digitals to the Octave the depth would be 18 inches, and for 72 it would be 21 inches, which may be considered the extreme for non-cyclical temperaments. For joining on the top tones to the bottom Mr. Bosanquet uses 84 tones in his cycle of 53, and 43 tones in Huyghens's cycle of 31.

It is evident, from inspection, that the fingering would be precisely *the same in all duodenes*, although different from the ordinary fingering. Hence it is only necessary to learn the fingering of one duodene, and to use the duodenal as before to mark the duodene.[1] This dispenses with

strument of at least 48 digitals to the Octave, tuned in practically just intonation (p. 656), when it can be obtained at so low a price.

[1] The duodene is readily picked out on the board. The root being selected (say C), the major Third *above* (say E) is on the *fourth* column to the *right* (or eighth to the left), and one Fifth *lower* in rank, that is, it is on a level with the *lower* half of the digital of the root; and the major Third below (say †A♭) is on the *fourth* column to the *left* (or eighth to the right) and one Fifth *higher* in rank, that is, it is on a level with the *upper* half of the digital of the root. Having obtained this trine, the Fifths above (say G and D from C; B and F♯ from E; and †E♭ and †B♭ from †A♭) rise one and two ranks, being on the same level with the *middle* part of the digital of the tonic (say C, E, and †A♭ respectively), and in the *seventh* and *second* column to the *right* (or the fifth and tenth to the left respectively), and the Fifths *below* (say F from C, A from E, and D♭ for †A♭) lie on the same level with the *lip* of the digital of the tonic (say C, E, and †A♭) respectively, and on the *seventh* column to the *left* (or fifth to the right) respectively. These directions may be easily verified on Table IX., and imme-

Mr. Bosanquet's degree marks (, ,, ,,) on the staff itself. But considering the ease with which it is possible on this keyboard to touch a tone one skhistic comma higher or lower than that required in the duodene itself, it may be convenient when the true is used for the false minor chord on the second of the key, to indicate the motion by writing the mark of an ascending or descending degree before the required notes (meaning one tempered comma higher or lower than the tone in the current duodene), as *d* ,*f* ,*a* or ,*d f a* in lieu of the more correct method of marking the dominant or subdominant duodenation. Similarly it becomes easy to play the chord of the dominant Seventh as *g b d′* ‡*f′* in place of *g b d′ f′*, and thus more closely to imitate the natural Seventh (p. 539), as Mr. Bosanquet generally prefers. Now this form necessarily does not enter into a duodene because it does not belong to tertian harmony proper, but is a tempered imitation of septimal harmony. We may, then, in any duodene use the mark of a descending degree to shew that this extra duodenary tone is to be played, thus: in the duodene of C, *g b d′* ,*f′*; in that of F, *c e g* ,*b*♭.

For equal temperament Mr. Bosanquet's board would reduce to 12 digitals forming a uniform row of the same height and length, with the ends of the oblong in the same straight line, and giving apparently the same fingering in all keys. But it is known that such fingering would be impracticable. If, however, we took the 27 tones of mean intonation, as shewn by the italics on Table IX., and, retaining the digitals, tuned the vibrators belonging to them in each column in unison (as *b*♯, *c*, in the first column, *c*♯ and *d*♭ in the second, and so on), it is evident that the same fingering would apply to equal as to mean temperament. We should thus get a means of fingering all keys in the same way even in equal temperament. This use of two digitals, differently situated, for the same tone, for facilitating fingering, was suggested to me by the practice of tuning English concertinas in equal temperament (as explained on p. 681), and is in fact similar to the duplication of digitals for cyclical temperaments already mentioned. Of course such a fingerboard is not likely to be adopted (unless for the experimental convenience of introducing the contrast of equally tempered intonation on a skhismatic instrument by means of a stop, retaining the fingerboard), but its existence shews how one of the difficulties of ordinary fingering could be overcome.

diately give the proper fingering for every possible scale. A line drawn from 3 on the left to 3 on the right in Table IX., passes just above the end of the oblong C, and hence $\frac{3}{4}$ inch above the end of its lip, just below the end of the oblong D and hence $\frac{1}{4}$ inch above the end of its lip, through E and F, at the end of G, and hence $\frac{1}{2}$ inch above the end of its lip and through A and B. Hence the major scale of any tone lies entirely on the line passing through the end of the oblong, that is $\frac{1}{2}$ inch above the end of the lip of its dominant digital. The fingering of the minor scales then becomes easy by considering them as relative minors of major scales (p. 667).

The large space here devoted to a description of Mr. Bosanquet's 'generalised keyboard' is justified by the importance of this invention to singers and to composers who wish to study the full effects of just intonation, in the close approximation thus rendered accessible, and by the impossibility of fully appreciating the application of the fundamental propositions of the present work to the complicated harmonies and modulations of modern music, without an instrument on which they can be fully realised.

SECTION H.

MISCELLANEOUS NOTES ON RECENT ACOUSTICAL AND MUSICAL RESEARCHES AND INSTRUMENTS.

Art. 1. *Professor Mayer's Acoustical Researches.*

Mr. Alfred M. Mayer, Ph.D., Member of the National (American) Academy of Sciences, and Professor of Physics in the Stevens Institute of Technology, Hoboken, New Jersey, United States, has recently published a series of acoustical researches, of which Nos. I. to V. have already appeared in the 'Philosophical Magazine,' and No. VI., which will probably also appear there, may be read in 'Silliman's Journal.' The following is an abstract of some parts of these.

(1a.) *New Objective Analysis of Compound Sounds.*—The analysis of compound sounds by resonators has two disadvantages; first, that it is subjective, inasmuch as but one observer at a time is capable of hearing the results; and, secondly, that the range of pitch reinforceable by a resonator is too great for extreme accuracy in the estimation of the actual component sound present. Both of these disadvantages were thus overcome. ('Phil. Mag.,' Oct. 1874, vol. 48, pp. 271–3, with a figure.)

A. Grenié's free-reed pipe, of the pitch $C = 128$, had part of its wooden chamber removed and replaced by morocco leather, at one point of which 8 silk cocoon fibres were attached, having their opposite extremities attached to 8 tuning-forks tuned to C, c, g, c', e' g', $\flat b'$, c'', at the point of the upper node in each where it divides into segments when giving its upper harmonic, so that this harmonic was eliminated. The cocoon fibres were stretched till they made no visible ventral segments when vibrating. The reed was tuned accurately to the C fork (of 64 vibrations) by means of the g fork. The forks were placed on proper resonance boxes. When the reed was sounded each fork 'sang out' loudly, but if the prongs of any fork were only slightly loaded, the fork was mute, and was so rapidly affected that Prof. Mayer estimates (same vol. p. 519) that the effect of intervals such as 2000 : 2001 (or ·0087 equal Semitones) can be rendered sensible to the ear. On ceasing to sound the

reed, the forks continued to sound, and produced a tone of so nearly the same quality as that of the reed, that it was easy to feel that the difference was due to the absence of partials higher than the eighth.

By this means, then, the analysis and synthesis of a compound tone can be shewn to a large audience at once, and all doubt as to its objective reality removed. At the same time the air in the resonance chamber of the reed acts on the leather cover, as in hearing it would have acted on the drumskin of the ear, and the conduction of that vibration by the cocoon fibres replaces the complicated arrangements in the interior of the drum and the fluid of the labyrinth of the ear, while the forks themselves serve as the organs in the cochlea. Prof. Helmholtz's physiological theory of audition is thus perfectly exhibited in a 'working model.' The action of the resonance chambers of the forks is simply to make the effects heard at a distance.

This is the last of six methods of resolution mentioned in 'Phil. Mag.' January Supplement, 1875, vol. 48, pp. 514-521, where some additional particulars are given. The other five methods are, 1) by resonators applied to the ear, 2) by resonators applied to Koenig's manometric flames (p. 581 above), 3) by resonators, with their nipple stopped, successively brought before the compound tone, and hence successively reinforcing its partial tones, 4) by resonant boxes (carrying tuning-forks, and accurately tuned to the partial tones), which, by the action of the air, excite their forks, and these remain sounding after removal, 5) by the beating of the harmonics successively with a series of forks slightly out of tune with it, managed generally by flattening the tone to be analysed. Another method is adduced below, art. 9 of this section. The agreement of so many different methods of analysis removes all doubt as to the effective compound nature of musical sounds. The following experiments, completing the laws of beats, lead to the same conclusion.

(1b.) *Limits of the Damping Powers of the Ear at Different Pitches, or of the Rapidity of Audibly Separable Impulses* ('Silliman's American Journal of Science and Arts,' 3rd series, vol. 8, Oct. 1874, pp. 241-255, with a figure; a paper read before the National Academy of Sciences at Washington, April 23, 1874).—See App. XV., p. 626–9.

Mode of experiment: A tuning-fork was placed before a resonator which was connected with one ear (the other plugged) by an india-rubber tube; between the fork and resonator, revolved with known speed a perforated disk, with apertures equal to the opening of the resonator, and blanks between them of twice that width. Sometimes a gutta-percha funnel was used in place of the resonator, and a stopped organ-pipe in place of a tuning-fork. The sound was thus heard and cut off at definite intervals. The object was to determine how rapidly the strokes of sound succeeded each other before they completely blended into one continuous tone. The following were the results: —

Simple Tones experimental on	Vibrational Numbers = N	Duration of Residual Sensations, or reciprocal of the number of beats producing a continuous sensation of sound = D	Number of Wave-lengths contained in each one of the separate beats when continuous sensation was produced = L
C	64	$\frac{1}{16}$ = ·0625 seconds	4·0
c	128	$\frac{1}{26}$ = ·0384 ,,	4·9
c'	256	$\frac{1}{47}$ = ·0212 ,,	5·4
g'	384	$\frac{1}{60}$ = ·0166 ,,	6·4
c''	512	$\frac{1}{78}$ = ·0128 ,,	6·5
e''	640.	$\frac{1}{90}$ = ·0111 ,,	7·1
g''	768	$\frac{1}{109}$ = ·0091 ,,	7·0
c'''	1024	$\frac{1}{135}$ = ·0074 ,,	7·6

The law connecting D and N in these experiments (exclusive of *e''*, which lies outside of it) is

$$D = \left(\frac{53248}{N + 23} + 24\right) \times \cdot 0001.$$

The diminution of D as N increases is important, (roughly but by no means accurately D × N is constant.) All observers make the number of beats producing maximum dissonance to be a fixed fraction of the number of beats giving continuous sensations at each part of the scale The fraction varies from 0·3 to 0·6 in different individuals, and is possibly 0·4 on an average. Hence the interval producing the maximum dissonance will be found by adding 0·4 times the denominators of the vulgar fractions in column D to the vibrational number in N, for the higher tone and using N for the lower; and the interval for which beats will be inaudible, because the audible impression of the first will not have died out before the next following beat is heard, will be founded by adding the whole denominators in D to the corresponding N. In this way the following Table may be formed, which is partly given by Prof. Mayer and is partly here calculated, and in especial the interval is given in equal semitones.

From the Note	Interval of Continuity of tone	Its No. of Equal Semitones	Interval of Maximum dissonance	Its No. of Equal Semitones
C	64 : 64 + 16	3·86314	64 : 64 + 6·4	1 65005
c	128 : 128 + 26	3·08185	128 : 128 + 10·4	1·35240
c'	256 : 256 + 47	2·91808	256 : 256 + 18·8	1·23483
g'	384 : 384 + 60	2·51344	384 : 384 + 24	1·04956
c''	512 : 512 + 78	2·45405	512 : 512 + 31·2	1·02408
e''	640 : 640 + 90	2·27789	640 : 640 + 36	0·94741
g''	768 : 768 + 109	2·29766	768 : 768 + 43·6	0·95296
c'''	1024 : 1024 + 135	2·14398	1024 : 1024 + 54	0·88970

This Table shews 'why it is that the middle portion of the scale is best adapted for musical composition; for while in the lowest octave the available consonant intervals are few on account of the spaces separating

them, in the higher Octaves the consonances are so contracted that these higher consonant intervals lose their sharpness of definition.' Compare above, p. 259-261.

These results enable us to make more exact such statements as that made by me on p. 291, note 1, all of which depend upon the pitch. It is seen that the interval of a minor Third as the limit of continuity applies only to the Octave c to c'. For g'''', supposing the law connecting D and N, given above, to hold for such a high pitch, the interval of continuity would have the ratio 3072 : 3072 + 225, or 1·22469 equal semitones, and the interval of maximum dissonance would be 0·49443 equal semitones, so that the interval of one semitone is near the limit of continuity; hence it is not surprising that no beats were heard in the first case referred to on p. 262, note 1. But for c'''' the interval of continuity would have the ratio 2048 : 2219, giving 1·62080 equal semitones, and the interval of maximum dissonance would be 0·66836 equal semitones, and hence the beats of the second case, there noted, should be quite conspicuous, agreeing with observations. This is a simple instance of the explanation which this law gives of apparent anomalies.

Referring to p. 215 above, Prof. Mayer says: 'If we apply the law to vibrations below 40 per second, when they do not produce a continuous sound, but explosive sensations in the ear, we reach a remarkable result. Thus the residual sensation corresponding to 30 vibrations in a second should remain in the ear $\frac{1}{10}$ of a second after the vibrations outside the ear have ceased; then we at once ask why is it, if the residual sensation last $\frac{1}{10}$ of a second, that 30 beats or pulses in a second do not blend? Do not 3 distinct impulses fall on the ear in each $\frac{1}{10}$ of a second? For they follow one another at $\frac{1}{30}$ of a second. This abrupt breaking down of the law can only be explained by the highly probable supposition that co-vibrating bodies in the ear, tuned to vibrations below 40 per second, do not exist, and therefore . . . the inner ear . . . can only vibrate *en masse*, and the durations of these oscillations of the ear, as a whole, are far too short to remain $\frac{1}{30}$ of a second.' For very high tones he conceives that the vibrations of the inner ear vibrating as one mass, might last long enough to make the tone continuous, but not to discriminate pitch.

Revolving a perforated disk before a compound sound, the beats being conducted to the ear by funnel and tube, as the rapidity of revolution diminished, the partials, beginning with the highest, fall into beats and become discontinuous in succession, beginning with the highest, but when they are reduced thus to beat all in unison, the effects of the beats are different, because those of the highest approach most nearly to the rate of greatest dissonance. This analysis of sound is similar to that of light. Prof. Mayer refers to a passage in Helmholtz's 'Physiological Optics' (German Edition, in Karstens's 'Encyklopädie der Physik,' 1867, vol. 9, p. 380, French translation, 1867, p. 500), in which a disk of black and

white sectors is rotated, at first slowly and then with increasing rapidity, and is observed by an eye which takes care not to follow the moving figure In this case the white appears reddish on the first advancing, and bluish on the following edge, and the colour alters with intensity of light and speed of rotation. The general conclusion is that 'when a point of the retina is illumined by white light and darkened in rapid rotation, so that the retina is in an alternate state of exalted and depressed excitement, *the moment of maximum excitement is not the same for all colours*, but that excitement for red and violet occurs earlier than for green.'

The quality of a tone immediately on its cessation, continually changes the persistence of the sensation of the lower partials over the upper. This is analogous to the instantaneous action of light on the eye, and the succession of sensations of colour after the eye is closed.

Prof. Mayer draws attention to the fact that his beats were all tones of the same pitch, whereas the beats of imperfect consonances are tones of variable pitch (App. XIV., p. 625).

(1c.) *Observations on the Sensitive Hairs of the Antennæ of the Culex Mosquito.*—In a paper in the 'Philosophical Magazine,' 1874, vol. 48, p. 376, Prof. Mayer has established that the fibrils of the antennæ of the male *Culex Mosquito* are acoustic organs vibrating to sounds extending through the middle and next higher Octave of the scale, c' to c''', about the compass of the song of the female. Moreover, these fibrils do not vibrate if the front of the wave of sound is parallel to its length, so that the insect can determine direction by the motion of his antennæ, and thus direct himself to the female in the dark. But Prof. Mayer does not think that either this vibration or that of the hairs of the *Mysis* (p. 225 above) prove Prof. Helmholtz's hypothesis of the action of Corti's organs, which he considers to be a different arrangement.

(1d.) *Sharpening and Flattening Tones by Motion.*—Compare p. 247 above. In the 'Philosophical Magazine,' 1872, vol. 43, p. 278, Prof. Mayer gives the result of simple experiments shewing that translation of a vibrating body affects its wave length, sharpening the sound by approach and flattening by retreat. And in the paper cited in (1b) he mentions incidentally that by rapidly rotating a vibrating tuning-fork he was able to raise its pitch by nearly a minor Third.

(1e.) *Harmonic Curves* (see p. 600).—In the 'Philosophical Magazine,' Supplement for January 1875, vol. 48, pp. 520–525, Prof. Mayer gives curves compounded by six curves of sines (p. 35), representing six partial tones, where, for convenience, the amplitudes are taken to vary as the wave lengths, and to have the same initial phase. They are combined by taking the algebraical sum of their ordinates, which law would of course not hold true for the amplitudes chosen (about one-third of the length of the wave). The resulting figure bears a most remarkable resemblance to fig. 25, p. 134 above. There is the same sudden rise on the left,

and step-like descent on the right; but the steps are more rounded, and the upper crest more pointed, and there are five steps in addition to the crest. Prof. Mayer then combines two such compound curves, and thus produces the resulting curves of two compound tones forming an Octave (with one high and one low crest, and also one high and one low trough, and the steps uneven and reduced in number), a Fifth (with four crests, two with long and two with short descents, the shorter having only one step), and a major Third (with eight crests, two extremely small, two moderate, three intermediate between the two last kinds, and one high, the ascents being abrupt as before, the descents rather wavy than stepped). These had all been drawn on a large scale with several hundred ordinates, and were reduced photographically. They form an excellent practical illustration of the nature of harmonic motions.

Art. 2. *Mechanical Description of Combined Vibrations.*

(2a.) *Mr. Donkin's Machine for Compounding Parallel Vibrations.*—In the Proceedings of the Royal Society for 19 February, 1874, vol. 22, pp. 196–199, Mr. A. E. Donkin, M.A., Fellow of Exeter College, Oxford, gives an account of 'an instrument for the composition of two harmonic curves,' placed so that the vibrations are in the same straight line, with a drawing of the machine, which I have seen at work at the makers', Messrs. Tisley and Spiller, 172, Brompton Road, South Kensington. By an arrangement of toothed wheels, which can be varied, all the usual ratios can be used, and any alteration of phase chosen. The curves are usually much compressed, the amplitudes being much greater than the wave lengths; but there is an arrangement for increasing the wave length. The effects are very interesting. Mr. Donkin kindly sent me examples of the ratios 54 : 55, 8 : 9, 1 : 2, and 1 : 3, the last in three forms with different relations of amplitude, but all shewing one high and two low crests, which by their arrangement over some 50 waves bring out intermediate forms of great beauty. The imperfect unisons 54 : 55 and 8 : 9 bring out the beats very clearly, the crests regularly increasing to a maximum and then decreasing almost to nothing. The machine can of course only combine two waves representing simple tones, and hence does not shew the results of harmony proper, as in Prof. Mayer's drawings (Art. 1e).

(2b.) *Mr. Tisley's Machine for Compounding Rectangular Vibrations.*—This is a most successful combination of two pendulums swinging at right angles and simultaneously actuating a pen, which draws all the figures of Lissajous (fig. 23, p. 131 above) to any ratio, commensurable or incommensurable, producing beautiful results, which instead of being fleeting, and when a spot of light is made to move by mirrors attached to tuning-forks, are preserved for permanent study. The machine is figured and described in 'Engineering,' 6 February, 1874, vol. 17, No. 423, p. 101.

Art. 3. *Reference to Proofs of Fourier's Theorem.*

The most accessible demonstration of this theorem is in Donkin's 'Acoustics,' pp. 60–66. Mr. James O'Kinealy, of the Bengal Civil Service, communicates to the 'Philosophical Magazine,' August 1874, vol. 48, p. 95, a new proof of the theorem by an analytical process for periodic functions, making it depend upon the solution of an exponential differential equation, treated symbolically. There is also a full investigation of these series in Prof. M. Ohm's 'System der Mathematik,' vol. 9, 1852, pp. 286-357.

Art. 4. *M. Terquem's Tonometer.*

M. Terquem has constructed a new tonometer, by means of Lissajous's vibroscope and tuning-forks, with runners giving the absolute vibrational numbers to the second decimal place. It is described in the 'Comptes Rendus,' January 12, 1874, and the 'Philosophical Magazine,' March 1874, vol. 47, p. 212.

Art. 5. *Dr. W. H. Stone's Instrumental Researches.*

Referring to p. 267 above, respecting the deepest tones which can be heard, and also to Prof. Mayer's suggestions, as to the cause of our hearing tones of 30 vibrations and under as more or less discontinuous pulses (p. 702), and knowing the efforts which Dr. Stone had made to restore deep tones to the orchestra, I requested him to give me an account of his researches, which he has obliged me with as follows in (5a) and (5b). Much of the success of Dr. Stone's double basso is due to his method of strengthening the resonance of his instrument. His contrafagotto has been used as an orchestral instrument for some years.

(5a). *New $C_{,}$ Contrabasso and Tension Bars for Viols.*—Dr. Stone has for some years endeavoured to restore to the orchestra the lower notes of the 16-foot Octave, which appear to have been neglected of late. It seemed a contradiction that, while the organ possessed that Octave, and another, the 32-foot, below it, and while even an instrument of so comparatively feeble a tone as the pianoforte could obtain these deep notes, they should be absent from the full band. Most of the great composers have employed them, especially Beethoven and Onslow. Many passages of their compositions had to be partially transposed, often (as in the C minor Symphony of Beethoven) much to the detriment of the general effect. In the Trio of this great work a scale passage occurs several times for the double basses alone, beginning on the 16-foot $G_{,}$. But this note being entirely absent on the ordinary three-string basses, as used in England, it was there customary to take it either altogether or in part in the Octave above. Some players, indeed, were in the habit of letting down the $A_{,}$ or lowest string, by a Tone to $G_{,}$ for this special passage: but the resonance of a string thus slackened was far inferior to what could be

obtained by more legitimate means. The fine part for the contrafagotto in the same symphony, descending to $C_{,}$, was usually omitted, or played an Octave higher by the ophicleide. The *Pastoral Symphony* likewise frequently contains $F_{,}$ natural, a note quite unattainable except on the four-string basses, whose lowest note is $E_{,}$ (p. 267).

It was obvious, in attempting to remedy this defect, that of the three modes by which vibrations in a stretched string may be slackened, two, namely, length and thickness, were inadmissible. The first renders the instrument so large as to be unwieldy and out of the reach of an ordinary arm. The second was found to cause rotation of the string under the impulse of the bow acting at its periphery, and thus to generate false notes. The third remained, in increasing the specific gravity of the string without enlarging its diameter. This was satisfactorily accomplished by covering a gut string with heavy copper wire such as is used for the lowest strings of pianos. The note $C_{,}$ was obtained, and an instrument thus strung was exhibited in London in the International Exhibition of 1872.

But it became clear that to give the new notes full power, and to prevent the danger of shaking the instrument to pieces, a means of strengthening the belly in the direction of strain was required, which should not unduly increase the weight of the sound-board. This requisite was ingeniously fulfilled by Mr. Meeson:—Four strips of white pine are glued on to the back of the belly, running its whole length, one on one side of the ordinary bassbar, and three on its other side, thus corresponding in number, and to a certain degree in position, to the increased number of strings. Two of them cross and intercept the usual *f*-shaped sound-holes, thus removing a weak place in the belly, and causing it to vibrate more homogeneously. They appeared on trial to add great power to the instrument throughout, and to remove the inequality and varying intensity of vibration which exists on most old instruments even by celebrated makers, and which musicians usually designate by the term 'wolf.' The bars are curved to an elliptical shape to fit the hollow of the belly, and to give the greatest resistance to compression with the smallest quantity of material. Even in a double bass the quantity of wood required is very small, and from its lightness when perfectly dry it hardly exceeds an ounce in weight. From their shape and function they are termed *elliptical tension bars*.

It appeared from subsequent experiments that the same system was applicable to the smaller members of the viol and violin family, giving an increased sonority and firmness to the tone. It succeeds best with violins of sweet but feeble quality, and in some of the older Italian instruments, where the progress of decay had to a certain extent diminished the volume of sound.

(5b). *New $C_{,}$ Contrafagotto.*—The contrafagotto or double bassoon as made on Dr. Stone's designs by Herr Haseneier of Coblenz, consists of a tube 16 feet 4 inches long, truly conical in its bore, and enlarging from

$\frac{1}{4}$ inch diameter at the reed to 4 inches at the bell or lower extremity. It is curved four times on itself for convenience of manipulation; so that in actual length it is about equal to the ordinary bassoon.

Its extreme compass is from $C_{,}$ upwards to the c', three octaves above that note. Its ordinary range, however, should be limited to the g below the highest c'; the notes above this being rather difficult to produce. It possesses every semitone of the diatonic scale throughout its compass, and is therefore able to play in any key with moderate facility.

The scale is founded on the Octave harmonic, and continued by means of the Twelfth. From $C_{,}$ to $F_{,}$ only a single sound is obtained by each key. Between the latter note, $F_{,}$ and its double Octave, f, the same fingering produces two sounds of the same name at the interval of an Octave, simply by a change of *embouchure* and greater pressure of wind. With the four foot $f\sharp$ a new harmonic sound begins, using the fingering of the eight foot $B\natural$, and by again increasing wind-pressure. Seven semitones thus procured carry the tone up to the c' above, which is the fourth C inclusive from the foundation note.

The holes from which the sound issues are of graduated size, increasing downwards with the size of the bore. They are placed as a rule in their correct positions, so as to cut off the proper portion of tube corresponding to the elevation of the note. Mechanism is adapted to them so as to bring them within reach of the fingers. To enable the player to distinguish what are called 'open' from closed holes, a different shape is given to the terminations of the levers. The first three fingers of each hand, which have to keep closed the six open notes of the scale in the ordinary bassoon, fall into saddle-shaped recesses worked in the brass of the key; whereas the two little fingers and the left thumb touch the cushion-shaped surface of keys similar to those used on other wind instruments. It is in consequence very easy for any person accustomed to the ordinary bassoon to adapt his playing to this. The saddle shape of the key also serves to support the upper joints of the finger, and to throw the labour of closing the hole more on the powerful muscles of the forearm than on the weaker fabric of the hand itself.

The Contrafagotto is not by any means a new instrument. Haydn gives it a part in *The Creation*, and some other of his works; Mozart uses it occasionally; Beethoven frequently writes for it; and Mendelssohn has employed it in his overture *the Hebrides*, in the *Reformation Symphony*, and elsewhere. The older instruments were of feeble rattling tone, rendered unwieldy by an unsuccessful attempt to obtain the $B\flat$ of the 32-foot Octave. Herr Haseneier considerably improved on these, and on his excellent foundation the present instruments, two in number, were made.

(5c). *Pressure on the Lungs in Playing Wind Instruments; and Effect of Galvanism on the Pitch of Strings.*—Dr. Stone, in papers read before the Physical Society, 18 April and 9 May, 1874, reported in the 'Philoso-

phical Magazine,' Aug. 1874, vol. 48, pages 113-115, has shewn by the application of a windgauge actuated by breath driven through a tube at the same time as through the instrument played, that in blowing wind instruments the pressures of the wind are mostly 'small, not exceeding, or, indeed, attaining the pressure of a fit of sneezing or coughing,' and 'are therefore very unlikely to injure the lungs;' and also that by passing a galvanic current through a strained wire the pitch of its note was materially lowered.

Art. 6. *Mr. Hermann Smith and Herr H. Schneebeli on the Source of Tone in Flute Organ Pipes.*

On pp. 138–143 above, Prof. Helmholtz has given what is now the ordinary theory, (which supposes a flute organ pipe to reinforce one of the sounds produced by the breaking of the stream of air directed from the mouth hole against the sharp lip,) without entering upon the resulting difficulties. Very recently an entirely different theory has been proposed by Mr. Hermann Smith, who has been practically conversant with organ pipes for twenty years, in a series of letters, first in the 'English Mechanic,' and afterwards in 'Nature,'[1] and subsequently, but to all appearance quite independently, by Herr Schneebeli in 'Poggendorff's Annals.'[2] Referring to the original papers for complete information, I will endeavour to give a short account of their views, which, however, I do not feel sure of having always completely understood, and which I have had no opportunity of properly testing by experiment.

Herr Schneebeli had an experimental pipe constructed in the usual way; with glass back and a movable lip and slit or windway, through which was driven air impregnated with smoke, as is frequently done to make it visible. When he so placed the lip and slit that the stream of air passed *entirely outside* of the pipe, no sound occurred; but if he blew gently upon this sheet of air, at right angles, the pipe sounded, and the tone continued until he blew through the other end of the pipe; neverthe-

[1] See the *English Mechanic* for January 1867. This periodical also contains a practical treatise on the harmonium by Mr. Hermann Smith, in the course of which his conception of an aëro plastic reed was first formulated. His recent papers on the subject in *Nature* are: 1) 'On Approach caused by Velocity, and Resulting in Vibration,' 3 May, 1873, vol. 8, p. 25; 2) 'On some Errors of Statement concerning Organ-pipes in recent Treatises on Natural Philosophy,' 15 May, 1873, vol. 8, p. 45; 3) 'Harmonic Causation and Harmonic Echoes,' 11 Sep., 1873, vol. 8. p. 383; 4) 'On the Variability of the Node in Organ-pipes,' 19 Feb., 1874, vol. 9. p. 301; 5) 'On the Physical Action taking place at the Mouth of Organ-pipes,' 2 July, 1874, vol. 10, p. 161; 6) 'On the Process of Tone-making in Organ-pipes,' 15 October, 1874, vol. 10, p. 481; 7) 'On the Building up of the Tone in "Gamba" Organ-pipe,' 25 Feb., 1875, vol. 11, p. 325. The following account has been condensed from the letters in *Nature* aided by personal explanations from Mr. Hermann Smith.

[2] *Zur Theorie der Orgelpfeifen. Poggendorff's Annalen*, 1874, vol. 153, pp. 301–305.

less under these circumstances it was very rare indeed to find that any smoke penetrated into the pipe. If the sheet of smoked air passed *entirely inside* of the pipe, there was also no sound; but then on blowing through the open end, so as to force some of it out, sound was produced, and it was stopped by blowing against the slit. This case was therefore the converse of the last. He concludes: 'That the stream of air which issues from the slit forms a species of air-reed (*Luft-Lamelle*, aërial lamina), and that this plays in the generation of vibrations in the mass of air, a part analogous to that of reeds in reed pipes.' He states that the vibrational nature of motion of the air between the slit and the lip can be shewn by attaching a piece of silk-paper to the edge of the lip or the split, and pressing a point against it. He further proposes a theory founded on Helmholtz's hydrodynamical investigations in the 'Berichte der Berliner Akademie,' 1868, 23 April, Crelle 60, and states it to this effect: When the split is in the normal position the air-reed strikes the lip, a portion of the stream enters, and produces a compression as in reed pipes; the reaction of this compression affects the air-reed and bends it outwards; on the pressure ceasing the air-reed returns to its original position and the process begins afresh.

Mr. Hermann Smith states that the air from the bellows is *not* directed 'against the edge of the lip,' and that, if it is so directed, the pipe does not speak. He also states that the sharpness of the lip is immaterial to mere speaking, and that a pipe that speaks well may have the edge of its lip half an inch thick. (Compare p. 102 above, where the wind which excites the sound in the bottle is blown *across* its mouth, and the edges of the opening are rounded, not sharp.) The statement on p. 139 above, that the wave of air in an open pipe is *twice* as long as the tube, is well known to be only approximative, and to require a correction, not yet determined with theoretical precision, dependent on the other dimensions or 'scale' of the pipe. Mr. Hermann Smith states that a closed pipe does not give a note which is an Octave, but one which is only a major to a minor Seventh, below the note it would give as an open pipe.

The source of tone, according to both Mr. Hermann Smith and Herr Schneebeli, is the formation of what the former calls an 'aëro-plastic reed,' and also simply an 'air reed,' and the latter a 'Luft-Lamelle,' which is produced outside of the pipe, and bends partly within it. For the formation of this reed both agree that it is essential for the exciting air to pass the lip, certainly not to enter the pipe. The existence of the reed is shewn by Mr. Hermann Smith by interposing a thin lamina, a shaving, or crisp tissue paper, which is caught by the air and vibrates as a reed[1], and by Herr

[1] Press a piece of crisp, but very light, thin paper firmly against the outside of the windway by means of a card or piece of wood. Let the paper project upwards till it nearly covers the mouth, but is quite clear of both lip and ears. The paper then resembles a free reed. On now blowing in the usual way through the slit, the paper

Schneebeli by the smoke mixed with air which enables the experimenter to see its motion directly, and also by a piece of silk-paper.

Herr Schneebeli supposes this air-reed to act by producing condensation, but Mr. Hermann Smith's theory of its origin seems to be as follows. The air driven rapidly and closely from the slit past the mouth of the pipe, in a flat stream, just and only just avoiding the edge of the lip, creates a vacuum, precisely as in the tubes for ether spray or perfume spray in common use, or in ordinary chimney pots. The air in the pipe under the action of the atmospheric pressure at the upper open end immediately descends to supply this vacuum, and by so doing not only bends the flat exciting stream of air outwards, but also of course produces a rarefaction in the tube, which by extending from the mouth upwards, necessarily weakens the force of the outward rush of wind. The external (not the exciting) air, taking advantage of this relaxation of force, enters the tube at the lip, causing a condensation in the lower part of the pipe, and the resulting wave of condensation before it has proceeded half-way meets the former wave of rarefaction, which continued to proceed from the further end of the tube, and thus forms a node. The node is consequently always nearer the mouth than the end of the pipe. The ratio of the length of the segment further from the mouth to that nearer to it varies from 4 : 3 to 7 : 6 according to the diameter or scale of the pipe and the strength of wind. In the meantime the exciting stream of air rights itself, passes over the vertical, bends inwards, and a small portion of it enters the pipe with the external air, to be cast out again by the returning wave of rarefaction, and by this time the exciting stream of air has been converted into a vibrating air-reed. That the first (but momentary) effect of the upward rush of the exciting stream of air is to abstract air from the pipe Mr. Hermann Smith considers to be demonstrated by inserting on the 'languid,' just within the mouth, some filaments of cotton, fluff, or down, which, in larger pipes, are shot out with energy. He supposes the air-reed afterwards to abstract and admit air in constant succession, thus producing the necessary stimulus for the sound heard, which would on this theory depend, among other conditions, upon the force of blast, its inclination to the mouth, the size and form of the mouth and ears, the interposition of obstructions between the reed and outer air (as the shading bar of the *Gamba*), the capacity and length of the pipe, and whether a node has to be formed in the pipe in the mode explained for open pipes, or in the mode used for stopped pipes, which are acknowledged to speak more readily than the former. The external air of course passes continually, but intermittently, through the mouth of the pipe.

The law of vibration of the air-reed as stated by Mr. Hermann Smith, but unobserved and probably unobtained by Herr Schneebeli, is: '*As its*

will commence its vibrations, which Mr. Hermann Smith considers to correspond to those of his air-reed. The effect is easily observed.

arcs of vibration are less, its speed is greater,' or '*the times (of vibration) vary with the amplitude,*' being different from the usual law of a vibrating reed in which the time is independent of the amplitude of vibration. If by extraneous influence the pitch of the pipe is flattened, as by partly shading the mouth from the external air, Mr. Hermann Smith states that the path of the air-reed is lengthened, and conversely. When an organ pipe speaks the tones of the air-reed and pipe are distinct and may be separated or combined; and when a pipe is said 'to fly off to its Octave,' he says that the air-reed leaps back to its Octave speed, compelling the pipe to follow, and that this can be made visible. The natural pitch of the air-reed is also, as he states, far higher than the pitch of the tones of the pipe.

The precise theory of the action of this air-reed does not appear to have been formulated. Difficulties will occur to the mind of every investigator, but the above views due to Mr. Hermann Smith's practical experience suggest numerous points for experimental investigation which may lead to most valuable results, and it is desirable that he should himself put them together in a connected scientific form for convenient reference.

Art. 7. *On the Use of Striking and Free Reeds.*

As respects reeds used in organs, which on p. 144 are stated to be at present 'free' and not 'striking' reeds, it should be observed that fig. 29, B, p. 145, really represents a striking reed, and even in fig. 29, A, although the reed is drawn as free, the form of the tuning wire is one only adapted to striking reeds; for that of a free reed see J. G. Toepfer's 'Orgelbaukunst' (1855), figs. 135–40. On this point I have consulted some of our principal organ-builders, who have obligingly sent me the following information. Mr. Willis tells me that he never uses free reeds, that no power can be got from them, and that he looks upon them as 'artificial toys.' Messrs. J. W. Walker & Sons say that they have also never used free reeds for the forty or more years that they have been in business, and consider that free reeds have been superseded by striking reeds. Mr. Thomas Hill informs me that free reeds had been tried by his father, by M. Cavaillé Coll of Paris, and others, in every imaginable way, for the last thirty or forty years, and were abandoned as 'utterly worthless.' But he mentions that Schulze (of Paulenzelle, Schwartzburg) told him that he never saw a striking reed till he came over to England in 1851, and that Walcker (of Ludwigsburg, Wuertemberg) had little experience of them, as Mr. Hill learned from him about twenty years ago. It was probably this old practice of Schulze and Walcker which led Prof. Helmholtz to suppose that striking reeds were no longer in use. Mr. Hill adds, however, that both these builders speedily abandoned the free reed, after seeing the English practice of voicing striking reeds. This is corroborated by what Mr. Hermann Smith tells me, namely that Schulze, in 1862, built the great organ at Doncaster with 94 stops, of which only

the Trombone and its Octave had free reeds (see Hopkins on the Organ, p. 117, for an account of this organ); and that two years ago he built an organ of 64 stops and 4,052 pipes for Sheffield, with not one free reed; also that Walcker built the great organ for Ulm cathedral, with 6,500 pipes and 100 stops, of which 34 had reeds, and out of them only 2 had free reeds; and that more recently he built as large a one for Boston Music Hall, without more free reeds; and again that Cavaillé Coll quite recently built an organ for Mr. Hopwood of Kensington of 2,252 pipes and 40 stops, of which only one—the *Musette*—had free reeds. He also says that Lewis, and probably most of the London organ-builders not previously mentioned, have never used the free reed.

The harshness of the striking reed is obviated in the English method of voicing, as explained to me by Mr. Hermann Smith, by so curving and manipulating the metal tongue, that instead of coming with a discontinuous 'flap' from the fixed extremity down on to the slit of the tube, it 'rolls itself' down, and hence gradually covers the aperture. The art of curving the tongue so as to produce this effect is very difficult to acquire; it is entirely empirical, and depends upon the keen eye and fine touch of the 'artist,' who notes lines and curves imperceptible to the uninitiated observer, and foresees their influence on the production of quality of tone. Consequently, when an organ-builder has the misfortune to lose his 'reed-voicer,' he has always great difficulty in replacing him. The quality of tone produced by the free reed can be greatly modified by comparatively slight changes. If the reed is quite flat, the end not turning up, as it does in fig. 28, p. 144, no tone can be produced. If the size of the slit round the edges be enlarged, by forcing a thin plate of steel between the spring and the flange, and then withdrawing it, the quality of tone is permanently changed. Another change is produced by curving the middle part up and then down in a curve of contrary flexure. Another change results from curving the ends of the reed up as in 'American organs'—a species of harmonium.

One of the earliest free reed instruments is the Chinese 'shang,' one of which Mr. Hermann Smith shewed me, and, having taken it to pieces, examined its construction, and measured its pipes, he has kindly furnished the following description: 'The body of the instrument is in the form and size of a teacup with a tightly fitting cover, pierced with a series of holes, arranged in a circle, to receive a set of small pipe-like canes, 17 in number, and of various lengths, of which 13 are capable of sounding and 4 are mute, but necessary for structure. The lower end of each pipe is fitted with a little free reed of very delicate workmanship, about half an inch long, and stamped in a thin metal plate, having its tip slightly loaded with beeswax, which is also used for keeping the reed in position. One peculiarity to be noticed is that the reed is quite level with the face of the plate, a condition in which modern free reeds would not speak. But this singular provision is made to ensure speaking either by blowing or suction. The corners of the reeds are rounded off, and thus a little space is left

between the tip of the reed and the frame for the passage of air, an arrangement quite adverse to the speaking of harmonium reeds. In each pipe the integrity of the column of air is broken by a hole in the side, a short distance above the cup. By this strange contrivance not a single pipe will sound to the wind blown into the cup from a flexible tube, until its side hole has been covered by the finger of the player, and then the pipe gives a note corresponding to its full speaking length. Whatever be the speaking length of the pipe the hole is placed at a short distance above the cup. Its position has no relation to nodal distance, and it effects its purpose by breaking up the air column and preventing it from furnishing a proper reciprocating relation to the pitch of the reed.'[1]

[1] The instrument described in the text is the 'sing' of Barrow ('Travels in China,' 1804, where it is well figured as 'a pipe, with unequal reeds or bamboos'), and 'le petit cheng' of Père Amiot ('Mémoires concernant l'histoire . . . des Chinois' . . ., 1780, vol. 6, where a 'cheng' of 24 pipes is figured, and two kinds with 19 pipes described, and the reed is shewn in position as free, but not of the correct shape). Amiot describes the 12 *lu* or degrees of later Chinese music, (according to the Chinese rules of B.C. 2637 on his p. 215,) to consist of a succession of Fourths (or Fifths) reduced to the same Octave, which he makes *f* to *f'*, thus: *f c' g d' †a †e' †b †f♯ †c'♯ †g♯ †d'♯ ††a♯*. Arranged in order of pitch with the Octave of the first, thus: *f †f♯ g †g♯ †a ††a♯ †b c' †c'♯ d' †d'♯ †e' f'*, these form a series of 12 Semitones, 5 of them generated by 7 just Fifths up (as *f* to *†f♯*), and 7 by 5 just Fifths down as (*†f♯* to *g*, or *†b* to *c'*); compare p. 646, commatic series, Nos. 6 and 7. These are the 12 Semitones mentioned on p. 398, l. 5, which did not become *equal* Semitones till Tsai-yu's time, A.D. 1573. Then Amiot says that the 13 pipes of his 'little cheng' contain these 12 *lu*, and the Octave of the first, as above, arranged in the order: *f †a †c'♯ f' †d'♯ †b g †g♯ c' †e' d' ††a♯ †f♯*. But the measurements of the speaking lengths of the pipes on Mr. Hermann Smith's own 'shang' seem to shew that they give only 8 tones, formed from perfect Fifths (the first 8 of Amiot, when reduced to his Octave) with the Octaves of four of them and a unison repetition of one, for convenience of playing, so that the compass is: *†A †B c d †e f †f♯ g †a †b c' d'*, where *†e* is the repeated note. (The actual pitch assigned by Mr. Hermann Smith is about two Octaves higher than this.) These notes may be compared with the later Greek scale on p. 414, having, like that, a conjunct tetrachord, and modulating from Greek Eolic, 7. †A. 5, into Greek Phrygian or 7. †A. 4, both of Table III., B. The 13 pipes are arranged thus on this instrument, where the 0 marks the position of a mute pipe: *0 0 g d d' †b c' 0 †a †e †B †A c †e f †f♯ 0*. Barrow (*ib.* p. 314) says of this instrument, 'the tones are far from being disagreeable, but its construction is so wild and irregular, that it does not appear to be reducible to any kind of scale.' As the scale just determined is truly Greek, it is probable that ancient Greek music might produce the same effect on modern European ears. The compliment is returned, for Barrow says, 'they (the Chinese) affected to dislike the Embassador's band, which they pretended to say produced no music, but a confusion of noises.' Compare the last quotation from Aristotle, suprà p. 362, note 1, and Barrow's statement, agreeing with Amiot's: 'a Chinese band generally plays or endeavours to play in unison, and sometimes an instrument takes the Octave; but they never attempt to play in separate parts, confining their art to melody only . . . they have not the least notion of counterpoint, or playing in parts,' p. 315. Mr. Hermann Smith says there is a 'Siamese Mouth Organ,' on the same principle, with free reeds speaking when the side-holes are closed, and with a speaking length determined by a back 'slot,' and hence less than the real length. The Japanese have also a similar instrument.

Art. 8. *The String Organ.*

Mr. John Farmer, music master and organist of Harrow School, experimented for several years on the excitement of strings by a blast of air for the purpose of obtaining tones fit for practical instruments. Starting from the Eolian harp, he passed over stages already tried by others, such as inclosing the string in flanges of its whole length, limiting the flanges and the air-slit to a portion of the length, and flattening the portion of the string over the air-slit, to improve the 'bite' of the air upon it. Finally he hit upon the new conception of attaching one end of his string to the free end of an harmonium reed made to vibrate outwards, and allowing the wind to act on the reed alone. By this means he obtained several notes of good quality of tone, determined by stopping his single string as on a violin. Though the range of pitch was necessarily limited he played simple airs with so much effect that a great desire was created for a complete musical instrument with similar qualities of tone. His experiments attracted the attention of Mr. J. Baillie Hamilton, then a boy at Harrow School, who, after entering University College, Oxford, endeavoured to carry them out to a successful issue. After many fruitless attempts made, at least partly, in conjunction with Mr. Farmer, he finally decided on modifying the relations of reed and string, and has since pursued his experiments with so much enthusiasm, perseverance, and success that the single string and reed excited by wind and playing very simple melodies, has expanded to the dimensions of a complete string organ with numerous stops.

Near the extremity of a free reed there is attached a small rod or pin, which is in turn fastened to a point near the middle of a steel pianoforte wire properly stretched above it in the same linear direction, and the reed is then excited by an harmonium bellows. The reed and string are therefore no mere continuations one of the other, but are in some sort independent instruments, which modify each other's motions through the agency of the connecting-rod. The whole is a combination of two distinct sets of vibrations, following different laws, so that the pitch of the combination is different from that of either reed or string separately. When the reed is excited, the string is easily seen to vibrate sometimes as a whole, and at others to divide itself into segments, depending on the various relations of tension, weight, and length of string, and so on. The middle or reed segment is generally shorter than the others. The tone produced is sustained and fine, without any tinkling or jarring of high partial or inharmonic secondary tones. It can, indeed, be made closely to resemble that of a metal diapason organ pipe. This experiment, which has been repeated in extremely diversified forms and combinations, shews that a quality of tone rivalling that of an organ can be produced from a simple string, set in motion by a reed which is excited by wind. It

therefore shews the possibility of constructing a *string organ*, that is an instrument possessing fine sustained tones like a pipe organ, but with all the advantages of the delicate action of strings, and of a common sounding board.[1]

Several practical questions of much importance have, however, required solution before this great result could be accomplished. They bear upon the relations of the pitch of the reed and string, when unconnected, to the pitch of the combination, the alteration of the quality of both tones by the interaction of the two vibrating bodies, of which the reed alone is immediately affected by the air; and the effect of altering the length or tension of the string, either by the tuning-key or the action of temperature and prolonged vibration. The following points have received special attention: the shape and loading of the reed, the point where the connecting-rod should be attached, and its distance from its connection with the string, the formation of the string (for which a beautiful hollow coil has been used at times with the view of giving increased weight to a string of comparatively short length, and of counteracting the effects of an alteration of tension by temperature), the degree of tension of the string, the reinforcement of the reed tone by a resonance chamber, the reinforcement of the string tone by a sounding-board, the relations between weight and tension of string and elastic power of the tongue of the reed, the conditions of rapid speech and of persistent pitch, the conditions of determinate quality of tone, with many others, which the extreme complexity of the mechanical problem renders it necessary to attack empirically. A few results of interest in relation to the present work can alone be mentioned.

Generally it is not possible to produce by the combination of reed and string, any tone which shall be more than an Octave, or at most a Tenth above the pitch of the unconnected reed. On increasing the length of the string so as to give, when it is unconnected and plucked, a pitch deeper than that which the reed can sharpen, the reed breaks the string up into segments, and the pitch of the combination rises and becomes that which would be produced by an unconnected string of the same weight and tension, and having the same length as one of the vibrating segments which is not attached to the reed—the segment attached to the reed itself, as before observed, is shorter. In this case, then, the string may be 'bridged' at one of the nodes between the segments without altering the pitch, so that (except for other practical purposes bearing on the stiffness of short

[1] Mr. Hamilton's feeling of the relative advantages of string and pipe instruments is thus expressed: string instruments possess, '1) a simple mode of reinforcement, 2) economy of space to produce a given note, 3) blending of harmonies upon a common sound-board, 4) sympathy;' and pipe instruments have, '1) special reinforcement by a column of air, 2) volume of tone, 3) a special quality, 4) sustained tone;' and experiment seems really to shew that the latter advantages can be combined with the former. *Choir*, No. 421, 16 January, 1875, vol. 19, p. 36.

strings), a string of the length of that short segment to which the reed is attached, would suffice to produce the pitch. But up to the point where the string breaks into segments, it is possible, by increasing the length of the string, to make the pitch of the combination lower than that of the reed, but higher than that of the string, by about a Fifth below the pitch of the reed. The point of least hesitation in speech is near to that where the string breaks up into segments. Suppose the pitch of the unconnected reed to be *d*♯, and that of the unconnected string to be *G*, the pitch of the combination will be about *A*. But by shortening the string without touching the reed (as by shifting the bridge), this combination-pitch can be heightened easily to *f*, and with difficulty to *d*♯, or by constraint to *f*♯. It is easy to raise the pitch until the bridge comes too near the connecting-rod of the reed. These results differ considerably according as the wind acts by pressure or suction, but under both conditions the reed may be so constrained that its tongue cannot descend to close the aperture between the flanges. The tongue then becomes merely a vibrating spring set in action by the wind, but not functioning as a free reed, and the pitch of the combination can be raised to a height limited only by the possibility of increasing the force of wind.

The effect of great tension with a light string is shewn by connecting a large reed in the 32-feet series with a fine string about a foot long, producing, when unconnected, a tone many Octaves higher than the unconnected reed. The result of the combination is a deep note of the 16-feet series, which speaks without hesitation. An alteration in the tension of the string alters the quality but not the pitch of the combination. The reed seems, therefore, to have under such circumstances so much control over the string that a slight change in the latter is of little consequence.

By alterations, then, in the weight and tension of the string, even without changes in the reed, great alterations of quality can be produced, and Mr. Baillie Hamilton already anticipates securing sufficient variety of quality of tone to furnish more than 20 stops on his proposed string organ. The complete instrument has, indeed, yet to be constructed, but it is advancing rapidly. Each harp of strings forming a 'stop,' is arranged in a vertical rectangular frame, bearing the reeds and wind chest below with the terminal fixed bridge beyond the reed attachment, and by a very simple arrangement these frames can be moved on a hinge out of the general case to facilitate tuning. Each stop, therefore, with the space between it and the next, need not occupy more than six inches in depth, adding greatly to the compactness of the instrument. The strings are stretched over a beautiful resonance box, on the principle of the bass viol, but of a rectangular shape, with orifices corresponding to the *f* holes of the viol, and extremely sonorous. The movable bridge which communicates the vibrations of the string to the resonance box, is placed on the

front of this sounding box. The air is led to the windchest by proper ducts, and the action of the trackers is direct and altogether much simpler than for the pipe organ. The force of wind can be varied without altering the pitch, a result which makes this instrument differ essentially from the ordinary organ.

The above description was drawn up after paying several visits to Lord Lindsay's laboratory, where the instrument is in process of construction and where Mr. Baillie Hamilton and his coadjutors obligingly explained all the details mentioned above, letting me hear the effects produced, and correcting my first appreciations when at fault. Although the incomplete state of the instrument prevented me from hearing passages played in harmony and melody, I have no reason to doubt that they will produce an excellent effect. Should the remaining difficulties be satisfactorily overcome, as there is good reason to hope, the string organ will be well qualified by its splendid tones to take a very high position among musical instruments.

As is manifest from the preceding notes, the mechanical problem to be solved is one of great complexity, and it cannot be reduced to calculation without many simplifications, which make it amount to a mere approximation. These evidently are of two kinds, according as we consider the string to be modified by the reed, or the reed by the string, the modification being in both cases an elastic constraint. But the first process is liable to overlook the effect of a more or less intermittent stream of air, and the second to omit the separate acoustic effect of a vibrating string. Both, in short, are liable to pass over the fact that two musical tones are produced by two musical instruments, which modify each other by interaction, one of these instruments being set in motion by the other. The reality of this view is clearly shewn by an arrangement very recently adopted. The string is stretched over the sound board, and of the rest of the apparatus only the point of the connecting-rod is visible, inclosed in a small air-tight leathern purse, so that no wind can escape on the same side as the string. The outlet for the wind is by a tube and pallet at the back, through which the tone of the reed issues. By placing the ear first near the front of the sounding board, and then near the reed behind it, the two musical tones can be heard distinctly separate.

In the mathematical investigation made by Mr. Bosanquet, and contained in the 'Philosophical Magazine,' No. 323, Feb. 1875, vol. 49, pp. 98–104, he regards the instrument 'as a string loaded at the point of attachment to the reed, subject also at the same point to a force tending to the position of rest, and varying directly as the displacement of the point from that position; this force represents the elasticity of the reed.' The method he employs is substantially that in Donkin's 'Acoustics,' p. 139, duly modified. The final equation at which he arrives is—

$$2\pi(\Lambda^2-\lambda^2)\lambda_0 \,.\, \sin\frac{2\pi b}{\lambda} \,.\, \sin\frac{2\pi(l-b)}{\lambda} = \Lambda^2\lambda \,.\, \sin\frac{2\pi l}{\Lambda},$$

where Λ is the wave length on the string of the note of the reed alone, λ the complete wave length on the string of the vibration absolutely sounded (both Λ and λ are double the lengths of the single segments); λ_0 is the length of the string, having a weight equal to the load; b is the distance of point of attachment from one end; l the length of the string. The determination of some of these data involve extreme difficulty, nor does there seem to be a means of finding λ_0 at all.

The following particular case, which Mr. Bosanquet was able to contrast with an actual experiment, will serve to shew the meaning of the last equation.

Suppose the point of attachment of the reed to the string, to be at the middle of the string, so that $2b = 2(l-b) = l$, in which case putting $L = l \div \lambda$, the above equation becomes

$$\pi(\Lambda^2-\lambda^2)\lambda_0 \,.\, \tan \pi L = \lambda\Lambda^2$$

which gives an infinite number of values of λ when Λ and λ_0 are assigned. The ratio L gives the pitch of the note sounded with reference to the Octave of the string.

Mr. Bosanquet assumes $\Lambda = l$ (or the reed to give the Octave of the string) and $4\pi\lambda_0 = 1$, as a pair of values which might easily occur, and are convenient for calculation, and thus puts the last equation in the form—

$$\tan \pi L = \frac{4L}{L^2-1}.$$

The first five roots of this equation are given in the following table, column L, 'proceeding further, we should find a root lying between every consecutive pair of integers.' Column P gives the corresponding pitch of the note referred to $\frac{1}{2}l$ in equal Semitones, and column Q, the same referred to the lowest note of the combination also in equal Semitones. Column R gives the calculated ratio of the length of the middle to any other segment, for the number of segments in column S.

L	P	Q	R	S
0·5868	−9·230		1·1736	
1·441	+6·326	15·556	·882	3
2·357	16·132	25·362	·714	5
3·295	20·646	29·876	·590	7
4·25	25·052	34·282	·50	9

On reducing P and Q to the same Octave by deducting multiples of 12, and comparing with Table II. and Table VI., B, the precise position of

the tone in the scale is readily estimated. The central segment is seen to diminish as the number of segments increases. The following observations, made by Mr. Bosanquet in Mr. Baillie Hamilton's laboratory, confirm these results generally. The pitch of the tone due to the formation of any number of segments was observed roughly by an harmonium in the room.

First Experiment			Second Experiment		
No. of Segments	Equal Note observed	No. of equal Semitones in the intervals observed	No. of Segments	Equal Note observed	No. of equal Semitones in the intervals observed
1	*A*	0	1	*G*	0
3	*b*	14	3	*b*·	16
5	*a*′b	23	5	*a*′b	25
7	between *d*″ & *e*″b	29·5			

These are seen to follow the general course of the secondary tones in column Q of the last table.

'The note employed,' says Mr. Bosanquet, 'may be either the fundamental [primary] or any one of the overtones [secondary]. As these are all inharmonious to each other, only one can be used at a time. But it is probable that, in particular cases, some two or more may become harmonious; and they would then be capable of combining in a true periodic motion.'

Lord Rayleigh, observing on this treatment of the subject ('Nature,' 18 Feb., 1875, vol. 11, p. 309), believes that 'the instrument ought to be regarded rather as a modified reed instrument than a modified string instrument. . . . The string and reed together form a system capable of vibrating in a number theoretically infinite, of independent fundamental modes (in the *mechanical*, not the *musical* sense) whose periods are calculated by Mr. Bosanquet. The corresponding series of tones could only by accident belong to an harmonic scale, and certainly cannot coexist in the normal working of Mr. Hamilton's instrument, one of whose characteristics is great sweetness and smoothness of sound. . . . One effect of the string, therefore, and probably an important one, is to intensify the gravest tone [prime] of the compound note given by the intermittent stream of air [in the reed].' And he mentions that he has noticed 'on one or two occasions an immediate falling off in the sound when the wind was cut off, although the string and reed remained in vibration for a second or two longer. A resonator tuned to one of the principal overtones [upper partials] was without effect when held to the string, but produced a very marked alteration in the character of the sound when held to the reed.' Lord Rayleigh finally gives it as his opinion that the 'office of the string [is] mainly to convey the

vibration of the reed itself (as distinguished from the wind) [that is to convey the vibration of the *tongue* of the reed] to the resonance-board, and secondly, to limit the excursions of the tongue to the reed. Lord Rayleigh therefore recognises the vibration of the tongue of the reed, independently of the sound produced by the puffs of air allowed to pass through the mortice hole, and supposes the string to convey that vibration to the sound board, and, of course, to limit its amplitude. The theories of Mr. Hermann Smith (pp. 708–12) tend to connect the joint action of the metal reed and string in the present case, with that of the metal reed and air column in reed organ pipes, and also with that of the air reed and air column in flute organ pipes. There are, of course, material differences in all three cases, but the complete investigation of the first may tend to throw more light on the other two[1] In the meantime the fact that the other two, which have long been worked, are still not completely investigated, will serve to shew that we have not to wait for a theoretical solution of the problem involved in the string organ before we are able to reduce it empirically to a working condition.

Art. 9. *New Musical Tones.*

Besides the string organ just explained, other attempts have been made to sustain the tones of strings in keyed instruments. One consists of bringing a resined cylinder against the string, which I heard in action nearly forty years ago at the inventor's, Herr Kaufmann, then of Dresden, and which has been improved, and recently patented in France, but does not seem likely to establish itself. Another is the subject of an Italian patent by Signor Caldera, and consists of a constantly reiterated striking of the string by a mechanical hammer, liberated by altering pressure on the digital, so that the effects of this reiteration and of a common piano action can be alternated at pleasure. The effect, which I heard on a fine instrument at Messrs. Kirkman's, Soho Square, is very like an incessant dropping of water, and although much admired by many people was soon distressing to my ear.

M. Fr. Kastner has endeavoured to utilise the explosive action of gas burned in a tube, which is well known to produce 'singing flames'; but his principle is new. His memoirs on these *Flammes Chantantes* are in the 'Comptes Rendus' for 17 Mar., 1873, and 7 Dec., 1874, and a complete account of his instrument, with a discussion, is contained in the 'Journal of the Society of Arts' (19 Feb., 1875, No. 1161, vol. 23, p. 293). His principle is that 'if two flames of a certain size be introduced into a tube made of glass, and if they be so disposed that they reach the third part of the tube's height (measured from the base), the flames will vibrate

[1] See Mr. Hermann Smith's letter in *Nature*, for 1 April, 1875, No. 283, vol. 11, p. 425.

in unison. This phenomenon continues as long as the flames remain apart; but as soon as they are united the sound ceases' (ibid. p. 295). By an ingenious mechanism the flames are united and separated by means of digitals, so that an actual air can be played. The tone is good, but there seems to be some uncertainty in the pitch, the pressure on the gas being important; and this, with other mechanical difficulties, have to be overcome before the *Pyrophone*, as it is called, can become a practical instrument.

Art. 10. *On the Resolution of Compound Tones by Distance and Reflection.*

(10a.) In the summary of M. Regnault's researches, written by himself, and given as an Appendix to Prof. Tyndall's 'Lectures on Sound,' 2nd. ed., 1869, pp. 323-332, he notes that according to the general complete formula the velocity of sound increases or diminishes with its intensity. He notes of a pistol-shot fired with one gramme of powder in tubes 0·108, 0·300, and 1·10 mètres (that is 0·3543, 0·9843, and 3·609 feet) in diameter, that the distances traversed before the sound was *extinguished* were 4055·9, 15240, and 19851·3 mètres (or 13307, 50001, and 65130 feet), and that the mean limiting velocities were respectively 326·66, 328·96, and 330·52 mètres (or 1071·74, 1079·28, and 1084·43 feet) in a second respectively at C. 0°. The mean velocity of sound in the open air is 1089·2 feet at C. 0°. This velocity is independent of the length of the wave, that is, of pitch. But the partials diminish in intensity while their vibrational numbers increase. Hence we must expect that the velocities of different partials will be different, that the prime will be heard before the second partial, and that the first and second partial will be heard together before the third partial, and so on, so that there will be a gradual, though rapid, alteration of quality of tone from that of the simple prime to that of the whole compound tone. Acute voices, as tenors or sopranos, are not so rich in audible partials as basses, and hence to hear a good compound tone, we must for great distances employ deep voices. In this light I read the following passage from M. Regnault's summary:

Experiments made with waves produced by the human voice and by wind instruments have demonstrated these principal facts. Acute sounds propagate themselves with much less facility than grave sounds. In very long conduits [tubes], to hear well it is necessary to employ a baritone; the fundamental sounds [primes] are heard before the harmonics [upper partials] which then succeed each other in the order of pitch [which is also the order of intensity]. The propagation of the sound changes its timbre [quality of tone, p. 36, note], which is due to the admixture of the harmonic [upper partial] sounds. In very long conduits, therefore, a tune embracing a certain extent of the gamut [scale] would change its character. (Ibid. p. 329.)

(10b.) The following two experiments were tried by me on a winter's night with an observer whose ear I could trust, in long streets at Kensington, representing a very wide and high cutting. I employed the g of a concertina (the lowest violin g) sounded for about half-a-second, at intervals of about half-a-second. The observer walked away from me and listened. At a distance of 255 (stepped) yards, the prime of g had become faint, but its second partial or Octave g' was still so clear that the observer was under the impression that I had struck both g and g' at once. At 310 (stepped) yards, when the wind was quite still, the prime g disappeared, and the Octave g' was heard alone, but a slight puff of wind, which set from me to the observer, restored the sound of the prime, which was thus heard to intermit. Such experiments require repetition, with much variation as to pitch and force of tone and many observers.

(10c.) The following is from Mr. W. Chappell's 'History of Music,' vol. i. p. 87:

If we hear the voices of men and women singing together in a room, the more rapid vibrations of a woman's voice seem to give it superior power; but if a chorus of men's and women's voices be heard singing the same subject at a distance, especially in the open air, the women's voices will seem to give brilliancy to the men's, and to die away in them.... [Adding in a note:] My learned friend, G. A. Macfarren ... tells me that he has often noticed this effect.

Has this observation anything to do with the last? Did the brilliancy given by the women's voices consist in reinforcing the Octave while the prime was enfeebled, giving a quality of tone resembling that in columns g', $C-c'$ of the table on p. 126, that is resembling that for the middle part of a pianoforte? There must at least be an enfeeblement of the primes of the men's voices. Compare Seiler's observations, p. 155, l. 23 above.

(10d.) In 'Nature,' 21 Aug., 1873, No. 199, vol. 8, p. 319, Lord Rayleigh gives an account of an echo at Bedgebury Park, the country residence of Mr. Beresford Hope, where

The sound of a woman's voice was returned from a plantation of firs, situated across a valley, with the pitch *raised an Octave*. The phenomenon was unmistakable, although the original sound required to be loud and rather high. With a man's voice we did not succeed in obtaining the effect.

Lord Rayleigh refers the phenomenon to the disturbance of waves of sound by obstacles which are small in comparison with the length of the wave, when, according to his mathematical investigation,

The reflecting, or rather diverting, power of the obstacle varies inversely as the fourth power of the wave-length. When a composite note, such as that proceeding from the human throat, impinges on the obstacle, its components are diverted in very different proportions. A group of small obstacles will return

the first harmonic [first upper partial tone] or Octave, sixteen times more powerfully than the fundamental [prime]. After this, it is not hard to understand how a wood, which may be considered to be made up of a great number of obstacles, many of which, in two or three of their dimensions, are small in comparison with the wave-length, returns a sound which appears to be composite. The increased reflection is of course at the expense of the direct sound. If we conceive a group of small obstacles to act on a train of plane waves of sound, the effect will be a *diffused* echo, which may be heard on all sides, appearing to proceed from the group, and the direct waves which maintain their direction. If the original sound be composite, the diffused echo contains the higher elements in excessive proportion, and for the same reason the direct wave, being shorn of these higher elements, will appear duller than the original sound. . . . What is here called the direct sound may itself be converted into an echo by *regular* reflection. For example, if a plane wall were covered by small projections, there would be a diffused echo, due to the projections in which the higher elements preponderated, and an ordinary echo, obeying the law of reflection, in which the wave elements would preponderate.

The last case, perhaps, applies to my two experiments in streets.

(10e.) Mr. Hermann Smith ('Nature,' 11 Sep., 1873, No. 202, vol. 8, p. 384) says:

At the bottom of my house there is a meadow, then a double row of houses with a high railway embankment at the end, and a wall rising beyond that. . . . Whilst looking over the meadow . . . the sound of a band in the distance came upon me, and, immediately following, the sound of another and more demonstrative band from an opposite direction . . . playing most noticeably different melodic phrases. . . . The music was *different*, so that, whilst listening, the illusion of a distinct band was difficult to dispel. . . . The distance of the place of echo was approximately 600 or 700 feet from the source of the sounds, my standing place being at one-fourth of the distance; between me and the band three houses intervened, over which the music came to me, whilst the terrace, on which the band was playing, opened freely on to the meadow; thus echo received the music earliest by reason of the unobstructed passage, and her rendering was that of natural selection, the most vigorous tones and the penetrating harmonics, whatever had most living power infused by the players and sustained by the characteristics of the instruments, whilst all the other notes, those of a low vibrating power and of inferior stamina, were lost by the way. . . . Observation . . . shewed that the course of the sound waves on their way to echo was in *front* of a detached line of cottage buildings, then passing into the enclosed space between the double row of houses up to the embankment, the recourse being by the *back* of the cottage buildings, across gardens and the meadow to the listener. The western sky [was] heavy with gorgeous clouds, and the air silent and sultry.

In all these cases, although in different ways, there was a clear resolution of compound tones without the use of resonators, and the phenomena do not easily admit of any other explanation. They therefore all corroborate the theory of this work.

Art. 11. *Observations on Herr Koenig's and Herr von Quanten's Remarks on Professor Helmholtz's Vowel Theory.*

(See pp. 153–172, 179–181.)

(11a.) *Koenig's Vowel Resonances.*—Herr Koenig (p. 580, l. 5), in a paper printed in the 'Comptes Rendus' for 25 April, 1870, and subsequently cited in his paper on manometric flames (p. 581, note), after numerous experiments on the vowels according to his own pronunciation, to determine the pitch of their resonance chambers by means of tuning-forks, specially constructed and furnished with runners, gives a series differing considerably (at least as respects U and I) from all those recorded on p. 163. Reducing his figures to the vibrational numbers here used, and taking c′ = 256, he finds that the resonances form a series of Octaves, having the following correspondences:

Vowel . .	U	O	A	E	I
Note . .	$\mathfrak{z}b\flat$	$\mathfrak{z}b'\flat$	$\mathfrak{z}b''\flat$	$\mathfrak{z}b'''\flat$	$\mathfrak{z}b''''\flat$
Vibrational No.	224	448	896	1792	3584

He notes particularly that the cavity of the mouth when arranged for U (*oo* in *pool*) resounded best for forks of between 220 and 230 vibrations in a second.[1] As respects I he notes that d'''' was reinforced by a cavity assumed by the mouth on its way from E to I, and that by constructing continually higher forks he found the best effect produced by $\mathfrak{z}b''''\flat$, but that acuter forks did not give so good an effect.

The reconciliation of the numerous differences in the resonance of the cavity of the mouth for different vowels is most probably to be found in a fact, which has been long familiar to me as a matter of observation, and a hindrance in my own investigations, that persons differ widely in their vowel pronunciation, and that the same person is not consistent in his own habits of speech. Herr Koenig has carefully worked out the appearance of the manometric flames for the vowels U O A E I as sung by himself upon each note of the just scale *C D E F G A B c d e f g a b c′*, with a laboriously preserved identity of resonance cavity, and having drawn up a table, compares the results of calculation with the diagrams of his flames. Thus for U, the characteristic resonance $\mathfrak{z}b\flat$ being near the third partial of D and E, and the second partial of A and B, and the primes of *a* and *b*, the groups of flames for D and E are divided into three, and those of A and B into two principal sections, while for *a* and *b* the great preponderance of intensity of the prime over the neighbouring partials is clearly shewn. In all these cases the tone reinforced is the just *a* or *b* (vibra-

[1] Observe that on p. 170 Prof. Helmholtz, in order to get an approximative U from a $b\flat$ reed pipe, applies the resonator $b\flat$ and not *f*.

tional numbers 213⅓ and 240), and the former is 0·845 equal Semitones flatter, and the latter is 1·195 equal semitones sharper than the resonance of the oral cavity, a point of considerable importance for what follows.

(11b.) *Table of Vowel Resonances.*—Herr Koenig's table has suggested to me the construction of the following (on p. 726) on a totally different principle, which may prove useful to investigators. It consists of two parts. Part I. contains the equally tempered tones of the Octave *C* to *c* expressed in letters and in the numbers of equal Semitones, by which each is distant from *C*, with their partial tones under them up to the 25th in order, with the 32nd annexed, numbered in the first column, and expressed by the number of equal Semitones that they are distant from *C* as determined by Table I., and division C, below. The notes *c*, *c′*, *c″*, *c‴*, *c⁗* in the margin shew the Octaves, and hence give the numbers of equal Semitones to be added to those in the table for finding the partials of higher tones. Thus as *c‴* is 48, and the 11th partial of F is 46·56, the 11th partial of *f‴* will be 48 + 46·56 = 94·56. The first part of the table therefore serves for any Octave.

In the second part of the table the first line represents the vowels as on p. 163 note, and under each is written in equal semitones the pitch to which the air in the cavity of the mouth most readily resounds, according to the different authorities cited, who, with the exception of Koenig, are supposed to give the pitch in equally tempered notes, and the difference between the pitches assigned should be carefully noted. Here observe that '45 to 50' means 'variable according to the speaker, sometimes as low as 45, sometimes as high as 50'; again '19, 31, or 38,' means that all three are given as heard at times; but '49 with 17' means that there are two resonance chambers suitable for 49 and 17 existing at once, and '30 or 31 with 38 or 39' means also that there are two resonance chambers, but that the pitch of their resonances are not determined within an equal Semitone.

This table much facilitates the comparison of the results. The resonances have been generally imperfectly determined by observers, and can seldom be relied upon within an equal semitone. Observe that German or Dutch U (which is probably deeper than the English *u* in *pull* or *poor*, itself much deeper than English *oo* in *pool*) has a much lower resonance (if we suppose Donders's result as given by Helmholtz to be erroneous by an Octave) than the possibly Frenchified OU of Koenig (a German domiciled in Paris), the difference varying from a Fourth to a minor Seventh. For O, A, E, Koenig practically agrees with Helmholtz, but Merkel gives resonances which are from one to two Octaves lower, or even more. The Octaves form great difficulties in the estimation of resonance, and such theories as led Professor Helmholtz to *d⁗* for I, by means of a 'minor chord of *g*' (p. 163, l. 1), or which led Koenig to

Part I.—Pitch of Partial Tones of Equally Tempered Notes, expressed in Equal Semitones.

No. of Partial		C	D	E	F	G	A	B	C'
	1	0	2	4	5	7	9	11	12
c	2	12	14	16	17	19	21	23	24
	3	19·02	21·02	23·02	24·02	26·02	28·02	30·02	31·02
c'	4	24	26	28	29	31	33	35	36
	5	27·86	29·86	31·86	32·86	33·86	35·86	37·86	38·86
	6	31·02	33·02	35·02	36·02	38·02	40·02	42·02	43·02
	7	33·69	35·69	37·69	38·69	40·69	42·69	44·69	45·69
c''	8	36	38	40	41	43	45	47	48
	9	38·04	40·04	42·04	43·04	45·04	47·04	49·04	50·04
	10	39·86	41·86	43·86	44·86	46·86	48·86	50·86	51·86
	11	41·51	43·56	45·56	46·56	48·56	50·56	52·56	53·56
	12	43·02	45·02	47·02	48·02	50·02	52 02	54·02	55·02
	13	44·41	46·41	48·41	49·41	51·41	53·41	55·41	56·41
	14	45·69	47 69	49·69	50·69	52·69	54·69	56·69	57·69
	15	46·88	48 88	50·88	51·88	53·88	55·88	57·88	58·88
c'''	16	48	50	52	53	55	57	59	60
	17	49·05	51·05	53·05	54·05	56·05	58·05	60·05	61·05
	18	50·04	52·04	54·04	55·04	57·04	59·04	61·04	62·04
	19	50·98	52·98	54·98	55·98	57·98	59·98	61·98	62·98
	20	51·86	53·86	55 86	56·86	58·86	60·86	62·86	63·86
	21	52·10	54 10	56·10	57·10	59·10	61·10	63·10	64·10
	22	53·51	55·51	57·51	58·51	60·51	62·51	64·51	65·51
	23	54·28	56·28	58·28	59·28	61·28	63·28	65·28	66·28
	24	55·02	57·02	59·02	60·02	62·02	64·02	66·02	67·02
	25	55·63	57·63	59·63	60·63	62·63	64·63	66·63	67·63
c''''	32	60	62	64	65	67	69	71	72

Part II.—Pitch of the Resonance of the Oral Cavities for Vowels, expressed in Equal Semitones.

Authority	U	O	A	Ö	Ü	Ä	E	I
Reyher	12	15	21 or 24	—	—	15	29	36
Hellwag	12	13	18	20	22	21	23	24
Flöcke	12	19	24	28	31	31	33	36
Donders, after Helmholtz	29	26	34	19?	45	—	49	53
Donders, after Merkel	16 or 17	16	23	19	33	—	24	41
Helmholtz	17	34	46 to 50	49 with 29	55 with 17	55 to 56 with 38	58 with 29	62 with 17
Merkel	14 or 15	17, 18, or 19	21, 23, or 24	19, 31, or 38. 30 or 31, with 38 or 39	45 or 50. 33 with 36, 38, or 40	26, 31, or 38. 20 (? 32) or 33 with 38	29 with 38 and with 41	43, 45, or 47
Koenig	21·69	33·69	45·69	—	—	—	57·69	69 69

suspect a series of Octave resonances for his five vowels, may often prove misleading (see p. 159, note).

(11c.) *Vowel Resonance Cavities.*—On p. 161, note, I have pointed out that there are probably at least two resonances for each vowel. But I believe there are many more. Between the true and false vocal chords lies the ventricle, a small resonance chamber with a narrow exit somewhat resembling a bird-call. This possibly chiefly affects the general quality of tone produced. Between the false chords and the epiglottis lies another which can be very strictly limited by the almost total closure of the epiglottis, and the degree of closure of course materially influences the resonance of both these cavities; it is known to be great for the vowels U and O, and for deep pitches of the voice. Between the epiglottis and the uvula (supposed to be arranged so as to close the entrance to the nose) and the arches of the palate, lies the pharynx, which is greatly alterable by muscular action, by raising and lowering the larynx, and by greater or less contraction of the arches of the palate. As a matter of fact, first conspicuously brought out by Mr. Melville Bell, we now know that the variations in this cavity and its oral termination produce great changes in the vowels. The front and back arches of the palate form also a limited resonance, variable with the different degrees of contraction, but always imperfectly separated from the other chambers. The action of this passage has not yet been satisfactorily distinguished, but its peculiar shape (a kind of tunnel with bays between two arches, the front arch keystoned by the uvula, and the bays partly obstructed by the tonsils, while both arches can be separated, constricted, or expanded) points to considerable influence on resonance, more especially as it forms the gallery connecting the two principal resonance cavities. From the front arches to the teeth we have the oral cavity, with its variable plug, the tongue, dividing it most irregularly, and a variable but most active amount of saliva or mucus. On page 157, note, I have shown how Mr. Melville Bell distinguishes at least 9 forms of this cavity, and I might add that for each form he distinguishes at least 3 varieties. I have lately been led, by studying dialectal pronunciation of English, to believe that an entirely new series of forms dependent upon raised and even reverted tip, and depressed middle and back of the tongue, must be studied. It is in the forms of this cavity that the characteristic of vowels is generally sought, and it is the only cavity to which the principle of discovering resonance by tuning-forks can be directly applied, while its real separation by the tongue into a variety of resonance cavities each more or less distinctly limited, but at the same time more or less reacting on each other, must render the discovery of the principal and secondary resonances extremely difficult, and is quite sufficient to account for the diversities exhibited in the second part of the preceding table. But this oral cavity

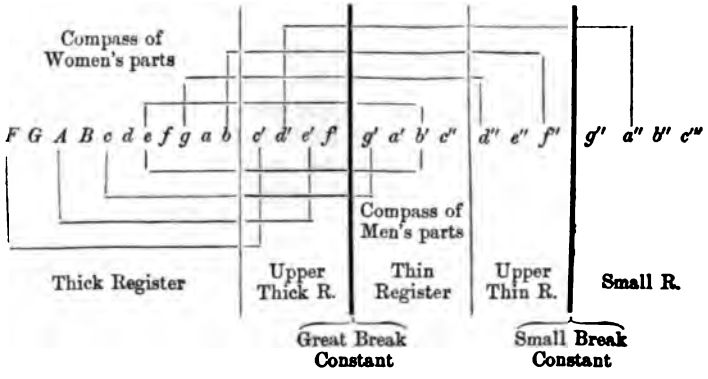
is limited in front and at the sides, first, by the hard teeth, which in numerous instances are more or less studded with gaps, (and when these are avoided, it is more or less artificially limited by hard false palates often not quite close to the real palate, but leaving resonant chinks); and next, by the soft cheeks and flexible lips. The degree of closure of the orifice of any resonance cavity most materially affects the pitch to which it speaks. It is possible by the action of the jaw to maintain the hard part of this limitation tolerably well fixed, but the cheeks, and principally the lips, are so very flexible, and move so rapidly and even unconsciously, that there is great difficulty in fixing them at all, and greater still in fixing them consciously at the same degree as would be usually assumed unconsciously in speech. Yet, as a matter of fact and observation, the purity of the vowel sounds depends greatly upon the degree of closure; and the Octave of the resonance can be entirely changed by the mere action of the lips. Hitherto only oral vowels have been considered. Now suppose the uvula not to bar the passage to the nose. Then the cavity above the epiglottis extends up to the pear-shaped upper pharynx, and is limited by the very mobile tremulous uvula towards the mouth, and by the back nostrils above. Then there are the complicated nasal passages with their bony and cartilaginous septa and distinct galleries, more or less impeded by mucus, and terminating in the front nostrils; the resonance of these passages is practically unknown, but we are aware of at least three or four distinct degrees of nasality. The effect of superadding nasal to oral resonance is well known to be an important part of the vowel system of many speakers, whether acknowledged (as in French and Portuguese, Polish and Gaelic) or unacknowledged (as in some American English, and Bavarian German).

Now there is no doubt physically that every variety of each of the cavities just alluded to, and their combination, must affect the general resonance of the whole, and that as the pulses of air in speech and singing must pass through them on their way to the outer air, the resulting sounds must be greatly modified in quality.[1] The two effects of speaking and singing must be distinguished. One possible source of difference is indicated on p. 155, but other and very important characteristics may be found, first, in the gliding nature of the *pitch* of speech sounds as distinguished from the holding pitch of singing; secondly, in the gliding nature of the *quality* of speech sounds and their origin or termination in unmusical windrushes or stops; and thirdly, supposing both the latter to be overcome, in the flaccidity of the walls of the resonance chambers for speech, and their continued tension for singing. This last distinction

[1] As acoustical conditions affecting resonance, the difference should be noted between transmitting pulses or puffs of air through a resonance chamber (as in reed instruments, including the voice) and forming vibrations in front of it (as in violins, and as for tuning-forks held in front of the orifice).

seems to me, from observation, to form a complete separation between the spoken and the sung vowel, and to render any comparison of the two extremely difficult. Then again, each individual speaker has his very distinctly marked national or local, modified by his idiosyncratic usual pitch and inflection of voice, for which he is accustomed to produce his vowels under different circumstances, intellectual or moral; whereas in singing he uses altogether different or extreme pitches and artificial inflections, for which he is unused to utter his vowels, and which attract his attention so much and require such exertion, that he seldom keeps his vowels pure, and even with the greatest care must be unable to do so, among other reasons, from want of any standard of purity in such cases. Any comparison between vowels spoken and vowels sung, and between vowels sung at usual or unusual, at medium or extreme pitches, is thus rendered extremely hazardous. Again, as we pass from the lowest to the highest singable notes in one voice or in different voices, we pass through different 'registers' (p. 166, note 3), the whole theory of which, including their physiological generation and vocal effect, is still under examination, but which are known roughly as 'chest,' 'throat,' and 'head' voice, or, as Mr. Curwen has called them, the 'thick,' 'thin,' and 'small' registers.[1]

[1] In Mr. Curwen's 'Standard Course,' pp. 29, 32, 33, 66, 67, 105–107, will be found much information on this subject, based partly on Mad. Seiler's work (suprà, p. 166, note 2), but also on a large amount of special experiment on a great variety of voices made by those Tonic Sol-fa teachers on whom Mr. Curwen can rely. For convenience of reference I here give some of the results indicated especially in the table on his p. 106, reduced to the notation of this work.



All registers *may* overlap downwards, that is, a few lower tones *may* be taken on upper registers, greatly altering their quality. 'Men commonly speak in their thick register. Tenor voices, however, use the pleasant higher thick register. Very rarely a man may be heard speaking in his thin register, with a thin squeaking quality.—Women commonly speak in their thin register; but some contraltos use their rich upper thick tones, and occasionally a woman may be heard to speak in the rough lower thick register.'—*Standard Course*, p. 107.

The registers are, however, known to result from peculiar modes of vibration of the vocal chords,[1] and their effect on quality of tone as well as pitch is always remarkable. They do not affect the resonance cavities themselves and their modifying power, but they affect the material to be modified to such a degree as entirely to change the audible result.

Now the first effect of all these circumstances is to alter the general quality of a speaker's or singer's voice to a degree which is not usually referred to a change of vowel, and secondly to alter the quality to a degree which portrays emotion of any kind, independently of pitch, and independently of recognised vowel change. It is only thirdly and chiefly that the effect is to produce vowel change. Those who have studied vowels however, (and on this point I must refer to the section 'On Vowel Quality and its Gradations,' pp. 1275–1307, in my 'Early English Pronunciation,' part IV., 1875) are aware that vowels 'gradate' into one another so imperceptibly that it is extremely difficult to separate them, and also that vowels, which detailed examination leads phonetists to distinguish 'specifically' are confounded 'generically' by speakers and listeners, so that they are to both 'appreciably the same' so far as the recognition of words, and hence the interchange of thought is concerned. But as this is the only circumstance in which listeners in general are interested they overlook the varieties. (See this principle well explained above in the excellent observations on pp. 95–100.) This is a matter which has occasioned me difficulties in my phonological researches for many years past, and has become unfortunately too familiar to me. It is contradictory to my own repeated observations to suppose that even the vowels U, O, A, E, I are identically the same for the same speaker, at all times, under all combinations, under all emotions, and at all pitches. Still less is it correct to suppose that different speakers pronounce what are acknowledged to be the same vowels in precisely the same way when under similar influences.

Now does all this diversity clash with Prof. Helmholtz's theory? Not in the least; to my mind it rather confirms it, while it explains the diversities shewn in Part II. of the table on p. 726. Every form of the resonance cavities produces a variation of quality of tone in the speaker. By the habits of different languages these various qualities, specifically different, are formed into groups, in which the elements are linguistically equivalent—that is, are divided generically. Ordinary observation of speech sounds is limited to the genus. Observers on resonance also heed only generic difference. They observe for their own vowel usage, or that of some one or two other persons. They observe only one principal resonance, whereas there are many reacting resonances at work, and the

[1] See Mad. Seiler's work (above, p. 166, note 2) and Merkel's *Funktionen des menschlichen Schlund- und Kehlkopfes* (Functions of the human pharynx and larynx), 1862, with numerous laryngoscopical illustrations; also Behnke in the *Lancet*, 8 Feb. 1873.

principal resonance cavity itself will, from its great irregularity, reinforce several tones lying at considerable intervals apart, and will therefore with more or less strength reinforce other tones lying at small intervals (such as 1 or 2 equal Semitones more or less) from their proper tones. The nature of the observation also throws great doubt on the Octave. Probably not much weight is to be attached to any observations recorded in Part II. of the table on p. 726, except those of Helmholtz, Merkel, and Koenig. Merkel constantly finds himself at a loss to fix the pitch to 1 or 2 equal Semitones, and sometimes to an Octave. By simply altering his Octaves we could bring him into close agreement with Helmholtz and Koenig. That the chief resonances of the principal vowels usually distinguished differ by very large intervals; that there are two Octaves, a little more or a little less, between the principal resonances for U and A, and for A and I, seems to me highly probable. Suppose we take 20, 45, and 70 equal Semitones as roughly representing the pitch of these cavities (assuming $C = 0$). Then we might expect the intermediate O and E to have resonances varying from about 28 to 35 for O, and 53 to 60 for E. In point of fact my own observations shew that even U, A, and I are variable, A being very variable indeed, and even U and I different in ordinary English from what they are in Italian. But O and E are so extremely variable that we are satisfied so far as the conveyance of thought is concerned with almost any intermediary between U and O or A and I; while fashion, education, and habits of comparatively small coteries, insist upon wondrous niceties, into which it would be here out of place to enter, but for which I may refer to almost every page of my 'Early English Pronunciation.'

(11d.) *Von Quanten's Objections to Helmholtz's Vowel Theory.*—It was not my intention to have added anything from my own special phonetic studies to the observations made in the notes to pp. 157–9, 161–7, 181, which I feared would be thought already too extensive. But the preceding remarks, containing opinions which I have long entertained, and have more or less expressed elsewhere, have been inserted as a necessary introduction to the consideration of an article in Poggendorff's 'Annalen,' 1875, No. 2, vol. 154, part 2, pp. 272–294, which appeared as the preceding sheet of this translation was passing through the press, and of which the conclusion is promised in part 4, too late for me to refer to before publication. This article is apparently a translation, communicated by the author, from a paper in the Transactions of the Swedish Academy, but these Transactions have not yet reached London. It is called 'Einige Bemerkungen zur Helmholtz'schen Vocallehre, von Emil von Quanten' (Some remarks on Helmholtz's vowel theory, by Emil von Quanten), and seems to me to be based upon a total misconception of the meaning, bearing, and application of that theory.

After a short introduction, giving the characteristic tones of Helmholtz as on p. 165, or in Part II. of the table on p. 726, Herr von Quanten says that in this case the vowels would constantly vanish as we ran up the scale. That for example the North German A with the characteristic tone *b″*♭ could not be produced on 9 of the equal Semitones of the scale, namely *c*♯ *d e f g g*♯ *a b c′*,[1] while the English and Italian A with the characteristic tone *d‴* (or 50 of my table) would vanish for *c*♯ *d*♯ *f f*♯ *g*♯ *a b c′* (Pogg. p. 280). Again he says that even if we allow Engel's statement that the pitch may vary by a whole tone one way or the other, the vowel being imperfect when the characteristic tone is not nearly hit, 'no one can justly say that A sounds more imperfectly on *c*♯ than on *c*, or less characteristically on *a* than on *f*♯, *b*♭, &c. . . . Generally, an unevenness in the quality of vowel tone arising in such a capricious manner, is too unnatural to be acknowledged as real' (Pogg. p. 281). And observing that no one tone can be an upper partial of all the tones in an Octave, he draws 'the general conclusion that the characteristic tones of vowels cannot have a constant pitch' (Pogg. p. 283), and hence that 'vowels are not formed on the principle conceived by Helmholtz' (Pogg. p. 287), which he had previously stated to be 'that first a vowel quality of tone is formed of a prime and a reinforced partial tone, and secondly that this upper partial is the same for every vowel' (Pogg. p. 283). Then taking a physiological view, and observing the motion of the larynx for varieties of pitch, he asserts that as 'a diminished volume gives a higher, and an augmented volume a lower pitch' (Pogg. p. 290, which, when no allowance is made for the form, and the size of external orifice, is certainly erroneous), the assumption of constant resonance for the same vowel quality is 'not correct' (Pogg. p. 287), and that hence 'absolute acoustic limits, as Helmholtz assumes, do not exist for the formation, but only relative physiological limits' (Pogg. 291), which, however, are not assigned in the part already printed in Poggendorff. And finally he says, 'Daily experience shews that none of the various kinds of voices are deficient in any vowel in either speaking or singing, notwithstanding Helmholtz's theory that U has the constant characteristic tone *f*, and O the constant characteristic tone *b′*, which would render it impossible to form these vowels on the middle tones *c″*—*g″*. And experience shews that *all* voices, bass, alto, soprano, &c., feel the same difficulty in producing the vowels upon their highest and lowest tones. . . . The absolute pitch of the fundamental tones has no influence whatever. All vowels are formed perfectly on fundamental tones

[1] Hence he apparently allows *b″*♭ to be an upper partial tone on *c*, *e*♭, *g*♭, *b*♭. But if we take these tones as equally tempered, we see by p. 726 that the 14th partial of *C* is 43·69, the 12th of *E*♭ is 46·02, the 10th of *G*♭ is 45·86, and the 8th of *B*♭ is 46, so no value of *b″*♭ will agree with all of these, and we may on his interpretation suppose that Helmholtz's theory would assign *B*♭ as the only pitch which would produce O in the Octave *C* to *c′*.

belonging to the middle compass of the voice, and less perfectly on those which are more or less distant from this middle region either way' (Pogg. 292). 'The number of characteristic tones is consequently for one set of vowels as numerous, and for the other set [with a twofold resonance] doubly as numerous as the fundamental tones existing in the compass of the human voice, with the slight exception of the deepest bass tones on which, for physiological reasons, the vowel I, and the highest tenor tones on which for similar reasons the vowel U cannot be produced' (Pogg. p. 294).

My previous remarks already serve to appreciate the incorrectness of the assumption here made that vowels are identically the same at all pitches. But what I hold to be the great misconception which pervades the whole of this paper, and which is the more deserving of notice that it has occurred to other and careful readers of this work, is that, if Prof. Helmholtz's theory be correct, a vowel could not be heard, or at least could only be imperfectly heard, unless the pitch of its characteristic resonance chamber were also the pitch of one of the upper partial tones of the note to which it was sung. Of course this leads to the absurdity (indicated in the last note) that scarcely any vowel could be sung, and it is certainly more logical to conclude that the critic had mistaken the theory than that such a writer as Prof. Helmholtz, checked by such observers as Donders, Merkel, and Koenig, should have enunciated a law which is 'self-contradictory' (Pogg. p. 283), or in other words, transparent nonsense. Of course Prof. Helmholtz's own enunciation (p. 165) has no resemblance to Herr von Quanten's simulacrum. The only possible ground for such a mistake which has been indicated to me, may perhaps be found in the author's taking as examples cases in which the resonance of the oral cavity had the same pitch as one of the partial tones of the note on which the vowel was sung (pp. 165, 172)—a mere matter of convenience depending on his own set of resonators. But again the question has been raised, how can *b'b* be 'characteristic' of the vowel O, when sung on *c'* or *d'*, which do not contain a trace of *b'b* among their upper partial tones, that is, 'how can *b'b* be "characteristic" of a vowel which can be perfectly formed without its co-operation?' This question seems to confound the existence of a partial tone *b'b* with the existence of a resonance chamber which is characterised by an ability to reinforce that tone primarily, and therefore specially, although it may, and in most cases must, be able to reinforce other and higher tones secondarily, as clearly stated in the enunciation on p 165. The fundamental difference between these two conceptions seems, however, to need further elucidation. In the meantime it will be at once apparent how such a view would allow a cavity having the primary resonance *b'b* to determine by its secondary resonances the vowel quality of such tones as *e'b* or *e''b*, &c., into which *b'b* does not enter, but only some of its Octaves.

(10e.) *Nature of Vowel Resonance, its Experimental Investigation, a Phonetic Significance.*—The action of a resonance chamber is twofold, fi general, and secondly particular. Both occur in the violin (on which s p. 137); but as the resonance chamber is there constant, the particul effect is rarely noticed, though it is certainly present. Still, with simp the general effect, rapid transitions from high to middle and low notes: *g*, *a* (on the *d* string), *e'''*, will produce results wonderfully simulating L AU, IU, UI. The two first can be obtained from *c*, *c''* on an harmoniun None of these sounds correspond to what would separately be recognise as pure vowels, and any sustained pure vowel rapidly ceases to retain it characteristic colouring. It is in rapid change of quality that vowel produce their proper effect.[1] But the mouth differs from the violin i having a movable resonance cavity, so that the particular effects becom

[1] Mr. Selley Taylor informs me that two friends of his could tell, the moment h struck one of Koenig's forks, giving his five gBb (p. 724), what vowel it suggested even when the forks were struck at random. On the other hand, many other people o whom he has tried the experiment 'failed to perceive any vowel character at all in th tones of the forks, or differed altogether in the vowels assigned to a particular fork. Some times one and the same listener would first call a particular fork tone A, for example and then, on the experimenter's repeating the same note, without saying it was th same, call it O or U, or some other vowel.' Rapidity of transition would probabl have here helped them. Isolated tones are but very rude imitations of the vowels, yet any boy imitating a fiddle will give some kind of O for the lowest or *g* string, various kinds of A for the next or *d'* string, various kinds of E for the next or *a'* string, and will not fail to squeeze out all kinds of I for the *e'* string, till he reaches an agony of simulated acuteness in the I he chirps out for the *e'''* harmonic. I have been accustomed to illustrate the action of the lungs, windpipe, and mouth in speech, by a small hollow indian-rubber ball with a hole in it (sold as a plaything), through which I pass the long tube of a small cup (used in one of Perry's inkstands, but a narrow glass tube with a bell expansion, as used by chemists, answers perfectly). Pressing the ball we have the wind rush through the small orifice of the tube, reinforced by the bell-like expansion of the cup, forming a very high, imperfect note. Covering the top of the bell more or less by the hand, the note is greatly flattened, and when a very small opening is thus left it is very dull. Rapidly covering and uncovering, which is easily managed, on passing from the small to the large orifice—although everything is very imperfect—the sound of *oo-ee* or *we* is immediately suggested to the listener, and reversing the motion *ee-oo* or *you* is recognised. Years ago I heard the same experiment much more completely performed with a bellows, reed-pipe, bell expansion, and tight-fitting cover, by Sir Charles Wheatstone. An ordinary jar tuned (by pouring in water) to *c''*, when excited by a *c'* fork struck strongly, gives *c''* the Octave, but if nearly covered by a sliding lid, produces *c'*. Pass rapidly from one position to the other, and the effects of *we*, *you*, are again produced. Even when water is admitted forcibly from a tap into a jug the transition from an U through an A to an I sound is unmistakable, as the size of the resonance cavity diminishes. But of course when notes are prolonged no similarity to vowels is heard. And differen people actuated by attention differently directed will hear vowels in notes as differ ently, perhaps, as they see faces in the burning coals or on a wall-paper pattern This seems to me to be at the bottom of the differences of Mr. Taylor's listeners.

obtrusive, and the general effects are not so much considered. Yet the tone produced in the larynx is also altered by the general action of the resonance cavity, which throughout all the varieties of form assumed, bears such a constant general resemblance to itself for the same individual speaker, as to characterise his quality of voice, oftentimes sufficiently for it to be recognised amid a hundred others. When a singer has 'a good voice' there must be a general and proper reinforcement of those partials in each note which are usually reinforced in good musical tones. When during the utterance of a note the form for a vowel is assumed, the resonance (which for any cavity, and especially any irregular cavity, is manifold) reinforces some of its partial tones more than others, and damps some more than others, according to the law laid down (p. 165, l. 3).

This does not mean that the resonance necessarily or frequently exalts one particular partial into predominance and damps the remainder into insignificance, but that it makes those partials which are nearest in pitch to the pitch of *any one of the proper tones* of this resonance cavity, louder than they otherwise would have been, and those which have no relation to any such proper tone, and are, perhaps, more than 4 equal Semitones distant, less loud than they would have been had no such resonance cavity intervened. In short, the particular form alters the comparative loudness of the partials, by favouring those nearest, for a given vowel, to a given pitch; a pitch which is independent of the pitch of the note itself, or, rather, would be independent of the pitch of the note uttered if it were easy, or even possible, to pronounce the same vowel accurately at all degrees of pitch; that is, to give precisely the same form to the resonance chambers, the same degree of tension to their walls, and of opening to the connecting passages. Take, for example, from Part II. of the table on p. 726, the cavity having a principal resonance 34, with other resonances, which, though unknown, must not be forgotten. This corresponds to the vowel O, according to Helmholtz and Koenig, and is two Octaves or more higher than the resonance assigned to this vowel by other observers. But keep to 34. For *C* the 7th partial (33·69) would be unnaturally strong, but the 6th (31·02) and 8th (36) would be also rather reinforced than damped. Would the 7th be stronger than the 6th? Perhaps about as strong. The 2nd (12), 3rd (19·02), and 4th (24) would be enfeebled, but would they be less prominent than the 7th? It is in the highest degree improbable. On *D* the 6th (33·02), would be more favoured than the 7th (35·69), on *E*, *F*, and *G* the 5th (31·86, 32·86, 33·86), but on *A* and *B* the 4th (33, 35). We conclude, then, that the effect of the vowel would be better felt on the higher parts of this Octave than the lower. And, in point of fact, no vowels are worth much below *F*, even if there. But at *A* a good O can be made, much deeper, however, than the ordinary O. Now proceed to the next Octave. The simplest way to do so in the Table on p. 726 is to diminish the assumed pitch of the resonance

by 12, and then to use the result in Part I. as it stands, remembering th the tones are in reality to be considered as higher by an Octave, or l equal Semitones. In this way, on reducing 34 to 22, we see that the r sonance will reinforce the 3rd and 4th partials of c[1], and the 3rd of d, very fairly, so that these tones will be good for O. But f will not giv this O so distinctly as g, a, b, which would have the 2nd partial reinforced For the next Octave we take 10 as the resonance pitch in Part I., p. 726 and we find c' almost the only note reinforced till we come to a', b'. Bu now comes another point into play. At b we have reached the ordinar limits of the thick register for bass voices (p. 720, note), and hence c', d are often difficult notes to strike. I doubt much whether the cavity with a resonance 34 could be preserved for c', d' without closing the mouth to such an extent that the quality of tone would be seriously injured through the absence of upper partial tones. The singer in all such cases sacrifices his vowel quality to his general musical quality. With bass singers, there-

[1] Mr. Sedley Taylor says that when he sang O upon c, he found g' the 3rd partial much stronger than c'' the 4th partial, though the former, by the table on p. 726 is 3·02, and the latter only 2 equal Semitones, from the characteristic resonance pitch $b'\flat$. But this resonance somewhat strengthens both. Naturally, i.e. without this resonance, the 3rd partial is much stronger than the 4th. It might remain much, though not so much, stronger. But the loudness of the 3rd and 4th partial being both greater than they would otherwise have been, and both in the neighbourhood of the characteristic resonance pitch, the O sound would be quite well distinguished. When Mr. Taylor sang A on c he found g'', the 6th partial, extremely prominent and much louder than $b''\flat$, although $b''\flat$ is the characteristic resonance pitch of the vowel. Taking 46 as the resonance pitch, diminishing it by 12, and using 34 in Part I. of p. 726, we find that the 5th, 6th, 7th, and 8th partials of c are respectively 6·14, 2·98, 0·31, and 2 equal Semitones distant. The 5th should be rather damped than otherwise, but would still possibly be louder than any of the other 3, although all these would be reinforced. That the 6th should be extremely prominent may be a peculiarity of the singer's natural quality of voice, and, if so, that it should be much louder than the 7th (which is, perhaps, naturally very weak), even when the latter were reinforced by resonance, would not be surprising. The vowel effect is due to the reinforcement of all those three partials in the neighbourhood of $b''\flat$. But it is notorious that A is of all vowels the one least distinctly differentiated from an ordinary good musical quality of tone, and is hence best adapted and most commonly employed for singing exercises. Singers, however, actually alter its quality most materially as they ascend or descend the scale. The above experiments were tried with a series of resonators; the following was tried with Koenig's manometric flames (p. 581):—'A trained treble voice sang the scale on O in the neighbourhood of $b'\flat$, both diatonically and by slurring or *portamento*, but the saw-edge on the mirror remained perfectly steady. The passage through the characteristic [resonance pitch] $b'\flat$ or one of its undertones [subtones] produced no alteration whatever.' That is, in all cases only the prime was reinforced, compare Koenig's remarks for a and b sung on U, p. 724. 'On the other hand, the least change of vowel *pronunciation* [that is, vowel genus] produced an instantaneous change of form,' due to the complete change of resonance whereby the prime ceased to be reinforced. As I conceive it, such experiments ar thoroughly consistent with Prof. Helmholtz's theory, as laid down on p. 165.

fore, the O on *c′* is changed, and made nearer in sound to A. But for the tenor the matter is different. Singing from *c* to *f′* in his thick, or upper thick, register he can get out a good tone on *c′* and a good O with the 2nd partial (36) well reinforced; but as he goes up his O fails him, so far as it depends on 34. The O heard would depend either upon the *higher* proper tones of this principal resonance cavity, or on the other unexplored and unrecorded resonance cavities of O (compare p. 161, note), or would change its quality in the direction of A. Take the soprano singing from *b* to *f″*. Here, since *b* and *c′* are near the lower extremity of the voice the muffled effect will readily give an U or O effect (see p. 734, note), and the O effect is in fact obtained by taking a higher resonance on the way to A

By the help of the tables on pp. 726 and 729, note, the above can be worked out in fuller detail, and all the other vowels can be examined. The following is suggested as a method of study:

Take the English words *loo*, *lo! lah! lay*, *lee*, which contain at least one form of the vowels U, O, A, E, I, preceded by the most vocal of oral consonants, which, however, necessitating a real contact of the tongue with the hard palate, allows the vowel to be 'attacked' firmly.

First endeavour to sing each vowel on the whole compass of the voice. All the different kinds of voices should be tried, beginning with the experimenter's own, and different singers having the same range of pitch. Begin with the middle note and sing the diatonic scale first down, then return and proceed to the top and end on the middle. Observe carefully whether any change of vowel quality occurs during the process; whether for any vowel the effect of *descending* is not to approximate the quality to the next on the left of the series *loo*, *lo*, *lah*, *lay*, *lee*, and of ascending to approximate it to the next on the right; and whether the vowel quality does not tend to become almost unrecognisable at the extremes. Observe also whether the vowel sounds when *sung* agree with the vowel sounds when *spoken* at approximatively the same pitch. As a further preliminary experiment, take some well-known air, involving considerable changes of pitch, but presenting no vocal difficulties, and sing it, or get it sung, with each note called *loo*, and then with each note called *lo*, or *lah*, or *lay*, or *lee*, and observe the remarkable difference of vocal effect.[1] And, finally,

[1] The air *Gentle Zitella*, popular some five and forty years ago, and not exceeding a Ninth in compass, will serve very well. It is here noted for a bass voice, and may be sung an Octave higher by an alto. Tenors may take it a Fifth, and sopranos a Twelfth higher. But it would be advisable to begin it with the lowest note on the compass, and gradually increase the pitch till the highest note reaches the upper limit of the voice. Each letter represents a quaver, a hyphen continues it for another quaver, and 0 is a quaver rest.

G c d	*e- g*	*g d g*	*e- G*	*G c d*	*e g* 0	*g d e*	*c-* 0
e e e	*a- e*	*e e d*	*c- c*	*d d d*	*g- d*	*d c A*	*G-* 0
G c d	*e- g*	*g d g*	*e- G*	*G c d*	*e g* 0	*g d e*	*c-* 0

See also p. 313, text and note.

sing the scale from the middle down, up, and down, to the middle, already suggested, repeating each note five times, and singing it each tim to a different word of the series *loo*, *lo*, *lah*, *lay*, *lee*. This last experimen should be conducted slowly, and the order should be varied, and especiall the extremes and the middle should be taken in gliding succession wit the others, and hence omitting the second *l*; thus, *loo-ah*, *loo-ee*, *lee-oo*, *lee-ah*, *lah-oo*, *lah-ee*. It will be found at first very difficult to maintain both pitch and approximate purity of vowel quality in such changes.

These experiments, which require no apparatus, are purely preliminary, for the purpose of accustoming the ear to observe, and the organs to produce, the vowel sounds at different pitches, and thus to enable the observer to understand the subsequent experiments. Another and useful preliminary experiment tending to shew the effect of altering one principal resonance, is to sing each vowel steadily for as long a time as the breath will hold, and, maintaining the tongue, pharynx, teeth, and pitch immovable, to vary the lips most fantastically, narrowing them to an almost imperceptible round hole, opening them wide, stretching them longitudinally but keeping them flat, pouting them, making the opening on one side of the mouth or the other; in short, making all kinds of grimaces, and changing rapidly from one form to the other. The audible effect is wild and absurd, but immensely instructive as to the action of the shape of the orifice on resonance, and the action of resonance on vowel quality.

Acoustical experiments requiring apparatus may then be conducted thus. A vowel is *spoken*, and the voice ceasing, but the position of the cavities being maintained, the principal or oral resonance is ascertained as accurately as possible by a tuning-fork furnished with runners. This requires to be repeated many times to ensure moderate success, and after the pitch is ascertained, the vowel must be again spoken to ascertain that the form of the cavities has not changed. Then the cavities should be adjusted to the resonance, without speaking, and afterwards the singer should sing a note to the corresponding vowel at different pitches for a short space of time, (merely sufficient to ascertain and recognise the vowel quality and compare it with the former,) and the resonance of the cavity should be immediately tested by the fork. It will probably be quite perceptibly changed. This experiment should be conducted for the whole compass of each kind of voice, and for the five vowels in question, which are a very small number indeed in comparison with those in use.[1]

[1] In my 'Early English Pronunciation,' p. 1294–7, it is shewn by the identification of vowel sounds in 45 European languages, made for that work by Prince Louis Lucien Bonaparte, that these languages distinguish 67 vowels, inclusive of nasals, and that at least 15 vowel signs (inclusive of nasal vowels) are required for an approximatively accurate representation of their vowel qualities. As respects the numbers of recognised vowels, only 3 languages (modern Greek, Spanish, and Illyrian) are content with the 5 in the text; and to take only well-known languages, 7 vowels are

Next the singer, keeping his mouth so as to give the same resonance (which, on account of the extreme difficulty of preserving it, must be constantly tested,) corresponding to that of one of his known vowels, already ascertained, should sing it to notes of which the partials correspond to the resonance of sets of resonators in the possession of the observer. The comparative loudness of the upper partials, should be noted for each vowel, and having been approximatively entered in a table similar to that on p. 181, the changes of loudness for changes of vowel should be compared, and then, by means of another table specially constructed for the pitch selected, and arranged in the same way as that on p. 726, these changes of loudness should be compared with the influence of resonance. In doing so we must bear in mind the natural quality of the singer's voice at different pitches, that is, the natural comparative loudness of the upper partial tones independently of the peculiar vowel resonance (p. 313), by making him sing on a neutral vowel, as *u* in *cur*, and tabulating the relative loudness of his partial tones. We may, however, assume that the partials decrease rapidly in force as they ascend in pitch, and that hence a reinforcement of a comparatively high partial, making it nearly of equal force with a lower one, though perhaps still of inferior loudness, will be very effective on vowel quality in rapid transitions; also that a resonance pitch lying between two partials, at nearly equal intervals, may reinforce both so equally as not to render any change of their mutual relation perceptible, but having reinforced both, will render both louder than they would have otherwise been, and hence will still characterise vowel quality.

(11f.) *Distinctive Character and Phonetic Value of Helmholtz's Vowel Theory.*—But even after all these experiments have been undertaken, conducted, tabulated, and analysed, it must be remembered that only one principal resonance has been used, and that many resonances are actually at work; and hence that the results are too imperfect to be conclusive, and can only lead to the formation of an hypothesis, which has to be verified synthetically, and which has been so verified approximatively, with a degree of success far exceeding what might have been expected, by the experiments of Willis and Helmholtz with simple resonance (p. 170, above) and of Helmholtz with tuning-forks (p. 180) and organ-pipes (p. 187). As the repetition of all these experiments requires special apparatus not easy to procure or manage, the following may be tried by the voice only, although the effect is necessarily imperfect from the compound character of its tones.

found in Welsh, 8 in Russian, 9 in Italian, 10 in High German, 12 in Dutch and Scotch, 13 in Icelandic, 16 in French and Danish, 17 in Swedish, 19 in Portuguese and English, and 21 in Gaelic. This is exclusive of varieties which occur provincially, and which would largely increase the numbers. This enumeration will give some notion of the enormous difficulties and extent of the concrete problem to be solved. Key words to all the forms mentioned are given in my book.

Obtain six voices which will sing *B*♭, *b*♭, *f′*, *b′*♭, *a″*, *f″*, and make them all sing the neutral vowel *u* in *cur*, but vary the *force* with which they produce it according to the indications of the first part of the table on p. 181, note. Of course the intonation must be very precise. Compare also the pianoforte experiment, pp. 104, & 189, note.

After such experiments, and my own phonetic observations, I can feel no doubt of the general accuracy of the principle announced by Helmholtz (p. 165). Perhaps then we may take the following to represent the state of the case so far as we at present know.

Vowel quality from all time, and especially since the experiments of Kratzenstein (1780) and Kempelen (1791), has been associated with the form of the cavities above the vocal chords, and that form has been assumed to be constant for each vowel. The various forms of those cavities, distinguishing the effects of the tongue, the contraction and expansion of the pharynx, and the closure of the lips, have been minutely studied by Melville Bell (p. 157, note). That the action of these cavities was due to resonance was perhaps first formulated by Wheatstone (p. 153, note). The pitch of the resonance was roughly ascertained 250 to 150 years ago (p. 162, note 1), and recently with more accuracy by Donders, Helmholtz, Merkel, and Koenig. The fact of constant double resonance, which could be arranged in two series, the front resonance being high while the back was low, and conversely, was brought out approximatively and made audible by Graham Bell (p. 161, note). The facts that the resonance is rather manifold than merely double, and that the vowel is constantly variable, representing a genus rather than a species, although very obvious deductions, were perhaps (I am not at all certain) first formulated by myself ('On the Physical Constituents of Accent and Emphasis,' in the 'Transactions of the Philological Society,' 1873-4, pp. 177-8, and 'Early English Pronunciation,' 1875, pp. 1277-84).

The great advance made by Prof. Helmholtz on all his predecessors, establishing him as the author of the theory, is the recognition of the pitch or pitches of the resonance chambers, and not their forms, as the effective marks of the vowels, and the indication of the means by which this result was effected, namely, by strengthening the partials in the neighbourhood of the pitch of the resonance, and weakening those at a distance. This indication was not an isolated statement, but a strict deduction from his discoveries concerning the compound nature of musical sound, and it could not have possibly been made before those discoveries. Speaking after more than thirty years of phonetic research (p. 170, note), I consider this indication one of the most remarkable advances in phonology which I have had the good fortune to encounter. To mention only a single important point, it explains what so long puzzled mere phonetists, how recognisably identical vowel sounds can be produced by different forms and capacities of the oral cavities, because resonance may be practi-

TABLE I.

NOTATION AND CALCULATION OF PITCH.

See App. XIX., Section A, pp. 642–4, and Table VI., A., p. 761.

A. Notations, Just Intonation. (See p. 642.)

Sign.	Name.	Value	Logarithm of Value
		C multiplied by	log *C*, added to
C	Tonic of *C*	1	0
D	major Second	$9 \div 8 = 3^2 \div 2^3$	·05115 25224 47381 28895
E	major Third	$5 \div 4 = 5 \div 2^2$	·09691 00130 08056 41436
F	Fourth	$4 \div 3 = 2^2 \div 3$	·12493 87366 08299 95313
G	Fifth	$3 \div 2$	·17609 12590 55681 24209
A	major Sixth	$5 \div 3$	·22184 87496 16356 36749
B	major Seventh	$15 \div 8 = (3 \times 5) \div 2^3$	·27300 12720 63737 65644
c	Octave	$2 \div 1$	·30102 99956 63981 19521
Sub-joined	subjoined	full value	full logarithm
♯	sharp	$135 \div 128 = (3^3 \times 5) \div 2^7$	·02312 37988 47137 75017
♭	flat	$128 \div 135 = 2^7 \div (3^3 \times 5)$	·97687 62011 52862 24983–
Pre-fixed	Prefixed	full value	full logarithm
†	high	$81 \div 80 = 3^4 \div (2^4 \times 5) = 1{\cdot}0125$	·00539 50318 86706 16354
‡	low	$80 \div 81 = (2^3 \times 5) \div 3^4 = 0{\cdot}98765\ 4320{,}9\ 876\ldots$	·99460 49681 13293 83646 – 1
ℓ	super	$64 \div 63 = 2^6 \div (7 \times 3^2) = 1{\cdot}01587\ 30158\ 7\ldots$	·00683 94245 30305 46598
ℓ	sub	$63 \div 64 = (7 \times 3^2) \div 2^6 = {\cdot}984375$	·99316 05754 69694 53402 – 1
¶	skhismic	$32805 \div 32768 = (3^8 \times 5) \div 2^{15} = 1{\cdot}00112\ 91503\ 90625$	·00049 01071 33600 37504
♭	hyposkhismic	$32768 \div 32805 = 2^{15} \div (3^8 \times 5) = 0{\cdot}99887\ 21231\ 51960$	·99950 98928 66399 62496 – 1
σ	skhistic	$\sqrt[9]{¶} + 1$	·00006 12633 91700 04688
♭	hyposkhistic	$1 + \sqrt[9]{¶} = \sqrt[9]{♭}$	·99993 87366 08299 95312 – 1
Approximations			
to ¶		$\frac{3}{2} + \sqrt[13]{2^7} = \sqrt[13]{(†¶)}$	·00049 04282 51692 21155
,,		$886 \div 885$	·00049 04511 89225 32310
,,		$\sqrt[11]{(81 \div 80)} = 1{\cdot}00112\ 9959$	·00049 04574 44246 01487
,,		$881 \div 880$	·00049 32362 61879 28635
to σ		$7001 \div 7000$	·00006 12286
to †		σ^{88}	·00539 11785

B. Logarithms for Musical Calculations. (See p. 644.)

No.	Logarithm	No.	Logarithm
1	0	25	1·39794 00086 72037 60957
2	0·30102 99956 63981 19521	26	1·41497 33479 70817 96442
3	0·47712 12547 19662 43730	27	1·43136 37641 58987 31189
4	0·60205 99913 27962 39043	28	1·44715 80313 42219 22114
5	0·69897 00043 36018 80479	29	1·46239 79978 98956 08733
6	0·77815 12503 83643 63251	30	1·47712 12547 19662 43730
7	0·84509 80400 14256 83071	31	1·49136 16938 34272 67967
8	0·90308 99869 91943 58564	32	1·50514 99783 19905 97607
9	0·95424 25094 39324 87459	37	1·56820 17240 66994 99681
10	1·0	41	1·61278 38567 19735 49451
11	1·04139 26851 58225 04075	43	1·63346 84555 79586 52641
12	1·07918 12460 47624 82772	47	1·67209 78579 35717 46441
13	1·11394 33523 06836 76921	53	1·72427 58696 00789 04563
14	1·14612 80356 78238 02593	59	1·77085 20116 42144 19026
15	1·17609 12590 55681 24208	61	1·78532 98350 10767 03389
16	1·20411 99826 55924 78085	67	1·82607 48027 00826 43415
17	1·23044 89213 78273 92854	71	1·85125 83487 19075 28609
18	1·25527 25051 03306 06980	73	1·86332 28601 20455 90107
19	1·27875 36009 52828 96154	79	1·89762 70912 90441 42799
20	1·30102 99956 63981 19521	83	1·91907 80923 76073 90383
21	1·32221 92947 33919 26801	89	1·94939 00066 44912 78472
22	1·34242 26808 22206 23596	91	1·95904 13923 21093 59992
23	1·36172 78360 17592 87887	97	1·98677 17342 66244 85178
24	1·38021 12417 11606 02294	101	2·00432 13737 82642 57428

C. Mutual Conversions of Tabular Logarithms and Numbers of Equ Semitones.

An interval may be accurately given by the numerical ratio of th vibrational numbers of its tones, or by the logarithm of that ratio, as i division A. But musicians are more accustomed to reckon by th number of equal Semitones and parts of an equal Semitone in the interva

1. As ·02 eq. sem. is almost exactly a skhisma, and ·22 almost exactl a comma, it is seldom necessary to proceed beyond three places of decimal: These may be found, within a unit in the last place, from the numeric ratios in division A of this Table and Table II., and, for all just interval by the following table:

Numbers	1	2	3	5	7
Eq. sem.	0	12	19·020	27·863	33·688

adding and subtracting the equal Semitones where the numbers are multiplied. Thus for

$$D = \tfrac{5}{2} = (3 \times 3) \div (2 \times 2 \times 2)$$

we have

$$2 \times 19{\cdot}020 - 3 \times 12 = 2{\cdot}040 \text{ equal Semitones.}$$

But for tempered intervals we must first find the tabular logarithm to five places, multiply it by 40, and subtract its 300th part. Thus log $\frac{9}{8} = {\cdot}05115 = l$; $40l = 2{\cdot}04600$, of which the 300th part is ·00682, the difference being 2·039, when these places only are kept, which is a little more correct than 2·040. Five places of decimals may be obtained correctly from logarithms of seven places by subtracting also the 300th and 5000th part of the original logarithm thus:

$l = \log. \frac{9}{8}$	= ·05115 25		Reverse Process.	
	40		s =	2·03910
$a = 40l$	= 2·0461 000		$f = s \div 40$ =	·05097 75
$b = a \div 300$	= ·0068 203		$f \div 300$ =	·00016 99
$c = l \div 300$	= ·0001 705		$f \div 10000$ =	·00000 51
$d = l \div 5000$	= ·0000 102		l =	·05115 25
$e = b + c + d$	= ·0070 010			which is correct to seven places.
$s = a - e$.	= 2·0390 990			
or to five places	= 2·0391 0	which is correct.		

These rules are deduced from the relations, first that the number of equal Semitones = tabular logarithm multiplied by M, where

$$M = 12 \div \text{tab. log } 2 = 39{\cdot}86314 \text{ nearly,}$$
$$= 40 - (40 \div 300) - (1 \div 300) - (1 \div 5000) \text{ nearly;}$$

and secondly, that the tabular logarithm = number of equal Semitones multiplied by

$$S = \text{tab. log } 2 \div 12 = \tfrac{1}{40} \times (1 + \tfrac{1}{300} + \tfrac{1}{10000}).$$

2. For very great accuracy, which is occasionally required, we must take

$$S = {\cdot}02508 \quad 58329 \quad 71998 \quad 43293 \quad 42$$
$$M = 39{\cdot}863 \quad 13713 \quad 86483 \quad 48175$$

and actually multiply by M and S or divide and multiply by S. For the last process the following table of multiples of S will be useful. It is at the same time a table of the tab. log. of the ratios of the twelve notes of equal temperament. The second table of the numbers of equal Semitones corresponding to all the prime numbers under fifty will allow of the calculation of the number of equal semitones in all intervals, up to the 50th

partial tone or harmonic, with extreme accuracy. It was used for the calculations in Table VI., A. and B.

Equal Notes	No. of Multiple.	Multiples of S = log. 2 ÷ 12 or tab. log. of the ratios of the 12 notes of Equal Temperament	Prime Nos.	Numbers of Equal Semitones corresponding to the Prime numbers under 50, that is, in the intervals of the Partial Tones thus numbered.
C♯	1	·02508 58329 71998 43293 42	1	0·0
D	2	·05017 16659 43996 86586 84	2	12·0
D♯	3	·07525 74989 15995 29880 26	3	19·019 55000 86538 74178
E	4	·10034 33318 87993 73173 68	5	27·863 13713 86483 48175
F	5	·12542 91648 59992 16467 10	7	33·688 25906 46912 49290
F♯	6	·15051 49978 31990 59760 52	11	41·513 17942 36475 67075
G	7	·17560 08308 03989 03053 94	13	44·405 27661 76931 05925
G♯	8	·20068 66637 75987 46347 36	17	49·049 55409 50040 72900
A	9	·22577 24967 47985 89640 78	19	50·975 13016 13230 25926
A♯	10	·25085 83297 19984 32934 20	23	54·282 74347 26841 54468
B	11	·27594 41626 91982 76227 62	29	58·295 77194 15308 65449
C	12	·30102 99956 63981 19521 04	31	59·450 35572 46425 02506
The numbers in the next columns were obtained by dividing the corresponding tab. log. by S without any contraction, and the system of checks adopted leads me to trust the result to the last figure.			37	62·513 44038 75473 97383
			41	64·290 62445 40483 75705
			43	65·115 17705 64251 75268
			47	66·655 06622 01316 48467

D. Fractions of a comma and skhisma and multiples of a comma in tabular logarithms and equal Semitones, to facilitate the approximate estimation of small intervals.

	Comma		Skhisma			Comma (only)	
	Tab. Log.	Equal Sem.	Tab. Log.	Equal Sem.		Tab. Log.	Equal Sem.
1/4	·001 3488	·05377	·000 1225	·00488	1/11	·000 4905	·01955
1/2	·002 6975	·10753	·000 2451	·00977	2/11	·000 9809	·03910
3/4	·004 0464	·16130	·000 3676	·01465	3/11	·001 4714	·05865
					4/11	·001 9618	·07820
1/2	·001 7983	·07169	·000 1634	·00651	5/11	·002 4523	·09775
3/2	·003 5967	·14338	·000 3267	·01302	6/11	·002 9427	·11730
					7/11	·003 4332	·13685
1/3	·001 0790	·04301	·000 0920	·00391	8/11	·003 9236	·15640
2/3	·002 1580	·08602	·000 1840	·00782	9/11	·004 4141	·17595
4/3	·003 2370	·12903	·000 2760	·01173	10/11	·004 9046	·19551
5/3	·004 3160	·17204	·000 3680	·01564	1	·005 3950	·21506
1/7	·000 7707	·03072	·000 0700	·00279	2	·010 7900	·43012
2/7	·001 5414	·06145	·000 1400	·00558	3	·016 1850	·64518
3/7	·002 3121	·09217	·000 2100	·00837	4	·021 5800	·86024
4/7	·003 0828	·12289	·000 2800	·01116	5	·026 9750	1·07530
5/7	·003 8535	·15361	·000 3500	·01395	6	·032 3700	1·29036
6/7	·004 6243	·18433	·000 4200	·01674	7	·037 7650	1·50542
					8	·043 1600	1·72048
1	·005 3950	·21506	·000 4901	·01954	9	·048 5550	1·93554

TABLE II.

VALUE, NOMENCLATURE, AND NOTATION OF INTERVALS.

See the Table pp. 516–19, and also Section B, p. 644, and Table VI. p. 76 , of this Appendix.

No.	Name	Example	Ratio	Logarithm	Equa Semito
		i. Tertian Intervals			
		Imperfect Unisons			
	Equal Unison				0
1	Skhisma	$c \times^{8} +$ †B♯	¶ = 32805 ÷ 327608	·000 4901	0·0195
2	Diaskhisma, or low comma	$c \times_{4} +_{2}$ Dbb	†♭ = 2048 ÷ 2025	·004 9049	0·1955
3	Comma	$c \times^{4} +_{1}$ †c	† = 81 ÷ 80	·005 3950	0·21506
4	Pythagorean, or high comma	$c \times^{12}$ ††B♯	†¶ = 531441 ÷ 524288	·005 8851	0·23460
5	Diesis	$c +_{8}$ †dbb	††♭ = 128 ÷ 125	·010 3000	0·41059
		Semitones or Minor Seconds			
	Equal Semitone	Equal c to c♯		·025 0858	1
6	Low Sharp, or Small Semitone	$c \times_{1} +^{2}$ ‡c♯	‡♯ = 25 ÷ 24	·017 7287	0·70673
7	Pythagorean Low Semitone (')	$c \times_{8}$ ‡db	♭♯ = 256 ÷ 243	·022 6337	0·90225
8	Sharp	$c \times^{3} +$ c♯	♯ = 135 ÷ 128	·023 1238	0·92179
9	Semitone (.)	$c \times_{1} +_{1}$ db	†♭♯ = 16 ÷ 15	·028 0287	1·11731
10	High Sharp	$c \times^{7}$ †c♯	†♯ = 2187 ÷ 2048	·028 5188	1·13685
11	High Semitone (:)	$c \times^{3} +_{2}$ †db	††♭♯ = 27 ÷ 25	·033 4238	1·33237
		Tones or Major Seconds			
	Equal Tone	Equal c to d		·050 1717	2
12	Low Tone or minor Tone (..)	$c \times_{2} +$ ‡d	10 ÷ 9	·045 7575	1·82404
13	Tone or major Tone (...)	$c \times^{2}$ d	9 ÷ 8	·051 1525	2·03910
14	Low augmented Tone (∵)	$c \times_{8} +^{3}$ ‡‡d♯	125 ÷ 108	·063 4862	2·53076
15	Augmented Tone (∴)	$c \times +^{2}$ ‡d♯	74 ÷ 64	·068 8813	2·74583

TABLE II.—*continued.*

No.	Name	Example	Ratio	Logarithm	Equal Semitones
		Thirds			
	Equal minor Third	Equal *c* to *e*♭		·075 2575	3
	Equal major Third	Equal *c* to *e*		·100 3433	4
16	Diminished Third or flat minor Third	$c \times_2 +_2$ †*e*♭♭	256÷225	·056 0575	2·23463
17	High diminished Third or high flat minor Third	$c \times^2 +_3$ ††*e*♭♭	144÷125	·061 4525	2·44969
18	Pythagorean or low minor Third (|)	$c \times_3$ *e*♭	32÷27	·073 7862	2·94135
19	Minor Third (—)	$c \times +_1$ †*e*♭	6÷5	·079 1813	3·15641
20	Major Third	$c +$ *e*	5÷4	·096 9100	3·36314
21	Pythagorean or high major Third (+)	$c \times^4$ †*e*	81÷64	·102 3050	4·07820
		Fourths			
	Equal diminished Fourth	Equal *c* to *f*♭		·100 3433	4
	Equal Fourth	Equal *c* to *f*		·125 4292	5
	Equal Tritone	Equal *c* to *f*♯		·150 5150	6
22	Diminished Fourth or high flat Fourth	$c +_2$ †*f*♭	32÷25	·107 2100	4·27372
23	Fourth	$c \times_1$ *f*	4÷3	·124 9387	4·98045
24	High Fourth	$c \times^2 +$ †*f*	27÷20	·130 3338	5·19552
25	Superfluous Fourth or low sharp Fourth	$c \times_2 +^2$ ‡*f*♯	25÷18	·142 6675	5·68717
26	Tritone or false Fourth or sharp Fourth or 'pluperfect' Fourth	$c \times^2 +$ *f*♯	45÷32	·148 0625	5·90224
		Fifths			
	Equal diminished Fifth	Equal *c* to *g*♭		·150 5150	6
	Equal Fifth	Equal *c* to *g*		·175 6008	7
	Equal superfluous Fifth	Equal *c* to *g*♯		·200 6867	8
27	Diminished Fifth or flat Fifth	$c \times_2 +_1$ *g*♭	64÷45	·152 9675	6·09777
28	High diminished Fifth or high flat Fifth	$c \times^2 +_2$ †*g*♭	36÷25	·158 3625	6·31283
29	Low Fifth	$c \times_3 +$ ‡*g*	40÷27	·170 6962	6·80449
30	Fifth	$c \times$ *g*	3÷2	·176 0913	7·01955
31	Superfluous Fifth or low sharp Fifth	$c +^2$ ‡*g*♯	25÷16	·193 8200	7·72627
32	High superfluous Fifth or sharp Fifth	$c \times^4 +$ *g*♯	405÷256	·199 2151	7·94134
		Sixths			
	Equal minor Sixth	Equal *c* to *a*♭		·200 6867	8
	Equal major Sixth	Equal *c* to *a*		·225 7725	9
	Equal superfluous Sixth	Equal *c* to *a*♯		·250 8583	10
33	Pythagorean or low minor Sixth	$c \times_4$ *a*♭	128÷81	·198 7250	7·92180
34	Minor Sixth	$c +_1$ †*a*♭	8÷5	·204 1200	8·13687
35	Major Sixth	$c \times_1 +$ *a*	5÷3	·221 8487	8·84359
36	Pythagorean or high major Sixth	$c \times^3$ †*a*	27÷16	.227 2438	9·05865
37	Superfluous Sixth or low sharp Sixth	$c \times_2 +^3$ ‡*a*♯	125÷72	·239 5775	9·55031
38	High superfluous Sixth or sharp Sixth	$c \times^2 +^2$ *a*♯	225÷128	·244 9725	9·76537

TABLE II.—*continued.*

No.	Name	Example	Ratio	Logarithm	Equal Semito
		Sevenths			
	Equal diminished Seventh	Equal *c* to *b*bb		·225 7725	9
	Equal minor Seventh	Equal *c* to *b*b		·250 8583	10
	Equal major Seventh	Equal *c* to *b*		·275 9441	11
39	Diminished Seventh or flat minor Seventh	*c* $\times_1 +_2$ †*b*bb	128 ÷ 75	·232 1487	9·2541
40	High diminished Seventh or high flat minor Seventh	*c* $\times^2 +^2$ ††*b*bb	216 ÷ 125	·237 5438	9·469[illegible]
41	Minor Seventh	*c* $\times_2$ *b*b	16 ÷ 9	·249 8775	9·9609
42	High minor Seventh.	*c* $\times^2 +_1$ †*b*b	9 ÷ 5	·255 2725	10·1759
43	Low major Seventh	*c* $\times_2 +^2$ ‡*b*	50 ÷ 27	·267 6062	10·6676
44	Major Seventh	*c* × + *b*	15 ÷ 8	·273 0013	10·8826
		Octave			
	Equal diminished Octave	Equal *c* to *c'*b		·275 9441	11
45	Diminished Octave or high flat Octave	*c* $\times +_2$ †*c'*b	48 ÷ 25	·283 3013	11·2932
		ii. SEPTIMAL INTERVALS			
	(See p. 283, note 2; p. 296, and note; p. 330, and p. 534, note; and Section D, No. 4, p. 660.)				
46	Supersecond	𝄾*c* 𝄾 × *d*	8 ÷ 7	·057 9920	2·3117
47	Subminor Third	*c* 𝄾 | *e*b	7 ÷ 6	·066 9467	2·6687
48	Supermajor Third	𝄾*c* 𝄾 + †*e*	9 ÷ 7	·109 1445	4·3508
49	Subfifth	*c* 𝄾 × 𝄾*g*b	7 ÷ 5	·146 1280	5·8251
50	Subminor Seventh	*c* 𝄾 $+_2$ 𝄾*b*b	7 ÷ 4	·243 0380	9·6882

iii. NUMBER OF INTERVALS IN AN OCTAVE.

An Octave contains—

614·222 skhismas ¶, No. 1.
61·373 diaskhismas †♭, No. 2.
55·798 commas †, No. 3.
51·151 Pythagorean commas ¶†, No. 4.
29·226 dieses ††♭, No. 5.
16·980 low sharps ‡♯, No. 6
13·300 low Semitones (') or ♭♯, No. 7.
13·018 sharps ♯, No. 8.
12·000 equal Semitones.
10·757 diatonic Semitones (.) or †♭♯, No. 9.
10·555 high sharps †♯, No. 10.
9·006 high Semitones (:) or††♭♯, No. 11.
6·579 minor Tones (..), No. 12.
6·000 equal Tones.
5·885 major Tones (...), No. 13.
4·742 low augmented Tones (·.·), No. 14.
4·370 augmented Tones (.·.), No. 15.

TABLE III.

SCALES OF QUINTAL AND TERTIAN HARMONY.

See Section D. of this Appendix, pp. 660–665.

See Notation of Intervals at beginning of Section B. p. 644.

A. *Five-Fifths Scales, Quintal, Gaelic, and Chinese.*

(See pp. 398–403, 660)

5 . C . 1. Fifths c × g × d × †a × †e
Scale *c . . . d . . . †e | g . . . †a | c′.*

This is No. 4 of p. 399. The example No. 4 on p. 402 is in 5. F. 1, with the Fifths f × c × g × d × †a. This may be reduced to harmony in the same way as 7 . C . 1, 7 . C . 2, or 7 . C . 3.

5 . C . 2. Fifths f × c × g × d × †a.
Scale *c . . . d | f . . . g . . . †a | c′.*

This is No. 1 of p. 399. The example No. 1 on p. 400 is in 5. D. 2, with the Fifths, g × d × †a × †e × †b. This may be reduced to harmony in the same way as 7 . C . 2, 7 . C . 3, 7 . C . 4.

5 . C . 3. Fifths b♭ × f × c × g × d
Scale *c . . . d | f . . . g | b♭ . . . c′.*

This is No. 3 of p. 399. The example No. 3 on p. 401 is in 5. E. 3, with the Fifths ‡d × a × e × b × f♯. This may be reduced to harmony in the same way as 7 . C . 3, 7 . C . 4, 7 . C . 5.

5 . C . 4. Fifths e♭ × b♭ × f × c × g
Scale *c | e♭ . . . f . . . g | b♭ . . . c′.*

This is No. 2 of p. 399. The example No. 2 on p. 401 is in 5. D. 4, with the Fifths f × c × g × d × †a This may be reduced to harmony in the same way as 7 . C . 4, 7 . C . 5, 7 . C . 6.

5 . C . 5. Fifths a♭ × e♭ × b♭ × f × c
Scale *c | e♭ . . . f | a♭ . . . b♭ . . . c′.*

This is No. 5 of p. 399. The example No. 5 on p. 402, when rendered pure by using *b′* for *c″*♯, is in 5. F♯ . 5, with the Fifths ‡d × a × e × b × f♯. The introduction of a sixth Fifth c♯ in the example, to serve as a leading note to ‡d, is a mere modern error, and destroys the character of the music. This may be reduced to harmony in the same way as 7 . C . 5, 7 . C . 6, or 7 . C . 7.

B. *Seven-Fifths Scales, Quintal, Greek.*

Made harmonic by flattening or sharpening some tones by a comma or two. Glarean's forms are all tempered. See pp. 403–427, 661.

No. 1, No. 2, &c., refer to division C. of this Table, p. 752.

7 . C . 1. Fifths c × g × d × †a × †e × †b × †f♯
Scale *c*...*d*...†*e*...†*f*♯ ' *g*...†*a*...†*b* ' *c*′.

Hypolydian or Syntonolydian, p. 411. Mode of the Fifth, p. 413. Rejected as unmelodic, and not found in use, p. 418. Direct harmonic form No. 1; other forms Nos. 34, 46, 51. These are some just forms of Glarean's *Lydian.*

7 . C . 2. Fifths f × c × g × d × †a × †e × †b
Scale *c*...*d*...†*e* ' *f*...*g*...†*a*...†*b* ' *c*′

Lydian, pp. 411. Mode of the First, Major, p. 413. Direct harmonic forms No. 3. Other forms Nos. 29, 48, 53. These are just forms of Glarean's *Ionic.*

7 . C . 3. Fifths b♭ × f × c × g × d × †a × †e
Scale *c*...*d*...†*e* ' *f*...*g*...†*a* ' *b*♭...*c*′.

Hypophrygian or Ionic, pp. 411 and 413. Direct harmonic form No. 5, which on p. 422 is called the mode of the Fourth; other forms No. 31, (which is the form adopted on pp. 461 and 480), and Nos. 43, 55. On p. 459 the two forms, Nos. 5 and 31, are amalgamated (see p. 460, note). These are just forms of Glarean's *Mixolydian.* It is the Arabic Uschak of p. 433.

7 . C . 4. Fifths e♭ × b♭ × f × c × g × d × †a
Scale *c*...*d* ' *e*♭...*f*...*g*...†*a* ' *b*♭...*c*′.

Phrygian, of pp. 411 and 413. Direct harmonic forms, No. 7. Other forms are Nos. 33, 45, 50. These are just forms of Glarean's *Doric.* No. 45 is considered the regular form of the mode of the minor Seventh, being No. 4 of p. 421 and No. 3 of p. 461. In No. 3, p. 459, we have two forms assigned, amounting to Nos. 33 and 45.

7 . C . 5. Fifths a♭ × e♭ × b♭ × f × c × g × d
Scale *c*...*d* ' *e*♭...*f*...*g* ' *a*♭...*b*♭...*c*′.

Hypodoric, Eolic or Locrian, of pp. 411 and 413. The direct harmonic form is No. 2. Other forms are Nos. 35, 47, 52. These are just forms of Glarean's *Eolic.* No. 52 or mode of the minor Third is No. 3 of pp. 421 and 480. In No. 4 of p. 460, Nos. 47 and 52 are amalgamated. Arabic Newa, p. 433.

7 . C . 6. Fifths ‡d♭ × a♭ × e♭ × b♭ × f × c × g
Scale *c* ' ‡*d* ... *e♭* ... *f* ... *g* ' *a♭* ... *b♭* ... *c'*.

Doric, of pp. 411 and 413. The direct harmonic form is No. 4. Other forms are Nos. 30, 49, 54. These are just forms of Glarean's *Phrygian*. No. 4 is the form adopted in No. 5, of p. 461, for the mode of the minor Sixth, but in No. 5, of p. 460, this form is amalgamated with No. 54, which is adopted as No. 7 of p. 422 for the same mode of the minor Sixth. On p. 473 we have again an amalgamated form, as either A mimipí or C matáma. But in the harmony mentioned on p. 478 we have a tetrachordal ambiguous scale, the A mimipí-ma, of Section D., VIII., of this table (p. 758), which has no foundation in the original Greek scale of 7. †E. 6.

7 . C . 7. Fifths ‡g♭ × ‡d♭ × a♭ × e♭ × b♭ × f × c
Scale *c* ' ‡*d♭* ... *e♭* ... *f* ' ‡*g♭* ... *a♭* ... *b♭* ... *c'*.

Mixolydian, of pp. 411 and 413. The direct harmonic form is No. 6. Other forms are Nos. 32, 44, 56. These are some other just forms of Glarean's *Lydian*. This is called the mode of the minor Second by Professor Helmholtz, and rejected as unmelodic, p. 418. Arabic Buselik, p. 433.

The *Chinese* 12 *lu* (p. 713 note) were used for 5 and 7 Fifths scales only.
The *Arabic* and *Persian* 17 tones (pp. 432, note 3) may be arranged thus

‡G♭	‡D♭	A♭	E♭	B♭	F	C	G	D	†A	†E		
						‡♭C	‡♭G	‡♭D	♭A	♭E	♭B	
					=	‡‡D♭♭	‡A♭♭	‡E♭♭	‡B♭♭	‡F♭	‡C♭	

They then form 12 tones in successive Fifths, some being lowered by a Pythagorean comma †¶. The scales of p. 435, note, may then be rearranged thus, with the tones in order of Fifths. The * marks a conjectural alteration of Kiesewetter's notes, and for the small letter his Arabic form is selected. The letters in parenthesis shew the four cases in which two forms of the Fifth are given. The symbol incloses in a parenthesis the numbers of the lowered Fifths, and, when one number is accented, the Fifth appears in both forms.

Name	Symbol	Notes in order of Fifths						
1. Uschak	7 . B♭	B♭	F	C	G	D	†A	†E
2. Newa	7 . A♭	A♭	E♭	B♭	F	C	G	D
3. Buselik	7 . ‡G♭	‡G♭	‡D♭	A♭	E♭	B♭	F	C
4. Rast*	7(6,7) . B♭	B♭	F	C	G	D	♭A	♭e*
10. Iszfahan	7(3',6,7) . B♭	B♭	F (‡♭C	C)	G	D	♭A	♭E
5. Husseini	7(6,7) . A♭	A♭	E♭	B♭	F	C	‡♭G	‡♭D
8. Sengule	7(4,6,7) . B♭	B♭	F	C	‡♭G	D	♭A	♭E
11. Büsürg	7(3',4,6,7) . F	F	C (‡♭G	G)	♭D	†A	♭E	♭B
6. Hidshaf	7(5,6,7) . E♭	E♭	B♭	F	C	‡♭D	‡♭D	♭A
7. Rahewi*	7(4,5,6,7) . B♭	B♭	F	C	‡♭G	‡♭D	♭A*	♭E
9. Irak	7(3',4,5,6,7) . B♭	B♭	F (‡♭C	C)	‡♭G	‡♭D	♭A	♭E
12. Zirefkend*	7(3',4,5,6,7) . F	F	C (‡♭G	G)	‡♭D	♭A	♭c*	♭B

B. *Seven-Fifths Scales, Quintal, Gr*

Made harmonic by flattening or sharpening some tones
Glarean's forms are all tempered. See pp. 40

No. 1, No. 2, &c., refer to division C. of this '

7 . C . 1. Fifths c × g × d × †a × †e × '
Scale *c . . . d . . . †e . . . †f♯ ' g . .*

Hypolydian or Syntonolydian, p. 411. M
Rejected as unmelodic, and not found in us
form No. 1; other forms Nos. 34, 46, 51.
of Glarean's *Lydian*.

7 . C . 2. Fifths f × c × g × d
Scale *c . . . d . . . †e ' f*

Lydian, pp. 411. Mode of the Fir .1. T
forms No. 3. Other forms Nos. 2? ..)
Glarean's *Ionic*.

7 . C . 3. Fifths bb × f × *g . †ab . . . †b*
Scale *c . . . d .* is form of 7 . C . 6.

Hypophrygian or Ionic, pp.
which on p. 422 is called the *f . . g . . a . bb . . . c'*
(which is the form adopted or No. 5 of p. 422, and
the two forms, Nos. 5 and ? fourth. See No. 31.)
are just forms of Glarean's

7 . C . 4. Fifths
Scale *f . . g♭ . . . †ab . . bl*
harmonic form of 7 . C . 7.

Phrygian, of pp.
Other forms are Nos.
No. 45 is consider *f . . †g . . . a . bb . . .*
being No. 4 of p. harmonic form of 7 . C . 4.
two forms assigne

7 . C . 5. *do + mi — so + ti —*

Hypodoric
nic form is
forms of Gl *f . . g . . . †a . . b . c'*
pp. 421 and
Arabic Ne *f♯ . †g♯ . a . . . b . c'*

IV. Mimima. Chords.

ti — *re* | *fa* — *lo* + *do* — *mo* + *so* + *ti* — *re* | *fa* — *lo*.

22. G pímima.

c . . . *d* . †*e*♭ . ∴ *f*♯ . *g* . . . †*a* . †*b*♭ . . *c′*.

23. E tímima.

c ∴ ‡*d*♯ . *e* . . . *f*♯ . *g* . . *a* . . . *b* . *c′*.

24. C mipíma.

c . . . *d* . †*e*♭ . . *f* . . . *g* . †*a*♭ ∴ *b* . *c′*.

(The 'instrumental' minor scale of p. 442; the 'modern' ascending minor scale of p. 465.)

25. A mitíma.

c . . ‡*d* . . . *e* . *f* ∴ ‡*g*♯ . *a* . . . *b* . *c′*.

26. F mimipá.

c . *d*♭ ∴ *e* . *f* . . . *g* . †*a*♭ . . *b*♭ . . . *c′*.

27. D♭ mimitá.

c . *d*♭ . . . †*e*♭ . †*f*♭ . . *g*♭ . . . †*a*♭ . . †*b*♭♭ ∴ *c′*.

28. B♭ mimiquá.

c . *d*♭ . . *e*♭ . . . *f* . *g*♭ ∴ *a* . *b*♭ . . . *c′*.

V. Mamami. Chords.

to + *re* | *fa* + *la* — *do* + *mi* — *so* — *to* + *re* | *fa* + *la*.

29. G pámami.

c . . . *d* . . *e* : †*f* . . *g* . . . †*a* . . *b* . *c′*.

(An harmonic form of 7 . C . 2.)

30. †E♭ támami.

c : †*d*♭ . . †*e*♭ . . . †*f* . . *g* . †*a*♭ . . . †*b*♭ . . *c′*.

(An harmonic form of 7 . C . 6.)

31. C mapámi.

c . . . *d* . . *e* . *f* . . . *g* . . *a* : †*b*♭ . . *c′*.

(An harmonic form of 7 . C . 3. The mode of the Fourth of pp. 461 and 480.)

32. †A matámi.

c . *d*♭ . . . †*e*♭ . . *f* : †*g*♭ . . . †*a*♭ . . . †*b*♭ . . *c′*.

(An harmonic form of 7 . C . 7.)

33. F mamapí.

$c \,..\, \ddagger d : \dagger e\flat \,..\, f \,...\, g \,..\, a \,.\, b\flat \,...\, c'.$

(An harmonic form of 7 . C . 4. No. 6 of p. 422, and there considered as a variant of the mode of the minor Seventh. See No. 45.)

34. ‡D mamatí.

$c \,..\, \ddagger d \,...\, e \,..\, \ddagger f\sharp \,.\, \ddagger g \,...\, a \,..\, \ddagger b : c'.$

(An harmonic form of 7 . C . 1.)

35. B♭ mamaquí.

$c \,..\, \ddagger d \,.\, e\flat \,...\, f \,..\, \ddagger g : \dagger a\flat \,..\, b\flat \,...\, c'.$

(An harmonic form of 7 . C . 5.)

VI. MIMAMI. Chords.

$to + re \mid fa - lo + do + mi - so - to + re \mid fa - lo.$

36. G pímami.

$c \,...\, d \,.\, \dagger e\flat \,...\, \dagger f \,..\, g \,...\, \dagger a \,..\, b \,.\, c'.$

37. E tímami.

$c \,...\, d \,..\, e \,...\, f\sharp \,..\, \ddagger g\sharp \,.\, a \,...\, b \,.\, c'.$

38. C mipámi.

$c \,...\, d \,..\, e \,.\, f \,...\, g \,.\, \dagger a\flat \,...\, \dagger b\flat \,..\, c'.$

39. †A♭ mitámi.

$c \,.\, d\flat \,...\, \dagger e\flat \,.\, \dagger f\flat \,...\, \dagger g\flat \,..\, \dagger a\flat \,...\, \dagger b\flat \,..\, c'$

40. F mimapí.

$c \,.\, d\flat \,...\, \dagger e\flat \,..\, f \,...\, g \,..\, a \,.\, b\flat \,...\, c'.$

41. ‡D mimatí.

$c \,..\, \ddagger d \,...\, e \,..\, \ddagger f\sharp \,.\, \ddagger g \,...\, a \,.\, b\flat \,...\, c'.$

42. B♭ mimaquí.

$c \,..\, \ddagger d \,.\, e\flat \,...\, f \,.\, g\flat \,...\, \dagger a\flat \,..\, b\flat \,...\, c'.$

VII. MAMIMI. Chords.

$to + re \mid fa + la + do - mo + so - to + re \mid fa + la.$

43. G pámimi.

$c \,...\, d \,..\, e : \dagger f \,..\, g \,...\, \dagger a \,.\, \dagger b\flat \,..\, c'.$

(An harmonic form of 7 . C . 3.)

44. †E♭ támimi.

$$c : \dagger d\flat \,..\, \dagger e\flat \,...\, \dagger f \,.\, \dagger g\flat \,..\, \dagger a\flat \,...\, \dagger b\flat \,..\, c'.$$

(An harmonic form of 7 . C . 7.)

45. C mapími.

$$c \,...\, d \,.\, \dagger e\flat \,..\, f \,...\, g \,..\, a : \dagger b\flat \,..\, c'.$$

(An harmonic form of 7 . C . 4. No. 4 of p. 421, mode of the minor Seventh of p without the leading tone of p. 469.)

46. A matími.

$$c \,..\, \ddagger d \,...\, e \,..\, \ddagger f\sharp : g \,..\, a \,...\, b \,.\, c'.$$

(An harmonic form of 7 . C . 1.)

47. F mamipí.

$$c \,..\, \ddagger d : \dagger e\flat \,..\, f \,...\, g \,.\, \dagger a\flat \,..\, b\flat \,...\, c'.$$

(An harmonic form of 7 . C . 5.)

48. ‡D mamití.

$$c \,..\, \ddagger d \,...\, e \,.\, f \,..\, \ddagger g \,...\, a \,..\, \ddagger b : c'.$$

(An harmonic form of 7 . C . 2.)

49. B♭ mamiquí.

$$c \,.\, d\flat \,..\, e\flat \,...\, f \,..\, \ddagger g : \dagger a\flat \,..\, b\flat \,...\, c'.$$

(An harmonic form of 7 . C . 6.)

VIII. Mimimi. Chords.

$$to + re \mid fa - lo + do - mo + so - to + re \mid fa - lo.$$

50. G pímimi.

$$c \,...\, d \,.\, \dagger e\flat \,...\, \dagger f \,..\, g \,...\, \dagger a \,.\, \dagger b\flat \,..\, c'.$$

(An harmonic form of 7 . C . 4.)

51. E tímimi.

$$c \,...\, d \,..\, e \,...\, f\sharp \,.\, g \,..\, a \,...\, b \,.\, c'.$$

(An harmonic form of 7 . C . 1.)

52. C mipími.

$$c \,...\, d \,.\, \dagger e\flat \,..\, f \,...\, g \,.\, \dagger a\flat \,...\, \dagger b\flat \,..\, c'.$$

(An harmonic form of 7 . C . 5. No. 3 of p. 421, mode of the minor third of p. ordinary form of the descending minor scale.)

53. A mití.

$$c \,..\, \ddagger d \,...\, e \,.\, f \,...\, g \,..\, a \,...\, b \,.\, c'.$$

(An harmonic form of 7 . C . 2.)

54. F mimipí.

c . d♭ ... †e♭ .. f ... g . †a♭ .. b♭ ... c′.

(An harmonic form of 7 . C . 6. No. 7 of p. 422, the mode of the minor Sixth of pp. 453, 461, 473–478.)

55. ‡D mimití.

c .. ‡d ... e . f .. ‡g ... a . b♭ ... c′.

(An harmonic form of 7 . C . 3.)

56. B♭ mimiquí.

c . d♭ .. e♭ ... f . g♭ ... †a♭ .. b♭ ... c′.

(An harmonic form of 7 . C . 7.)

D. *Tetrachordal Scales, Tertian, Modern.*

See p. 667.

To indicate the fourth triad add *ma-* or *mi-* to beginning, and *-mi* or *-ma* to any trichordal. Use seven consecutive tones forming Thirds, as the scale.

I. MA − / MI − } MAMAMA { − MA / − MI

ta + ra − / ta − ro + } *fa + la − do + mi − so + ti − re* { *+ fi − le / − fo + le.*

1. Ex. C *ma-mapáma.*

b♭ + ‡d − f + a − c + e − g + b − d.

The first 7 tones give C *mamapá* or F *mapáma*, last 7 tones give C *mapáma*. The intermediate 7 give the tetrachordal scale, which is also A *mitími*, No. 53 above.

2. Ex. C *mapáma-mi.*

f + a − c + e − g + b − d − †f + †a.

First 7 tones give C *mapáma*; last 7 give G *mápami* or C *pámami.* The intermediate 7 give the new tetrachordal scale,

c ... d .. e: †f .. g .. a ... b . c′.

II. MA − / MI − } MIMAMA { − MA / − MI

ta + ra − / ta − ro + } *fa − lo + do + mi − so + ti − re* { *+ fi − le / − fo + le.*

III. $\left.\begin{matrix}\text{MA} -\\ \text{MI} -\end{matrix}\right\}$ MAMIMA $\left\{\begin{matrix}-\text{MA}\\ -\text{MI}\end{matrix}\right.$

$\left.\begin{matrix}ta + ra -\\ ta - ro +\end{matrix}\right\} fa + la - do - mo + so + ti - re \left\{\begin{matrix}+ fi - le\\ - fo + le.\end{matrix}\right.$

IV. $\left.\begin{matrix}\text{MA} -\\ \text{MI} -\end{matrix}\right\}$ MIMIMA $\left\{\begin{matrix}-\text{MA}\\ -\text{MI}\end{matrix}\right.$

$\left.\begin{matrix}ta + ra -\\ ta - ro +\end{matrix}\right\} fa - lo + do - mo + so + ti - re \left\{\begin{matrix}+ fi - le\\ - fo + le.\end{matrix}\right.$

Ex. A *mipimá-ma.*

$$\ddagger d - f + a - c + e + \ddagger g\natural - b + \ddagger d\natural - f\natural .$$

The first 7 of the above notes give E *mimipá*, the last 7 give E *mipáma* the intermediate 7 give the new tetrachordal scale with 3 semitones and 2 augmented tones.

$$e \,.\, f \,.\cdot.\, \ddagger g\natural \,.\, a \ldots b \,.\, c' \,.\cdot.\, \ddagger d'\natural \,.\, e.'$$

V. $\left.\begin{matrix}\text{MA} -\\ \text{MI} -\end{matrix}\right\}$ MAMAMI $\left\{\begin{matrix}-\text{MA}\\ -\text{MI}\end{matrix}\right.$

$\left.\begin{matrix}ta + ra -\\ ta - ro +\end{matrix}\right\} fa + la - do + mi - so - to + re \left.\begin{matrix}+ fi - le\\ - fo + le.\end{matrix}\right\}$

VI. $\left.\begin{matrix}\text{MA} -\\ \text{MI} -\end{matrix}\right\}$ MIMAMI $\left\{\begin{matrix}-\text{MA}\\ -\text{MI}\end{matrix}\right.$

$\left.\begin{matrix}ta + ra -\\ ta - ro +\end{matrix}\right\} fa - lo + do + mi - so - to + re \left\{\begin{matrix}+ fi - le\\ - fo + le.\end{matrix}\right.$

VII. $\left.\begin{matrix}\text{MA} -\\ \text{MI} -\end{matrix}\right\}$ MAMIMI $\left\{\begin{matrix}-\text{MA}\\ -\text{MI}\end{matrix}\right.$

$\left.\begin{matrix}ta + ra -\\ ta - ro +\end{matrix}\right\} fa + la + do - mo + so - to - re \left.\begin{matrix}+ fi - le\\ - fo + le.\end{matrix}\right\}$

VIII. $\left.\begin{matrix}\text{MA} -\\ \text{MI} -\end{matrix}\right\}$ MIMIMI $\left\{\begin{matrix}-\text{MA}\\ -\text{MI}\end{matrix}\right.$

$\left.\begin{matrix}ta + ra -\\ ta - ro +\end{matrix}\right\} fa - lo + do - mo + so - to + re \left\{\begin{matrix}+ fi - le\\ - fo - le.\end{matrix}\right.$

Ex. A *mimipí-ma*

$$\ddagger d - f. + a - c + e - g + b + \ddagger d\natural - f\natural .$$

The first 7 of the above tones give E *mimipí*, the last 7 give E *mipíma* the intermediate 7 give the new tetrachordal scale :

$$e \,.\, f \ldots g \,..\, a \ldots b \,.\, c' \,.\cdot.\, \ddagger d'\natural \,.\, e'.$$

taken as a variation of the mode of the minor Sixth on p. 478.

TABLE IV.

LIMITS OF DUODENATION AND NUMBER OF TONES.

See Section E, pp. 668–671.

	1.	2.	I. 3.	II. 4.	III. 5.	IV. 6.	V. 7.	VI. 8.	9.
l	†††B♭♭	††D♭	††F	††A	†C♯	†E♯	G♯♯	B♯♯	‡D♯♯♯
m	†††E♭♭	††G♭	††B♭	†D	†F♯	†A♯	C♯♯	E♯♯	‡G♯♯♯
n	†††A♭♭	††C♭	††E♭	†G	†B	D♯	F♯♯	A♯♯	‡C♯♯♯
p	††D♭♭	††F♭	††A♭	†C	†E	G♯	B♯	‡D♯♯	‡F♯♯♯
q	††G♭♭	††B♭♭	†D♭	†F	†A	C♯	E♯	‡G♯♯	‡B♯♯
r	††C♭♭	††E♭♭	†G♭	†B♭	D	F♯	A♯	‡C♯♯	‡E♯♯
s	††F♭♭	††A♭♭	†C♭	†E♭	G	B	‡D♯	‡F♯♯	‡A♯♯
t	††B♭♭♭	†D♭♭	†F♭	†A♭	C	E	‡G♯	‡B♯	‡‡D♯♯
u	††E♭♭♭	†G♭♭	†B♭♭	D♭	F	A	‡C♯	‡E♯	‡‡G♯♯
w	††A♭♭♭	†C♭♭	†E♭♭	G♭	B♭	‡D	‡F♯	‡A♯	‡‡C♯♯
x	†D♭♭♭	†F♭♭	†A♭♭	C♭	E♭	‡G	‡B	‡‡D♯	‡‡F♯♯
y	†G♭♭♭	†B♭♭♭	D♭♭	F♭	A♭	‡C	‡E	‡‡G♯	‡‡B♯
z	†C♭♭♭	†E♭♭♭	G♭♭	B♭♭	‡D♭	‡F	‡A	‡‡C♯	‡‡E♯

The tones in the *small* central oblong form the duodene of which C is root. The tones in the *large* central oblong form all the duodenes wl have at least one tone in common with the central duodene of C, form the limits of radical duodenation from C.

The complete Table of 117 tones contains all the 70 duodenes wl have at least one tone in common with original duodenes whose roots tones in the duodene of C, forming the limits of general radical duodenat

The 48 tones in columns I. to VI., between the dotted lines, are tl considered sufficient for instruments with fixed tones in skhistic or unequ just intonation, and are further treated in the middle section of Table

This is the first table of modulations adapted to Just Intonation i has yet been constructed. By omitting the marks †, ‡, it reduces to a ta of modulations for tempered music. Compare Gottfried Weber's *Vers einer geordneten Theorie der Tonsetzkunst*, 1830–2, vol. ii. § 180, p. for equal temperament, and M. Emile Chevé's theoretical appendix Mme. Chevé's work, p. 327 (supra p. 638, n. 2), for his cycle of 29.

TABLE V.

Skhistic Tones and their Synonyms.

See pp. 652, 675.

The synonyms are printed in small letters; the top sections of columns II. to VI. are synonyms to the middle sections of columns I. to V. respectively; and the bottom sections of columns I. to V. are the synonyms of the middle sections of columns II. to VI. respectively. By omitting the signs †, ‡, and placing bars over or under the letters, as shown by those over or under the upper column numerals I. and VI., the notation will become that of Herr von Oettingen (see p. 425, note), and by prefixing an inverted acute accent in place of putting a bar over, and prefixing an inverted grave accent instead of placing a bar under (as in Mr. Bosanquet's notation) we obtain a typographically convenient substitute for Herr von Oettingen's notation, having the advantage of being applicable to the staff notation. This is shown by the marks prefixed to the lower column numerals. See pp. 695, 764.

		I.	II.	III.	IV.	V.	VI.	
Top Section	1t	—	††g♯	††b♯	†d♯ ♯	†f♯ ♯ ♯	†a♯ ♯ ♯	1t
	2t	—	††c♯	††e♯	†g♯ ♯	†b♯ ♯	d♯ ♯ ♯	2t
	3t	—	††f♯	††a♯	†c♯ ♯	†e♯ ♯	g♯ ♯ ♯	3t
	4t	—	††b	†d♯	†f♯ ♯	†a♯ ♯	c♯ ♯ ♯	4t
	5t	—	††e	†g♯	†b♯	d♯ ♯	f♯ ♯ ♯	5t
	6t	—	††a	†c♯	†e♯	g♯ ♯	b♯ ♯	6t
	7t	—	†d	†f♯	†a♯	c♯ ♯	e♯ ♯	7t
	8t	—	†g	†b	d♯	f♯ ♯	a♯ ♯	8t
Middle Section	1m	††Ab	†C	†E	G♯	B♯	‡D♯ ♯	1m
	2m	†Db	†F	†A	C♯	E♯	‡G♯ ♯	2m
	3m	†Gb	†Bb	D	F♯	A♯	‡C♯ ♯	3m
	4m	†Cb	†Eb	G	B	‡D♯	‡F♯ ♯	4m
	5m	†Fb	†Ab	C	E	‡G♯	‡B♯	5m
	6m	†Bbb	Db	F	A	‡C♯	‡E♯	6m
	7m	†Ebb	Gb	Bb	‡D	‡F♯	‡A♯	7m
	8m	†Abb	Cb	Eb	‡G	‡B	‡‡D♯	8m
Bottom Section	1b	dbb	fb	ab	‡c	‡e	—	1b
	2b	gbb	bbb	‡db	‡f	‡a	—	2b
	3b	cbb	ebb	‡gb	‡bb	‡‡d	—	3b
	4b	fbb	abb	‡cb	‡eb	‡‡g	—	4b
	5b	bbbb	‡dbb	‡fb	‡ab	‡‡c	—	5b
	6b	ebbb	‡gbb	‡bbb	‡‡db	‡‡f	—	6b
	7b	abbb	‡cbb	‡ebb	‡‡gb	‡‡bb	—	7b
	8b	‡dbbb	‡fbb	‡abb	‡‡cb	‡‡eb	—	8b
		,,I.	,II.	III.	,IV.	,,V.	,,,VI.	

TABLE VI.

JUST AND TEMPERED INTONATION COMPARED.

A. Elements of Calculations of the division B. in this Table.

1. For Logarithms.

Tone	Just	Skhistic	Pythagoras	Skhismic	Bosanquet	Mean	Huyghens
C	·0	·0	·0	·0	·0	·0	·0
D	·051 1525	·051 0300	·051 1525	·051 1525	·051 1183	·048 4550	·048 5532
E	·096 9100	·096 9100	·102 3050	·096 4199	·096 5568	·096 9100	·097 1065
F	·124 9387	·125 0000	·124 9387	·124 9387	·124 9558	·126 2875	·126 2384
G	·176 0913	·176 0300	·176 0913	·176 0913	·176 0741	·174 7425	·174 7916
A	·221 8487	·221 9100	·227 2438	·221 3586	·221 5126	·223 1975	·223 3448
B	·273 0013	·272 9400	·278 3963	·272 5112	·272 6309	·271 6525	·271 8981
¶	·000 4901	·0	·005 8851	·0	·0	−·010 3000	−·009 7106
†	·005 3950	·005 1500	·0	·005 8851	·005 6798	·0	·0
††	·010 7901	·010 3000	·0	·011 7702	·011 3596	·0	·0
†††	·016 1851	·015 4500	·0	·017 6553	·017 0394	·0	·0
♯	·023 1238	·022 9400	·028 5188	·022 6337	·022 7192	·019 0775	·019 4213
♯♯	·046 2476	·045 8800	·057 0376	·045 2674	·045 4384	·038 1550	·038 8426
♯♯♯	·069 3714	·068 8200	·085 5564	·067 9011	·068 1576	·057 2325	·058 2639

2. For Equal Semitones.

Tone	Just	Skhistic	Pythagoras	Skhismic	Bosanquet	Mean	Huyghens
C	0·0	0·0	0·0	0·0	0·0	0·0	0·0
D	2·03910	2·03422	2·03910	2·03910	2·03774	1·93157	1·93548
E	3·86314	3·86314	4·07820	3·84360	3·84906	3·86314	3·87097
F	4·98045	4·98289	4·98045	4·98045	4·98113	5·03422	5·03226
G	7·01955	7·01711	7·01955	7·01955	7·01887	6·96570	6·96774
A	8·84359	8·84603	9·05865	8·82405	8·83019	8·89736	8·90323
B	10·88269	10·88025	11·09775	10·86315	10·86792	10·82893	10·83871
C	12·0	12·0	12·0	12·0	12·0	12·0	12·0
¶	0·01954	0·0	0·23460	0·0	0·0	−0·41059	−0·38710
†	0·21506	0·20530	0·0	0·23460	0·22642	0·0	0·0
††	0·43013	0·41059	0·0	0·46920	0·45283	0·0	0·0
†††	0·64519	0·61589	0·0	0·70380	0·67925	0·0	0·0
♯	0·92179	0·91447	1·13685	0·90225	0·90566	0·76050	0·77419
♯♯	1·84357	1·82894	2·27370	1·80450	1·81132	1·52099	1·54838
♯♯♯	2·76536	2·74341	3·41055	1·70675	2·71698	3·28149	2·32357

For the notes in Bosanquet's cycle the notation is here assumed to be the same as that of the next adjacent skhismic notes in division B. The above data result immediately from the conditions of the systems as shewn in section C. To find the logarithm or number of equal semitones for any other note, add the values against the letters and against †, ♯; and for ‡, ♭ subtract the values against † ♯. The pitch of any note in the following Table B can thus be separately calculated and its value there verified. Differences will inevitably occur in the last places of decimals, but they are of no importance. Table I. enables the calculation to be made for logarithms to twenty places of decimals and for equal semitones to seventeen places of decimals. The cycles may also be calculated by taking multiples of the values of one degree in each cycle. To ten places of decimals we find for

One degree of the Cycle of	Expressed in Logarithms	Expressed in equal Semitones
12	0·02508 58330 = log 2 ÷ 12	1·0 = 12 ÷ 12
31	0·00971 06450 = log 2 ÷ 31	0·38709 67742 = 12 ÷ 31
53	0·00567 98112 = log 2 ÷ 53	0·22641 50943 = 12 ÷ 53

The logarithms for equal temperament are given to twenty-two places of decimals in Table I. C. p. 745.

As few ears distinguish intervals of less than half a skhisma, the numbers of equal semitones need not practically exceed two places of decimals, and hence the second set of values given above may be conveniently reduced to the following approximate, but practical because audible, number of equal semitones in the principal theoretical intervals.

Tone	Just	Skhistic	Pythagoras	Skhismic	Bosanquet	Blair	Burghers	Equal
C	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0
D	2·04	2·03	2·04	2·04	2·04	2·05	2·04	2
E	3·86	3·86	4·08	3·84	3·85	3·86	3·85	4
F	4·98	4·98	4·98	4·98	4·94	4·95	4·95	5
G	7·02	7·02	7·02	7·02	7·05	7·05	7·05	7
A	8·84	8·85	9·06	8·82	8·81	8·80	8·80	9
B	10·88	10·88	11·10	10·86	10·87	10·86	10·86	11
c	12·0	12·0	12·0	12·0	12·0	12·0	12·0	12
♯	·02	0·0	0·23	0·0	0·0	−0·02	−0·02	0
†	·22	·21	0·0	0·23	0·23	0·24	0·24	0
♯	·92	·91	1·14	0·90	0·91	0·79	0·79	1

B. Just and Tempered Tones, expressed in Vibrational Numbers, Equal Semitones, and Logarithms, in order of Pitch.

Column *Logarithm*. This column contains the logarithm (to seven places of decimals) of the ratio of the vibrational number of any tone in the following columns, to the vibrational number of the tonic in the same

Octave. The initial decimal point is omitted. The first three places are not repeated as long as all of them remain unaltered.

The *sixth* and *seventh* places are added to show the mathematical divisions and to enable us to obtain the five first places correctly under all circumstances of accumulated intervals. They have no acoustic value, and tones separated only by them are appreciably identical.

The *difference of two logarithms* shews the *interval* between the corresponding tones, which can be immediately found in this Table as an interval from C, or, with its name and actual ratio, for just tones, in Table II. When not there, the number corresponding to this difference in a table of logarithms, gives the ratio of the vibrational numbers of the tones forming that interval.

Column *Just.* This column contains every just tone mentioned in Tables IV. and V., together with the unevenly numbered partial tones of C up to the 47th reduced to the same Octave, each placed in a line with its logarithm. The *Capital Letters* (either Roman or Italic) indicate the 72 tones in lines *p* to *x*, together with the 3 tones in col. 1, lines *l*, *m*, *n*, and the 2 tones in col. 9, lines *y*, *z* of Table IV., or 77 tones in all, which are all those necessary to play Table IV. in *practically just intonation* (p. 656). The *Italic Capitals* represent the 48 tones in the middle section of Table V., that is, all the tones considered *generally* necessary in practically just intonation. The *Roman small letters* give the other 40 tones of Table IV., and the *Italic* small letters those 55 other tones of Table V. which do not occur in Table IV. A tone written as a small letter will be found to be sharper or flatter than some tone written as a capital by not more than a skhisma (¶ of Table I.), and will, in practically just intonation, be played as that *adjacent* tone. The simple figures refer to the number of the partial tone, reduced to the same Octave with the other notes. There are 16 of these (namely, the 7, 11, 13, 17, 19, 21, 23, 29, 31, 33, 35, 37, 39, 41, 43, and 47th partial tones) which do not refer to any of the tones previously mentioned, and only 8 (namely, the 1, 3, 5, 9, 15, 25, 27, and 45th partial tones) which do. The total number of tones in this column is therefore 188, namely, the 77 tones of practically just intonation, the 95 played as synonyms, and the 16 partial tones (below the 48th) not used in tertian harmony.

Column *Skhistic.* This column also contains all the tones mentioned in Tables IV. and V., supposing the intonation to be skhistic (p. 652), and hence shews their close approximation to the just tones which they represent. The capitals have the same range as before. But the tones in small letters, which in practically just intonation were merely *played as* those in capitals, now become actually identical with those tones; so that

if skhistic intonation could be easily produced in practice, there wou no inequality in the intonation.

Column *Pythagoras*. This column contains a series of 57 tones, for 56 perfect Fifths with each other as in Seven-Fifths or Pythagorean sc Table III. p. 750; the capitals point out the 17 tones of the Arabic sc pp. 432, 751, and the small letters the others. The number of t has been chosen in reference to the cycle of 53. Observe that logarithms for the four tones, bbbbbb, fbbbb, cbbbb, gbbbb, di only by ·000 9069, and the number of equal Semitones only ·03615 from those of d♯♯♯, a♯♯♯, e♯♯♯, and b♯♯♯, respectively; co pare the equations on p. 657. Observe that generally a tone in Bosanque cycle of 53 comes close to one of these of Pythagoras, and that in th four cases, answering to Bosanquet's Nos. 28, 6, 37, and 15 respective they come between the two exceptionally close Pythagorean tones.

Column *Skhismic*. Here the skhismic intonation causes similar ide tifications of tones to those in the skhistic column, and the capitals ha similar meanings; but each skhismic tone is the same as a Pythagore tone, with the name changed, so as to make a descent of 8 Fifths (whe reduced to the same Octave by rising 5 Octaves), the same as an asce of a major Third. It is thus seen how very close the skhismic intonatic is to that of Bosanquet's cycle.

Column *Bosanquet*. The cycle of 53, which Mr. Bosanquet has mad his own (p. 655), is so nearly identical with the skhismic system th advantage has been taken of the circumstance to introduce Mr. Bosanquet own notation in place of mine, which would in each case be identical with th nearest skhismic note, presenting the same identities in 53 cases, an in the 4 cases corresponding to Bosanquet's Nos. 6, 15, 28, 37, makin all the tones identical which Bosanquet's tones divide. Thus Bosanquet No. 6 would be my ‡‡c♯ = ‡B♯♯ = †Db b, for his cycle, which may b verified by the data in p. 761. Mr. Bosanquet makes c his No. 4. Th tones are divided into groups of 12, each tone having the same nam and synonyms as in equal intonation, but in the present column only tho printed in *capitals* in the column *Equal* are used, and the other names (n the tones of course) must be supplied from that column according to th principles explained on p. 694. The degree-marks represent the elev tion or depression of a tone by one division corresponding to one comm of that cycle. Hence on continuing the number of tones to 84 (as in th Table) instead of 53, certain identities will arise. These identiti depend on the fact that in this cycle a major tone consists of 9 comma and that 4 commas above c is c♯ = db, which is therefore 5 comm below d. This makes ,,,,c = c♯ = db, and ,,,,,db = d, whence as ,, destr

each other, ,,c = ,,,,d♭ as in No. 6, and ,,d♭ = ,,,,d as in No. 10, and so on in all cases. Mr. Bosanquet uses these marks in the staff notation also. When Mr. Bosanquet's notation is applied to other temperaments which are not cyclical, no such identities occur, and when applied to other cyclical temperaments the identities follow other laws which must be determined for each temperament separately.

Columns *Mean* and *Huyghens*. The mean tone temperament is here given to 35 tones (7 naturals, 7 sharps, 7 flats, 7 double sharps, and 7 double flats) instead of the usual 27 (including only 3 double sharps and 3 double flats), for comparison with Huyghens's cycle of 31. It will be seen that the logarithms of the mean tones b♯♯, d♯♯, e♯♯, a♯♯ differ from those of the mean tones d♭♭, f♭♭, g♭♭, c♭♭ by ·001 5225, and the numbers of equal semitones by ·06068, while Huyghens's pairs are identical, as Nos. 1, 9, 14, 27 respectively. Mr. Saunders's 'tilting action' (p. 659) enables 24 tones of this temperament to be played with the usual fingering; and Mr. Bosanquet's generalised keyboard is applicable with great ease to play Huyghens's cycle; see section G, No. 15, p. 694, and Table IX., p. 794.

Column *Equal*. The 12 tones of the equal temperament are here given with their synonyms, the names in capitals are those used in the col. *Bosanquet*, but are there always supposed to represent any one of the synonyms in this column. Between these tones, which have their number of Semitones prefixed, is placed, in a line with each logarithm, the number of equal Semitones which it represents, so that the pitch of any tone mentioned in this table can be read off either as a logarithm or as a number of equal Semitones. The numbers are given to five places of decimals, but practically two places suffice; see p. 762. The differences between two such numbers of equal semitones give the numbers of equal semitones in the corresponding intervals.

Column *Vibrations*. To facilitate the calculation of beats and combinational tones this column contains the vibrational number of each note in the Octave c' to c'', for $c' = 264$, the theoretical pitch used throughout the text. The whole series is readily found by logarithms for any other value of c'. But for $c' = 256$, the English theoretical pitch, subtract $\frac{1}{33}$ part; that is, divide the number in the table by 3 and the quotient by 11, and subtract the last quotient from the tabular number. Thus for just D♭ = 281·6, $\frac{1}{3}$ of 281·6 = 93·8666, $\frac{1}{11}$ of 93·8666 = 8·5333, and 281·6 − 8·5333 = 273·0666, the value of D♭ when C = 256. To find the numerical value of the ratio of any pitch to any other, divide the vibrational numbers. To find the numerical value of the ratio of any pitch to that of $c' = 264 = 3 \times 11 \times 8$, divide by 3, then 11, and then 8; thus for D♭ it is $\frac{1}{8}$ of 8·5333 = 1·0666.

Logarithm	Just	Skhistic	Pythagoras	Skhismic	Rosanquet	Mean	Huyghens	Equal	Vibrations
000 0000	1.*C*	‡*d*bb *C* †b♯	C	‡dbb C †b♯	4. c	c	0. c	0 dbb C b♯	264 000
4901	†*b*♯	—	—	—	—	—	-	0·01934	·200
004 9049	dbb	...	...	...	...	...	...	0·10553	267 000
005 1500	—	*d*bb †*C* ††b♯	—	—	—	...	—	0·20530	·140
3950	†*C*	...	...	...	...	...	...	0·21500	·300
6798	—	—	—	—	5. c	--		0·22642	·475
8851	††*b*♯	...	b♯	dbb †C ††b♯	...	...	...	0·23460	·602
008 7775	—	—	—	—	—	b♯ ♯	—	0·31093	269·800
009 7106	...	...	...	...	...	...	1. b♯ ♯ dbb	0·38710	·900
010 3000	†Dbb	†Dbb	—	—	—	dbb	- -	0·41050	270·886
8634	...	...	fbbbb	‡‡c♯ ‡B♯ ♯	...	...	...	0·43305	·086
011 3596	—	—	—	—	6. c db			0·45283	·096
7703	...	...	a♯ ♯ ♯	†Dbb	...	...	...	0·46920	271·268
012 3337	‡‡c♯	—	—	—	-	—	—	0·49167	·005
6400	...	‡‡c♯ ‡B♯ ♯	...	...	...	...	...	0·50388	·700
8238	‡B♯ ♯	—	—	—	—	-	—	0·51120	·911
013 3639	33.	...	...	...	...	...	...	0·53278	272 260
015 4500	—	††Dbb	—	—	—	..	—	0·61587	273 661
6949	††Dbb	...	...	...	...	...	...	0·62565	·710
016 7485	—	—	ebbb	‡‡db ‡C♯ b♯ ♯	—	—	—	0·66765	·880
017 0394	...	...	...	...	7. c db	...	...	0·67925	·864
2386	‡‡*d*b	—	—	—	—	—	--	0·68719	·690
7287	‡*C*♯	...	...	...	...	...	...	0·70078	275 000
7900	—	‡‡*d*b ‡*C*♯ *b*♯ ♯	—	—	—	—	—	0·70917	·030
018 2188	b♯ ♯	...	...	...	...	...	...	0·72526	·310
019 0775	—	—	—	—	—	c♯	—	0·76050	·855
4212	...	...	...	...	...	...	2. c♯	0·77419	276·074
022 6337	‡db	—	Db	‡db C♯ †b♯ ♯	—	—	—	0·90326	

023	1238	C♯	...	...	...	...	...	...	0·92179	278·437
	6139	†b♯ ♯	—	—	—	—	—	—	0·94132	·752
025	0858	...	...	...	...	...	...	...	1. c♯ Db	279·698
026	3289	17.	—	—	—	—	—	—	1·04955	280·500
027	5386	ebbb	...	...	...	...	...	...	1·09778	281·282
028	0287	Db	—	—	—	—	—	—	1·11731	·600
	0900	...	ebbb Db †c♯	...	...	...	...	...	1·11975	·640
	3991	—	—	—	—	9. ,db	—	—	1·13208	·840
	5188	†c♯	...	c♯	ebbb Db †c♯	...	...	...	1·13685	·918
029	1319	—	—	—	—	—	—	3. db	1·16129	282·316
	3775	...	...	...	...	...	db	...	1·17108	·476
032	9336	†ebbb	—	—	—	—	—	—	1·31284	284·798
033	2400	...	†Db ††c♯	...	...	...	...	...	1·32505	285·000
	4238	†Db	—	—	—	—	—	—	1·33237	·120
	9139	††c♯	...	...	...	...	...	...	1·35191	·442
034	0789	—	—	—	—	10. „db ,,,d	—	—	1·35849	·550
	4040	...	...	b ♯♯	†ebbb †Db ††c♯	...	...	...	1·37145	·764
035	4575	‡‡C♯ ♯	—	—	—	—	—	—	1·41346	286·458
	5800	...	‡‡C♯ ♯	...	...	...	...	...	1·41835	·539
038	1550	—	—	—	—	—	c♯ ♯	—	1·52100	288·243
	3287	††Ebbb	...	...	...	...	...	...	1·52790	·358
	3900	—	††Ebbb ††db	—	—	—	—	—	1·53035	·399
	8188	††db	...	...	...	...	...	...	1·54743	·684
	8426	—	—	—	—	—	—	4. c♯ ♯	1·54839	·700
	9181	35.	...	...	...	...	...	...	1·55140	·750
039	3822	—	—	fbbb	‡‡d ‡C♯ ♯	—	—	—	1·56990	289·059
	7587	...	...	...	...	11. ,,,db ,,d	...	...	1·58491	·309
040	3624	‡‡d	—	—	—	—	—	—	1·60898	·712
	7300	...	‡‡d ‡C♯ ♯	...	...	...	...	...	1·62364	·957
	8525	‡C♯ ♯	—	—	—	—	—	—	1·62851	290·039
04[illegible]	2674	‡ebb	...	Ebb	‡ebb ‡D c♯ ♯	...	...	...	1·80451	293·002
	4385	—	—	—	—	12. ,,,,db ,d	—	—	1·81132	·118
	7575	‡D	...	...	...	...	...	...	1·82404	·333

Logarithm	Just	Skhistic	Pythagoras	Skhismic	Bosanquet	Mean	Huyghens	Equal	Vibrations
045 8800	—	‡ebb ‡D c♯♯	—	—	—	—	—	1·82893	293·416
046 2476	c♯♯	...	...	...	...	...	...	1·84357	·665
048 4550	—	—	—	—	—	d	—	1·93157	295·161
5532	...	...	...	...	...	...	5. d	1·93548	·221
050 1717	—	—	-	—	—	...	—	2. c♯♯ D ebb	296·330
6624	ebb	...	...	...	...	...	...	2·01957	·665
051 0300	—	ebb D †c♯♯	—	—	—	—	—	2·03422	·916
1183	...	...	...	...	13. d	...	...	2·03774	·976
1525	9. D	—	D	ebb D †c♯♯	—	—	—	2·03910	297·000
6426	†c♯♯	...	...	...	...	...	...	2·05863	·335
056 0575	†Ebb	—	—	—	—	—	—	2·23463	300·373
1800	...	†Ebb †d	...	...	...	...	...	2·23952	·458
5476	†d	—	—	—	—	—	—	2·25416	·713
7981	...	...	...	...	14. d	...	...	2·26415	·886
057 0376	—	—	c♯♯	†Ebb †d	—	—	—	2·27370	301·052
058 2639	...	...	...	...	...	...	6. ebb	2.32258	·903
7550	—	—	—	—	—	ebb	—	2·34214	302·245
061 3300	...	††Ebb	...	...	...	...	...	2·44481	304·042
4525	††Ebb	—	—	—	—	—	—	2·44969	·128
062 0161	...	...	gbbbb	‡‡eb ‡‡D♯ ‡c♯♯♯	...	...	...	2·47215	·523
4779	—	—	—	—	15. „d „eb	—	—	2·49057	·847
9228	...	...	b♯♯♯	††Ebb	...	...	...	2·50830	305·160
9961	‡‡eb	—	—	—	—	—	—	2·51123	·211
063 0517	37.	...	...	...	...	...	...	2·51344	·250
4862	‡‡D♯	—	—	—	—	—	—	2·53076	·555
6700	...	‡‡eb ‡‡D♯ ‡c♯♯♯	...	...	...	...	...	2·53809	·685
9763	‡c♯♯♯	—	—	—	—	—	—	2·55030	·900
066 4800	...	†††Ebb	...	...	...	...	...	2·65009	307·670
8475	†††Ebb	—	—	—	—	—	—	2·66476	·930
067 5325	...					d			

067 9011	—	—	fbb	‡eb ‡D♯ c♯♯♯	—	—	—	2·70675	308·677
9745	...	...	...	..	...	...	7. d♯	2·70968	·730
068 1577	—	—	—	—	16. ,,,d ,eb	—	—	2·71698	·860
3912	‡eb	...	...	...	...	...	...	2·72629	309·026
8200	—	‡eb ‡D♯ c♯♯♯	—	—	—	—	—	2·74339	·331
8813	‡D♯	...	...	...	...	...	...	2·74583	·374
069 3714	c♯♯♯	—	—	—	—	—	—	2·76536	·724
073 2961	‡fbb	...	...	...	...	...	...	2·92181	312·536
7862	Eb	—	Eb	‡fbb Eb d♯	—	—	—	2·94135	·890
8375	...	...	...	...	17. eb	...	...	2·94340	·926
9700	—	‡fbb Eb d♯	—	—	—	—	—	2·94867	313·021
074 2763	d♯	...	...	...	...	...	...	2·96089	·242
6336	19.	—	—	—	—	—	—	2·97513	·500
075 2575	...	...	...	...	...	...	...	3. d♯ Eb	·951
077 6852	—	—	—	—	—	—	8. eb	3·09677	315·710
8325	...	...	...	...	...	eb ...		3·10265	·817
078 6912	fbb	—	—	—	—	—	—	3·13687	316·443
079 1200	...	fbb †Eb †d♯	...	...	...	...	...	3·15397	·755
1813	†Eb	—	—	—	—	—	—	3·15641	·800
5174	...	...	...	...	18. ,eb	...	...	3·16981	317·045
6714	†d♯	—	d♯	fbb †Eb †d♯	—	—	—	3·17595	·158
084 0862	†Fbb	...	...	...	...	...	...	3·35193	320·398
2700	—	†Fbb ††eb	—	—	—	—	—	3·35926	·534
5763	††eb	...	...	...	...	...	...	3·37147	·760
085 1972	—	—	—	—	19. ,,eb ,,,e	—	—	3·39623	321·219
5565	...	...	c♯♯♯	†Fbb ††eb	...	...	...	3·41055	·485
9146	39.	—	—	—	—	—	—	3·42483	·750
086 6100	‡‡D♯♯	‡‡D♯♯	...	...	...	d♯♯	...	3·45255	322·265
087 3958	—	—	—	—	—	—	9. d♯♯ fbb	3·48387	·850
088 1325	...	...	...	...	...	fbb	...	3·51323	323·397
089 4200	—	††Fbb	—	—	—	—	—	3·56456	324·358
4813	††Fbb	...	...	...	...	...	...	3·56699	·403
090 5347	—	—	gbbb	‡e ‡D♯♯	—	—	—	3·60900	325·191

Logarithm	Just	Skhistic	Pythagoras	Skhismic	Bosanquet	Mean	Huyghens	Equal	Vibrations
090 8770	...	...	...	...	20. ,,,eb ,,e	...	...	3·62264	325·448
091 5150	‡e	—	—	—	—	—	—	3·64808	·926
7600	...	‡e ‡D♯ ♯	...	...	...	...	...	3·65785	326·110
092 0051	‡D♯ ♯	—	—	—	—	—	—	3·66762	·294
096 4199	‡f b	...	Fb	‡fb E d♯ ♯	...	...	...	3·84360	329·628
5568	—	—	—	—	21. ,,,eb ,e	—	—	3·84906	·732
9100	5. E	‡f b E d♯ ♯	...	...	...	e	...	3·86314	330·000
097 1065	—	—	—	—	—	—	10. e	3·87097	·142
4001	d♯ ♯	...	...	...	...	...	...	3·88268	·373
100 3432	—	—	—	—	—	—	—	4. d♯ ♯ E fb	332·620
101 8149	f b	...	...	...	...	...	...	4·05866	333·748
102 0600	—	fb †E† †d♯ ♯	—	—	—	—	—	4·06843	·936
2366	...	...	...	...	22. e	...	...	4·07547	334·072
3050	†E	—	E	fb †E† †d♯ ♯	—	—	—	4·07820	·125
7951	†d♯ ♯	...	...	...	...	...	...	4·09774	·502
106 8170	-		—	—	—	—	11. fb	4·25806	337·614
107 2100	†Fb	†Fb †††e	...	...	...	fb	...	4·27372	·920
6339	41.		-	—	—	—	—	4·29062	338·250
7001	†††e	...	...	...	...	...	...	4·29326	·301
9164	-	—	—	—	23. ,e	—	—	4·30189	·470
108 1902	...	...	d♯ ♯	†Fb †††e	...	...	...	4·31280	·684
109 2437	‡‡E♯	—	—	—	—	—	—	4·35480	339·506
5500	...	‡‡E♯	...	...	...	...	...	4·36702	·746
112 1040	†gbbb		—	—	—	—	—	4·46925	341·750
3600	...	†gbbb †††Fb	...	...	...	...	...	4·47902	·951
6050	†††Fb			—	—	—	—	4·48879	342·144
118 1684	...	...	abbbb	‡‡f ‡E♯ ‡d♯ ♯ ♯	...	...	...	4·51125	·588
5062					24. ,,e ,,f	—	—	4·52830	·926
115 1480	††f								

114	7000	...	‡‡*f*	‡*E*♯	‡d♯♯♯	...		...		...	...		...	4·57232	343·799
115	1288	‡d♯♯♯		—		—		—		—	—		—	4·58840	344·138
	9875	...		...		...		...		...	e♯		...	4·62364	·819
116	5277	—		—		—		—		—	—	12.	e♯	4·64516	345·248
118	0993	21. g f		...		...		...		...	...		...	4·70781	345·600
119	0536	—		—		gbb	‡f	E♯	d♯♯♯	—	—		—	4·74585	347·262
	2760	...		...		...		...		25. ,,,e ,f	...		...	4·75472	·440
	5437	‡f		—		—		—		—	—		—	4·76539	·654
	8500	...	‡*f*	*E*♯	*d*♯♯♯	...		...		...	...		...	4·77761	·900
120	0338	*E*♯		—		—		—		—	—		—	4·78493	348·047
	5239	*d*♯♯♯		...		...		...		...	...		...	4·80447	·440
124	4486	‡*g*bb		—		—		—		—	—		—	4·96091	351·603
	9387	*F*		...		*F*	‡*g*bb	F	†e♯	...	...		...	4·98045	352·000
	9558	—		—		—		—		26. f	—		—	4·98113	352·014
125	0000	...	‡*g*bb	*F*	†e♯	...		...		...	...		...	4·98289	·047
	4288	†e♯		—		—		—		—	—		—	4·99999	·397
	4292	...		...		...		...		...	...		...	5. e♯ F gbb	·398
126	2384	—		—		—		—		—	—	13.	f	5·03226	353·055
	2875	...		...		...		...		...	f		...	5·03422	·095
128	3185	43.		—		—		—		—	—		—	5·11518	354·750
129	8437	gbb		...		...		...		...	...		...	5·17598	355·998
130	1500	--	*g*bb	†*F*	††e♯	—		—		—	—		—	5·18819	356·248
	3338	†*F*		...		...		...		...	...		...	5·19552	·400
	6356	—		—		—		—		27. ,f	—		—	5·20755	·649
	8239	††e♯		...		e♯	gbb	†F	††e♯	...	...		...	5·21505	·802
135	0650	—		—		—		—		—	e♯♯		—	5·38413	360·304
	2387	†Gbb		...		...		...		...	...		...	5·39104	·448
	3000	—		†Gbb	††f	—		—		—	—		—	5·39348	·499
	7286	††f		...		...		...		...	...		...	5·41058	·855
	8021	—		—		bbbbbb		‡E♯♯		—	—		—	5·41350	·916
	9490	...		...		...		...		...	...	14.	e♯♯ gbb	5·41936	361·038
136	3155	—		—		—		—		28. ,,f ,,gb	—		—	5·43396	·343
	5875	...		...		...		...		...	gbb		...	5·44480	·571

Logarithm	Just	Skhistic	Pythagoras	Skhismic	Bosanquet	Mean	Huyghens	Equal	Vibrations
136 7090	—	—	d♯ ♯ ♯	†Gbb ††f	—	—	—	5·44965	361·670
137 6400	...	‡E♯ ♯	...	...	...	...	...	5·48678	362·446
7625	‡E♯ ♯	—	—	—	—	—	—	5·49165	·551
138 3034	11.	...	...	...	...	...	...	5·51318	363·000
140 4500	—	††Gbb	—	—	—	—	—	5·59876	364·800
6338	††Gbb	...	...	...	...	...	...	5·60610	·954
141 6873	—	—	abbb	‡‡gb ‡F♯ e♯ ♯	—	—	—	5·64810	365·840
9953	...	...	...	...	20. ,,,f gb	...	...	5·66038	366·100
142 1774	‡‡gb	—	—	—	—	—	—	5·66763	·337
6675	‡F♯	...	...	...	...	...	...	5·68717	·667
7900	—	‡‡gb ‡F♯ e♯ ♯	—	—	—	—	—	5·69207	·770
143 1576	e♯ ♯	...	...	...	...	...	...	5·70671	367·080
145 3650	—	—	—	—	—	f♯	—	5·79471	368·951
6597	...	...	...	...	...	...	15. f♯	5·80645	369·202
147 5724	‡gb	—	Gb	‡gb F♯ †e♯ ♯	—	—	—	5·88270	370·831
6751	...	...	...	...	30. ,,,f gb	...	...	5·88679	·919
9400	—	‡gb F♯ †e♯ ♯	—	—	—	—	—	5·89736	371·145
148 0625	45. F♯	...	...	...	...	...	...	5·90224	·250
5526	†e♯ ♯	—	—	—	—	—	—	5·92178	·670
150 5150	...	...	...	...	...	...	...	6. f♯ Gb	373·352
152 4774	abbb	—	—	—	—	—	—	6·07823	375·043
9675	Gb	...	...	...	...	...	...	6·09777	·467
153 0900	—	abbb Gb †f♯	—	—	—	—	—	6·10266	·573
3549	...	...	...	...	31. gb	...	...	6·11321	·802
4576	†f♯	—	f♯	abbb Gb †f♯	—	—	—	6·11731	·890
155 3703	...	...	...	...	...	...	16. gb	6·19355	377·550
6650	—	—	—	—	—	gb	—	6·20530	·806
157 6079	23.	...	...	...	...	...	...	6·28374	[illegible]
158 2400	—	†Gb ††f♯	—	—					

158	8526	††*f*♯	—	—	—	—	—	—	6·33237	380·590
159	0347	...	...	...	...	32. ,,gb ,,,g	...	...	6·33962	·749
	3427	—	—	e♯♯	†Gb ††f♯	—	—	—	6·35190	381·019
160	3962	‡‡F♯♯	...	...	...	...	...	...	6·39390	·944
	5800	—	‡‡F♯♯	—	—	—	—	—	6·40124	382·105
163	2675	††Abbb	...	...	...	...	...	...	6·50835	384·478
	3900	—	††Abbb ††gb	—	—	—	—	—	6·51324	·586
	7576	††gb	...	...	...	...	...	...	6·52789	·912
164	3210	—	—	bbbbb	‡‡g ‡F♯♯	—	—	—	6·55035	385·412
	4435	...	...	...	...	...	f♯♯	...	6·55521	·520
	7145	—	—	—	—	33. ,,,gb ,,g	—	—	6·56604	·761
165	0810	...	...	...	...	...	...	17. f♯♯	6·58065	386·087
	3012	‡‡*g*	—	—	—	—	—	—	6·58942	·283
	7300	...	‡‡*g* ‡*F*♯♯	...	...	...	...	...	6·60654	·664
	7913	‡*F*♯♯	—	—	—	—	—	—	6·60896	·719
166	9479	47.	...	...	...	...	...	...	6·65507	387·750
170	2061	‡*abb*	—	—	—	—	—	—	6·78495	390·670
	3943	...	...	...	...	34. ,,,,gb ,g	...	...	6·79245	·840
	6962	‡*G*	—	—	—	—	—	—	6·80449	391·111
	8800	...	‡*abb* ‡*G* *f*♯♯	...	...	...	...	...	6·81181	·277
171	1863	f♯♯	—	—	—	—	—	—	6·82403	·643
174	7425	...	...	...	...	...	g	...	6·96579	394·772
	7916	—	—	—	—	—	—	18. g	6·96774	·817
175	6008	...	...	...	...	...	...	...	7. f♯♯ G abb	395·553
	6012	*abb*	—	—	—	—	—	—	7·00001	·554
176	0300	...	*abb* *G* †*f*♯♯	...	...	...	...	...	7·01711	·944
	0741	—	—	—	—	35. g	—	—	7·01887	·984
	0913	3.*G*	...	G	abb G †f♯♯	...	...	...	7·01955	396·000
	5814	†*f*♯♯	—	—	—	—	—	—	7·03909	·447
180	9962	†*Abb*	...	...	...	...	...	...	7·21507	400·498
181	1800	—	†*Abb* †g	—	—	—	—	—	7·22241	·667
	4863	†*g*	...	...	...	...	...	...	7·23461	·950
	7540	—	—	—	—	36. ,g	—	—	7·24528	401·193

Logarithm	Just	Skhistic	Pythagoras	Skhismic	Bosanquet	Mean	Huyghens	Equal	Vibrations
181 9764	...	...	f♯♯	†Abb †g	...	...	...	7·25415	401·403
184 5023	—	—	—	—	—	—	10. abb	7·35484	403·744
185 0425	...	...	...	...	...	abb	...	7·37637	404·247
186 3300	—	††Abb	—	—	—	—	—	7·42770	405·447
3913	††Abb	...	...	...	...	...	...	7·43013	·504
9546	—	—	cbbbb	‡‡g♯ ‡F♯♯♯	—	—	—	7·45260	406·030
187 4338	...	...	...	...	37. ,,g ,,ab	...	...	7·47170	·479
8616	—	—	e♯♯♯	††Abb	—	—	—	7·48875	·879
188 4250	‡‡g♯	...	...	...	...	...	...	7·51121	407·407
6700	—	‡‡g♯ ‡F♯♯♯	—	—	—	—	—	7·52099	·637
9151	†F♯♯♯	...	...	..	...	...	...	7·53075	·867
191 4800	—	†††Abb	—	—	—	—	—	7·63300	410·283
7862	†††Abb	...	...	...	...	...	...	7·64520	·573
192 8399	—	—	bbbb	‡ab ‡G♯ f♯♯♯	—	—	—	7·68720	411·570
193 1136	...	...	...	...	38. ,,g ,ab	...	...	7·69811	·830
3299	‡ab	—	—	—	—	—	—	7·70674	412·035
8200	25.‡*G*♯	‡*a*♭ ‡*G*♯ f♯♯♯	...	...	...	g♯	...	7·72627	·500
194 2129	—	—	—	—	—	—	20. g♯	7·74194	·878
3101	*f*♯♯♯	...	...	...	...	...	...	7·74581	·966
198 7250	ab	—	Ab	ab G♯ †f♯♯♯	—	—	—	7·92180	417·184
7934	...	...	...	...	39. ab	...	...	7·92453	·252
9700	—	ab *G*♯ †*f*♯♯♯	—	—	—	—	—	7·93157	·420
199 2151	*G*♯	...	...	...	...	...	...	7·94134	·656
7052	†*f*♯♯♯	—	—	—	—	—	—	7·96088	418·128
200 6867	...	...	...	...	...	...	...	8. g♯ Ab	419·074
203 6299	*b*bbb	—	—	—	—	—	—	8·11733	421·923
9235	...	...	...	...	...	...	21. ab	8·12903	422·209
204 1200	†*A*b	*b*bbb †*A*b †*g*♯	—	—	—	ab	--	8·13687	·400
4732	...	...	...	...	40. ab	...	...	8·15094	·744
8101	†*g*♯	—	g♯	bbbb †Ab †g♯					

209	0249	†bbbb		...		...		...		...		...		8·33239	427·198
	2700	—	†bbbb	††*A*b	††*g*♯	—		—		—		—		8·34218	·439
	5150	††Ab		...		...		...		...		...		8·35193	·680
210	0051	††*g*♯		—		—		—		—		—		8·37147	426·163
	1530	...		...		...		...	41.	,Ab		...		8·37736	·309
	4952	—		—		♯♯♯	†bbbb	††Ab	††g♯	—		—		8·39005	·646
	8534	13.		...		...		...		...		...		8·40528	420·000
211	5487	‡‡G♯♯		—		—		—		—		—		8·43300	·687
	6100	...		‡‡G♯♯		...		...		...		...		8·43546	·748
212	8975	—		—		—		—		—	g♯♯	—		8·48677	431·024
213	6342	...		...		...		...		...	...	22.	g♯♯	8·51613	·756
214	4200	††Bbbb		††Bbbb		—		—		—	—		—	8·54745	432·538
215	4735	...		...		cbbb	‡a	‡G♯♯		...	...		...	8·58945	433·588
	8328	—		—		—		—	42.	,,,ab ,,a	—		—	8·60377	·947
216	4537	‡a		...		...		...		...	...		...	8·62853	434·568
	7600	—	‡*a*	‡*G*♯♯		—		—		—	—		—	8·64074	·875
	9438	‡*G*♯♯		...		...		...		...	...		...	8·64806	435·059
221	3586	‡*b*bb		—		Bbb	‡bbb	A	g♯♯	—	—		—	8·82405	439·504
	5126	...		...		...		...	43.	,,,,ab ,a	...		...	8·83019	·660
	8487	*A*		—		—		—		—	—		—	8·84359	440·000
	9100	...	‡*b*bb	*A*	*g*♯♯	...		...		...	...		...	8·84603	·062
222	3388	g♯♯		—		—		—		—	—		—	8·86312	·493
223	1975	...		...		...		...		...	a		...	8·89736	441·368
	3448	—		—		—		—		—	—	23.	a	8·90323	·518
225	7725	...		...		...		...		...	...		...	9. g♯♯A bbb	443·903
226	7537	bbb		—		—		—		—	—		—	9·03910	444·994
227	0600	...	*b*bb	†*A*	†*g*♯♯	...		...		...	...		...	9·05133	445·311
	1924	—		—		—		—	44.	a	—		—	9·05660	·447
	2438	27. †*A*		...		A	bbb	†A	†g♯♯	...	...		...	9·05865	·500
	7339	†*g*♯♯		—		—		—		—	—		—	9·07819	446·003
232	1487	†*B*bb		...		...		...		...	...		...	9·25416	450·560
	2100	—	†*B*bb	††*a*		—		—		—	—		—	9·25662	·624
	6388	††a		...		...		...		...	...		...	9·27370	451·069

Logarithm	Just	Skhistic	Pythagoras	Skhismic	Bosanquet	Mean	Huyghens	Equal	Vibrations
232 8723	—	—	—	—	45. a	—	—	9·28302	451·311
233 0554	...	...	...	...	...	...	24. bbb	9·29032	·501
1289	—	—	g♯ ♯	†Bbb ††a	—	—	—	9·29325	·578
4975	...	...	...	...	...	bbb	...	9·30794	·961
237 0537	†cbbb	—	—	—	—	—	—	9·44970	455·678
3600	...	†cbbb ††Bbb	...	...	...	...	...	9·46192	456·000
5438	††Bbb	—	—	—	—	—	—	9·46924	·192
238 1072	...	...	dbbbb	‡‡bb ‡A♯ ‡g♯ ♯ ♯	...	...	...	9·49170	·784
5521	—	—	—	—	46. „a „bb	—	—	9·50943	457·252
239 0874	‡‡♭b	...	...	...	...	...	...	9·53077	·816
5775	‡A♯	—	—	—	—	—	—	9·55031	458·333
7000	...	‡‡♭b ‡A♯ ‡g♯ ♯ ♯	...	...	...	...	...	9·55520	·463
240 0676	‡g♯ ♯ ♯	—	—	—	—	—	—	9·56985	·851
242 2750	...	...	...	...	...	a♯	...	9·65788	461·181
5100	—	†††Bbb	—	—	—	—	—	9·66720	·438
7661	...	...	...	...	...	...	25. a♯	9·67742	·711
9388	†††Bbb	—	—	—	—	—	—	9·68430	·894
243 0380	7. gbb	...	...	...	...	...	...	9·68826	462·000
9923	—	—	cbb	‡bb A♯ g♯ ♯ ♯	—	—	—	9·72630	463·016
244 2319	...	...	...	...	47. „a „bb	...	...	9·73585	·272
4824	‡♭b	—	—	—	—	—	—	9·74583	·539
8500	...	‡♭b A♯ g♯ ♯ ♯	...	...	...	...	...	9·76050	·932
9725	A♯	—	—	—	—	—	—	9·76537	464·062
245 4626	g♯ ♯ ♯	...	...	...	...	...	...	9·78491	·586
249 3874	‡cbb	—	—	—	—	—	—	9·94137	468·804
8775	Bb	...	Bb	‡cbb Bb †a♯	...	...	...	9·96091	469·333
9117	—	—	—	—	48. bb	—	—	9·96226	·370
250 0000	...	‡cbb Bb †a♯	...	...	—	...	...	9·96578	·466
3676	†a♯	—	—	—	...	—	—	9·98045	·863
8583	...	...	...	...	...	...	10. a♯ Bb	10·00452	470·394
252 4768	—	—	—	—	—	—	26. bb	10·00843	472·150
5759	...	...	...	...	...	bb	...		

254	7824	cbb		—		—		—		—		—	10·15643	474·664
255	1500	...	cbb	†Bb	†††a♯	...		...		...		...	10·17109	475·066
	2725	†Bb		—		—		—		—		—	10·17597	·200
	5915	...		...		...		...	49. ,bb	...		...	10·18868	·549
	7626	†††a♯		—		a♯	cbb	†Bb	†††a♯	—		—	10·19551	·737
258	2780	29.		...		...		...		...		...	10·29577	478·500
260	1775	†Cbb		—		—		—		—		—	10·37149	480·705
	3000	...		†Cbb	†††bb	...		...		...		...	10·37637	·733
	6676	†††bb		—		—		—		—		—	10·39103	481·140
261	2713	...		...		...		...	50. ,,bb ,,,b	...		...	10·41509	·810
	3525	—		—		—		—		a♯♯		—	10·41835	·900
	6477	...		...		g♯♯♯		†Cbb	†††bb	...		...	10·43010	482·227
262	1874	—		—		—		—		—		27. a♯♯ cbb	10·45161	·827
	6400	...		‡A♯♯		...		...		...		...	10·46967	483·330
	7013	‡A♯♯		—		—		—		—		—	10·47210	·399
	8750	...		...		...		...		...		cbb ..	10·47901	·592
265	4500	—		††Cbb		—		—		—		—	10·58167	486·467
	5725	††Cbb		...		...		...		...		...	10·58656	·605
266	6260	—		—		dbbb	‡‡cb	‡B	a♯♯	—		—	10·62855	487·786
	9511	...		...		...		...	51. ,,,bb ,,b	...		...	10·64151	488·152
267	1161	‡‡cb		—		—		—		—		—	10·64808	·337
	6062	‡B		...		...		...		...		...	10·66762	·889
	7900	—	‡‡cb	‡B	a♯♯	—		—		—		—	10·67495	489·096
268	0963	a♯♯		...		...		...		...		...	10·68716	·441
271	6525	—		—		—		—		—		b —	10·82893	493·465
	8981	...		...		...		...		...		... 28. b	10·83871	·744
272	5112	‡cb		—		Cb	‡cb	B	†a♯♯	—		—	10·86314	494·442
	6309	...		...		...		...	52. ,,,,bb ,b	...		...	10·86792	·578
	9400	—	‡cb	B	†a♯♯	—		—		—		—	10·88025	·930
273	0013	15. B		...		...		...		...		...	10·88269	495·000
	4914	†a♯♯		—		—		—		—		—	10·90222	·559
275	9442	...		...		...		...		...		...	11. a♯♯ B cb	498·366
277	4161	‡dbbb		—		—		—		—		—	11·05867	500·058
	9062	Cb		...		...		...		...		...	11·07821	·622

Logarithm	Just	Skhistic	Pythagoras	Skhismic	Bosanquet	Mean	Huyghens	Equal	Vibrations
278 0900	—	‡dbbb Cb †b	—	—	—	—	—	11·08555	500·834
3107	...	...	...	...	53 = 0. b	...	...	11·09434	501·089
3963	†b	—	b	‡dbbb Cb †b	—	—	—	11·09775	·187
281 6087	...	...	...	...	...	...	29. cb	11·22581	504·908
9525	—	—	—	—	—	cb	—	11·23950	505·308
283 2400	...	†Cb ††b	...	...	...	...	...	11·29084	506·809
3013	†Cb	—	—	—	—	—	—	11·29327	·880
7914	††b	...	...	...	...	...	...	11·31281	507·452
9906	—	—	—	—	1. ,b ,,,c	—	—	11·32076	·685
284 2815	...	...	a♯♯	†Cb ††b	...	...	...	11·33235	508·025
285 3350	‡‡B♯	—	—	—	—	—	—	11·37435	509·259
5800	...	‡‡B♯	...	...	...	...	...	11·38413	·546
287 2417	31.	—	—	—	—	—	—	11·45036	511·500
288 2062	†Dbbb	...	...	...	...	...	...	11·48880	512·637
3900	—	†Dbbb ††cb	—	—	—	—	—	11·49614	·854
6963	††cb	...	...	...	...	...	...	11·50834	513·216
289 2597	—	—	ebbbb	‡‡c ‡B♯	—	—	—	11·53080	·882
6704	...	...	...	...	2. ,,b ,,c	...	...	11·54717	514·368
290 2399	‡‡c	—	—	—	—	—	—	11·56987	515·043
7300	‡B♯	‡‡c ‡B♯	...	...	...	b♯	...	11·58941	·625
291 3194	—	—	—	—	—	—	30. b♯	11·61290	516·325
295 1448	...	...	Dbb	‡c B♯ †a♯♯♯	...	...	...	11·76540	520·893
3502	—	—	—	—	3. ,,,b ,c	—	—	11·77359	521·140
6350	‡c	...	...	...	...	...	...	11·78493	·481
8800	—	‡c B♯ †a♯♯♯	—	—	—	—	—	11·79471	·776
296 1251	B♯	...	...	...	...	...	...	11·80447	522·070
6152	†a♯♯♯	—	—	—	—	—	—	11·82401	·660
300 5399	‡dbb	...	...	...	...	...	...	11·98047	527·404
301 0300	C	‡dbb C †b♯	C	‡dbb C †b♯	4. c	c	31 = 0. c	12. b♯ C dbb	528·000

C. Absolute Pitch.

Given the vibrational number of any tone, just or tempered, to find the vibrational number of the corresponding C in the same Octave. Find the log. of the given number by a table of logarithms, subtract the log. opposite the given tone in (B), the result is the log. of the vibrational number of C. Example, given the vibrational number of $a' = 440$.

	Just	Skhistic	Skhismic	Bosanquet	Mean	Huyghens	Equal
log 440	2·643 4527	2·643 4527	2·643 4527	2·643 4527	2·643 4527	2·643 4527	2·643 4527
log for a'	·221 8487	·221 9100	·221 3586	·221 5126	·223 1975	·223 3448	·225 7725
log c'	2·421 6040	2·421 5427	2·422 0941	2·421 9401	2·420 2552	2·420 1079	2·417 6802
c'	264·000	263·962	264·298	264·204	263·181	263·092	261·626

Having found log. C in any Octave, add that to the log. for any tone in (B), and the result is the log. of the vibrational number of that tone in that Octave.

The following are Scheibler's determinations (the most accurate known) of the exact vibrational numbers of the tones of a' tuning-forks. ('Tonmesser,' p. 52, A.D. 1834.) He selected 440 as the best for singers and instruments, and that is adopted in this work, where the intonation is assumed to be just. It was also intended to be selected by the Society of Arts in England for orchestral use, but instead of taking $a' = 440$, which, as above shewn, would give for the orchestral equal temperament, $c' = 261·626$, the Society assumed that c' would be $= 264$, as in just intonation, and hence gave a pitch 2·374 vibrations sharper than Scheibler's for c', and 3·993 sharper for a'. This raised the whole pitch by the interval between equally tempered a' and just a', that is, $\frac{8}{11}$ of a comma, or ·15630 equal Semitones. See p. 26, note 2.

The last columns of the following lists have been calculated for this table, and shew the logarithm and the number of equal Semitones for the interval by which each pitch is sharper (marked ♯) or flatter (marked ♭) than Scheibler's selected pitch of $a' = 440$.

	Vibrations	Logarithms	Equal Sem.
No. 1. French Opera, Academy of Music, $a'=$	426·75	♭ ·013 2792	♭ ·52934
No. 2. „ „ (most trustworthy)	433·75	♭ ·006 1532	♭ ·24529
No. 3. French Conservatoire, Concerts, Italian Opera	434·95	♭ ·005 0134	♭ ·19986
No. 4. French Conservatoire, Concerts, Italian Opera	440·7	♯ ·000 6904	♯ ·02753
No. 5. French Conservatoire (older fork)	435·05	♭ ·004 9135	♭ ·19587
Berlin Orchestra (best obtained: these forks vary by 3 or 4 vibrations)	441·625	♯ ·001 6010	♯ ·06382
I. Vienna Orchestra	433·665	♭ ·006 2983	♭ ·25107
II. „ „	436·335	♭ ·003 6326	♭ ·14481
III. „ „	439·15	♭ ·000 8398	♭ ·03348
IV. „ „	440·10	♯ ·000 0987	♯ ·00394
V. „ „ (most trustworthy)	440·87	♯ ·000 8578	♯ ·03419
VI. „ „ ("an excrescence")	444·87	♯ ·003 7804	♯ ·15070

In the Journal of the Society of Arts, 6 June, 1860, vol. 8, pp. 57[illegible] foll., the following pitches are given, the two last columns being added as before:

	Vibrations	Logarithms	Equal Sem.
Handel's tuning fork, about 1740, $a' =$	416	b ·024 3594	b 0·97104
Philharmonic Society, 1812–42	433	b ·006 9648	b 0·27754
French Diapason Normal, 1859	435	b ·004 9634	b 0·19785
Stuttgard Congress, 1834 (Scheibler's pitch)	440	0	0
Italian Opera, London, 1859	455	♯ ·014 5587	♯ 0·58043

The theoretical English pitch $c' = 256$, gives

	Just	Skhistic	Skhismic	Bosanquet	Mean	Huyghens	Equal
$a' =$	$426\frac{2}{3}$	426·727	426·185	426·336	427·994	428·149	430·54

As Handel used mean intonation, his $a' = 416$ implied $c' = 248·816$. Those of the following pitches of a', which are older than 1800, should probably be thus estimated. They are reduced from the report of the French commission, 1 Feb. 1859, as given in Journal of Society of Arts, vol. 7, pp. 492–8, the two last columns being added as before.

Place and Year	Observer	Vibrations	Logarithms	Equal Sem.
Paris, Grand Opera				
1699	Sauveur $a' =$	404	b ·037 0713	b 1·47777
1704	"	405·3	b ·035 6761	b 1·42220
1713	"	405·85	b ·035 0871	b 1·39868
1810	Drouet	423	b ·017 1123	b 0·68215
1823	Fischer	431·35	b ·008 6229	b 0·34383
1830	Drouet	435·75	b ·004 2153	b 0·16813
1836–9	Delezenne	441	♯ ·002 9859	♯ 0·03931
1858	Lissajous	448	♯ ·007 8253	♯ 0·31194
Berlin				
1752	Marpurg	421·875	b ·018 2689	b 0·72826
1806–14	Wieprecht (the English equal theoretical pitch)	430·5	b ·009 4795	b 0·37789
1823	Fischer	437·32	b ·002 6534	b 0·10578
1830	Wieprecht	440	0	0
1834	Scheibler (as before)	441·625	♯ ·001 6010	♯ 0·06383
1858	Wieprecht	450·75	♯ ·010 4830	♯ 0·41789
Saint Petersburgh				
1796	Sarti	436	b ·003 9662	b 0·15811
1858	Lissajous	451·5	♯ ·011 2051	♯ 0·44666
Turin				
1845	Delezenne	440		
1858	Lissajous	444·75	♯ ·004 6633	♯ 0·18589
Milan				
1845	Delezenne	446·57	♯ ·006 4368	♯ 0·25660
1856	Lissajous	450·3	♯ ·010 0492	♯ 0·40059

Hence to determine the absolute pitch of the notes in a piece of music, it is necessary to prefix, first, the name of the system of intonation, and secondly, the vibrational number of *one* note. The vibrational numbers of the other notes must be calculated as above for all the other tones on that given system, otherwise great errors will be committed.

D. Examination of the two Consonant Triads in Just and Tempered Intonation.

The triads taken are $c' + e' - g'$, and $c' - \dagger e'\flat + g'$, with root $c' = 264$. The numbers of beats and the differential tones and their beats will differ for each variation in the root of the triad, and may be calculated separately by the vibrational numbers in B, col. *Vibrations*, or be deduced by multiplying the following numbers by the pitch of the new root and dividing by $264 = 3 \times 11 \times 8$. The intonations are arranged in the order of perfection resulting from the subsequent examination. The column *Unequal* refers to those imperfect chords which occur in practically just intonation (p. 657), arising from using the five imperfect Fifths, †C to †A♭♭, †E to C♭, G♯ to E♭, B♯ to ‡G, and ‡D♯♯ to ‡B, which are each one skhisma too flat, in place of the perfect Fifths †C × †G, †E × †B, G♯ × D♯, B♯ × F♯♯, ‡D♯♯ × A♯♯, generating eight imperfect triads,

4 Major					and 4 Minor				
†C	†E	†A♭♭,	G♯	B♯	E♭,	†E	†A♭♭	C♭,	B♯	E♭	‡G,
†E	G♯	C♭,	B♯	‡D♯♯	‡G,	G♯	C♭	E♭,	‡D♯♯	‡G	‡B,

but as none of these have the root C, they are not suited for comparison with the others. Hence the triads of c' e' $a'\flat\flat$, and c' $\dagger e'\flat$ $a'\flat\flat$ have been formed, in which $a'\flat\flat$ is considered as a falsely tuned g'. It must be remembered that there are only 4 major and 4 minor chords of this kind in practically just intonation, so that the comparison is thus made between the *general* character of the other intonations, and the *exceptional* errors of this particular intonation, which for all its 35 other major, and 35 other minor triads is absolutely just.

In the following table (p. 782) the first division (a) gives the vibrational numbers of the notes employed, the second division (b) gives the various intervals two and two—that is the *dyads*—which occur in all three positions of each triad, namely, in $c' + e' - g'$, $e' - g' \times_1 c''$, $g' \times_1 c'' + e''$, and $c' - \dagger e'\flat + g'$, $\dagger e'\flat + g' \times_1 c''$, $g' \times_1 c'' - \dagger e''\flat$, and gives the number of the beats they produce in a second, while the third division (c) gives the differential notes generated by their primes. In the position $c' + e' - g'$ the differential tones of the dyads $c' + e'$ and $e' - g'$, and in the position $g' \times_1 c'' + e''$ those of the dyads $g' \times_1 c''$ and $c'' + e''$, beat perceptibly, and these beats are given in the fourth division (d).

Ratio	Note	Just	Skhistic	Unequal	Bosanquet	Skhismic	Mean	Huyghens	Equal
				(a) *Vibrational Numbers of the Notes used.*					
—	c'	264	264	264	264	264	264	264	264
—	$\dagger e'\flat$	316·8	316·755	316·8	317·045	317·158	315·817	315·710	313·9
—	e'	330	330	330	329·732	329·628	330	330·142	332·6
—	g'	396	395·944	395·553	395·984	396	394·772	394·817	395·5
				(b) *Beats of the Dyads contained in the Triads.*					
	Dyads								
5/4	$c'+e'$	0	0	0	1·071	1·488	0	0·568	10·4
5/4	$c'+e''$	0	0	0	2·142	2·976	0	1·136	20·9
3/2	$c'\times g'$	0	0·112	0·894	0·032	0	2·456	2·366	0·89
6/5	$e'-g'$	0	0·28	2·235	1·528	2·232	6·14	7·767	17·95
8/5	$e'+_1 c''$	0	0	0	2·144	2·976	0	1·136	20·96
4/3	$g'\times_1 c''$	0	0·224	1·788	0·064	0	4·912	4·732	1·78
5/3	$g'-_1 e''$	0	0·28	2·235	1·528	2·232	6·14	6·767	17·95
6/5	$c'-\dagger e'\flat$	0	0·224	0	1·225	1·79	4·915	5·45	14·24
6/5	$c''-\dagger e''\flat$	0	0·448	0	2·450	3·58	9·83	10·90	28·49
5/4	$\dagger e'\flat+g'$	0	0	1·788	1·289	1·79	0	0·718	12·45
5/3	$\dagger e'\flat-_1 c''$	0	0·224	0	1·225	1·79	4·915	5·45	14·24
8/5	$g'+_1\dagger e''\flat$	0	0	3·676	2·578	3·58	0	1·436	24·91
				(c) *Differential Tones generated by these Dyads.*					
5/4	$c'+e'$	$C=$ 66	66	66	65·732	65·628	66	66·142	68·62
5/4	$c''+e''$	$c=132$	132	132	131·464	131·256	132	132·248	137·24
3/2	$c'\times g'$	$c=132$	131·944	131·553	131·984	132	130·772	130·817	131·55
6/5	$e'-g'$	$C=$ 66	65·944	65·553	66·252	66·372	64·772	64·675	62·93
8/5	$e'+_1 c''$	$g=198$	198	198	198·268	198·372	198	197·851	175·38
4/3	$g'\times_1 c''$	$c=132$	132·056	132·447	132·016	132	133·228	133·183	132·44
5/3	$g'-_1 c''$	$c'=264$	264·056	264·447	263·480	263·256	265·228	265·467	269·68
6/5	$c'-\dagger e'\flat$	$\dagger A_1\flat=$ 52·8	152·755	52·8	53·045	53·045	51·817	51·710	49·95
6/5	$c''-\dagger e''\flat$	$\dagger A\flat=105·6$	105·510	105·6	106·090	106·316	103·634	103·420	99·90
5/4	$\dagger e'\flat+g'$	$\dagger E\flat=$ 79·2	79·189	78·753	78·939	78·842	78·955	79·107	81·60
5/3	$\dagger e'\flat-_1 c''$	$\dagger a\flat=211·2$	211·245	211·2	210·955	210·842	212·183	212·290	214·04
8/5	$g'+_1\dagger e''\flat$	$\dagger b\flat=237·6$	237·566	236·047	238·862	238·316	236·862	236·603	232·34
				(d) *Beats of the Differential Tones in the Triads.*					
	Position								
—	$c'+e'-g'$	0	0·056	0·447	0·520	0·744	1·228	1·421	5·69
—	$g'\times_1 c''+e''$	0	0·056	0·447	0·552	0·744	1·228	0·835	4·79

1) *Just Intonation* stands quite alone, characterised by entire absence of beats (in the middle parts of the scale), and by differential tones which belong to the harmony (with the exception of those from one interval in each position of the minor triad).

2) *Skhistic Intonation* stands next to just, characterised by absence of beats for the major Third and minor Sixth; by such extremely slow beats in other cases that they could be perceptible only in the highest Octaves; and by differential tones so slightly different from the correct tones that the error could scarcely be noticed even if the tuner should succeed in making it precisely; the beats of these tones are also quite imperceptible.

3) *Unequal Intonation* (applying only to the 4 imperfect major and 4 imperfect minor triads of practically just intonation) seems to stand midway between skhistic and Bosanquet's; it is certainly not worse than the last, and it is certainly not equal to the former; but inasmuch as all the other 35 major and 35 minor triads formed with the 48 just tones of the middle section of Table V., are absolutely just, while every corresponding major and minor triad of these two other intonations have faults quite comparable with those of this unequal intonation, and the faults of all three intonations are so slight that the transition from such a faulty triad to a just triad and conversely, if noticed at all, would not be annoying to the ear; it would seem that practically just intonation as a whole is fully equal to skhistic, if not even superior to it, a fortunate circumstance considering how much easier it is for the tuner to approximate decently to this than to any other kind of intonation.

4) *Bosanquet's Intonation*, characterised by slowly beating major and minor Thirds, and practically no beats of the Fourth and Fifth, with scarcely perceptible errors and beats of the differential tones, is decidedly the best of the practicable cyclic temperaments, and where it is desirable to modulate infinitely in all directions, stands first; when, however, an instrument is not arranged to play more than 48 out of its 53 tones, it is slightly inferior to practically just intonation.

4') *Practical Cyclic Intonation* (p. 658, 14a) seems to belong here.

5) *Skhismic Intonation* exaggerates the errors of the Thirds in Bosanquet's, of which it is simply a worsened form, without any redeeming character, and it would be almost quite as difficult to tune; its value is purely historical.

5') *Skhismo-Cyclic Intonation* (p. 658, 14b) would not be worse than skhismic but for the variability of its major Thirds, and may probably be placed here.

6) *Mean Intonation*, characterised by strong beats of the Fifths, Fourths, minor Thirds, and major Sixths, and by the absence of beats in the major Thirds and minor Sixths, is decidedly the best of the commatic systems, though far inferior in every respect to the preceding intonations.

The beats of the differential tones are very distinct in the position $c' + e' - g'$, and serve even as a verification for the tuner. The major triads have not a bad effect; the minor triads are very harsh. Dr. Robert Smith, in his 'Harmonics, or the Philosophy of Musical Sounds,' p. 37, 2nd ed. 1759, says, 'This is usually called the vulgar temperament,' adding that Salinas (1513–90) claims to have discovered the reason of this temperament which he found at Rome, and that Zarlino (1519–90) published it soon after; and also adds that Dechales calls Guido d'Arezzo (XI th century) its inventor, 'but that opinion wants confirmation, especially as Dechales makes no mention of the claims of Zarlino and Salinas to it.' In my paper on Temperament (see p. 641) I have shewn that this system is decidedly superior to Dr. Smith's own temperament of 'Equal Harmony,' a commatic system (p. 649) for which the major Sixth and Fifth beat equally and in opposite directions, and log. V = ·174 5927.

7) *Huyghens' Intonation* is slightly inferior to the last in every respect, except in power to modulate infinitely in all directions; for which it is the first of the commatic cycles, and only inferior to Bosanquet's.

8) *Equal Intonation* is seen to be so bad in every respect, except the Fourth and Fifth (which produce the same results as in the unequal chords of practical intonation), that nothing but the fewness of the notes which it requires (as a cycle) to modulate infinitely, could have brought it into notice. The great harshness of its major Thirds and minor Sixths, and of its minor Thirds and major Sixths, its strikingly inharmonic differential tones, and their very distinct beats (all of which stand out in strong relief on the ordinary harmonium), render it quite unfit for the performance of any music with sustained tones and delicate harmonies except with qualities of tone for which the partial tones above the fourth are scarcely audible. It is totally unfit for part singing or for stringed quartetts, or 'double-stop passages' on a violin, or for the compound stops of organs. This refers only to its effect in harmonies. Its effect in melody is altogether different, because in that it can produce no beats. Those whose musical ear has grown to the melody of equal temperament necessarily find mean tone temperament, or just intonation, with flatter major Thirds, Sixths, and Sevenths, dull, heavy, and disagreeable, and probably in view of the melody they like, overlook the harmony, and do not remark its bad effects. A concertina-tuner, who tuned justly intoned chords for me, told me he did not like them; he missed the beat. This reminds one of the man who could not sleep at nights in the country, it was so dreadfully quiet. But a little habit produces a very different state of feeling. At the same time it is well to remember that our rules of harmony grew up under the mean tone temperament, and that although the Chinese prince Tsai-yu, in A.D. 1573, that is, two centuries before equal temperament was common in Europe, gave the exact lengths and bores of 12 pipes in correctly equal temperament (given on Chinese authority by Amiot on his p. 105), yet

the Chinese harmony never extended beyond the occasional use of an Octave, for which, of course, equal temperament is perfectly suitable. As mentioned on p. 656, strictly equal temperament is a thing unknown in practice owing to the extreme difficulty of tuning the Fifths by estimation of ear. 'Many years ago,' says Prof. de Morgan ('On the Beats of Imperfect Consonances,' Trans. of the Cambridge Phil. Soc. 9 Nov. 1857, vol. 10, p. 142), 'I had two *dulcimers*, as I suppose they must be called, of a couple of Octaves each. The notes were given by single strings, and the sound was produced by a hammer held in the hand; they stood exceedingly well in tune, and the sound was as pure as that of a tuning-fork. When I tuned one to equal temperament, as I thought, and then the other, I never found agreement, though each was satisfactory by itself. I soon left off, setting down the discordance to my own inexperience. But an old professional tuner, to whom I mentioned the subject, assured me that he did not believe either that any tuner *gained* equal temperament, or that any one tuner agreed with himself or with any other. He summed up by saying that "equal temperament was equal nonsense."' For all ordinary pitches of c' a very decent (although unequal) *imitation* of equal temperament may be effected by making the Fifths $c'\ g'$, $c'\sharp\ g'\sharp$, $d'\ a'$, $d'\sharp\ a'\sharp\ e'\ b'$ (lying within the Octave $c'\ c''$) beat *once* in *one* second, and the Fifths $f'\sharp\ c''\sharp$, $g'\ d''$, $g'\sharp\ d''\sharp$, $a'\ e''$, $b'\flat\ f''$, $b'\ f''\sharp$ (in which the higher note is in the Octave above) beat *three* times in *two* seconds; in this case $f'\ c''$ will beat about *once* in between *one* and *two* seconds. The order of tuning is $c'\ g'$, $g'\ d''$, $d'\ g'\ d''$, $d'\ a'$, $a'\ e''$, $e'\ a'\ e''$, $e'\ b'$, $b'\ f''\sharp$, $f'\sharp\ b'\ f''\sharp$, $f'\sharp\ c''\sharp$, $c'\sharp\ f'\sharp\ c''\sharp$, $c'\sharp\ g'\sharp$, $g'\sharp\ d''\sharp$, $d'\sharp\ g'\sharp\ d''\sharp$, $d'\sharp\ a'\sharp$, $a'\sharp\ f''$, that is $b'\flat\ f''$. The beats can be counted by a half second's pendulum of an ordinary clock, or by a weighted string having a length from centre of suspension to centre of bob, of 9·7848 inches, or nearly enough $9\frac{7}{8}$ inches. The following table will shew the extreme closeness of the approximation thus obtained. It must be remembered that an interval of ·01 equal Semitones is barely perceptible (p. 787, l. 17 from bottom), and that to distinguish an interval of ·0087 equal Semitones by means of delicate apparatus is considered to be a great feat (p. 699, last line but one). The vibrational numbers have been calculated from the formulæ,

$$2g' = 3c' - 1,\quad 4d' = 3g' - 1{\cdot}5,\quad 2a' = 3d' - 1,\quad 4e' = 3a' - 1{\cdot}5,$$
$$2b' = 3e' - 1,\quad 4f' = 3b' - 1{\cdot}5,\quad 4c'\sharp = 3f'\sharp - 1{\cdot}5,\quad 2g'\sharp = 3c'\sharp - 1,$$
$$4d'\sharp = 3g'\sharp - 1{\cdot}5,\quad 2a' = 3d'\flat - 1,\quad 4f' = 3a'\sharp - 1{\cdot}5,$$

taking c' first = 256, and next = 264, and the resulting numbers are contrasted with the corresponding exact numbers for equal temperament in adjoining columns, and a final column gives the error from the true equal tones expressed in equal Semitones for each case.

Note	Equal	Imitation	Too flat by equal Semitones
c'	256	256	·000
$c'\sharp$	271·23	271·09	·009
d'	287·35	287·25	·006
$d'\sharp$	304·44	304·23	·011
e'	322·61	322·41	·007
f'	341·72	341·51	·001
$f'\sharp$	362·04	361·96	·003
g'	383·57	383·50	·003
$g'\sharp$	406·37	406·14	·010
a'	430·54	430·38	·006
$a'\sharp$	456·15	455·84	·011
b'	483·26	483·11	·001

Note	Equal	Imitation
c'	264	264
$c'\sharp$	279·70	279·64
d'	296·33	296·25
$d'\sharp$	313·95	313·84
e'	332·62	332·53
f'	352·40	352·32
$f'\sharp$	373·35	373·25
g'	395·55	395·50
$g'\sharp$	419·07	418·96
a'	443·99	443·88
$a'\sharp$	470·39	470·26
b'	498·37	498·30

9) *Pythagoras' Intonation* exaggerates every fault of the last, and is harmonically, no redeeming quality. It was never meant for harmony by the Greeks, and never can have been used for harmony by any musician although its use has been advocated for violin playing, where its effect would be cuttingly painful. Its interest is purely historical, and it can be at most used for melody, for which alone it was intended. It should be observed that the differential tones in this intonation are precise fractions of the root; thus for $c' + \dagger e'$ it is $\frac{17}{64}c'$, for $c' \times g'$ it is $\frac{1}{2}c'$, for $\dagger e' - g'$ it is $\frac{15}{64}c'$, for $e' +_1 c''$ it is $\frac{47}{64}c'$, for $g' -_1 \dagger e''$ it is $\frac{33}{32}c'$, for $c - e$ it is $\frac{5}{27}c'$, for $e'\flat + g'$ it is $\frac{17}{27}c'$, for $e'\flat -_1 c''$ it is $\frac{13}{27}c'$, and for $g' +_1 e'$ it is $\frac{43}{27}c'$. Pythagoras is hardly to be credited with the invention of the scale. In very early times the Chinese had a Five-Fifths scale, determined by closed pipes of the lengths of 81, 54, 72, 48, 64 'musical lines' (Amiot, p. 118), of which 81 represented a length of $9\frac{7}{8}$ English inches or, very nearly indeed, the length of the half seconds pendulum, which gave the standard pitch, identified by Amiot with f', so that these notes were f', c'', g', d'', $\dagger a'$. It is clear that this scale could not have been continued without producing fractional numbers. But by B.C. 2637, *Ling lun*, one of the grandees of the court of *Hoang-ty*, carried the scale to 12 Fifths (which, reduced to the Octave f to f', are given above, p. 71 note), and the old Chinese rules by which these were formed are translated by Amiot on his p. 215, and leave no doubt as to the precise value of the notes. That is, 2000 years *before* Pythagoras, a very much more developed scale of this kind was known in China. The Chinese still call the original five notes the 'five tones,' the seven notes of the Greek scale (being the above five and the two *pien*, or $\dagger e''$ and $\dagger b'$, introducing Semi-tones) the 'seven principles,' and the five chromatic notes are the 'five complements.' Amiot says (on his p. 161), 'These seven principles which have been always known to Chinese sages, were ignored by the vulgar literati, . . . who would only admit the five *tones*, rejecting the two *pien*

being in their opinion new inventions.' The statement above, p. 398, referring this to Prince Tsai-yu, seems, therefore, to be slightly erroneous. The seven notes were known thousands of years before he existed. See, also, p. 713, note, and p. 784. The whole Chinese system is founded on the number 3 and the mystic trigrams called *fou-hi*; see Amiot, plate xix., fig. 15a. The history of the Arabic 17 Fifths has yet to be written.

E. Intervals actually played on Instruments with Unfixed Tones.

The preceding divisions give the principal and most widely accepted theoretical intonations hitherto invented. The present division contains the results of the investigations of Messrs. Cornu and Mercadier of the intervals actually in use. They will be found in the 'Comptes Rendus' for 8 and 22 Feb. 1869, vol. 68, pp. 301 and 427; 17 July, 1871, vol. 73, p. 178; 29 Jan. 1872, vol. 74, p. 321; and 17 Feb. 1873, vol. 76, p. 432, of which only the two first are referred to on p. 507 above. The method of observation at first was to bring the instruments near the parabolic reflector of a phonautograph, allowing the vibrations to be described on the smoked cylinder (p. 30 above) at the same time with those of a chronometric tuning-fork, and counting the number of waves in a second for each note, and taking the ratio to the fundamental note. But for the violin and violoncello, as described in 'Comptes Rendus,' vol. 73, p. 178, a thin piece of tin plate was placed under the bridge, and the vibrations were conducted by a wire direct to the inscribing style. The tones on the monochord were determined by the ratio of the length of the string sounded to its whole length, and the observers are reported to have perceived a difference of 1 in a 1000 in the length, corresponding to ·0173 equal semitones or about $\frac{2}{7}$ skhisma. Only the means of the results of the first series of experiments are given by Messrs. Cornu and Mercadier, but the number of observations is too small to make means trustworthy. In subsequent experiments they give every case as well as the mean, and the mean is here omitted. The *first* and *second* series were experiments made on the playing of amateurs; in the *third* series the players were professionals, namely, M. Léonard, a Belgian violinist, and M. Séligmann, violoncellist. In the *fourth* and *fifth* series the players were again amateurs, but in the *sixth* series the violinist was M. Ferrand of the Opéra Comique. The results are given by Messrs. Cornu and Mercadier in actual numerical ratios; I have here reduced these ratios to the corresponding number of equal Semitones in the intervals they designate, and have added the value of the same intervals in the same form for just, equal, and Pythagorean intonation. The nature of the divergence from equal temperament is shewn by the number of equal Semitones. In the columns for D, F, G, c, which are of the same pitch in both just and Pythagorean intonation, the

(=) shews that the note played had the same pitch, ♯ that it was shar
♭ that it was flatter. In the other columns where just is different f
Pythagorean intonation, the first of the two signs, ♯ and ♭, refers to
and the second to Pythagorean intonation; thus ♯ ♭ 3·9598 shews
the E was sharper than just and flatter than Pythagorean E. In no si
case did the major or minor Third or major Seventh actually obser
exactly agree with the just, the Pythagorean, or the equal interval.
major Seventh or leading note is always sharper than in just or eq
intonation, and frequently sharper than in Pythagorean intonation.

The minor scale of *A* in the original is here assumed to be *C*
readier comparison.

Messrs. Cornu and Mercadier conclude finally, vol. 76, p. 434, that

'Musical intervals belong to at least two different systems of differ
values:

'1) The intervals employed in melodies *which have no modulati*
agree with those of the *Pythagorean scale.*

'2) The intervals between two notes *sounded together*, in chords,
basis of harmony, have for their ratios the following numbers: 2
Octave, $\frac{3}{2}$ Fifth, $\frac{4}{3}$ Fourth, $\frac{5}{4}$ major Third, $\frac{6}{5}$ minor Third, $\frac{5}{3}$ major Si
$\frac{8}{5}$ minor Sixth, and $\frac{7}{4}$ Seventh, where the Fourth and Sixths were dedu
from observation of the Fifth and Thirds, and the Seventh from
dominant chord.'

Thus for unaccompanied harmony of two tones (chords more t
two tones were not tried) just intonation *alone* was used. For mel
the major Thirds, major Sixths, and especially the major Sevenths (lead
notes) were much sharpened, and the minor Thirds and minor Si
generally much flattened. But did this arise from the custom of eq
temperament? (as M. Guéroult thinks, ibid. 9 May, 1870, vol. 70, p. 10
to which Messrs. Cornu and Mercadier replied on 30 May, 1870, vol.
p. 1170) or really from the feeling of Fifths? The latter was impos
for the leading note, which is sometimes much sharper than in Pyt
gorean intonation, and the Fifths played were by no means always t
Messrs. Cornu and Mercadier say that the divergence from the mean c
reaches $\frac{1}{3}$ of a comma, that is, ·0717 of an equal Semitone; but as
Pythagorean major Sixth, major Third, and major Seventh differ f
the corresponding equal tempered intervals by only $\frac{3}{11}$, $\frac{4}{11}$, and $\frac{8}{11}$
comma respectively, that is, by only ·0587, ·0782, and ·0978 of an e
semitone, this uncertainty renders it impossible to decide whether
scale played was intentionally equal or Pythagorean, or whether eve
did not vary with the feeling of the moment. Taking into consider
that the pitches actually shewn in the tables vary considerably, that
very rarely repeat themselves, that the notes are sometimes flatter
sometimes sharper than either just or Pythagorean intonation, and
this uncertainty pervades even such intervals as the Fourth, Fifth,

Octave, I am inclined to adopt the hypothesis of an intentionally variable intonation. Whether founded on the feeling of Pythagorean or equal temperament, it is difficult to decide. But it is certainly not founded on any feeling of just intonation for harmony. If then these players, as Messrs. Cornu and Mercadier assert from first to last in the unmistakable terms already cited, adopt just intonation of intervals for harmony, a serious question arises as to how they treat the relations of tonality. The first part may lead and the others may be adapted to it, or the bass may determine the intonation of the other parts. In either case there would be a great variability, through which modulation, and even the adjustment of parts without much previous combined practice, would become extremely difficult; see pp. 483, 490, 506, 507, text and notes. But how about the return to the same key after modulations (p. 512)? Huyghens ('Cosmotheōros,' lib. 1, p. 77, as cited by Dr. Smith, 'Harmonics,' 2nd ed. p. 228) suggests that as 'erring from the pitch first assumed... would greatly offend the ear of the musician, he naturally avoids it by his memory of pitch, and by tempering the intervals of the intermediate sounds, so as to return to it again.' But how accurately does he remember the original pitch? In some cases in the following experiments the Octave (and hence the original pitch) will be seen to be sharpened or flattened by more than a skhisma. In the course of a few modulations this skhisma might easily become a comma. Where is the guarantee for remembering the original pitch? Would an alteration of a whole comma in passing through a series of modulations be noticed? We know indeed that unaccompanied singers constantly 'flatten' by much more than a comma. Messrs. Cornu and Mercadier stated, on 17 Feb. 1873, that they were engaged in examining the influence of modulation on the value of melodic intervals, and the nature of the intervals in chords of more than two notes, but after looking through the 'Comptes Rendus' from that date onwards, I cannot find that they have presented any memoirs on the subject, which is much to be regretted, as probably no scientific determinations are in existence. Table IV. simply shews what sounds ought to be played in modulations, and would be played on instruments with fixed tones properly arranged; not what intervals are *now* played and sung, and mere memory of ear does not suffice.

MELODIC INTERVALS RECKONED FROM C, AN
AS ACTUALLY PLAYED ON INSTRUMEN

MAJOR	No.	D	E	F	
First Series					
AMATEURS					
Monochord	1	—	♯ ♯4·1516	—	=7
Voice	2	—	♯ b4·0011	—	b6
Organ pipes	3	—	♯ ♯4·0969	—	b6
Violin	4	—	♯ b4·0559	—	♯7
Violoncello	5	—	♯ ♯4·0833	—	♯7
Second Series	1	—	♯ b4·0148	♯ 5·0539	b7
AMATEURS	2	♯ 2·1159	♯ ♯4·1243	b 4·9761	♯7
Violin	3	♯ 2·1159	♯ b3·9873	♯ 4·9891	♯7
	4	b 1·9842	♯ b3·9598	—	♯7
	5	♯ 2·1005	♯ b4·0560	♯ 5·0539	=7
	6	b 1·8845	♯ b4·0148	—	=7
	7	b 2·0083	♯ b4·0560	b 4·8980	♯7
	8	—	♯ ♯4·1107	b 4·8980	
	9	b 2·0083	♯ b3·9598	b 4·9501	♯7
	10	b 1·9465	♯ ♯4·1117	—	♯7
	11	—	♯ ♯4·1107	—	
	12	—	♯ b4·0423	—	=7
Violoncello	1	—	♯ ♯4·1516	—	♯7
	2	—	♯ b3·9460	—	
	3	♯ 2·0820	♯ b3·9735	♯ 5·0669	♯7
	4	—	♯ ♯4·0833	—	
	5	—	♯ ♯4·1107	—	
	6	—	♯ b4·0148	—	♯7
	7	—	♯ b4·0696	—	♯7
	8	—	♯ b4·0423	—	
	9	= 2·0391	♯ ♯4·1516	♯ 5·1055	b7
	10	b 2·0083	—	♯ 5·1055	=7
	11	b 1·9774	♯ ♯4·1516	♯ 5·0021	
	12	♯ 2·0698	♯ ♯4·1379	b 4·9761	b7
	13	b 2·0237	♯ b4·0285	♯ 4·9891	♯7
Third Series	1	—	♯ ♯4·1107	b 4·8980	
Violin	2	—	♯ ♯4·1651	b 4·9501	♯7
M. LEONARD	3	—	♯ ♯4·1651	—	
(PROFESSIONAL)	4	—	♯ ♯4·1107	—	
	5	♯ 2·0698	—	—	
	6	♯ 2·1618	—	—	
	7	—	—	—	
	8	♯ 2·0698	♯ b4·0696	—	
	9	—	♯ ♯4·1107	—	
	10	—	♯ b4·0148	—	
	11	—	♯ b4·0148	—	
	12	♯ 2·0698	♯ b4·0148	—	
	13	b 1·9929	♯ b4·0696	—	b6
	14	♯ 2·0852	♯ b4·0423	—	

MAJOR	No.	D	E	F	G	A	B	c
loncello	1	—	♯ b3·9322	b 4·8719	= 7 0196	—	—	
SÉLIGMANN	2	—	♯ b4·0696	—	—	—	—	
OFESSIONAL)	3	—	♯ ♯4·1923	b 4·9631	♯ 7·1575	—	—	
	4	—	♯ ♯4·1107	—	—	—	—	
	5	—	♯ b3·9873	b 4·9111	= 7·0196	—	—	
	6	—	—	b 4·9631	--	—	—	
	7	—	♯ ♯4 1379	—	b 6·9502	♯ b8·9196	—	
	8	—	♯ b4·0696	—	b 6·9617	♯ b8·9609	—	
	9	—	♯ b4·0285	—	b 6·9733	♯ ♯9·0638	—	
t		2·0391	3·8631	4·9805	7·0196	8·8436	10·8827	12·0
ıal		2·0	4·0	5·0	7·0	9·0	11·0	12·0
hagorean		2·0391	4·0782	4·9805	7·0196	9·0587	11·0978	12·0

MINOR	No.	*d*	*eb*	*f*	*g*	*ab*	*b*	*c'*
rth Series	1	—	b b2·9240	—	b 6·9733			♯ 12·0797
lins	2	—	b b2·8067	—	♯ 7·0772			—
ATEURS	3	—	b ♯2·9532	—	= 7·0196			♯ 12·1036
	4	—	b ♯2·9532	—	—			—
	5	—	b b2·9386	—	—			b 11·9567
	6	—	b ♯3·0115	—	♯ 7·0541			♯ 12·0605
	7	—	b ♯2·9824	—	= 7·0196			♯ 12·0346
	8	b 2·0237	b ♯2·9970	—	b 7·0080			—
	9	b 1·9620	b ♯2·9970	♯ 5·0539	—			—
	10	b 1·9465	b b2·9240	b 4·9761	♯ 7·0657			—
	11	—	b b2·9240	—	—			—
	12	--	b ♯3·0261	—	—			—
h Series	1				♯ 7·0541	b b7 8202	—	—
lins	2				—	b b7·8312	—	—
ATEURS	3				♯ 7·0887	b ♯8·0174	♯ ♯ 11·1847	b 11·9566
	4				—	b ♯8·0174	—	—
	5				b 6·9617	b ♯7·9847	♯ ♯ 11·1120	♯ 12·0864
	6				♯ 7·1231	b ♯7·9847	♯ ♯ 11·1847	—
	7				♯ 7·0426	b b7·8053	♯ b 11·0115	b 11·9654
	8				♯ 7·1231	b b7·8312	♯ b 11·0206	—
th Series	1	b 1·9929	b ♯2·9532	b 4·9371	b 6·9965	b ♯8·1044	—	—
lin	2	♯ 2·0696	b b2·1121	♯ 4·9891	b 7·0080	b ♯7·9301	♯ ♯ 11·1302	—
FERRAND	3	= 2·0391	b ♯2·9824	♯ 5·0280	b 6·9841	b ♯7·9301	♯ ♯ 11·1607	♯ 12·0087
OFESSIONAL)	4	♯ 2·0545	b ♯3·0551	b 4·9241	b 6·9965	b ♯7·9410	—	
	5	= 2·0391	b ♯2·9824	—	—	—	—	
t		2·0391	3·1564	4·9805	7·0196	8·1369	10·8827	12·0
ıal		2·0	3·0	5·0	7·0	8·0	11·0	12·0
hagorean		2·0391	2·9487	4·9805	7·0196	7·9218	11·0978	12·0

TABLE VII.

LIST OF TRINES AND DUODENES IN ORDER OF FIFTHS.

The capitals point out the 48 tones in the middle section of Table the small letters are skhistic synonyms. See p. 675.

Name.	No.	Trine.			Name.	No.	Trine.			Name.	No.	
†Bb	1	††Ab	†C	†E	††A♯	1	††g♯	††b♯	†d♯♯			
†Eb	2	†Db	†F	†A	†D♯	2	††c♯	††e♯	†g♯♯			
†Ab	3	†Gb	†Bb	D	†G♯	3	††f♯	††a♯	†c♯♯			
Db	4	†Cb	†Eb	G	†C♯	4	††b	†d♯	†f♯♯			
Gb	5	†Fb	†Ab	C	†F♯	5	††c	†g♯	†b♯			
Cb	6	†Bbb	Db	F	†B	6	††a	†c♯	†e♯			
Fb	7	†Ebb	Gb	Bb	†E	7	†d	†f♯	†a♯			
Bbb	8	†Abb	Cb	Eb	†A	8	†g	†b	d♯			
Ebb	9	dbb	fb	ab	D	9	†C	†E	G♯	†C♯♯	9	††b♯
Abb	10	gbb	bbb	‡db	G	10	†F	†A	C♯	†F♯♯	10	††c♯
‡Dbb	11	cbb	ebb	‡gb	C	11	†Bb	D	F♯	†B♯	11	††a♯
‡Gbb	12	fbb	abb	‡cb	F	12	†Eb	G	B	†E♯	12	†d♯
‡Cbb	13	bbbb	‡dbb	‡fb	Bb	13	†Ab	C	E	†A♯	13	†g♯
	14	ebbb	‡gbb	‡bbb	Eb	14	Db	F	A	D♯	14	†c♯
	15	abbb	‡cbb	‡ebb	Ab	15	Gb	Bb	‡D	G♯	15	†f♯
	16	‡dbbb	‡fbb	‡abb	‡Db	16	Cb	Eb	‡G	C♯	16	†b
†E♯♯	17	†d♯♯	†f♯♯♯	†a♯♯♯	‡Gb	17	fb	ab	‡c	F♯	17	†E
†A♯♯	18	†g♯♯	†b♯♯	d♯♯♯	‡Cb	18	bbb	‡db	‡f	B	18	†A
D♯♯	19	†c♯♯	†e♯♯	g♯♯♯	‡Fb	19	ebb	‡gb	‡bb	E	19	D
G♯♯	20	†f♯♯	†a♯♯	c♯♯♯	‡Bbb	20	abb	‡cb	‡eb	A	20	G
C♯♯	21	†b♯	d♯♯	f♯♯♯	‡Ebb	21	‡dbb	‡fb	‡ab	‡D	21	C
F♯♯	22	†e♯	g♯♯	b♯♯		22	‡gbb	‡bbb	‡‡db	‡G	22	F
B♯	23	†a♯	c♯♯	e♯♯		23	‡cbb	‡ebb	‡‡gb	‡C	23	Bb
E♯	24	d♯	f♯♯	a♯♯		24	‡fbb	‡abb	‡‡cb	‡F	24	Eb
A♯	25	G♯	B♯	‡D♯♯						‡Bb	25	ab
‡D♯	26	C♯	E♯	‡G♯♯						‡Eb	26	‡db
‡G♯	27	F♯	A♯	‡C♯♯						‡Ab	27	‡gb
‡C♯	28	B	‡D♯	‡F♯♯						‡‡Db	28	‡cb
‡F♯	29	E	‡G♯	‡B♯						‡‡Gb	29	‡fb
	30	A	‡C♯	‡E♯							30	‡bbb
	31	‡D	‡F♯	‡A♯							31	‡ebb
	32	‡G	B	‡‡D♯							32	‡abb

TABLE VIII.

MANUALS FOR DUODENARY INSTRUMENTS. See pp. 676, 690.

The capital letters indicate broad and low, small letters narrow and high, digitals

Names of Duodenes with Synonyms.				No.	Digitals, containing the tones of the Duodenes displayed horizontally.											
					1	2	3	4	5	6	7	8	9	10	11	12
†Bb	††a♯			1	†C	†db	D	†eb	†E	†F	†gb	G	††ab	†A	†bb	††B
†Eb	†d♯			2	C	†db	D	†eb	††E	†F	†gb	G	†ab	†A	†bb	††B
†Ab	†g♯			3	C	db	D	†eb	††E	F	†gb	G	†ab	††A	†bb	††B
Db	†c♯			4	C	db	†D	†eb	††E	F	gb	G	†ab	††A	bb	††B
Gb	†f♯			5	C	db	†D	eb	††E	F	gb	†G	†ab	††A	bb	†B
Cb	†b			6	†C	db	†D	eb	†E	F	gb	†G	ab	††A	bb	†B
Fb	†e			7	†C	‡db	†D	eb	†E	†F	gb	†G	ab	†A	bb	†B
Bbb	†a			8	†C	‡db	D	eb	†E	†F	‡gb	†G	ab	†A	†bb	†B
ebb	D	†c♯♯		9	†C	‡db	D	†eb	†E	†F	‡gb	G	ab	†A	†bb	B
abb	G	†f♯♯		10	C	‡db	D	†eb	E	†F	‡gb	G	†ab	†A	†bb	B
‡dbb	C	†b♯		11	C	db	D	†eb	E	F	‡gb	G	†ab	A	†bb	B
‡gbb	F	†e♯		12	C	db	‡D	†eb	E	F	gb	G	†ab	A	bb	B
‡cbb	Bb	†a♯		13	C	db	‡D	eb	E	F	gb	‡G	†ab	A	bb	†B
	Eb	d♯		14	‡C	db	‡D	eb	†E	F	gb	‡G	ab	A	bb	†B
	Ab	g♯		15	‡C	‡db	‡D	eb	†E	‡F	gb	‡G	ab	†A	bb	†B
	‡Db	c♯		16	‡C	‡db	D	eb	†E	‡F	‡gb	‡G	ab	†A	‡bb	†B
	‡gb	F♯	†e♯♯	17	‡C	‡db	D	‡eb	†E	‡F	‡gb	G	ab	†A	‡bb	B
	‡cb	B	†a♯♯	18	C	‡db	D	‡eb	E	‡F	‡gb	G	‡ab	†A	‡bb	B
	‡fb	E	d♯♯	19	C	‡‡db	D	‡eb	E	F	‡gb	G	‡ab	A	‡bb	B
	‡bbb	A	g♯♯	20	C	‡‡db	‡D	‡eb	E	F	‡‡gb	G	‡ab	A	bb	B
	‡ebb	‡D	c♯♯	21	C	‡‡db	‡D	eb	E	F	‡‡gb	‡G	‡ab	A	bb	‡B
		‡G	f♯♯	22	‡C	‡‡db	‡D	eb	‡E	F	‡‡gb	‡G	ab	A	bb	‡B
		‡C	b♯	23	‡C	‡db	‡D	eb	‡E	‡F	‡‡gb	‡G	ab	‡A	bb	‡B
		‡F	e♯	24	‡C	‡db	‡‡D	eb	‡E	‡F	‡gb	‡G	ab	‡A	‡bb	‡B
		‡bb	A♯	25	‡C	‡db	‡‡D	‡eb	‡E	‡F	‡gb	‡‡G	ab	‡A	‡bb	B
		‡eb	‡D♯	26	‡‡C	‡db	‡‡D	‡eb	E	‡F	‡gb	‡‡G	‡ab	‡A	‡bb	B
		‡‡ab	‡G♯	27	‡‡C	‡‡db	‡‡D	‡eb	E	‡‡F	‡gb	‡‡G	‡ab	A	‡bb	B
		‡‡db	‡C♯	28	‡‡C	‡‡db	‡D	‡eb	E	‡‡F	‡‡gb	‡‡G	‡ab	A	‡‡bb	B
		‡‡gb	‡F♯	29	‡‡C	‡‡db	‡D	‡‡eb	E	‡‡F	‡‡gb	‡G	‡ab	A	‡‡bb	‡‡B

TABLE IX.

MR. BOSANQUET'S GENERALISED KEYBOARD. See pp. 692–9.

Fifths	2	9	4	11	6	1	8	3	10	5	12
10		BLUE					BLUE		BLUE		
9											
8											
7											
6											
5											
4											
3	WHITE		WHITE		WHITE			WHITE		WHITE	
2		†D♭	c♯♯♯	BLACK		WHITE	†G♭		†††A♭	g♯♯♯	BLACK
1											
12	b♯	10	†E♭♭	d♯	†F♭	e♯	32	f♯♯♯	41	†B♭♭	a♯
11											
10											
9											
8	†C	c♯	14	†E♭	23	†F	f♯	†A♭♭	g♯	45	†B♭
7											
6											
5											
4											
3											
2	5	D♭	d	18	e	27	G♭	36	†A♭	a	49
1											
12	c	9	D	†E♭	†E	f	31	g	40	†A	b♭
11											
10											
9											
8	C	d♭	13	e♭	22	F	g♭	G	a♭	44	B♭
7											
6											
5											
4											
3											
2	4	C♯	e♭♭	E♭	f♭	26	F♯	a♭♭	G♯	b♭♭	48
1											
12	B♯	8	‡D	17	E	E♯	30	‡G	39	A	A♯
11											
10											
9											
8											
7											
6											
5											
4											
3											
2	3	‡C♯	12	‡D♯	21	25	‡F♯	34	‡G♯	43	47
1											
12											
11											
10											
9											
8											
7											
6											
5											
4											
3											
2	‡B♯	7	‡C♯♯	‡‡D♯	‡D♯♯	‡E♯	29	‡F♯♯	38	‡G♯♯	‡A♯
1		BLACK					BLACK		BLACK		
12											
11											
10											
9											
8											
7											
6											
5											
4											
3											
2											
1	RED		RED		RED	RED		RED		RED	
12											
11				BLUE							BLUE

7 to 1	12 to 6	16 to 10	21 to 15	25 to 19	30 to 24	34 to 28	38 to 32	43 to 37	47 to 41	52 to 46

The numbers in the oblongs and in the lines at the bottom of the Table are assigned to the tones of Bosanquet's cycle of 53, in Table VI., B., col. Bosanquet, an distribution on this keyboard, which was invented for playing them. The capital le 48 skhistic tones, or the corresponding 48 tones in practically just intonation, formin section of Table V. The small *italic* letters are the 27 tones of mean temperame colours see p. 696. Mr. Bosanquet uses only white and black. See a woodcut of t in 'Proceedings of Royal Society,' vol. 23, p. 404.

TABLE X.

CHORDS ESTIMATED AS ARTIFICIAL QUALITIES OF TONE.

See Chaps. X., XII., and pp. 447–8, 454–5, 538.

This table is arranged for making experiments with the Just Harmonium (p. 681), shewing the effect of Mayer's laws (p. 701), which had not been published when pp. 286–297 were printed.

Part A. *Relative Pitch* shews the notes of the seven just major triads, and the dominants of five, on the Just Harmonium, with the relative pitch superscribed. For convenience some notes in the next Octaves above are given, distinguished by being underlined. These may generally be reached by taking the lowest note of the combination one or two Octaves lower. Thus 3, 15, 20 would be *g b″* [*e‴*], or *c′* [*e‴ a‴*], for which play *G b′ e″* or *c e″ a″*. By descending the greater harshness and loudness of the dissonant intervals 8 : 9, 9 : 10, 15 : 16, 24 : 25 are brought into play according to Mayer's laws. Each combination should be taken at all the pitches which the instrument allows, and the different effects noted.

Part B. *Beating Distances and Amplitudes.* The fifth column NP or 'number of partial,' gives 32 partials of *C*. In the first column PP or 'pitch of partial,' the names and vibrational numbers of such of these partials as have their primes on the instrument are given. The second column ES or 'equal Semitones,' gives the number of equal Semitones by which any one is distant from the prime, and hence, by subtraction, the interval between each; these intervals remain the same whatever be the pitch of the prime. Column BD or 'beating distance,' gives the number of equal Semitones which form the limiting interval for perception of beats of simple tones at that absolute pitch, and MD or 'maximum dissonance' the number of equal Semitones which form the interval for greatest harshness of these beats; the numbers MD are very nearly $\frac{5}{12}$ of the numbers BD. These numbers refer to absolute pitch, and are placed against the lowest tone of the beating combination. For intermediate tones take proportional parts as near enough. Hence, when a combination is taken at a different pitch of the prime, the pitch of the dissonant tones must be observed. Especially note that when that pitch is lowered by an Octave the dissonance is harsher for the same interval.

The columns headed by the letters *C*, *c*, *g*, &c., and figures 1, 2, 3, &c., contain the assumed 'amplitudes' of vibration for each partial tone in the compound tones bearing those names. The quality of tone is here assumed to be such that the amplitude of any partial is inversely proportional to its number. The amplitude of the prime of *C* is assumed to be 100, and the primes of all the other compounds are assumed to decrease in power, and hence in amplitude, as their pitch ascends, according to a rough estimate on an harmonium. Of course in separate instruments the

degrees of force might vary extremely, and the qualities also. H this is merely an example to shew how the calculations could be ma these details could be precisely given. The results differ greatly for e quality of tone.

Part C. *Artificial Qualities of Tone.* The columns NP give 'number of the partial' in the chord or artificial quality of tone produ by simultaneously sounding two natural qualities of tone. In the otl columns the numbers at the head refer to Part A, and give the relati pitch of the tones combined. The numbers against the partials shew t *intensities* or degrees of loudness, (not the *amplitude* or excursions of t particles of any of these artificial partials,) calculated by adding togeth the amplitudes in Part B of the corresponding partials of the compou tones combined, squaring the result, dividing by 100, and taking the qu tient to the nearest unit. Thus, on combining tones 3 and 4 in Part I we find the amplitudes of the partials against 12 in NP to be 23 and 2 their sum is 50, the square of which is 2500, and this divided by 10 gives 25 for the intensity of the 12th artificial partial of the combinatio of 3 and 4 in Part C. When an artificial partial is composed of a differ ential combinational tone only, D is written in its place without any esti mate of its intensity (always small), and it is not noticed when it merel increases the intensity of another artificial partial. These numbers giv approximately the relative intensities of the artificial partials, assumin that of the prime of $\dot{C}$ to be 100, and hence the artificial qualities of tone

The first column of the first series of combinations or *Fundamenta Qualities* gives the natural quality of 1, the second of 1 and 2, the third o 1, 2, 3, 4, or 1 to 4. These can be readily struck by the fingers. Fo the fourth combination 1, 2, 3, 4, 5, 6, 8, or 1 to 8, and the fifth 1 to 16 I find it convenient to drive nails at proper intervals into a lath in tw rows so as to press upon the digitals

C e g c′ e′ c″

d″ e″ g″ b″ c‴.

Placing them over the finger-board, the nails of the second row may b kept free of the digitals, or brought over them at pleasure, while the nai of the first row strike the notes in both cases. The effect of these qual ties of tone should be especially studied.

The second series of combinations are the *Dyad Qualities*, the lowe tone not being 1, and the effect of adding 1 to all these should be tried, tending to make the fact of an artificial quality of tone more evident supplying the prime, and as shewing the meaning of a 'fundamental ba by transforming these into fundamental qualities.

The third series of combinations are a few *Triad Qualities*, exclud those having 1, which should also be added to each subsequently to fo fresh fundamental qualities.

By selecting from Part A such examples of combination as contain F or †F in any Octave, the effect of *disturbed consonance* is shewn by replacing it by †F or F respectively. The beats in *F* †*F*, *f* †*f*, *f′* †*f′*, *f″* †*f″* increase in rapidity and diminish in force and in harshness; the beats in *F* †*F*, *F* †*f*, *F* †*f′*, *F* †*f″* are respectively as numerous as the last, and also diminish in intensity, but being disguised by the lower partials of F which do not beat, are less distinctly heard. The beats in *f″* †*f″*, *f′* †*f″*, *f* †*f″*, *F f″* are all equally rapid, but become more and more disguised by the non-beating partials. This is an excellent illustration of the meaning of partial tones and Mayer's laws.

By touching one of the digitals very lightly, so that the note only just sounds, we flatten the note very slightly, and hence produce a disturbance of the consonance less than by the alteration proposed, and obtain the effects of temperament.

By selecting combinations containing †B♭, †E♭, †A♭, or B, E, A in any Octave, and then using one for the other respectively, as *b♭* †*e′♭* *g′* for †*b♭* †*e′♭* *g′*, we obtain effects of discord, where the disturbances are greater than for temperament. Similarly we may interchange B and C, E and F, D and †E♭, G and †E♭, C and D♭, for the usual Semitone, which does not necessarily produce dissonance.

Part D. *Chords.* This has combinations of 10 major and 8 minor chords, either as triads or tetrads, the triads arranged in order of sonorousness, as given on pp. 333–335. To the simple forms the pitch of the first differential tones is added, to shew when they act as disturbing the chord, if not by producing beats, at any rate by introducing unrelated tones, which tend to obscure the quality of tone, and render it difficult to be seized by the ear. One 'letter form' of the triads is given, but all should be played with the different notes furnished for the numbers in Part A, and the triads being played first, the effect of the different 'duplications' introduced to complete the triads, in improving, and occasionally injuring the artificial quality of tone, by filling up gaps and strengthening artificial partials formerly weak, should be especially noted. The effect of adding the fundamental 1 in all cases in reducing the whole to a fundamental artificial quality of tone should also be observed. In the minor chords note that on account of the high number of the artificial partials there is more difficulty in appreciating them as artificial qualities embracing all the three tones, and that the effect in general is that of an artificial quality resulting from the *closest* or *lowest*, or *loudest* dyad, *disturbed* by the third tone. Thus in 12 . 15 . 20, we feel 4 . 5 disturbed by $\frac{20}{3}$; in 10 . 12 . 15, we feel 5 . 6 disturbed by $\frac{15}{2}$; and in 5 . 12 . 15, we feel 4 . 5 disturbed by $\frac{5}{3}$. This forms a transition to the disturbances of dissonances, for which the cases in pp. 538–543 are readily used in all the forms of Part A.

Part A. *Relative Pitch.*

1	2	3	4	5	6	8	9	10	12
C	*c*	*g*	*c′*	*e′*	*g′*	*c″*	*d″*	*e″*	*g″*
D♭	*d*♭	†*a*♭	*d′*♭	*f′*	†*a′*♭	*d″*♭	†*e″*♭	*f″*	†*a″*♭
†*E*♭	†*e*♭	†*b*♭	†*e′*♭	*g′*	†*b′*♭	†*e″*♭	†*f″*	*g″*	†*b″*♭
F	*f*	*c′*	*f′*	*a′*	*c″*	*f″*	*g″*	*a″*	*c‴*
G	*g*	*d′*	*g′*	*b′*	*d″*	*g″*	—	*b″*	*d‴*
†*A*♭	†*a*♭	†*e′*♭	†*a′*♭	*c″*	†*e″*♭	†*a″*♭	†*b″*♭	*c‴*	†*e‴*♭
†*B*♭	†*b*♭	†*f′*	†*b′*♭	*d′*	†*f″*	†*b″*♭	—	*d‴*	†*f‴*

Part B. *Beating Distances and Amp*

PP		ES	BD	MD	NP	*C* 1	*c* 2	*g* 3	*c* 4	*e′* 5
C	66	0	3·86	1·65	1	100				
c	132	12	3·08	1·35	2	50	90			
g	198	19·02			3	33	—	85		
c	264	24	2·92	1·23	4	25	45	—	80	
e′	330	27·86			5	20	—	—	—	75
g′	396	31·02	2·51	1·05	6	17	30	43	—	— 7
		33·69			7	14				
c″	528	36	2·45	1·02	8	13	23	—	40	— —
d″	594	38·04			9	11	—	28	—	— —
e″	660	39·86	2·28	0·95	10	10	18	—	—	38 —
		41·51			11	9				
g″	792	43·02	2·30	0·95	12	8	15	23	27	— 3
		44·41			13	7				
		45·69			14	7	13			
b″	990	46·88			15	7	—	17	—	25 —
c‴	1056	48	2·14	0·89	16	6	11	—	20	— —
		49·05			17	6				
d‴	1188	50·04			18	6	10	14	—	— 2
		50·98			19	5				
e‴	1320	51·86			20	5	9	—	16	19 —
		52·10			21	5	—	13		
		53·51			22	5	8			
		54·28			23	4				
g‴	1584	55·02			24	4	7	11	13	— 1
		55·63			25	4	—	—	—	15
		56·41			26	4	7			
		57·06			27	4		10	—	— —
		57·69			28	3	7	—	12	
		58·30			29	3				
b‴	1980	58·88			30	3	6	9	—	14 1
		59·45			31	3				
c⁗	2112	60	1·62	0·66	32	3	5	—	10	— —

Part C. *Artificial Qualities of Tone.*

Fundamental Qualities.					NP	Dyad Qualities. Better							Worse				NP	Triad Qualities.					
		4 to 1	8 to 1	16 to 1		3/2	5/2	4/3	5/3	5/4	6/5	8/5	12/5	8/3	10/3	16/5		5/3	5/4	6/5	8/5	6/5	8/6
1	3/1																	3/2	3/2	3/2	3/2	4/3	5/3
10	100	100	100	100	1	D	—	D	—	D	D	—	—	—	—	—	1	D	D	D	—	D	D
10	25	196	196	196	2	81	81	—	D	—	—	—	—	—	—	—	2	81	—	81	D	D	D
11	139	139	139	139	3	72	D	72	72	—	—	D	—	72	72	—	3	72	72	D	72	—	D
10	4	225	225	225	4	20	20	64	—	64	—	—	—	—	—	—	4	20	64	—	—	64	—
4	4	4	90	90	5	—	56	—	56	56	56	56	56	D	—	56	5	56	56	56	56	56	56
12	36	81	256	256	6	53	9	18	18	—	49	—	—	19	19	—	6	53	19	100	19	50	49
2	2	2	2	2	7	—	—	—	—	—	—	—	D	—	D	—	7						
16	2	58	199	199	8	40	5	16	—	16	—	42	—	42	—	—	8	5	16	5	42	16	42
1	15	15	15	98	9	8	—	7	8	—	—	—	—	8	8	—	9	8	8	—	8	—	—
8	1	8	44	146	10	3	31	—	14	14	14	14	14	—	30	14	10	25	—	31	14	14	14
1					11	—	—	—	—	—	—	—	—	—	—	D	11						
6	10	53	117	250	12	14	2	25	5	7	12	—	50	5	5	—	12	12	25	25	5	38	12
4	—	4	4	4	14	2	2	—	—	—	—	—	—	—	—	—	14	2	—	2	—	—	—
—	6	6	24	88	15	3	6	3	18	6	6	6	6	3	3	6	15	18	18	6	18	6	6
3	—	14	49	121	16	1	1	4	—	4	—	11	—	11	—	16	16	1	4	—	11	4	11
3	4	9	28	69	18	6	1	2	2	—	5	—	—	2	2	—	18	22	2	11	2	5	5
2	—	9	24	59	20	1	8	3	4	12	4	4	4	—	8	4	20	8	12	8	4	12	4
—	3	3	3	3	21	2	—	2	4	—	—	—	—	2	2	—	21	—	—	—	2	—	—
2	—	2	2	2	22	1	1	—	—	—	—	—	—	—	—	—	22	1	—	—	—	—	—
1	5	5	56	67	24	3	—	6	1	2	3	5	2	11	7	—	24	3	6	6	11	10	16
—	—	—	4	4	25	—	2	—	—	2	2	2	2	—	—	2	25	2	2	2	2	2	2
1	—	—	1	1	26	—	—	—	—	—	—	—	—	—	—	—	26	1	1	—	1	—	—
—	2	2	2	12	27	1	—	1	1	—	—	—	—	1	1	—	27	1	1	—	—	1	
1	—	5	5	5	28	—	—	1	—	1	—	—	—	—	—	—	28	—	1	—	—	—	—
1	1	3	21	76	30	2	4	1	5	2	8	2	2	1	7	2	30	5	5	12	5	8	8
1	—	3	12	29	32	—	—	—	—	1	—	3	—	3	—	4	32	—	1	—	—	1	3

Part D. *Chords.*

Triads.			Diff. Tones.			One Letter Form.			Duplications.										
									Major Triads.										
1	3	5	1	2	3	*c*	*g*	*c′*	2	3	4	5	2	3	5	8	2	3	5	6
1	4	5	1	1	2	*g*	*c*	*e*	3	4	5	6	3	4	5	8	3	4	5	10
1	5	6	1	3	4	*c*	*e′*	*g′*	2	4	5	6	2	5	6	8	2	5	6	10
1	5	6	1	1	2	*c′*	*e′*	*g′*	4	5	6	8	4	5	6	10	4	5	6	12
1	5	8	2	3	5	*g*	*c′*	*c″*	3	5	6	8	3	5	8	10	3	5	8	16
1	6	8	1	2	3	*e′*	*g′*	*c″*	5	6	8	10	5	6	8	12	5	6	8	16
1	4	10	1	6	7	*g*	*c′*	*e″*	3	4	6	10	3	4	8	10				
1	8	10	2	5	7	*g*	*c″*	*e″*	3	6	8	10	3	8	10	16				
1	5	12	1	7	8	*c*	*e*	*g′*	4	5	8	12	4	5	10	12				
1	8	12	3	4	7	*e*	*c′*	*g′*	5	8	10	12	5	8	12	16				
									Minor Triads.										
1	15	20	3	5	8	†*e′*b	*g′*	*c″*	12	15	20	24	15	15	20	40				
1	12	15	2	3	5	*c′*	†*e′*b	*g′*	10	12	15	20	10	12	15	24				
1	12	15	3	7	10	*c*	†*e′*b	*g′*	5	10	12	15	5	12	15	24				
1	15	40	3	25	28	†*e*b	*g*	*c″*	12	15	24	40	12	15	30	40				
1	15	20	5	9	14	†*e*b	*g′*	*c″*	6	12	15	20	6	15	20	30				
1	20	24	4	5	9	*g*	*c′*	†*e′*b	15	20	24	30	15	20	24	40				
1	10	15	4	5	9	†*e*b	*c′*	*g′*	6	10	12	15	6	10	15	20				
1	15	24	5	9	14	*c′*	*g′*	†*e″*b	10	15	20	24	10	15	24	30				

Additional Note on Professor Mayer's Researches.—Professor M has reprinted his researches, abstracted on pp. 700–703, in the 'Ph sophical Magazine' for May 1875 (4th ser. No. 326, vol. 49, pp. 352–3 with two woodcuts). In doing so he has given the limits of continuit tone as determined by the very sensitive ears of Mme. Seiler (p. 1 note 2) and her son, Dr. Carl Seiler, now in America. These have him greatly to extend some of the intervals for continuity of tone given p. 701, and those which he now assigns probably mark a very rare deg of auditory sensitiveness exercised under the most favourable circumstanc Even those on p. 701 are possibly greater than would be noticed ordinary ears, but they have been retained on pp. 795–8, as most genera useful. The interval of maximum dissonance varies still more from i dividual to individual, and the numbers on p. 701 probably mark average amount, although it is often much smaller. If deduced from t extended intervals, probably only 0·3 of the denominators mentioned p. 701 should be added, as calculated in the table below.

Keeping the previous notation, Professor Mayer's new results, i dicating an extremely rare amount of sensitiveness of ear, may be rep sented as follows, in the form adopted in both the tables and in t equation on p. 701:—

$$D \times (N + 31) = 3{\cdot}2 + {\cdot}0022 \times (N + 31).$$

Note.	N.	D.	Interval of Continuity in equal Semitones.	Interval of Max. Diss. in eq. Sem.	L.
C	64	1 + 26	5·00	1·92	2·5
c	128	1 ÷ 45	5 22	1·73	2·8
c'	256	1 + 70	4·19	1·37	3·6
g'	384	1 ÷ 102	4·08	1·33	3·7
c''	512	1 ÷ 130	3·92	1·27	3·9
e''	640	1 + 152	3·69	1·18	4 1
g''	768	1 ÷ 166	3·39	1 09	4·6
c'''	1024	1 ÷ 180	2·80	0·89	5·6
c''''	2048	1 ÷ 270	2·62	0·59	7·6

In the 'Philosophical Magazine' (ib. p. 358) Professor Mayer a mentions that on revolving the perforated disk (p. 700 above) t secondary tones are generated, proceeding continuously upwards a downwards from the tone of the fork as the velocity increases, till, a certain velocity, they are separated by a major Sixth, generating a d ferential tone. At that moment four tones are heard forming a ma chord with the relative pitches 1, 3, 4, 5; namely, 1 differential, 3 lo secondary, 4 fork, 5 upper secondary, tone. On increasing the velo the differential and secondary tones vanish, and the tone of the fork al is heard with perfect continuity.

INDEX.

The numbers refer to the pages, unless it is otherwise stated. Square brackets [ ] refer to the Translator's additions.

A

A (as *a* in *father*), its resonance cavity, 157, and pitch of the same, 158.
A, Ä, E, I, the resonance of their cavities forms an ascending minor chord, 162.
Ä (as *a* in *bat*), its resonance cavity, 159.
Ä, E, I, have double resonance cavities, 160.
A° (as *au* in *caught*), its resonance cavity, 158.
Abdul Kadir, his scale, 432. his intonation of the three first *Makamat* is Pythagorean, 433. his rules, 566.
Abrupt change of vibrational form implies sensible high upper partial tones, 54.
Absolute consonances, Octave, Twelfth and double Octave, 295.
[Absolute pitch, in use in different countries, 779.]
Accidental scales, defined, 410.
Accidentals, little used in old music, 377.
Accompaniments, difficult to invent, 375, but now a necessity, 450.
Acoustics, Physical and Physiological, 1, 4–5.
Actors ὑποκριταί, 363.
Age of violins, its advantage, 136.
Air chamber of violins, its effect on the quality of tone, 138.
[Air reed, of Schneebeli and Hermann Smith, 709.]
Algebraical sum of waves of water, 41, and of air, 43.
Ambiguity of minor chord, 454.
[Ambiguous scales of tonal modes, 460, note, 667, 757].
Ambrosch, History of Music, gives Chinese melodies, 398.

ARISTOTLE

Ambrose of Milan, his four authentic scales, 372, 416, are essential, 417.
[Amiot on Chinese music, 713 note, 784.]
Amplitude of vibration, 52.
Ampullae of the semicircular canals of the internal ear, 202. their supposed office, 217.
Analysis of any sensation, always difficult, 78.
Analysis of compound waves of water, 43.
Anche libre, or free reed, 144.
Ancient Greek songs had very small compass, 414.
Ansa, or Indian tonic, 372.
Antony's book on Gregorian Church Music, 365.
Anvil of ear, 192.
Apparatus for forming artificial qualities of tone, 176, 178, 604.
Appoggiatura, always a semitone, 440.
Appun, maker of conical tin resonators, 581.
Aquaeductus vestibuli, 202.
Arabesques in architecture, 388.
Arabic conquest of Persia, 430. music homophonic, 361. Quarter tones, 408. tonal system, 430, [661, 787]. tonal modes with just intonation, 433. with 8 tones, 434. [scales rearranged, 751. temperament, 651.]
[Arabo-Persian and Arabic systems compared, 435 note.]
Arabs refer musical tones to the elements and constellations, 347.
Architectural principles in Greece and Italy, 359, in the North, 360.
Archytas, his rules, 566.
Aristotle, his problems and *de audibilibus*, on music, asks why Octaves alone are sung, and that men's and boys' voices

form Octaves, 362. on the μέση, supposed to indicate the key note, 368, 412. on the greater euphony in descending than ascending, 370. on the relation of music to feeling, 385. on descending leading notes, 439.

Aristoxenus, in his time enharmonic scale disused, 407, 416. his circle of Fifths, 416.

Artificial Vowels, apparatus for, 175–178, 604. produced by tuning-forks, 179, and by organ pipes, 187.

Ascending scales, 421.

Attack and release of tones are characteristic, 106.

Audibility of beats, depends upon the magnitude of the interval, 259, [700].

Auditory hairs of crustaceans, 225, [of mosquito, 703].

Auditory passages, their natural resonances e'''' to g'''', 167.

Auditory sand, 203. its supposed office, 217.

Auditory stones, 206.

Auld Robin Gray, air, 402.

Authentic scales, 410. of Ambrose of Milan, 372, 417.

B

Bach, Emanuel, in 1753, required the pianos to be equally tempered, 500. says that a correctly toned piano has the most perfect intonation of all instruments, 503.

Bach, Sebastian, does not use minor chords at end of pieces, but only of preludes, 458. his cadences in the mode of the minor Sixth, 470. his 'Doric cadences,' 476. uses sharpened Thirds, 500. his upward resolution of the dominant Seventh, 548. often uses hidden Fifths and Octaves and false relations in his chorals, 562.

Barrow, his example of Chinese music, 400. [on Chinese music, 713 note.]

Basevi on intercalated tones in Italian music, 547.

Basilar membrane, hears musical sounds, 218. intonation of its different parts, 218. breaks easily in the radial direction, not in the other, 219. its vibrations mathematically investigated, 613.

Basilicæ, 359.

Bass notes with tinkling upper partials, 168, 271.

Bass shakes always rough, through defect in the ear, 214, 215, [702].

Bass voices, their tinkling, 168, 271.

Basses figured, 381.

Bassoon tongues, 145.

Bassoon, its partial tones, 148.

Baisch's violins, give good vibra forms, 136. pitch of their air bers, 137.

Beating distance of tones at diff pitches, according to Mayer, 701, 8

Beating stops on organ and harmoni 253.

Beating Tones, their variation in pi 625.

Beats, 6, 7. how distinct from combi tional tones, 240. their nature, 2 experiments on, with polyphonic sir 247. with fluctuating pitch, 248, 6 the law of their number, 248. render visible on sympathetically vibrati bodies, 249. shewn by vibrating flam 250, 581. on auditory nerves, 25 audible limit to their rapidity, 25 more than 30 in a second can be di tinctly heard, 252. 4 to 6 in a secon easily counted, 253. [cheap apparat for shewing, consisting of two pit pipes, 253, note.] imitated by oth intermittences of sound, 254. the mental effect due to physiological effe of intermittent excitement of the nerve 255. their roughness diminishes as t interval increases after a Semitone, 25 [701]. their discrimination when rap is difficult owing to the nature of se sation, 261. among the very highe partials of compound tones, 269. pr mote audibility, 270. of upper parti tones, 272. of mistuned consonance 275. their distinctness when arisir from upper partials, 279. of 33 in second, produced by all intervals different pitches, 291. of combin tional tones, 300. of differential ton of compound tones, 301. of combin tional tones of simple tones, 30 generally arise from combination tones, 309. calculations of their inte sity at different intervals, 626, [701 of combinational tones mathematical investigated, 629.

[Bedgebury Park echo, 721.]

Beethoven, master of instrumental, not vocal music, 511. his Doric cadenc 477. [writes for deep contrabasso, 70 writes for deep contrafagotto, 707.]

[Bell, Graham, his examination of t pitch of the double resonance cavities vowels, 161, note, 740.]

[Bell, Melville, his vowel arrangement resonance cavities, in 'Visible Spee 157, note. on effect of the pharyn vowel resonance, 727. on vowel p tion, 740.]

Bell-shaped glasses broken by singers, 62.
Bells, have inharmonic upper partials, 117. of glass, their proper tones, 118. of metal, their best proper tones, 118. when unsymmetrical give beating tones, 118. hand, in music, their tinkling detestable effect, 119.
[Berlin pitch, 779, 780.]
Bernouille, D., on motions of strings, 23.
Binocular vision, 97.
Blindness of one eye, unobserved for years, 96.
Blowing, effect of its force on tones of flute or flue pipes, 140, [708]. effect of various styles of blowing on quality of tone, 153.
Blythe, blythe, and merry, air, 401.
Boethius on tuning the lyre, 393.
Bois-Raymond, du, senior, his vowel arrangement, 157.
[Bosanquet's cycle of 53, p. 655, 673, 675, note. pitch of its tones, 764. practical intonation for, 657. his theory of the division of the Octave, 692, note. his generalised keyboard described, with theory, 692–9, 794. his notation applied to just intonation as a substitute for von Oettingen's, 695. exceptionally applied in duodenes to mark the imitation of the subminor Seventh, 698. his harmonium for the cycle of, 53. cost and maker, 696, note. his mathematical investigation of the combination of reed and string in the string organ, 717. the triads in his cycle of, 53, their character, 783.]
Bottle with long neck, has double resonance, 160.
Bottles with deep simple tones, blown across, how constructed, 102.
Bowed instruments, the piercing character of their tones, 315. why unfit for slow sustained chords, 315. their peculiar beats, 315. their scratchiness due to beats, which render their harmonies often unpleasant, 315. mobile and expressive, mark consonance and dissonance well, 315. how their quality of tone is affected by the resonance box, 318. the peculiarity of their intonation, 505. do not allow equal temperament in double stop passages, 506.
Bowing, its effect on even bad violins, 136.
Braes of Balquhidder, air, 402.
Brandt, proves correctness of Young's law for strings, 85.
Brass instruments commence heavily, 108.
Breguet watch-keys, their resemblance to articulation of hammer and anvil in the ear, 196.
[Brooks, makes Bosanquet's generalised keyboard, 696, note.]
Brumel's resolution of chord of dominant Seventh, 457.
Busalik, Arabic and Persian scale, 433, 751.
Büsürg, Arabic and Persian scale, 434, 751.
Byzantine architecture, 359.

C

Caccini invents recitative, 381.
Cadence, complete, plagal and imperfect, 452.
[Caldera's patent repeating hammer on pianos, 720.]
Canals, semi-circular, in the ear, 201.
Cancelling of sound by sound, 212.
Canonic imitation in the 12th cent., 374–5.
Catgut strings, their quality of tone, 128.
[Cavities producing vowel resonances, numerous, 727.]
[Cell, harmonic, 662. subdominant, tonic, dominant, 663.]
Changing notes, 547.
[Chappell, in his History of Music, says he cannot hear harmonics on a harmonium, 151, note. regards the Fourth as no consonance, 299, note. observes the dying away of women's voices in men's, 722.]
Charles II. and his court astonished at the *punctum caecum*, 97.
['Cheng,' Chinese, Amiot's name for 'shang,' 713, note.]
[Chevé, his table of modulations, 759.]
Chinese music, homophonic, 361.
Chinese scales, 398, [786].
Chinese examples of scale of five tones, 400, 402.
Chinese airs, some belong to the old Greek enharmonic system, 403.
[Chinese *Lu*, 713, note, 751, 786.]
[Chinese music, with scale of 12 Fifths, and equally tempered scale, 713, note. founded on the trigram *fou-hi*, 787.]
[Chinese Five-Fifths scales, 749, 786.]
[Chinese pitch pipe, its length, 786.]
Chladni, his sand figures, 65. his figures on elastic plates, 116.
Choice of instruments for a piece of music, 309.
Chord, defined, 35. on the second of the major scale, 455, [and note]. of dominant Seventh, its resolution in 15th century, 457. of the Italian Sixth, 478. of the diminished Seventh, 536, 542, almost insupportable in just intonation, 497. of the diminished Fifth, its use in marking the key, 528. of the dominant

Seventh, its dissonant tone, and resolution, 538. of the Seventh upon the Second of a major scale, its dissonant tone and resolution, 539. of the Seventh upon the Second of the minor scale, 540. of the Seventh on the Seventh of the major scale, its dissonant tone and resolution, 540. of the major Seventh in the direct system, 542. of the minor Seventh in the direct system, 542. of the Seventh on the tonic of the minor key, 543.
Chorda tympani, 195.
Chordal relationship in two degrees, 457.
Chordal resemblances, 575.
Chordal sequences, 551.
Chords, conditions of their consonance, 321. fundamental, of Third and Sixth and of Fourth and Sixth, 323. inversions, 324. consonant, exceeding the compass of an Octave, positions of, 329. of the natural Seventh, why not used in harmony, 330. of four parts, or tetrads, 336. their most perfect positions, 337–8. their independent significance not felt in the middle ages, 457. of the Sevenths, 530, their resolution, 555, of the direct system and reverted system, 531. of the reverted system mark the key precisely, 535. [estimated as artificial qualities of tone, 795.]
Chroai of the Greeks, 408.
Chromatic tetrachord, 404.
Chrysanthus of Dyrrhachium, Archbishop in 1832, rejects polyphonic music, 297.
Circular elastic plates, their proper tones, 117.
[*Clang*, its meaning in English, 36 note.]
Clarinet, its tongues or reeds, 145. its partial tones, has none even, 148, 152. peculiar effect of its quality of tone on consonances, 319. mutual effect of clarinet and oboe in playing consonances, 319. mutual effect of clarinet and harmonium in playing consonances, 320. its cylindrical resonance, 602.
Cochlea of ear, 201. its construction, 207. its supposed office, 217.
Cockle shells, air, 401.
Coincident upper partials in consonances, 277.
[Coll (Cavaillé) organ builder, does not use free reeds, 711.]
Colour and chiaroscuro, more easily observed by painters, 95.
'Colourings' of the Greeks, 408.
Combinational tones, 6, 7. depend on vibrations which are not infinitesimal, 229. proof that they are objective in polyphonic siren, 236. may exist objectively in the ear without existing

Thirds, 289. absolute, Octave, Twelfth and double Octave, 295. perfect, Fifth and Fourth, 295. medial, the major Sixth and major Third, 295. imperfect, the minor Third and minor Sixth, 295.
Consonant intervals calculated, 22. do not beat, 275. arranged in order of distinctness of their delimitation, 278. arising from mistuning other consonant intervals, 281.
Consonant triads, within the compass of an Octave, table of all their possible forms, 322. [in just and tempered intonation, examined, 781.]
Consonants, their nature, 107.
Continuous sounds, why heard as continuous, 221.
[Contrabasso, Dr. Stone's deep, 705. used by Beethoven and Onslow, 705.]
[Contrafagotto, Dr. Stone's deep, 706. used by Haydn, Mozart, Beethoven, Mendelssohn, 707.]
Cornu and Mercadier's experiments on violinists &c., 507, note. [observations on intervals actually played, 787.]
[Correlative decads, 667.]
Corti, Marchese, his membrane figured, 208. his organs, rods, or arches, 207, 208. figured and described, 209. with the basilar membrane they hear musical sounds, 218. they are absent in birds and amphibia, 218, 220. their probable office is to transmit the vibrations of the basilar membrane to the terminal appendages of auditory nerve, 221. their beats, 251. how they affect audibility of beats, 260. their damping power; is it less for high than for low pitch? not probable, 261. Helmholtz's theory concerning them has no connection with his explanation of consonance and dissonance, 343.
Coussemaker, history of harmony, 298, note. says that old German and Celtic melodies are like modern European, 373.
Crumples in vibrational forms of violin strings, 134.
Crustaceans, their auditory apparatus, 225.
[Culex mosquito, its sensitive hairs, examined by Mayer, 703.]
[Curwen, J., 'Standard Course,' treatment of mental effects, 429, note. on rejection of leading note in Wales and Scotland, 441, note. works on Tonic Solfa, 637. on registers of the voice, 729, note.]
Cutting tones, 173.
[Cycles defined and compared in octs, 653, and in logarithms, 654. of 30103, and 3010, p. 654; of 301, pp. 654, 655; of 53 and 31, p. 655; of 12, p. 656. of 29, p. 759.]
[Cyclic intonation, practical, 658.]
[Cyclic systems of uniform temperament, 653.]
Cylindrical tubes, their resonance, 602.
[Czermak, on whispering, 162, note.]

D

d'Alembert, his opposition of Rameau's theory of consonance, 351-3. his account of common French temperament in 1762, p. 500. his explanation of the old Greek heptachord, 546, note.
Damping, its rapidity for the sympathetically vibrating parts of the ear, 212. for *A*, about the second or third degree, 215. [at different pitches, determined by Mayer, 700.]
Dance music of 1529, oldest for instruments, 374.
[Decad, harmonic, 663. dominant, subdominant, relative, 666; correlative, 667.]
[Decadation, its nature and origin, 665.]
[Dechales says that Guido d'Arezzo discovered mean intonation, 784.]
Deepest audible tone? 264. answer, musical tone *E*, of the double bass, with 42 vibrations, 267.
Degrees of tones, definite, always felt to be necessary in music, 564.
Dehn treats the Fourth as a dissonance, 299.
Delezenne, his experiments shew that the best violinists play in just intonation, 506, 641. [his determination of pitch, 780.]
Déméter, hymn to, 371, note.
[De Morgan on tuning by equal temperament, 785.]
Descending leading note in old Greek Doric, 438.
Destruction of sound by sound, 292.
[Diapason normal, French, 780.]
Diaphony of Hucbald, 373.
Diatonic scale, 409, 566.
Diatonic tetrachord, 404.
Didymus's data for the diatonic tetrachord, 404.
Difference of pitch between exciting tone and sympathetically vibrating body, its effect on the sympathetic vibration, table, 213, 216.
Differential tones, or combinational tones, whose vibrational numbers are the differences of those of their generators, discovered by Sorge and Tartini, 230.

stronger than summational tones, 230. those of primes are loudest, 230. [experiments on, by translator, 231, note.] formerly believed to be subjective, and due to beats, 235. [experimental disproof by translator, 231, note.] table of, 231. examples of, 232. of compound tones, their beats, 301.
Diminished Fifth, chord of, its use in marking the key, 528.
Diminished Seventh, chord of, 536, 542.
Direct system, chords of, 531.
Discantus, first form of part music, 11th century, 373.
Dissonance, results from beats of upper partials, 295. its nature, 309.
Dissonances, their physiological roughness and psychological entanglement, 342. their frequency caused by equal temperament, 510. theoretically infinite in number, but practically limited by the scale, 515.
Dissonant chords as representing compound tones, 538. those which are essentially so, 548.
Dissonant intervals beat, 275.
Dissonant tone or note in a chord, 538. laws for its motion, 548. intruding on the body of other tones it moves diatonically, 548. rules for its introduction, 550.
Dissonating upper partials in the human voice, 168.
Distance, its effect on qualities of tone, vowels, and consonants, 111. [its effect in resolving compound tones, 722.]
Dominant chord, and its relation to the tonic chord, 451.
[Dominant decad, 666.]
Dominant Seventh, chord of, its dissonant tone and resolution, 538.
Donders first discovers that the cavities of the mouth are tuned to different pitches for the different vowels, 162. his determination of pitch of vowel resonances, compared with Helmholtz's, 163, note. [his phonautographic figures of R in his physiology of speech sounds, 254, note. on vowels, 740.]
Donkin's, Prof., 'Acoustics,' 585, 595, 600. [proof of Fourrier's theorem, 705.]
[Donkin, A. E., his machine for compounding parallel vibrations, 704.]
Doric scale of Greeks, in reference to Aristotle, 370, 411, [751]. of Glarean, 376, 413. tetrachord, 405. cadences, 475.
Double bass, its lower tone *E*, the lowest audible musical tone, 267, [705].
Double images in vision, 97.
[Double triolet, 689.]
Dove's siren, 20.
Driving machine for siren, 579
[Drouet's determinations of pit
Drum of ear, 191.
Drumskin of ear, 191. its
mechanical results, 200.
Drum gallery of the internal ea
Du Bois Reymond, senior, h
arrangement, 157.
Ductus cochlearis, 204.
[Duodenal, defined, its function, 6
[Duodenary manual described, 793.]
[Duodenary, small and great, 690.
[Duodenation, vertical and latera
its limits, Table IV., 759.]
[Duodene, how defined and limite
original, limits of the variation
pitch of its root, 670. how i
on Bosanquet's generalised key-
697, note.]
[Duodenes in order of Fifths, Table 792.]
[Duóni, two combined simplex
moniums, how arranged, 683.]

E

E (as *e* in *met*), its resonance cavity,
Ear, canine, sensitive to violin *e'''*,
human, how it apprehends qualiti
tone, 187. construction of, 190.
peculiar apparatus for sympat
vibration, 212. damping of its
pathetically vibrating parts, 213.
it resolves all motions into pen
vibrations, 222. peculiarly sens
to *f''''*♯, 270. natural resonances
passages, 167.
Ears differ in judgment of quality
ficially altered, 104.
Ecclesiastical mode, differed essent
from modern music, 380.
Ecclesiastical scales, of Glarean, 376.
Ambrose of Milan, 416.
[Echo, harmonic, 722–3.]
Egyptian origin of Pythagoras's lav
musical consonance, 2.
Egyptian flutes, and their scale accor
to Fétis, 416, note.
Elastic plates, proper tones when
cular, 117.
Electro-magnetic apparatus for for
artificial qualities of tone, 176,
driving machine for sirens, 579.
[Elements, harmonic, 662.]
Eleventh worse than Fourth, 297.
[Ellis, A. J., his notation of pitc
plained, 425. papers on music,
experiments on resolution of com

tones by distance, 722. on vowels, 740.]
Empty tones, 173.
Engel's vowel theory, 166, [732].
[English theoretical pitch, 26, 779.]
Enharmonic intervals, Greek, 547. tetrachord, 404. mode of the Greeks and Arabs, 407. Organ, Gen. T. Perronet Thompson's, 635. [its scale, 678].
Eolic mode, Greek, 411.
Eolic scale of Glarean, 376, 413.
[Equal semitones, their number in any interval, how calculated, 743. in the partial tones up to the 47th, table for finding, 745. logarithms of the first 12, to 22 places of decimals, 745.]
Equal temperament, its errors estimated too lightly through ignorance of just intonation, 498. now universal except for bowed instruments, 500. creates an 'awful din' on the harsh and mixture stops of an organ, 503. its four sources of beats and their effects, 501. [calculated, 650, 745. how realised, 659. reduces duodenations to alterations in the order of the tones in one duodene, 669 note. difficult to tune, 784. practically imitated, 785. known in China 200 years before it was known in Europe, 713, note, 784.]
[Equal triads, their character, 784.]
Equally tempered chords, their defects, 497, [784.]
Eratosthenes's data for the older chromatic tetrachord, 404.
Erfurt cathedral bell, 118.
Erse avoid seven tones in the scale, 398.
Essentially dissonant chords, 548.
Essential scales, 411.
Este's book of Psalms, 442.
Esthetic analysis of music, 569. law of the unconscious regularity of works of art, 569–571.
Esthetical and physical methods, 357.
Esthetics, scientifically considered, deal with the psychological, 358.
Esthonians reject leading note in minor scales, 441.
Etruscan architecture, 359.
Euclid, his definition of consonance and dissonance, 342 and note, [660, note]. his scale, 414.
Euler, his investigations, 23, 24. his theory of consonance and dissonance abstracted, 347. where defective and how differing from Helmholtz's, 350.
Europeans perhaps learned music from the Arabs at time of crusades, 437.
Eustachian tube, 191.
Even partial tones not so easily heard, 79.
Expressive instruments, 309.
Expression, dramatic, μίμησις, 363. in music different from that in speech, 577.
Extended intervals, when improved or worsened, 294.
Eye as affected by alternation of excitement, 256. compared with ear, as to power of appreciation of rapid intermissions of sensation, 262. [compared with ear and hand, in rapidity of recording observation, 262, note.]

F

"*f* holes" of a violin, 137.
False minor triad, 527, [and note].
'False relations,' what they are, and why they are forbidden, 562.
Farabi, his scale, 432. taught Greek music to Arabs and Persians, 437.
[Farmer, his experiments on moving strings by wind, 714.]
Feelings, best depicted by music, 385.
Fenestra rotunda et ōvālis, 191, 201.
[Ferrand, violinist, the intervals he played, 787.]
Fessel's resonance tubes, their dimensions, 585.
Fétis conjectures a Hebrew origin to cadences of reading, 365. says that old German and Celtic melodies are like modern European, 373. finds scales with intervals of a tone or a tone and a half among Chinese, Malays, Papuas, &c., 396–7.
Fifth, its nature, 285. a perfect consonance, 295. determined occasionally by the combinational tones only, 305. found in all musical scales, 389. not so pure a repetition as the Twelfth, 302. justly intoned, imperfect, false, and superfluous, examined, 521–2.
Fifths, consecutive, why forbidden, 557–560. not naturally disagreeable, 559. do not arise when consecutive chords are closely connected, 560.
Fifths, hidden, meaning of their prohibition, 561.
Figured basses, 381.
Fine wire has numerous upper partials, 127.
[Fischer's determination of pitch, 780.]
[Five-Fifths scales, 661, 749.]
Flörcke, on speech, 162, note.
Flue or Flute Pipes, their musical tones, 138. wood and metal, described and figured, 139. pipes speak quickly, 143. [on their source of tone, 709.]

Fluid of the labyrinth of the ear, 202, 205.
Flute concerto, joke upon, 311.
Flute performances at Pythian games, 362.
Flutes, Greek αὐλοί, like oboes, 362, note.
'Forbidden' progressions, meaning of, 554.
Force or loudness, 16, 17.
Form of vibration. See Vibrational Form.
Forte and *Piano*, on an organ, cannot be produced by stronger blowing, 143.
Fortlage notes Mozart's use of 'Doric cadence,' 477.
[*Fou-hi*, Chinese sacred trigram, on which their musical intervals are founded, 787.]
Fourier's law or theorem enunciated, 52, 54. [references to its demonstration by Donkin, O'Kinealy, and M. Ohm, 705.]
Fourth, a perfect consonance, 295. not admitted as a consonance, because of the progression of parts, 298. [Chappell's views on, 299, note.] determined occasionally by combinational tones only, 305. why unsuitable in the closing chord, 448. justly intoned, imperfect, false, and diminished, examined, 521–2.
Fourths found in all musical scales, 390.
Four-times accented Octave, sensitiveness of ear for, 169.
Franco of Cologne includes the Thirds as imperfect consonances in 12th century, 289. his arrangement of consonances, 298.
Free reed, 144. [how Helmholtz's belief concerning their present disuse in organs probably arose, 711. found first in the Chinese 'shang,' 713, note.]
French horn confused with the human voice at a distance, 111. its scale, 149.
French pitch, 26, [779].
Full tones, 173.
Fundamental bass of a chord defined, 479.

G

Gabrieli, a contemporary of Palestrina, 379. his treatment of chords, 457.
[Gaelic Five-Fifths scales, 749.]
Gaels avoid seven tones in the scale, 398.
Galileo, 23.
Galin-Paris-Chevé system, 638, note.
[Galvanism, its effect on pitch of strings, 708.]
Geigenprincipal, organ stop explai 141, [and note].
Gemshorn, organ stop, 142, [and note]
[Generic and specific vowel qualit 730.]
[*Gentle Zitella*, air noted for experim on vowel resonance, 737, note.]
Gerbert on medieval ecclesiastical mu cians, 298, note.
[German and English habits of speec compared, 107, note.]
[German pitch, 26, 779.]
Glarean, his *Dodecachordon*, 298, 376, 418. his six authentic ecclesiastical modes, 376, 418. attributes different keys to the tenor and bass, soprano and alto, in a composition, 376. his names of ecclesiastical modes or scales contrasted with the Greek, 418.
Glass bells, proper tones of, 118.
Gleitz on the Erfurt bells, 118.
Glottis is composed of membranous tongues, 147.
Goethe's talent for observation, 95.
[Got, pronounces *oui* audibly without voice, 110, note.]
Goudinel, master of Palestrina, 379.
Graham's 'Songs of Scotland' gives Gaelic melodies, 398.
[Grave harmonics, an inapplicable name for differential tones, 230, note.]
Greek architecture, 359.
Greek, music, ancient and modern, is homophonic, 361. epic melody, 366, [and note]. ancient airs ended on dominant, 371. tetrachords, 403. Quarter-tones, 408. 'colourings,' 408. ear more cultivated than ours for delicate distinctions of intervals, why? 409. scales of eight-stringed lyre, 411. songs, ancient and modern, have very small compass, 414. scales, later, the principle of their nomenclature, and list of their names, 415. [Seven-Fifths scales, 750.]
Greeks did not acknowledge Thirds as consonances, 297, [660, note]. had a feeling for tonality, 371.
Gregory the Great, settles the Roman liturgy, 365. his *cantus firmus* had no division of time, 374, 379. his plagal scales, 417.
[Grenié's free-reed pipe, 699.]
Guadanini's violin, gives steadiest vibrational forms, 136.
Guéroult (author of French translation of this work), his explanation of increased pitch in beats, 248, 625. [his double manual harmonium, simpler experimental instrument, its tona system, 682. his harmonium re-ar

ranged for one manual, with 4 stops, 687. his compass, how it can be played on Bosanquet's generalised keyboard, 697. he considers that the players observed by Cornu and Mercadier intended to play in equal temperament, 788.]
Guido d'Arezzo, his hexachord, 545–6. [is said to have discovered mean intonation, 784.]
Guitar, strings, 120.
Günther's pianofortes, 123.
Gut strings, their quality of tone, 128.

H

[Hamilton, J. Baillie, his experiments on moving strings by wind, as in the string organ, 714. his feeling of the relative advantages of pipes and strings, 715, note.]
Hammers, for harmonicas, soft, 116. sharp-edged; hard, 121. soft, 122.
Hammer of ear, 192. its articulation with the anvil like Breguet watch-keys, 196. the effect of this articulation on combinational tones, 197. its contact with anvil loosened when air is forced into the drum, and its effect on hearing weak and strong tones, 197. its clicking over anvil in ear, 238.
Hammer-struck strings, 121.
Hammer-Purgstall (Baron von), on Arabic music, 432.
Hāmulus of ear, 203.
Handbells in music, their tinkling detestable effect, 119.
Handel, his treatment of closing minor chords, 455. his close in minor chord, and chord without Third, or with major Third, 458. his guarded use of the minor close, 466. his Doric cadence, 475.
Hanslick on the musically beautiful, 2.
Harmonic, upper partial tones, 6, 7, 33. subtones, 70. tissues result from esthetical principles, 358. tissues not known till 16th century, 375–6. music, 361, 377. [scales, their principle, 661. elements, 662. cell or unit of concord, 662. heptad, or unit of chord-relationship, 662. decad, or unit of harmony, 663. trichordals and their scales, 664. hepta-decad, or unit of tonic decadation, 666. duodene, or unit of duodenation, 668. echo, 722–3. curves drawn by Mayer, 703. curves described by Donkin's machine, 704.]
Harmonic more lively than melodic impressions, 566.
Harmonic reaction on the construction of scales, 568.
Harmonicas of glass and wood, their quality of tone, 115.
Harmonics, usually so called [distinct from upper partials, 37, note]. not partial tones, but representing them, 73, 80. on piano strings, 81, 83, 85. [grave, old name for combinational tones, 230, note. of *C*, how to note, 642.]
Harmoniousness of intervals, shewn graphically, 292.
Harmonisation, the only point in which modern excels ancient music, 479.
Harmonium gives powerful differential tones, which are partly formed in the ear, 236. justly intoned, with single manual, as proposed by the author, 633. [as proposed by the translator. 686. just, simplex experimental instrument, its tonal system, 681.] vibrators described and figured, 144.
Harmoniums owe their defects to equal temperament, 504. their peculiar combinational tones, 632.
[Harmony, its four kinds, 660.]
Harp strings, 120.
[Haseneier's construction of Dr. Stone's contrafagotto, 706.]
Hasse's researches on basilar membrane, 218, 220.
Hauptmann, wrong in allowing a Pythagorean *a* in the ascending minor scale of *e*, 423, note. his notation of pitch, 424. his reason for the old avoidance of closing with minor chords, 456. his settlement of the intonation of the intercalary tones, 462. his minor-major mode, 472. his chords of the direct system, 531. his chords of the reverted system, 531. his theory of the prohibition of consecutive Fifths, 560.
[Haydn, writes for deep contrafagotto, 707.]
Heidenhain's *tetănomōtor*, 207.
Helicotrēma, 203.
Hellwag, on formation of speech, 162, note.
Helmholtz, his nomenclature for the modes, 413. [his double manual harmonium, simplex experimental instrument, its tonation system, 682. his complex harmonium with one manual, re-arranged for four stops, 686. his compass, how it can be played on Bosanquet's generalised keyboard, 697. his physiological theory of audition shewn in a 'working model' by Mayer, 700. his theory of the source of tone of the pipes, 708. his vowel theory, its dis-

tinctive character and phonetic value, 739–40.]
[Helmholtzian or skhistic temperament, 652.]
[Hemitonic temperament, 650.]
Hemony of Zütphen's determination of good tones in bells, 118.
Henle, on the basilar membranes, 219.
Henrici takes the upper partial tones of tuning-forks an Octave too high, 105.
Hensen, his figure of the inner zone of the *membrāna basilāris*, 208. his researches on basilar membranes, 218. his observations and experiments on the auditory apparatus of crustaceans, 225.
[Hephtharmonium, complex harmonium for playing in the heptadecad of C, 687.]
[Heptad, harmonic, 662.]
Herschel, Sir J., on Willis's experiments, 170.
[Herschel, Capt. J., his recurrent or skhismo-cyclic intonation, 658.]
Hexachord of Guido d'Arezzo, 645–6.
[Hexharmonium, complex harmonium for playing in six decads, and shewing double forms of the minor chord on the second of the major scale, 688.]
Hidden Fifths and Octaves, meaning of their prohibition, 561.
Hidschaf, Arabic and Persian scale, 434, 751.
High tones, heard in the *cochlea* of the ear, nearest to the round window, 220. excite more powerful sensations than low tones, with same *vis viva*, 264.
[Hill, organ builder, never uses free reeds, 711]
Hissing consonants, 109.
[Hoang-ty, Chinese emperor, when the 12 *Lu* were invented, 786.]
Hoarseness, whence it may arise, 154.
Holes, side, effect of on wind instruments, 153.
Hollow, quality of tone, 172.
Homophonic music, 361.
Horns, French, their tones and upper partial tones, 149.
Hucbald, 10th century, invents part music, called *organum* or diaphony, 373.
Hullah's pitch, 26.
Husseini, Arabic and Persian scale, 434, [751].
[Huyghens, his *Cyclus Harmonicus*, 655. pitch of its tones, 764. examination of its triads, 784.]
Hypo-Doric mode, Greek, 411, [750].
Hypo-Lydian mode, Greek, 411, [750].
Hypo-Phrygian mode, Greek, 411, [750].
Hypothesis of the sympathetic vibration of parts of the ear, sufficient to explain its apprehension of quality of tone, 190.

I

I (in *machine*), its resonance cavity, 164.
[Imitation of equal temperament for practical use, 785.]
Imperfect consonances of Franco of Cologne were the Thirds, 289. are the author's minor Third and minor Sixth, 295.
Incus of ear, 192, 196.
Indian music homophonic, 361. tonic or *ansa*, 372. melodies, like modern European, 373.
Indirect tetrads, 531.
Instrumental tones characterised by peculiar noises, 111.
Intensity of tone, how measured, 122.
Intercalary tones, their intonation undetermined, 427.
Intercalary notes, their intonation, 461.
Intercalated chromatic notes the sole remnants of attempts at graduated intervals, 565.
Intercalation of tones in Italian music, law for, 547.
Interference of sound, and its distinction from beats, 240. from two organ pipes on the same portvent, 242. from the two prongs of tuning-forks, 242. experiments on, with polyphonic siren, 246.
Intermittent sounds resembling beats, 254.
Interval, breadth of, how it affects the audibility of beats, 258.
Intervals, beating 33 in a second, 291. occasionally determined by beats of combinational tones, 308. effect of transposing their tones, by extension, 330. of definite extent are found in the music of all times and nations, 389. consonant, dissonant, and tempered with their ratios and degrees of roughness, enumerated, 516–519. the most clearly felt are not the most attractive, 575. [new notation for, 644. how to calculate from Tables IV. and V., 645. expressed as numbers of tempered Fifths, in commatic and skhismatic temperaments, 646. named, and expressed by numbers of equal Semitones, 646, 746. smallest distinguished, by Mayer, ·0087 equal Semitones, 699. table of their value, nomenclature and notation, 746. numbers of (less than Thirds), contained in an Octave, 748.]
Intonation of different parts of basilar membrane, 218. [Just and Tempered, tabulated, 761–778.]
Inversions of chords, 324.

Inwards and outwards striking tongues, 146.
Ionic scale and mode of Glarean, 376, 418. Greek, 411.
Irak, Arabic and Persian scale, 434, [751].
Irish scales, 398.
Irrational Tetrachords, 406.
Isafahan, Arabic and Persian scale, 434, [751].
Italian architecture, 359.
[Italian Opera, London, pitch, 780.]

J

Jean de Muris on consonances, 298
[Jennings, builder of Bosanquet's harmonium, estimates cost for harmoniums with similar keyboards, 696, note.]
Joachim, found by Helmholtz's experiments to play in just intonation, 507
Josquin's plagal cadence, 457
Just intonation, why attempts at it have hitherto failed, 501. in singing, 635. can be carried out in singing, as proved by tonic solfaists, 640. is natural for uncorrupted ears, is really observed, and is easier than tempered intonation, 640. [unequally and practically, 656. the sole foundation of the theory of harmony, 659. found universally in harmony of unfixed tones by Cornu and Mercadier, 788.]
Justly intoned chords, how they differ in effect from equally tempered and Pythagorean chords, 497.
[Just and tempered intonation tabulated, 761–778.]
[Just triads, their character, 783.]

K

Kaim's pianofortes, 123.
[Kastner, Fr., his *pyrophone*, or instrument formed with singing flames, 720.]
[Kaufmann's resined cylinder pressed against strings, 720.]
[Kempelen on vowels, 740.]
Keppler's musical imaginings, 347.
Kettle-drum, its proper tones, and tuning, 119.
Keynote, or tonic, 368.
Kiesewetter, inclined to deny what does not fit into the major and minor scale, 361. on Arabic music, 431, note. attributes Persian scale of 12 semitones to the Christian missionaries, 436. [prevented by equal temperament from thoroughly understanding Arabic music, 661.]
Kircher, Athanasius, finds the microcosm musical, 347.
Kissar, or five-stringed lyre of North Africa, 307.
['Klang' used for 'compound tone' by Helmholtz, 36 note, 90 note.]
[Knox's Psalter, 442, note.]
Koenig, his two large tuning-forks, with 24 to 61 vibrations, could scarcely be heard for 28 vibrations, 268, note. makes the author's resonators, 580. his manometric flames, 581. [determination of vowel resonances, 724. his forks for vowel resonances experimented on by Sedley Taylor, 734. on vowels, 740.]
Kölliker estimates that there are 3,000 of Corti's arches, 221.
[Kratzenstein on vowels, 740.]

L

L, its trill, 110.
Labyrinth of ear, 190. described, 201.
[Ladd sells Koenig's apparatus, 580.]
Lāmina spīrālis, 204.
Larynx and its membranous tongues, 146.
Lateral decads, 666.
[Latin epic melody, 367, note.]
'Leading note,' 437. existed in old Greek Lydian, 441.
[Léonard, violinist, the intervals he played, 787.]
[Lewis, organ-builder, never used free reeds, 712.]
Lichaon of Samos forms scale of disjunct tetrachords, 409–10.
[Liddle's pitch-pipes for beats, and fifes for differential tones, 253, note.]
Ligamentum mallei anterius, 194. *posterius*, 195.
Limits of audible pitch, 27.
[Lindsay, Lord, construction of string-organ in his laboratory, 717.]
[Ling-Lun arranges Chinese scale of 12 *Lu*, 786.]
[Linking of chords together, 552.]
Lips are membranous tongues, 147. their action in sounding brass instruments, 147.
Lissajous's mode of observation, 128. [his curves, mechanically described by Tisley's machine, 704. his determinations of pitch, 780.]
[Liston's organ, its tonal system, 676.]
Locrian mode, Greek, 411.
[Logarithms converted into numbers of equal semitones, 743. for musical

poor, rich, dull, bright, &c., 112. its general character, 172.
Musical tone, exemplified, 11. periodic, 12. differing by force, pitch and quality, 16. less marked in speaking than in singing vowels, 110. its quality, 172.
Musical tones without upper partials, 113. with inharmonic upper partials, 114. of strings, 120. of bowed instruments, 128. of reedpipes, 143. [new, 720.]
Musical vibrations visible and tangible, 13.
My Peggie is a young thing, air, 401.
Mysis (opossum shrimp) retains its hearing after its otoliths have been extirpated, 225.

N

N, its resonance cavities, 169.
Nasal quality of tone, due to absence of even partials, 173.
Nasals, as M, N, heard further than consonants, 111.
'Natural' does not imply beautiful, 352.
Natural chords often dissonant, 352.
Natural Seventh, why intervals involving it are not used as consonances, 296. why not used in harmony, 330. [chords of 534, note.]
Naumann on Hauptmann's Hegelianism, 424, note.
Navel of drumskin, 200.
Neidhard, one of the inventors of the equal temperament, 500.
Nerve of cochlea, its expansion, 207.
Nerves, their analogy to telegraph wires, 224.
Nerves of the vestibule, their terminations described, 206.
Newa, Arabic and Persian scale, 433, [750, 751].
Newton's investigations of the motion of strings, 23.
NG, its resonance cavities, 169.
Nicomachus on the comparison of the seven tones with the seven planets, 369, [and note]. quotes Philolaus on the divisions of the six-toned scale, 397.
Nodal lines on circular membranes, 65.
Noise exemplified, 11; non-periodic, 12.
Noises, irregular but characteristic, accompanying tones of musical instruments, 101, 109. they disappear at a distance, 111.
[Noises accompanying vowels are more marked in German than Englia note.]
Nomoi, melodies, themes, 363.
Northcote, Miss, blind organist of T. Perronet Thompson's Enhar Organ, 636.
Notes, musical, 7.
[Nucleus of decad, 663.]
[Number of just tones required for dern modulation, Table IV., 759.]

O

O (as the *o* in *more*), its resonance ca 157, and its pitch, 158.
O^a (nearly as the *o* in *cot*), its reson cavity, 158.
Objective combinational tones of p phonic siren, 236.
Objective existence of upper par tones and colours, 76.
[Objective analysis of compound sou 699.]
Oboe, its tongues or reeds, 145. its p tial tones, 148. Its canonical res ance, 603.
[Octs defined, 653.]
[Oc'taval harmony, 660.]
Octave, difficulty in distinguishing t Octave in which a vowel is sung, note played, 104. its nature, 2 and double Octave absolute cons ances, 295. found in all musi scales, 389. produces no partials heard in lower tone, 390.
Octaves, consecutive, why forbidden, 5 hidden, meaning of their prohibitio 561.
Odes sung in Greece, 363.
Oettingen, A. von, his notation of pit introduced, 425. his report of t Esthonian rejection of the leading no 441. carries out the analogy betwe the major mode and mode of min Sixth, 478. his phonic system musical composition, 567. [for pri ing purposes his notation may be co veniently replaced by Bosanquet's, 69
Ohm (G. S.), his law, 34. enunciat 51. and proved by experiment, his experiment with two violins, vari for two bottles, 102, 103. his te 'illusion of the ear' objected to Seebeck, 103.
[Ohm, M., his proof of Fourier's theore 705.]
[O'Kinealy's proof of Fourier's theor 705.]
Olivier, on speech, 162, note.
Olympos transformed the Greek D

into the old enharmonic, 397. his ancient enharmonic tetrachord, 404.
[Onslow writes for deep contrafagotto, 705.]
Opera, the most active cause in the development of modern harmony, 381. arose from a desire to rehabilitate Greek tragedy, 381.
Opossum shrimp (*mysis*), its auditory hairs, 225.
Order of harmoniousness of consonances independent of key or modulation, 297.
Organ, its pipes on the same portvent, their interference, 242. its flute pipes, do not define harmonics well, 311. its narrow stopped pipes, not good for the mass of harmony, 312. [its principal work explained, 141, note.] its principal work good for harmonics, 311. its reed pipes, 144. its reeds, described and figured, 144. its reed pipes and compound stops, only fit for accompanying a congregation, 312. its wide stopped pipes, unsuited for combinations of harmony, their use, 310, 311.
[Organ, the string, 714.]
Organ stops, *compound*, 143. *cornet*, 91. *Geigen principal*, 91, 141. *Gemshorn*, 142. *principal*, 91. *quintaten*, 50, 142. *Rohrflöte*, 142. *Salicional*, 142. *Sesquialtera*, 91. *Spitzflöte*, 142. *viola di gamba*, 141, [710]. *violonbass*, 141. *violoncello*, 141. [*violone*, 141, note.]
Organum or diaphony of Hucbald, 373.
Os orbiculāre of the stirrup of the ear, 198.
Oscillations, 12.
Ossicles of ear, 192.
Otoliths of ears of fish, 206. of ears of crustaceans, 225.

P

P, T, K, German and English, 107.
Palestrina, by the less perfect positions of chords attains an expression now coarsely given by discords, 340, 457. his choice of chords in *Stabat Mater*, 341, 379. commissioned to remodel Roman Catholic Church music, 378.
Parhypatē, a kind of descending leading note to the hypatē, 370.
[Paris pitch, 780.]
[Paris prize choir, how affected by a piano accompaniment, 314. note.]
Part music, its invention, 373.
Partial tones defined, 34. why so difficult to distinguish by the unassisted ear, 101. of strings plucked or struck by a hammer, table of their theoretical intensity, 126. [first 32 of C_r how to write, 642. pitch of the first 47 of C reduced to the Octave c' to c'', 766–778.]
Passing notes, 547.
Paul, conjectures that Hucbald's part music consisted of canonical responses, 373.
Pendular vibrations, 34 and note.
Perception, the mental image of external objects, deduced from sensation, 94. when simple, often depends on complicated sensations, 99.
Perfect consonances, Fifth and Fourth, 295.
Peri invents recitative, 364, 375, 381.
Periodic motion defined and exemplified, 12, 13.
Periodic motions combine to form new periodic motions, 46.
Periodicity is independent of form, 48.
Persian scale of 12 semitones, 436.
[Persian scales re-arranged on the principle of successive Fifths of which some are flattened, 751.]
Persian tonal system, 430, [751].
Petrous bone, in which the labyrinth of the ear is embedded, 202.
Pflueger's Archiv, 198, note.
Phases, effect of altering, 48. differences of, 52. do they affect quality of wire? 174. answer, No, 184, 186. their relation to loudness of resonance, 182. made visible, 185. of waves caused by resonance, 608.
[Philharmonic pitch, 780.]
Philipp de Vitry, on consonances, 298.
Philolaus quoted by Nicomachus on the divisions of the six-stringed lyre, 397.
Phrygian tetrachord, 405.
Phrygian mode, Greek, 411, [750]. Glarean's, 376, 413.
Phrynis's nine-stringed lyre, 414.
Physical and esthetical methods, 357.
Physics, scientifically considered, deal with the technical, 358.
Physiological effect of rapid intermittences of sensations, 254, 255.
Pianofortes, why the upper tones need but few partials, 317. peculiar nature of its quality of tone allows of dissonances which would be very harsh on other instruments, 317. and favours equal temperament, 503.
Pianoforte hammers, their nature, difference, and effect on tone, 123.
Pianoforte, its echo of vowels, 104, [189 note].

Piano-strings, their upper partials and harmonics, 81, 83, 85. where struck, 120, 124. their quality of tone examined by plucking and striking, 122. their vibrational forms mathematically investigated, 589.
Pitch depends on vibrational period or number, 17, 18.
Pitch, or relative height of tone, 16, 17. of membranous tongues, how altered, 147. of resonance cavities of mouth, how determined by Donders, 162. when different heard by different nerve fibres, 222. alters by intervals, and not continuously, 384. [notation of, for just intonation in ordinary typography, 642, 760. and in musical typography, 643. absolute, noted, 643. how to calculate, 644. table of its notation for just intonation, 742. table of its values in different intonations, 766–778. of *c'* and *a'* in use in different countries at different times as standards, 26, 643, 779.]
Pius IV. orders change in ecclesiastical music, 378.
Pizzicato, 120. very penetrating, 108.
Plagal scales, 410. of Pope Gregory, 417. cadence, 452.
Playford's 'Dancing Master,' 401.
Plectrum, 121.
Plucked strings, 121. their motion mathematically investigated, 582.
Plutarch, *dē Mūsicā*, 362. dwells on small compass in Greek music, 403.
Politzer's insertion of glass tube into the round window of the ear, 202.
Polyphonic music, 361, 373. closed on tonic note, 447.
Polyphonic siren, 22. gives powerful, differential, and summational tones, 235. figured and described, 243–246. apparatus for opening the holes described, 635.
Polyphony proper the result of discant, 374.
Poole, his organ, 635. [his tonal system, 677, 678, note. his compass, how it can be played on Bosanquet's generalised keyboard, 697. his double and triple harmony, 534, note.]
Poor tones, 173.
Pope John XXII. blames major Sevenths, 441.
Popular music mostly in the major mode, 453.
[Practical cyclic intonation for Bosanquet's cycle, 657–8.]
[Practical cyclic triads, their character, 783.]
[Practically just intonation, 656.]
Practice, its influence on perceptio
Praetorius on 'wolves,' 499. desc
universal cymbalum with 19 d
to the Octave, 499.
Preparation of dissonant note, 549.
Prime tone defined, 33.
Prime tones in deep musical note
often not so loud as the first upper
tial tones. 265.
Principalstimmen of organs expla
141.
Processus Foliānus of the hammer o
ear, 193. *lenticulāris* of the sti
198.
Prohibited progressions, meaning of.
Proper tones of circular elastic pl
117. of glass bells, 118.
Protestantism, its musical effect, 377.
Ptolemy, his data for the diatonic te
chord, 404. his equal diatonic te
chord, 406.
Punctuation, how sung according to
customs of Münster, 365.
Punctum caecum, or blind point of the
96. its size, 97. why not observed.
discovered by Mariotte, and shewn
Charles II., 97.
Purkinje, 95.
['Pyrophone,' Kastner's, 721.]
Pythagoras, his musical laws, 1, 2
382, 568. settles the eight degree
the scale, including the Octave,
his enigma: 'Why is consonance de
mined by the ratio of small w
numbers?' how solved, 346. his
mony of the spheres, 347. his a
renounced by Archytas and Al
Kadir, 566.
Pythagorean comparison of the s
notes to the seven planets, 369. i
nation of the Doric tetrachord, 40
Thirds, their melodic effect, 409.
tonation, 428. [intonation attrib
to violinists in melody, by Cornu
Mercadier, 788. temperament,
tones, their pitch, 764. triads exami
785.]
Pythian games give prizes for flute
sic, 362.

Q

Qualitative differences of tone redu
quantitative differences in the ex
ment of certain nerve fibres, 223.
Quality of tone, 6, 16. does not de
on amplitude or rapidity of vibra
but on form, 28, 32. hitherto cre
with all possible peculiarities, 106
say that it depends upon vibrat

form is a merely negative statement, 106. musical, defined, 109. the same for all simple tones, and its character, 113. altered for simple tones by the introduction of new simple tone, 103. of pianoforte strings in different parts of the scale, 123. of strings varies with place of excitement, 124. of violins, as depending on the air chamber, 138. of tones of different instruments, their general character, 172–3. independent of difference of phase, and depends solely on the relative loudness of the upper partial tones, 184, 186. the hypothesis of its apprehension by sympathetic vibration of parts of ear, 190. how perceived, 222. its effect on harmoniousness of intervals, 294. what kind of it favours tempered intonation, 502. [artificial, formed in chords, estimated, 795.]

[Quanten, von, his objections to Prof. Helmholtz's vowel theory, 731. his apparent misconception of the same, 733.]

Quartertones, in Greek tetrachords, 407. in Arabic singing, 408.

Quartetts of bowed instruments by players not used to play together, why often harsh, 316, [and note]. 506.

Quartetts of voices among amateurs, justly intoned, 509.

[Quartóni, four combined simplex harmoniums, how arranged, 685.]

[Quintal or Pythagorean temperament, 649. harmony, 660. its scales, 749–751.]

Quintaten, organ stop, means *quintam tenentes*, 142.

[Quintóni, five combined simplex harmoniums, how arranged, 685.]

R

R, its trill, 109–10. produces beats, 254, [and note].

Radical note of a chord defined, 480.

Rahawi, Arabic and Persian scale, 434, [751].

Rameau hears upper partials of human voice, 155. his theory of consonance, 351. first theoretically recognises the value of the different tones of a chord, 479. his temperament in 1726 described, 499. his fundamental bass, 545–6. his understood fundamental bass, 390. his resolutions of the chord of the added Sixth, 551. his rule for the progression of the fundamental bass, 552. his formulation of the ambiguity of the minor chord, 553.

Rast, Arabic and Persian scale, 434, [751].

Rational construction of diatonic scales, 418. degrees of major and minor scales, 419.

[Rayleigh, Lord, his observations on Mr. Bosanquet's mathematical investigation of the string organ, 719. his observations on the Bedgebury Park echo, 722.]

Recitative, its intention, 364.

Recussus ellipticus and *sphaericus*, 203.

Reed pipes of organs, 144. how influenced by resonance, 601. their quality of tone, due to the puffs of air as in sirens, and resonance, 150.

Reed stops of organs described and figured, 144.

Reeds free and striking, their construction, 144. [manufacture and use in organs, 711–12.]

Reeds of clarinets, oboes, and bassoons, 145. See also Tongues.

[Reeds and strings, their mutual action when combined, as in the string organ, 715.]

[Reflection, its effect in resolving compound tones, 721.]

Registers of the voice, 150. [their influence on vowels, 166, note. shewn by a diagram, 729, note.]

[Regnault's experiments on transmission of sound, and effect of intensity, 721.]

Reissner's membrane, 204.

Relationship of compound tones, direct, 544. indirect, 545. of chords, 450.

[Relative decads, 666.]

Release and attack of tones, 106.

Renaissance, architecture, 360.

Resemblance of features, indescribable but clearly felt, an illustration of resemblance between notes and chords, 574.

Resolution of dissonant note, 549. of chords of the Seventh, 555.

Resonance of the ear passages, 168. influence of on reed pipes, 601. sympathetic, relation of its strength to the length of time required for the tone to die out, 612. tubes change the tones of tongues, 152. bottle with membranous cover, 66.

Resonators, described, 68. use of in hearing upper partials, 82. spherical, applied as resonance chamber to a reed in unison, their effect, 152. their size and construction, 579. made by Koenig, 580. of tin, made by Appun, 581.

[Resultant tones, Tyndall's name for combinational tones, 230, note.]

Retrospect of physiological and physical results, 341.

Speaking and singing differ in the mode of putting on the voice, 155.
Spitzflöte, organ stop, 142.
Staff or ordinary notation, its inconveniences, 638.
Stapes, or stirrup, of the ear, 192.
Stereoscope, its nature, 98.
Stereoscopic vision, 98.
Stirrup of ear, 192, 198.
[Stone, astronomer, on chronographic registration of astronomical observations, 263, note.]
[Stone, Dr. W. H., researches on deep tones, 705. new *C*, contrabasso, 705. tension bars, 706. new *C*, contrafagotto, 706. experiments on pressure of lungs in playing wind instruments, 707. its effect of galvanism on strings, 708.]
Stones, auditory, 206.
Stradivario's violin, 137.
Straduarius or Stradivario, 137.
Strawfiddle, its quality of tone, 115.
Striking inwards and outwards, 146.
Striking reeds, 144. [how Helmholtz's belief concerning their disuse in organs probably arose, 711.]
[String organ, the, 714.]
Strings (stretched), their mode of vibration, 71. their upper partials and their harmonics, 73. their sympathetic vibrations, 74. plucked, their forms, and mode of action through a bridge, 85, 86. plucked, their motion, mathematically investigated, 582. struck or plucked have numerous upper partials, 120, 121. struck, with hard sharp edge, and soft hammer, 121. their different qualities of tone, according to the place of excitement, 124. table of the theoretical intensity of their partial tones, 126. effect of thickness on their upper partials 127. their sympathetic resonance, 589. of pianoforte, their vibrational forms mathematically investigated, 589. of violins, their vibrational forms, 131. of violins, analysis of their motion, 595. [combined with reeds on the string organ, 715.]
Struck strings, their quality of tone partly depends upon the rapidity with which it dies away, 108.
Style, freely selected principle, 566.
Subdominant chord and its relation to the tonic chord, 451. [decad, 666.]
Subjective phenomena, 95.
Subtones, harmonic, 70.
Suites of S. Bach and Handel, 375.
Summational tones, or combinational tones whose vibrational numbers are the sums of those of their generators, discovered by Helmholtz. 230 amples of, 234. weaker than di tial tones, 230.
Superimpositions of waves, 41. sounds, 42. of note and Octav of note and Twelfth, 49.
Supplementary notes of a scale, th tonation, 460, 462.
Suspension of dissonant note, 549.
Swings or oscillations, 12.
Sylvester establishes the Roman sch singing, 365.
Sympathetic resonance and vibratio illustrated by violin strings, 57. bell-ringing, 57. by ring pend 58. takes place when exciting v tions have a simple vibration co ponding to that of sympathising 60. of tuning-forks, 64. of str 586. its relation to the time of d out, 612.
Sympathetic vibration of pianoforte st with voice, 61, 104, [189, note]. s times possible when there is not e correspondence of periodic time, 62 tuning-forks, 63. of stretched str with experiments, 74. of ear, the pothesis of, would explain its ap hension of quality of tone, 190. the ear, how affected by differen pitch, 213, 216.
Sympathetical vibrating bodies vib in the pitch of the exciting tone. when that ceases, in their own t 215.
Synēmmenoi, tetrachord, 377.

T

Tartini estimates all combinational t an Octave too high, 105. [his *suoni* or combinational tones, 230.] theory of consonance, 351. his covery of combinational tones, 353
[Taylor, Sedley, on variation of pitc beating tones, 626. his experiment Koenig's forks for the vowel resona 734, note. his experiments on v resonance, 736.]
[Telegraph at night by vowels only, note.]
Telegraph wires, their analogy to ne 224.
Temperament, its effect on singers. its advantages historically in the provement of music, 498. fuses d ent intervals, 525.
[Temperaments, their object and fu mental relations, 647. uniform, commatic system. Pythagorea

quintal, mean or meantone or mesotonic or tertian, 649. commato-skhismatic system, 650. equal or hemitonic skhismatic system, skhismic or Arabic, 651. skhistic or Helmholtzian, comparative errors, 652. cyclic systems and table of chief cycles, 653. cycles of 30103, 3010, and 301, p. 654. Bosanquet's cycle of 53, commatic cycle of 53, Huyghen's cyclus harmonicus, 655. cycle of 12, unequal temperaments, unequally just intonation, practically just intonation, 656. practical intonation for Bosanquet's cycle, 657-8 how they are realised, 659. practical unequal imitation of equal temperament, 785.]

[Tempered consonant intervals compared, 652, 782.]

Tempered instruments mislead singers, 509.

[Tempered intonation, how to note, 643.]

Tenth, major, better than major Third, 297.

Tenuēs, consonants, 107.

Terpander, 382, 568. his *Cithăra* of six strings, composed of a tetrachord and a trichord, 397.

[Terquem's tonometer, 705.]

[Tertian harmony, 660. its scales, 752-758.]

[Tertian or mean tone temperament, 649.]

[Tetrachordal scales, 667, 757.]

Tetrachords, of the Greeks, connected and separate, 392. of Olympos, old chromatic, diatonic, of Didymus, 404. Doric or of Pythagoras, Phrygian, Lydian, 405. irrational, 406.

Tetrads, consonant, their most perfect positions, 337-8. dissonant, direct and indirect, 531.

Thebes in Egypt, flutes found at, 416, note.

[Theoretical English pitch, 780.]

Thickness and material of strings, their effect on quality of tone, 127.

Third, major and minor, their nature, 288. major, a medial consonance, 295. minor, an imperfect consonance, 295. major, between simple tones does not beat if mistuned, 304. occasionally determined solely by combinational tones, 306. and Sixth of simple tones not well defined when struck alone, but when struck with Fifths or Octaves well defined by combinational tones, 307. its invention not obvious, 393. justly intoned major and minor, and Pythagorean minor, examined, 519-20. diminished, examined, 524.

Thirteenth, major and minor, worse than Sixth, major and minor, 297.

Thompson, Gen. T. Perronet. [his life, 635, note.] his Enharmonic Organ, and work on just intonation, 635. [tonal system of his organ, 678. cost of its manual, 696, note. his use of the cycle of 53, p. 655.]

Thorough bass, 3.

[*Timbre*, original and derived meanings of, 364, note.]

Tinkling of string, how produced, 125. of bass and soprano voices, 168.

[Tisley and Spiller, makers of Donkin's machine for the mechanical description of compound parallel vibrations, and inventors and makers of a machine for compounding rectangular vibrations, 704.]

Tonality, 7. a term due to Fétis, 368. the feeling for it destroyed by equal temperament, 510, [669].

Tonal modes, the five melodic, 418. their tonic, dominant and subdominant chords, 453. arranged in Thirds to shew their major and minor chords, 459. distinguished by the major or minor character of their subdominant, tonic and dominant chords, in a table, 480. [this principle carried out by trichordal scales, 664.]

Tonal relationship of the first and second degree, 395. expressed by common partials in a table, 396.

Tonal relationship, principle of, 565. its cause, not an object of conscious perception, 572.

Tonal system, modern, its fundamental principle, 382. [practical, how reduced from complete, 673. of 48 tones, practically sufficient, 679.]

[*Ton*, German word, used exclusively for simple tone by Helmholtz, 36, note.]

Tone, a musical sound, defined, 35. [why the word is not used for a simple tone exclusively in this translation, 36, note.] used (in this translation) as an indefinite term applied to either a simple or a compound tone, 90, [and note].

Tone, an interval, major and minor, examined, 523. [when the word is used for an interval it has always a capital T in this translation, 36, note.]

Tones of major and minor scales arranged in Thirds to shew the intervals, 519.

Tongues of clarinets, oboes and bassoons, 145. striking inwards and outwards, 146. membranous, 146. of clarinets, and their action and mode of vibration, 148. of organs and harmoniums, compared with those of wooden instruments, 148. [free and striking, 711.]

Tonic or keynote defined, 368. was there any in Greek music? 412. in polyphonic music represented by a chord, 447.

Tonic chord, does not play a prominent

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